ME314 Final Project - Die in a Cup
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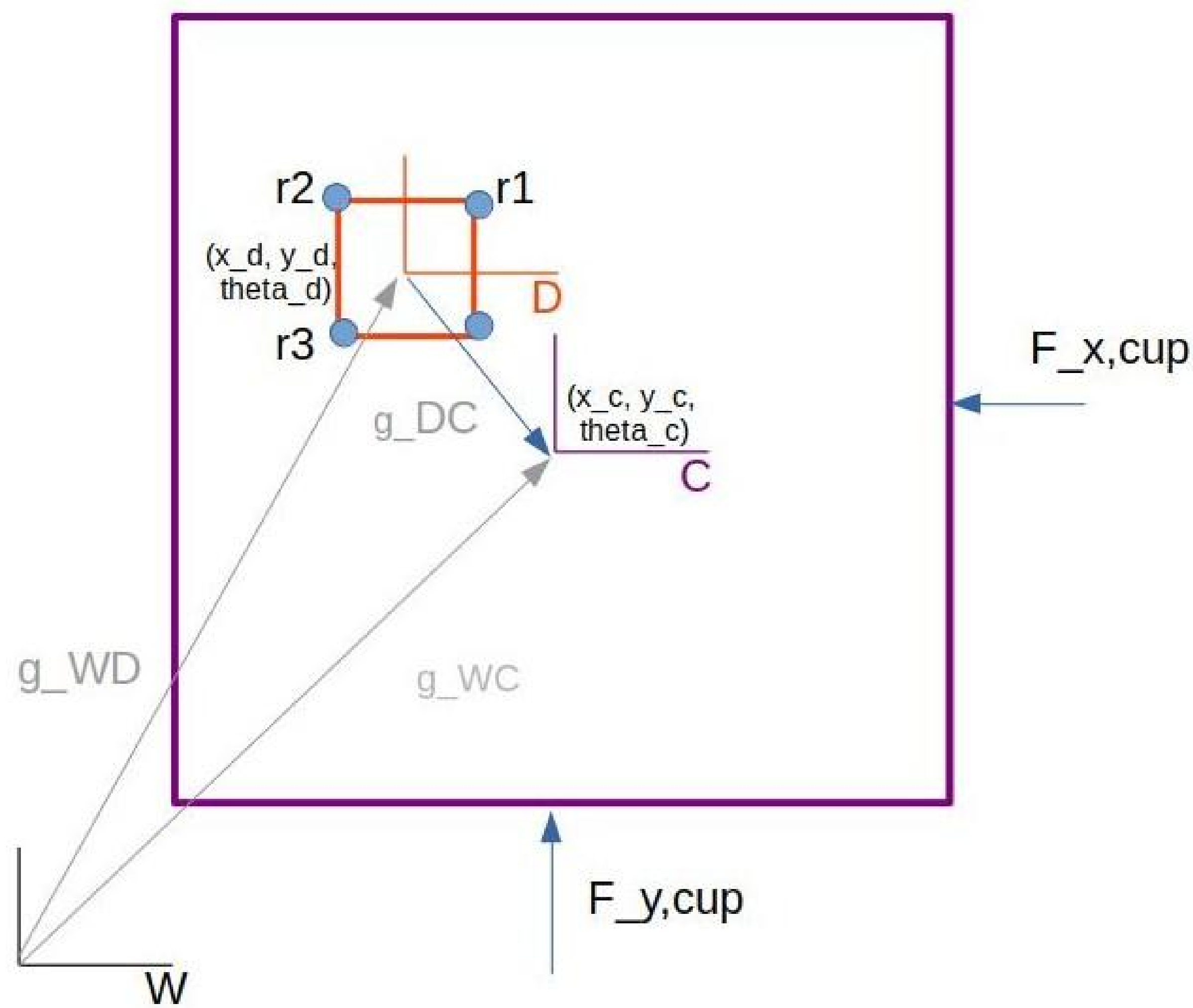
Project Description: Default Project

Model drawing



Approach to calculating Euler-Lagrange, the constraints, the external forces and the impact laws

- Imposed Oscillatory external forces
    - F_y,cup = -1000 * (y_c - sin(t/2)) + m_cup*g
    - F_x,cup = -1000 * (x_c - sin(t/2))

- Constraints/Impact conditions
    - For each corner of the die, r_i, the x-component in the body frame of the cup coincides with either side/vertical wall of the cup(in its body frame), during impact.
        - r_i [x_c] = L_cup/2 and r_i [x_c] = - L_cup/2.
    - Likewise, the y-component of r_i in the body frame of the cup coincides with the horizontal walls of the cup(in its body frame)
        - r_i [y_c] = L_cup/2 and r_i [y_c] = - L_cup/2
    - Thus the 4 impact conditions for each corner
        - ⇒ r_i [x_c] - L_cup /2
        - ⇒ r_i [x_c] - L_cup /2
        - ⇒ r_i [y_c] - L_cup /2
        - ⇒ r_i [y_c] - L_cup //2
        - Where r_i [x_c] and r_i [y_c] are the x and y components of corner, r_i, in the cup frame; L_cup is the height and width of the cup

- Euler Lagrange Equations and Impact Updates

Euler Lagrange Eqns

+ For motion away from impacts.

$$\frac{d}{dt}\left(\frac{dL}{d\dot{q}}\right) - \frac{dL}{dq} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ F_{x,cup} \\ F_{y,cup} \\ 0 \end{bmatrix}$$

where $q = (x_{dice}, y_{dice}, \theta_{dice}, x_{cup}, y_{cup}, \theta_{cup})$

$$L = \underset{total}{\text{Kinetic Energies}} - \underset{total}{\text{Potential energies}} = KE_{tot} - PE_{tot}$$

$$KE_{tot} = KE_{cup} + KE_{dice}$$

$$= \frac{1}{2} V_{cup}^T \tilde{I}_{cup} V_{cup} + \frac{1}{2} V_{dice}^T \tilde{I}_{dice} V_{dice}$$

$\cancel{PE_{tot} = mgm}$

$$PE_{tot} = \cancel{M_{cup}}\, PE_{cup} + PE_{dice}$$

$$= M_{cup}\, g\, y_{cup} + M_{dice}\, g\, y_{dice}$$

$$V_{cup} = \cancel{(g_{w,cup}^{-1})\, dg_{w,cup}} \left(g_{w,cup}^{-1} \frac{d}{dt}(g_{w,cup})\right)^{\vee}$$

$$\text{\& } V_{dice} = \left(g_{w,dice}^{-1} \frac{d}{dt}(g_{w,dice})\right)^{\vee}$$

Impact Update Laws

+ For updated $\dot{q}$s upon impact

$$\left.\frac{dL}{d\dot{q}}\right]_{\tau-}^{\tau+} = \lambda \nabla \phi$$

$$\left[\frac{dL}{d\dot{q}} \cdot \dot{q} - L(q, \dot{q})\right]_{\tau-}^{\tau+} = 0$$

Simulation Results

The die in the cup begins at rest and drops under gravity until one corner impacts the bottom of the cup, which is itself being driven by an external force. Both cup and die react upon impact and continue to collide multiple times during the simulation/animation as they both translate and rotate continually. The die only impacts the internal walls of the cup at its (the die’s) corners, per the initial conditions: q_die = (0, 0, pi/4), q_cup=(0, 0, -pi/15)