Intro to Computer Graphics
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Problem sheet for chapter 9: Camera models and the view pipeline

Problem 1:

Check that the projection matrix $\mathbf{Q}_{\vec{n},d}$ for parallel projection, as defined in the lecture, satisfies the relation

$$\mathbf{Q}_{\vec{n},d} \cdot \mathbf{Q}_{\vec{n},d} = d\mathbf{Q}_{\vec{n},d}$$

Lösung:

Here just the 2D case:

$$\mathbf{Q}_{\vec{n},d} \cdot \mathbf{Q}_{\vec{n},d} = \begin{pmatrix} d & 0 & 0 \\ 0 & d & 0 \\ -n_1 & -n_2 & 0 \end{pmatrix} \cdot \begin{pmatrix} d & 0 & 0 \\ 0 & d & 0 \\ -n_1 & -n_2 & 0 \end{pmatrix} = \begin{pmatrix} d^2 & 0 & 0 \\ 0 & d^2 & 0 \\ -n_1 d & -n_2 d & 0 \end{pmatrix} = d\mathbf{Q}_{\vec{n},d}$$

Problem 2:

Consider a vertex in a 3D scene with world space coordinates $(1, 2, 3)$. The camera is configured as follows:

```
const camera = new THREE.PerspectiveCamera(90, 2,
    2, 10);
camera.position.set(5,0,0);
camera.lookAt(0,0,0);
```

Which location (in pixel coordinates) is this vertex projected to on a canvas of width = 600 pixels and height = 300 pixel?

Lösung:

Since the camera is shifted by 5 units into the x-direction. It needs to be rotated by $\pi/2$ around the y axis to look at the origin. With the abbreviation $\mathbf{C} =$ `camera.matrix`, we get

$$\mathbf{C} = \begin{pmatrix} 0 & 0 & 1 & 5 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \Longrightarrow \mathbf{C}^{-1} = \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & -5 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

The camera space coordinates of the vertex are

$$\begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & -5 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \\ 3 \\ 1 \end{pmatrix} = \begin{pmatrix} -3 \\ 2 \\ -4 \\ 1 \end{pmatrix}$$

The projection matrix of the camera is

$$\mathbf{Q} = \begin{pmatrix} 1/2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -3/2 & -5 \\ 0 & 0 & -1 & 0 \end{pmatrix}$$

The normalized device coordinates of the vertex are

$$\begin{pmatrix} 1/2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -3/2 & -5 \\ 0 & 0 & -1 & 0 \end{pmatrix} \cdot \begin{pmatrix} -3 \\ 2 \\ -4 \\ 1 \end{pmatrix} = \begin{pmatrix} -3/2 \\ 2 \\ 1 \\ 4 \end{pmatrix} \overset{\text{p.d.}}{\rightarrow} \begin{pmatrix} -3/8 \\ 1/2 \\ 1/4 \\ 1 \end{pmatrix}$$

Finally, the x and y coordinates have to be scaled and shifted to canvas pixel coordinates:

$$\begin{aligned} x_s &= \frac{w_s}{2} x_{\text{ndc}} + \frac{w_s}{2} = 300 \cdot \left(-\frac{3}{8} + 1 \right) = 187.5 \\ y_s &= -\frac{h_s}{2} y_{\text{ndc}} + \frac{h_s}{2} = 150 \cdot \left(-\frac{1}{2} + 1 \right) = 75 \end{aligned}$$

The transition from world space to NDCs can also be done by a single matrix multiplication by

$$\mathbf{Q} \cdot \mathbf{C}^{-1} = \begin{pmatrix} 0 & 0 & -1/2 & 0 \\ 0 & 1 & 0 & 0 \\ -3/2 & 0 & 0 & 5/2 \\ -1 & 0 & 0 & 5 \end{pmatrix}$$