I'm still not getting what problems 11 & 12 want us to do.  Let me take Problem 11 (a) as an example.

S = {[1,0,0,0,0], [0,1,0,0,0], [0,0,1,0,0], [0,0,0,1,0], [0,0,0,0,1]}

A = {[1,0,0,0,0], [0,1,0,0,0]}

Therefore S - A = {[0,0,1,0,0], [0,0,0,1,0], [0,0,0,0,1]}

For 11 (a), z = [1,1,1,1,1]

So, the question is asking, find a vector 'w' that belongs in the "S - A" set, that when Span S is added with 'z' and removed with 'w', is the same as Span S.  Am I correct?

So we need to find a vector in the "S - A" to remove from S, and then add vector z into S.

So S1 = S - w + z

and Span S1 = Span S

I'll pay it forward....

We are given

S = {[1,0,0,0,0], [0,1,0,0,0], [0,0,1,0,0],[0,0,0,1,0], [0,0,0,0,1]}

and

A={[1,0,0,0,0], [0,1,0,0,0]}

and asked to find a vector w in S-A such that Span S = Span (S U {z} -w})

in case (a) : z = [1,1,1,1,1]

So how do we solve this?

Looking at S above, lets refer to each vector as v1, v2, ... v5.

Z is in the Span of S and can be written as 1v1+1v2+1v3+1v4+1v5

in case of (b) z=[0,1,0,1,0], this is also in Span of S and can be written as 0v1+1v2+0v3+1v4+0v5

OK. so back  to the problem at hand using Lemma exchange.

We want to add Z ([1,1,1,1,1])  to the Set S and remove a vector from S, such that Span S = Span S with Z added and w removed.

For problems  (a)-(c), when we add Z to S, we must remove a vector w from the set S, but the vector to be removed is limited to the set defined by S-A.

S-A = { [0,0,1,0,0],[0,0,0,1,0], [0,0,0,0,1]} so **any answer you choose for all problems can only be one of these choices.**

Key Gist of Lemma exchange:  If  a vector to be added (in our case Z) can be written as a linear combination of the other vectors in S, then Span of S = Span of S -v where v = vector that is part of the linear combination.  **In plain English:** When you add Z to S, if Z can be written as a linear combo of other vectors, then  one of the other vectors has to make room and get the hell out.

for (a), Z = [1,1,1,1,1] and is written as a linear combo of vectors in S giving us:

Z= 1v1+1v2+1v3+1v4+1v5

Z wants in, this is going to be a messy break up. A vector in  v1-v5 is going to have to  hit the road. But wait!

vector w **in S-A**, so **v1 and v2 didn't sign the prenup and can never be ejected.**

So one of the vectors  v3,v4, or v5 is out of luck. Removed from the set. Do not pass go. Do not collect $200.

Same logic for part (c)

However for part (c), there are 2 correct answers, but the grader only accepts 1.

Problem #12:

This problem is much easier if you draw out the picture version of this similar to the diagram in the problems above.

This is a crude idea of what your diagram will look like:

 A ------v1------ B -----v4------E

 |                       |

v3                    v2

 |                       |

 D                     C

Same concept: here our set of S is composed of vectors v1....v4, however A changes from problem to problem.

Lets look at (a) for an example.     A is given as v1 and v4, so remember, these vectors are the Untouhcables, they can't be removed.

So in our problem we have a 1 under d and e, meaning we have the vector connecting D to E.  Follow the yellow brick road to get from D to E gives you v3 -> v1 -> v4

That path from D -> E tells you the  vectors along the way  that form a linear combination that gives you z, so you know what that means! We can toss one out.  Now look at your Untouchables again.  v1 and v4. So if we had to toss out a vector that forms a linear combination of D-E, what is the only option left?