Find the expression of $(\vec{u}.\nabla)\vec{u}$ where

$$\vec{u}(t,x,y,z)=u_1(t,x,y,z)\vec{i}+u_2(t,x,y,z)\vec{j}+u_3(t,x,y,z)\vec{k}$$

$$\vec{u}.\nabla=u_1\frac{\partial}{\partial x}+u_2\frac{\partial}{\partial y}+u_3\frac{\partial}{\partial z}$$

$$(\vec{u}.\nabla)\vec{u}=(\vec{u}.\nabla)u_1\vec{i}+(\vec{u}.\nabla)u_2\vec{j}+(\vec{u}.\nabla)u_3\vec{k}$$

$$(\vec{u}.\nabla)u_1=\left(u_1\frac{\partial}{\partial x}+u_2\frac{\partial}{\partial y}+u_3\frac{\partial}{\partial z}\right)u_1 \;=u_1\frac{\partial u_1}{\partial x}+u_2\frac{\partial u_1}{\partial y}+u_3\frac{\partial u_1}{\partial z}$$

$$(\vec{u}.\nabla)u_2=\left(u_1\frac{\partial}{\partial x}+u_2\frac{\partial}{\partial y}+u_3\frac{\partial}{\partial z}\right)u_2 =u_1\frac{\partial u_2}{\partial x}+u_2\frac{\partial u_2}{\partial y}+u_3\frac{\partial u_2}{\partial z}$$

$$(\vec{u}.\nabla)u_3=\left(u_1\frac{\partial}{\partial x}+u_2\frac{\partial}{\partial y}+u_3\frac{\partial}{\partial z}\right)u_3 \;=u_1\frac{\partial u_3}{\partial x}+u_2\frac{\partial u_3}{\partial y}+u_3\frac{\partial u_3}{\partial z}$$

$$(\vec{u}.\nabla)\vec{u}=(\vec{u}.\nabla)u_1\vec{i}+(\vec{u}.\nabla)u_2\vec{j}+(\vec{u}.\nabla)u_3\vec{k}$$

$$=\left(u_1\frac{\partial u_1}{\partial x}+u_2\frac{\partial u_1}{\partial y}+u_3\frac{\partial u_1}{\partial z}\right)\vec{i}+\left(u_1\frac{\partial u_2}{\partial x}+u_2\frac{\partial u_2}{\partial y}+u_3\frac{\partial u_2}{\partial z}\right)\vec{j}$$

$$+\left(u_1\frac{\partial u_3}{\partial x}+u_2\frac{\partial u_3}{\partial y}+u_3\frac{\partial u_3}{\partial z}\right)\vec{k}$$

Particular case $u_1=4tx, u_2=-2t^2y, u_3=4xz$

$$(\vec{u}.\nabla)\vec{u}=\left(16t^2x\right)\vec{i}+\left(4t^4y\right)\vec{j}+\left(16txz+16x^2z\right)\vec{k}$$

Find the acceleration of a particle at (-1,1,0) in the following velocity field

$$\vec{u} = u_1\vec{i} + u_2\vec{j} + u_3\vec{k} \qquad u_1 = 4tx, u_2 = -2t^2y, u_3 = 4xz$$

$$\vec{a} = \frac{d\vec{u}}{dt} = \frac{\partial\vec{u}}{\partial t} + (\vec{u}.\nabla)\vec{u}$$

$$\frac{\partial\vec{u}}{\partial t} = 4x\vec{i} - 4ty\vec{j} \qquad (\vec{u}.\nabla)\vec{u} = (16t^2x)\vec{i} + (4t^4y)\vec{j} + (16txz + 16x^2z)\vec{k}$$

Local acceleration

Convective acceleration

$$\vec{a} = (4x + 16t^2x)\vec{i} + (-4ty + 4t^4y)\vec{j} + (16txz + 16x^2z)\vec{k}$$

