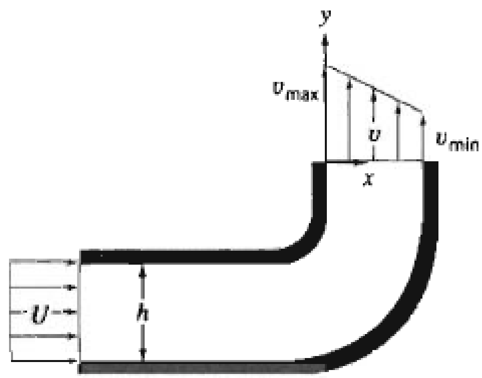
Fluid Mechanics and Rate Processes: Fluid Kinematics Tutorial: August 18, 2016

P1. (4.26, Fox and McDonald, 6^{th} Ed.) Water enters a two-dimensional channel (90^o bend) as shown in the Fig below. The inlet velocity profile is uniform while the outlet profile is assumed to be linear, as indicated in the Fig. evaluate v_{mim} for $v_{max}=2v_{min}$. U =7.5m/s, h =75.5mm



Flow through a 90^o bend

Given: Water flow in the two-dimensional square channel shown.

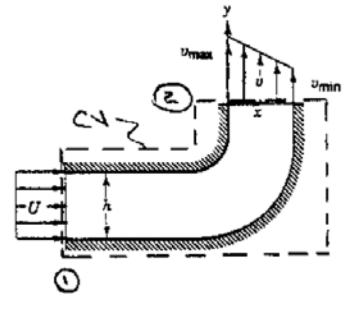
$v_{max} = 2v_{min}$, $U = 7.5$ m/s , $h = 75.5$ mm

Find: v_{min}

Solution: Apply conservation of mass to the CV shown.

Basic equation:

$$0 = \cancel{\frac{\partial}{\partial t}}^{\,0(1)} \int_{CV} \rho \, d\forall + \int_{CS} \rho \vec{V} \cdot d\vec{A}$$



Assumptions: (1) steady flow
(2) incompressible flow
(3) uniform flow at section ①

Then

$$0 = \vec{V}_1 \cdot \vec{A}_1 + \int \vec{V}_2 \cdot d\vec{A}_2$$

$$0 = -Uwh + \int_0^h v\, w\, dx$$

The velocity distribution across the exit at ② is linear

$$v_2 = v_{max} - (v_{max} - v_{min})\frac{x}{h} = 2v_{min} - v_{min}\frac{x}{h} = v_{min}\left(2 - \frac{x}{h}\right)$$

$$\therefore Uwh = \int_0^h v_{min}\left(2 - \frac{x}{h}\right) w\, dx = v_{min}\, w \left[2x - \frac{x^2}{2h}\right]_0^h$$

$$U \cancel{w} \cancel{h} = v_{min}\, w \left[2h - \frac{h}{2}\right] = \frac{3}{2} v_{min} \cancel{w} \cancel{h}$$

$$\therefore v_{min} = \frac{2}{3} U = \frac{2}{3} \times 7.5 \frac{m}{s} = 5.0 \text{ m/s} \longleftarrow v_{min}$$