Intro to Computer Graphics
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Problem sheet for chapter 7: Linear maps and transformation matrices

Problem 1:

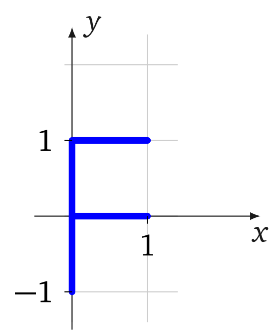
Consider the following maps $\mathbb{R}^2 \to \mathbb{R}^2$:

$$A_1 : \begin{pmatrix} x \\ y \end{pmatrix} \to \begin{pmatrix} -y \\ x \end{pmatrix} \qquad A_2 : \begin{pmatrix} x \\ y \end{pmatrix} \to \begin{pmatrix} x+1 \\ y \end{pmatrix} \qquad A_3 : \begin{pmatrix} x \\ y \end{pmatrix} \to \begin{pmatrix} x+y \\ y \end{pmatrix}$$

(a) Which of the maps are linear?

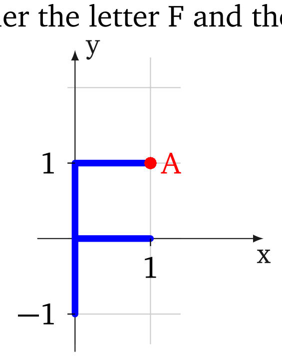
(b) For the linear maps: Write down the representation matrices.

(c) For the linear maps: Draw pictures how they transform the letter F:



Problem 2:

Consider the letter F and the two transforms **R** and **S** shown below:



$$\mathbf{S} = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & 2 \end{pmatrix}, \qquad \mathbf{R} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$$

(a) Draw the letter F after the transform $\mathbf{R} \cdot \mathbf{S}$ has been applied to it. What are the coordinates of the transformed point A?

(b) Draw the letter F after the transform $\mathbf{S} \cdot \mathbf{R}$ has been applied to it. What are the coordinates of the transformed point A?

Problem 3:

Consider the rotation matrix $\mathbf{R}(\varphi) = \begin{pmatrix} \cos(\varphi) & -\sin(\varphi) \\ \sin(\varphi) & \cos(\varphi) \end{pmatrix}$.

(a) Verify that it has unit determinant, i.e. $\det(\mathbf{R}(\varphi)) = 1$,

(b) Verify that it is orthogonal: $\mathbf{R}(\varphi)^T = \mathbf{R}(\varphi)^{-1}$

(c) Verify that the composition of two rotations is a rotation by the sum of the angles, i.e.

$$\mathbf{R}(\varphi) \cdot \mathbf{R}(\theta) = \mathbf{R}(\varphi + \theta)$$

Hint: Use the addition theorems $\sin(\varphi + \theta) = \sin(\varphi)\cos(\theta) + \cos(\varphi)\sin(\theta)$ and $\cos(\varphi + \theta) = \cos(\varphi)\cos(\theta) - \sin(\varphi)\sin(\theta)$.

Problem 4:

Consider the following two vectors in $\mathbb{R}^3$:

$$\vec{v} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \qquad \vec{w} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

(a) What is the angle between $\vec{v}$ and $\vec{w}$.

(b) Find a 3×3 rotation matrix $\mathbf{R}$ that rotates $\vec{v}$ into $\vec{w}$, i.e. satisfies $\vec{w} = \mathbf{R} \cdot \vec{v}$.

(c) Check that $\mathbf{R}$ is orthogonal and has a determinant of 1.

(d) Find a matrix that rotates $\vec{w}$ into $\vec{v}$.