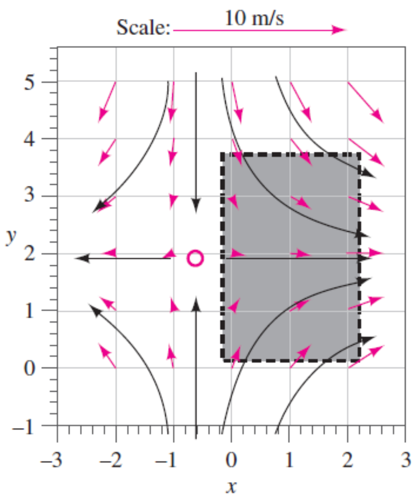
$$\text{at point } (-1,1,0) \quad \vec{a} = -4(1+4t^2)\vec{i} - 4t(1-t^3)\vec{j}$$

Flow Visualization

(Chapter 1 of F M White)

Visualize velocity, acceleration in the following flow field

$$\vec{u} = (.5 + .8x)\vec{i} + (1.5 - .8y)\vec{j} \ \text{m/s}$$



Pink arrows show velocity vector (magnitude and direction) at each point, such a plot is known as vector plot

Velocity vectors are tangent to the **long black lines**, known as **streamlines**

Shaded region resembles flow field at a bell-mouth inlet (not important for present discussion)

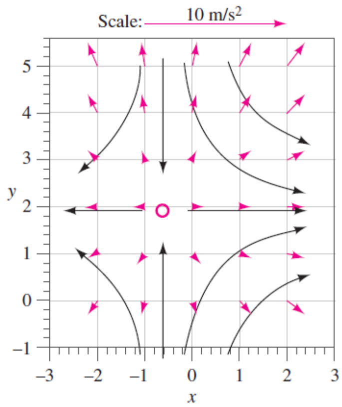
Visualize velocity, acceleration in the following flow field

$$\vec{u} = (.5 + .8x)\vec{i} + (1.5 - .8y)\vec{j} \ \text{m/s}$$

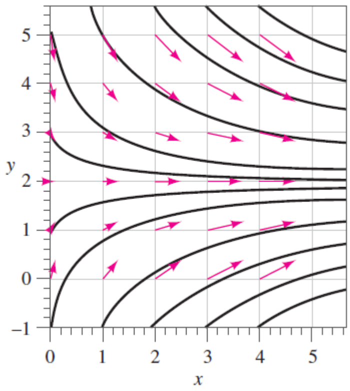
The acceleration of a particle is given by

$$\vec{a} = (.4 + .64x)\vec{i} + (-1.2 + .64y)\vec{j}$$

Pink arrows show acceleration vector (magnitude and direction) at each point, black lines are the **streamlines** (indicates velocity direction). The Figure shows velocity and acceleration fields simultaneously



Streamline: imaginary lines in the flow field; tangent at any point indicates velocity direction



Two streamlines **must not** cut each other

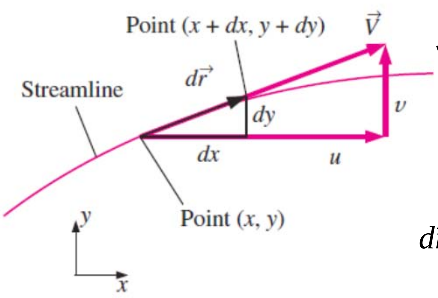
The Equation of streamlines will be given by (prove)

$$\frac{dx}{u}=\frac{dy}{v}=\frac{dz}{w}$$

Find the Equation of the streamlines for the flow field:

$$\vec{u}=(.5+.8x)\vec{i}+(1.5-.8y)\vec{j}$$

Streamline: imaginary lines in the flow field; tangent at any point indicates velocity direction



In a two-dimensional (2-D) velocity field $\vec{V}=u\vec{i}+v\vec{j}$

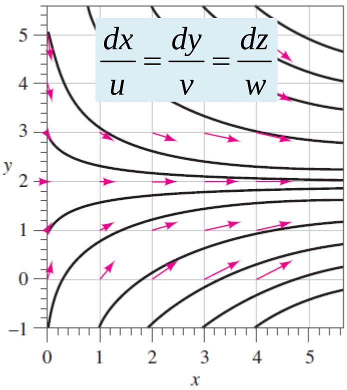
Streamline segment: $d\vec{r}=dx\vec{i}+dy\vec{j}$

$d\vec{r}$ is parallel to $\vec{V} \Rightarrow \dfrac{dx}{u}=\dfrac{dy}{v}$

Equation of a streamline

In 3-D, Equation of a streamline is given by $\dfrac{dx}{u}=\dfrac{dy}{v}=\dfrac{dz}{w}$

