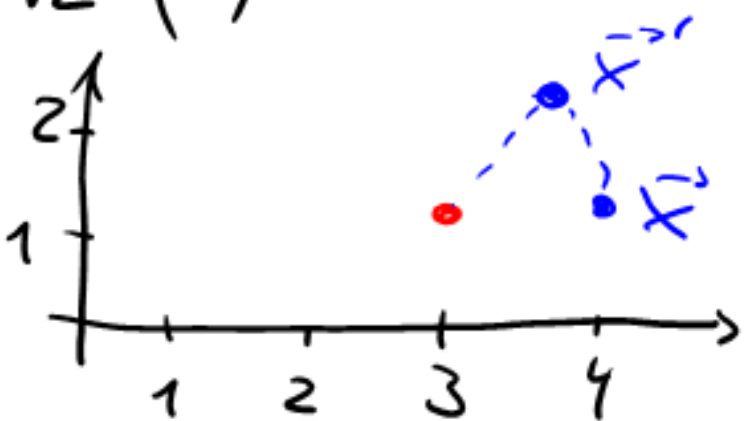
Exercise 1

Where does a rotation by 45° around the pivot $\underbrace{(3,1)}_{\vec{p}}$ map the point $\underbrace{(4,1)}_{\vec{x}}$?

$$R = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \Rightarrow R\vec{x} = \frac{1}{\sqrt{2}}\begin{pmatrix} 3 \\ 5 \end{pmatrix}$$

$$\vec{p} - R\vec{p} = \begin{pmatrix} 3 \\ 1 \end{pmatrix} - \frac{1}{\sqrt{2}}\begin{pmatrix} 2 \\ 4 \end{pmatrix}$$

$$R\vec{x} + \vec{p} - R\vec{p} = \begin{pmatrix} 3 \\ 1 \end{pmatrix} + \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \vec{x'}$$



Exercise 2

- Find the 3×3 matrix representing a rotation by 45° around the pivot $(3, 1)$.
- Where gets the point $(4, 1)$ mapped to?
- Write Javascript code to check this.

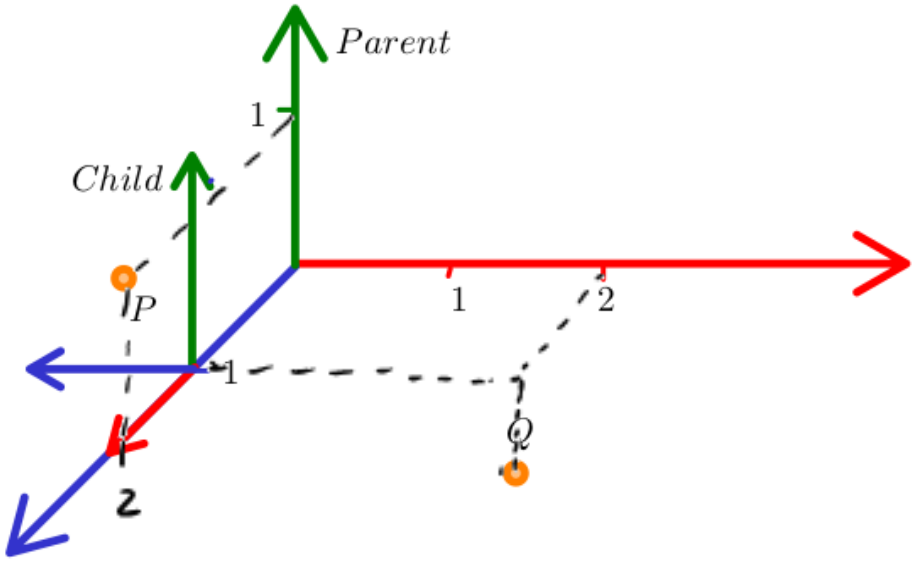
$$R = \begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 3 - 2/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} & 1 - 4/\sqrt{2} \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 4 \\ 1 \end{pmatrix} \hat{=} \begin{pmatrix} 4 \\ 1 \\ 1 \end{pmatrix} \rightarrow R \cdot \begin{pmatrix} 4 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{4}{\sqrt{2}} - 1/\sqrt{2} + 3 - 2/\sqrt{2} \\ 4/\sqrt{2} + 1/\sqrt{2} + 1 - 4/\sqrt{2} \\ 1 \end{pmatrix}$$

$$\hat{=} \begin{pmatrix} 1/\sqrt{2} + 3 \\ 1/\sqrt{2} + 1 \end{pmatrix}$$

Exercise 3

Consider the following parent child coordinate systems:



1. Write down the transformation matrix.
2. A point P has coordinates $(1, 1, 0)$ in the child frame. What are its coordinates in the parent frame?
3. Another point Q has coordinates $(2, -1, 1)$ in the parent frame. What are its coordinates in the child frame?

Child frame is obtained from parent frame by
- translation by 1 in z-direction
- rotation by $-90°$ around y-axis

$$M = \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$P_p = M \cdot \begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 2 \\ 1 \end{pmatrix} \hat{=} \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} \checkmark$$

$$M^{-1} = \begin{pmatrix} 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$Q_c = M^{-1} \cdot \begin{pmatrix} 2 \\ -1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \\ -2 \\ 1 \end{pmatrix} \hat{=} \begin{pmatrix} 0 \\ -1 \\ -2 \end{pmatrix} \checkmark$$