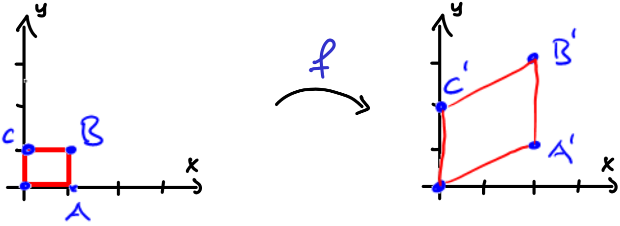
Exercise 1

For some linear map in $\mathbb{R}^2$ we know

$$f(\vec{e}_1) = \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \qquad f(\vec{e}_2) = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

- Find the matrix $\mathbf{A}_f$ of the linear map. $A_f = \begin{pmatrix} 2 & 0 \\ 1 & 2 \end{pmatrix}$
- What is $f(\vec{x})$ with $\vec{x} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$? $f(\vec{x}) = A_f \vec{x} = \begin{pmatrix} 2 & 0 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}$
- Draw the image of the unit square under f?

$$A_f \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$$



Exercise 2

Consider a rotation in $\mathbb{R}^2$ by 45°.

1. Write down the rotation matrix. $= R$
2. What is the result of rotating the vector $(2, 1)$ by 45°?
3. Draw the image of the square with vertices $(1, 0), (2, 0), (2, 1), (1, 1)$ under a rotation of 45°.

Hint: $\cos(\pi/4) = \sin(\pi/4) = 1/\sqrt{2}$

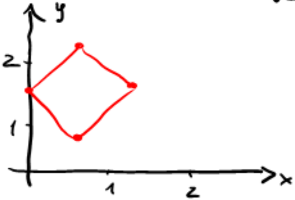
1.) $R = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$

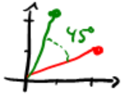
2.) $R \cdot \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 2 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 3 \end{pmatrix}$

3.) $\begin{pmatrix} 1 \\ 0 \end{pmatrix} \to \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$\begin{pmatrix} 2 \\ 0 \end{pmatrix} \to \frac{1}{\sqrt{2}}\begin{pmatrix} 2 \\ 2 \end{pmatrix}$

$\begin{pmatrix} 1 \\ 1 \end{pmatrix} \to \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ 2 \end{pmatrix}$

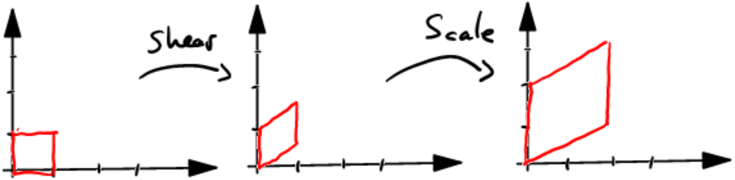




Exercise 3

Write the map of exercise 1 on slide 10 as a composition of two elementary transforms.

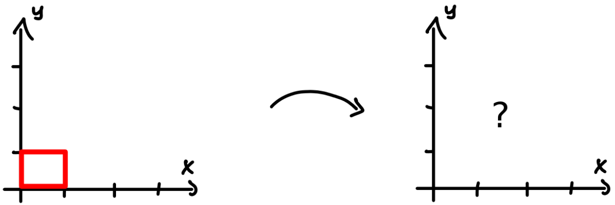
$$A_f = \begin{pmatrix} 2 & 0 \\ 1 & 2 \end{pmatrix} = \underbrace{\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}}_{\text{Scale}} \cdot \underbrace{\begin{pmatrix} 1 & 0 \\ 1/2 & 1 \end{pmatrix}}_{\text{Shear}}$$



Exercise 4

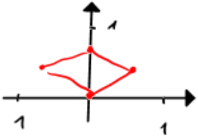
Let **R** be a 2D-rotation by 45° and **S** be a scaling by $\frac{1}{2}$ into the y-direction.

1. What happens to the unit square when it is first rotated by **R** and then scaled by **S**?
2. What happens to the unit square when it is first scaled by **S** and then rotated by **R**?
3. What happens to the unit square when it is first rotated by **R**, then scaled by **S** and then rotated back by $\mathbf{R}^{-1}$?