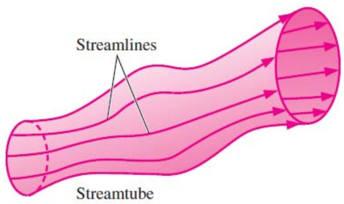
Streamline: imaginary lines in the flow field; tangent at any point indicates velocity direction



Two streamlines **must not** cut each other

A fluid particle cannot cross a streamline

Streamtube: tube made of streamlines



A fluid particle entering in a streamtube will remain inside the streamtube

Find the Equation of the streamlines for the flow field:

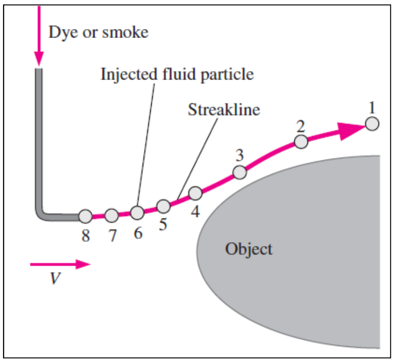
$$\vec{u}=(.5+.8x)\vec{i}+(1.5-.8y)\vec{j}$$

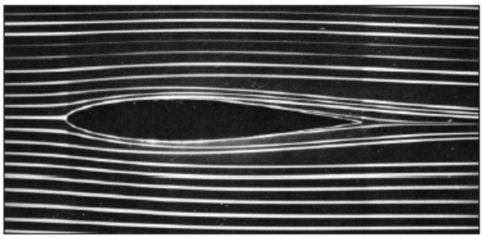
$$\frac{dx}{u}=\frac{dy}{v} \quad \Rightarrow \frac{dx}{.5+.8x}=\frac{dy}{1.5-.8y}$$

$$\Rightarrow (1.5-.8y)(.5+.8x)=C$$

Depending on the values of C, we will get a family of streamlines

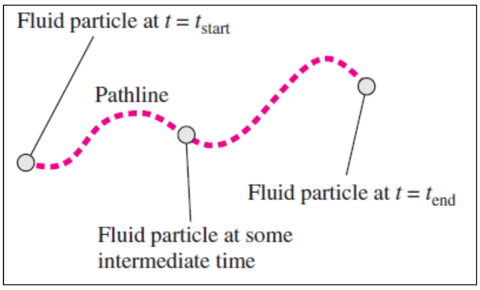
Streakline: locus of fluid particles that have passed sequentially through a prescribed point

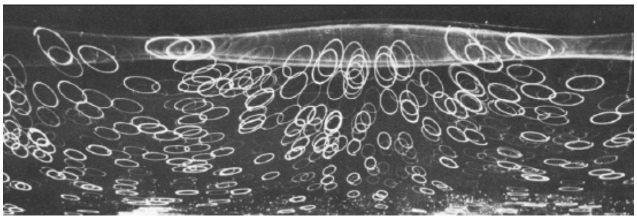




Streakline produced by dye visualization for flow past an aerofoil

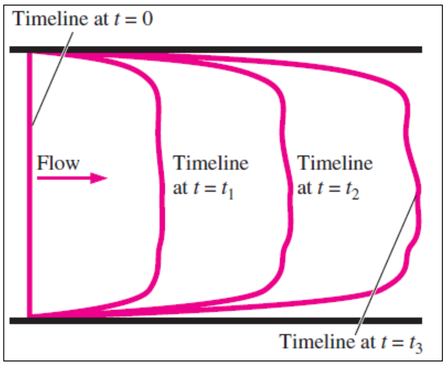
Pathline: actual path travelled by an individual fluid particle over a prescribed period of time

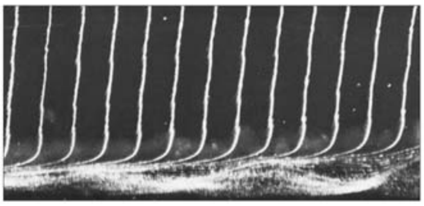




Wave in a free-surface flow; pathlines captured by long-exposure photographs of white tracer particles

Timeline: set of adjacent fluid particles that were marked at the same (earlier) instant of time





Timeline produced by, hydrogen bubble visualization for flow over flat plate