

Problem sheet for chapter 2: Vectors and Matrices

Problem 1:

A point A in three-dimensional space has coordinates $(4, 3, 8)$, another point B has coordinates $(2, 10, 5)$. What is the distance between the points?

Problem 2:

(a) Which of the following three vectors are perpendicular?

$$\vec{a} = (-8, 1, 2) \quad \vec{b} = (4, -3, 1) \quad \vec{c} = (-1, -2, -2).$$

(b) Calculate the angle between the vectors $\vec{a} = (1, -1, 1)$ and $\vec{b} = (-1, 1, -1)$.

Problem 3:

A straight line in \mathbb{R}^3 passes through the origin and the point $Q = (-4, 4, 2)$. Which point P' on this line is closest to point $P = (1, 2, 3)$?

Problem 4:

What is the area of the triangle defined by the following points?

$$A = (1, 2), \quad B = (7, 2), \quad C = (3, 6)$$

Hint: Think of the points as points in the x - y -plane of \mathbb{R}^3 and use the cross-product.

Problem 5:

Consider a plane in \mathbb{R}^3 that is positioned and oriented such that the three points

$$A = (1, 2, 3), \quad B = (-1, 1, 5), \quad C = (2, 2, 2)$$

are located within the plane. A particle with velocity $\vec{v}_{\text{in}} = (-1, 2, 3)$ hits the plane and is reflected specularly (i.e. incoming angle equals outgoing angle). Determine the velocity of the particle after reflection.

Problem 6:

Calculate the products $A \cdot B$, $A^T \cdot B^T$, $B \cdot A$ and $B^T \cdot A^T$ for the matrices A and B given below. What is the relation between $(A \cdot B)^T$ and the individual matrices A^T and B^T ?

$$A = \begin{pmatrix} 2 & -1 \\ 0 & 3 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

Problem 7:

Which matrices X_1 and X_2 satisfy the matrix equations $X_1 \cdot A = B$ as well as $A \cdot X_2 = B$ with

$$A = \begin{pmatrix} 2 & 1 \\ 1 & 0 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix}?$$