Exercise 1

Implement a ball reflected inside view frustum:

- (x_c, y_c, z_c): coordinate of ball in camera space!
- h_c, w_c: height and width of frustum at z_c:

$$h_c = 2z_c \tan\left(\frac{\theta_y}{2}\right) \ , \ w_c = a \cdot h_c$$

- Collision with frustum plane:

```
if(x_c > wc/2 - radius) {
  // change speed of ball
}
```

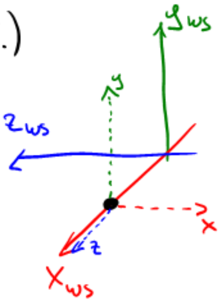
Exercise 2

Consider the following code:

```
const camera = new THREE.PerspectiveCamera(...) ;
camera.position.set(5, 0, 0);
camera.lookAt(0, 0, 0);
```

1. Work out `camera.matrix`
2. Verify with `three.js` : see ex2.js

1.)



Index ws: world space axes
Dashed: camera space axes

Camera axes are
- translated by 5 into x-dir.
- rotated by +90°

$$\hat{=} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{pmatrix}$$ as linear part
(see slide 7.35)

$$\Rightarrow M = \begin{pmatrix} 0 & 0 & 1 & 5 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$ = passive transform, thats maps coordinates from camera space to world space.

check: $M \cdot \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \\ 0 \\ 1 \end{pmatrix}$ ✓ (location of camera)

$M \cdot \begin{pmatrix} 0 \\ 0 \\ -5 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$ ✓ (Origin of world space)

Exercise 4

Consider perspective projection in $\mathbb{R}^2$ with camera at the origin onto a line passing through $(1, 0)$ and $(0, -1)$.

- Write down the 3×3 projection matrix.
- What are the projections of
  - $P = (1, 0)$
  - $Q = (2, -2)$
  - $R = (1, 1)$

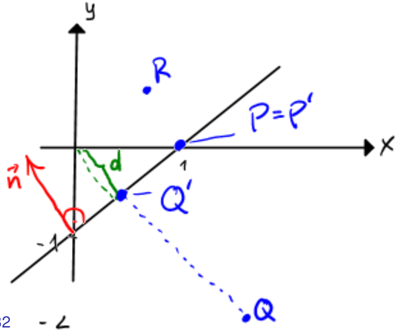
$$d = \frac{1}{\sqrt{2}}, \quad \vec{n} = \frac{1}{\sqrt{2}} \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$Q_{\vec{n},d} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & -1 & 0 \end{pmatrix}$$

$$P' = Q_{\vec{n},d} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \xrightarrow{p.d.} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

$$Q' = Q_{\vec{n},d} \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 2 \\ -2 \\ 4 \end{pmatrix} \xrightarrow{p.d.} \begin{pmatrix} 1/2 \\ -1/2 \\ 1 \end{pmatrix}$$

$$\Rightarrow Q' = \begin{pmatrix} 1/2 \\ -1/2 \end{pmatrix}$$



$$R' = Q_{\vec{u},d} \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \xrightarrow{\text{p.d.}} \begin{pmatrix} \infty \\ \infty \\ 1 \end{pmatrix}$$

R is projected to ∞ because $\overline{OR}$ is parallel to the projection line.

In general, quantities with a 0 in the bottom homogeneous coordinate can either be viewed as a point at ∞ or as a vector pointing in that direction.

Exercise 5

1. Verify that the function

$$g(z) = \frac{f+n}{f-n} + \frac{2nf}{f-n} \cdot \frac{1}{z}$$

   has the properties

   - $g(-n) = -1$
   - $g(-f) = 1$

2. Draw $g(z)$ in the intervall $[-f, -n]$ for $f = 3$ and $n = 1$.

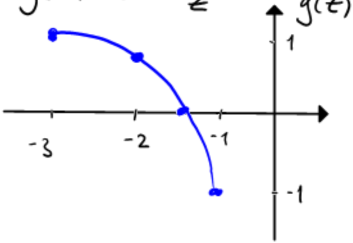
1.) $g(-n) = \frac{f+n}{f-n} + \frac{2\cancel{n}f}{f-n} \cdot \left(\frac{1}{-\cancel{n}}\right)$

$= \frac{f+n-2f}{f-n} = \frac{n-f}{f-n} = -1$

$g(-f) = \frac{f+n}{f-n} + \frac{2n\cancel{f}}{f-n} \cdot \left(\frac{1}{-\cancel{f}}\right)$

$= \frac{f+n-2n}{f-n} = \frac{f-n}{f-n} = 1$

2.: $g(z) = 2 + \frac{3}{z}$

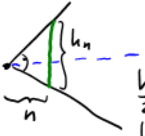


Exercise 6

1. Condsider the following code:

$a = \frac{W_n}{h_n}$

```
const cam = new THREE.PerspectiveCamera(90,2,2,10)
```



- Work out $\mathbf{Q}_{\text{ndc}}$
- Compare with `three.js` → see ex6.js

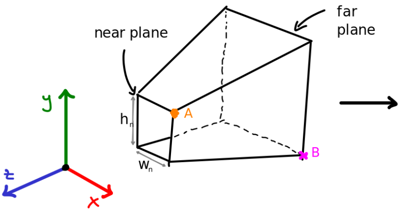
$\frac{h_n}{2} = n = 2$

$h_n = 4$

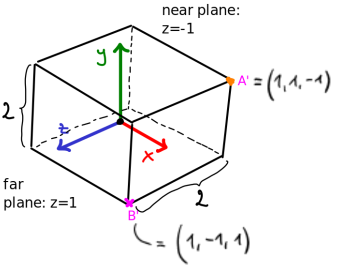
$W_n = 2h_n = 8$

2. Check that $\mathbf{Q}_{\text{ndc}}$ maps A to A' and B to B':

View frustum



NDC cube



$$\frac{2n}{w_n} = \frac{4}{8} = \frac{1}{2}, \quad \frac{2n}{h_n} = \frac{4}{4} = 1, \quad \frac{n+f}{n-f} = \frac{12}{-8} = -\frac{3}{2}, \quad \frac{2nf}{n-f} = \frac{40}{-8} = -5$$

$$Q_{ndc} = \begin{pmatrix} 1/2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -3/2 & -5 \\ 0 & 0 & -1 & 0 \end{pmatrix}$$

$$A = \begin{pmatrix} w_n/2 \\ h_n/2 \\ -n \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \\ -2 \end{pmatrix} \Rightarrow Q_{ndc} \cdot A = Q_{ndc} \begin{pmatrix} 4 \\ 2 \\ -2 \\ +1 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ -2 \\ 2 \end{pmatrix} \xrightarrow{\text{p.d.}} \begin{pmatrix} 1 \\ 1 \\ -1 \\ 1 \end{pmatrix} \checkmark$$

$$B = \begin{pmatrix} w_f/2 \\ -h_f/2 \\ -f \end{pmatrix} = \begin{pmatrix} 20 \\ -10 \\ -10 \end{pmatrix} \Rightarrow Q_{ndc} \cdot B = Q_{ndc} \begin{pmatrix} 20 \\ -10 \\ -10 \\ 1 \end{pmatrix} = \begin{pmatrix} 10 \\ -10 \\ 10 \\ 10 \end{pmatrix} \xrightarrow{\text{p.d.}} \begin{pmatrix} 1 \\ -1 \\ 1 \\ 1 \end{pmatrix} \checkmark$$

$$\frac{h_f}{2} = f \Rightarrow h_f = 20, \quad w_f = 2 \cdot h_f = 40$$