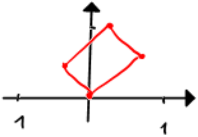


$$R = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & -1\\ 1 & 1\end{pmatrix}, \quad S = \begin{pmatrix}1 & 0\\ 0 & 1/2\end{pmatrix}$$

1.) $S \cdot R = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & -1\\ 1/2 & 1/2\end{pmatrix}$, $\quad S \cdot R\begin{pmatrix}1\\1\end{pmatrix} = \begin{pmatrix}0\\ 1/\sqrt{2}\end{pmatrix}$

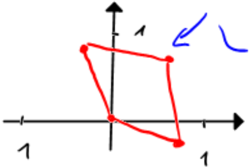


2.) $R \cdot S = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & -1/2\\ 1 & 1/2\end{pmatrix}$, $\quad RS\begin{pmatrix}1\\1\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}1/2\\ 3/2\end{pmatrix}$



3.) $R^{-1} = R^T = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ -1 & 1\end{pmatrix}$

$$R^{-1} \cdot S \cdot R = \frac{1}{2}\begin{pmatrix}1 & 1\\ -1 & 1\end{pmatrix}\begin{pmatrix}1 & -1\\ 1/2 & 1/2\end{pmatrix} = \frac{1}{2}\begin{pmatrix}3/2 & -1/2\\ -1/2 & 3/2\end{pmatrix} = \frac{1}{4}\begin{pmatrix}3 & -1\\ -1 & 3\end{pmatrix}$$

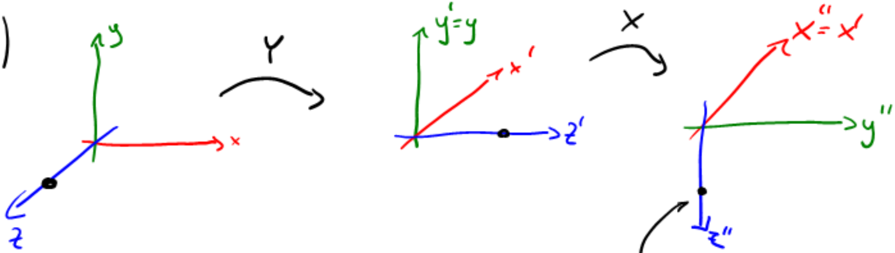


Scale in $\begin{pmatrix}1\\1\end{pmatrix}$ - Direction

Exercise 5

1. Do the example of the with order *YXZ*.
2. Check your result with `three.js`.

1.)



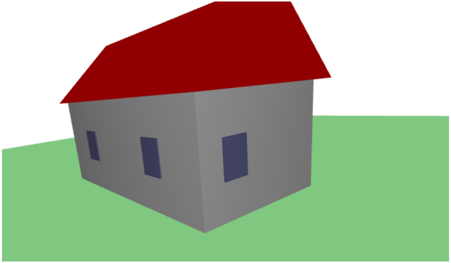
$\underline{\underline{\begin{pmatrix}0\\-1\\0\end{pmatrix}}}$ in xyz-frame

2.) see ex5.js

Exercise 6

see house directory

Add windows to the house:



- Implement windows as thin black box geometries.
- Position them correctly on the walls.

Hint: Add axes showing the *local* frame of any object

```
// len: length of axes
obj.add(new THREE.AxesHelper(len));
```

Exercise 7

1. Check that $\vec{u} = \vec{e}_z$ reproduces the correct result.
2. Work out the rotation matrix for a rotation by 60° around an axis pointing into direction $(1, 1, 1)$.
3. What do the vectors $(1, -1, 0)$ and $(2, 2, 2)$ rotate into?
4. Read the `THREE.Matrix4` documentation (look for `makeRotationAxis`) to create this rotation matrix in the browser. Use this to check the results of parts 2. and 3. Hint: Use the provided `printMat` function (see `lib` directory). → see ex7.js

1.) $u_1 = 0$, $u_2 = 0$, $u_3 = 1$ $\Rightarrow R_{\vec{e}_z,\theta} = \begin{pmatrix} c & -s & 0 \\ s & c & 0 \\ 0 & 0 & 1 \end{pmatrix}$ ✓

2.) $u_1 = u_2 = u_3 = 1/\sqrt{3}$

$c := \cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$

$s := \sin(\pi/3) = \frac{\sqrt{3}}{2}$

$(1-c)u_n^2 + c = \frac{1}{2}\cdot\frac{1}{3} + \frac{1}{2} = 2/3$

$(1-c)u_n u_m \pm s u_k = \frac{1}{2}\cdot\frac{1}{3} \pm \frac{\sqrt{3}}{2}\cdot\frac{1}{\sqrt{3}}$

$= \begin{cases} 2/3 \\ -1/3 \end{cases}$

$$\Rightarrow R = \frac{1}{3}\begin{pmatrix} 2 & -1 & 2 \\ 2 & 2 & -1 \\ -1 & 2 & 2 \end{pmatrix}$$

3.) $R\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$, $R\begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix}$ obvious because $\begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix}$ points along axis of rotation.