ESO204A, Fluid Mechanics and rate Processes

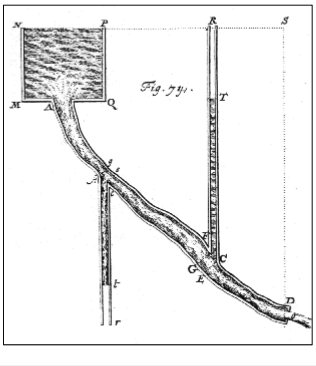
Momentum conservation along a streamline (Bernoulli Equation)
Energy conservation: integral formulation

Chapter 3 of F M White
Chapter 4 of Fox McDonald

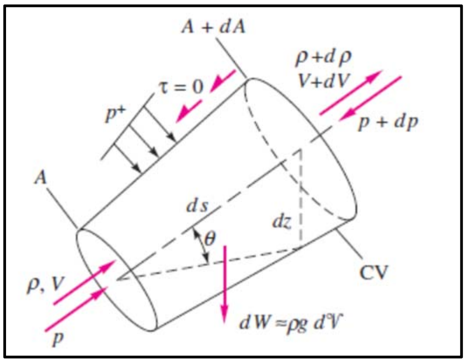
Momentum conservation along a streamline: Bernoulli Equation

Very useful for steady, frictionless flows; essentially a 'differential', not 'integral' formulation; often predicts real situation reasonably well

1700-1782



Assume steady, incompressible flow along a streamline



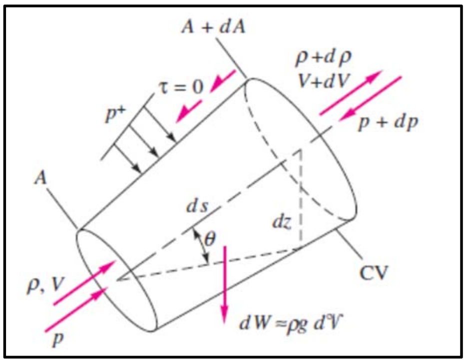
Mass conservation:

$$VA = (V + dV)(A + dA)$$

Momentum conservation:

$$F_{ps} + F_{bs} = -\rho V^2 A + \rho (V + dV)^2 (A + dA)$$

$$= -\rho V^2 A + \rho VA(V + dV) = \rho VAdV$$



$$F_{ps} = pA - (p + dp)(A + dA) + \left[\left(p + \frac{dp}{2}\right)A_p\right]_s$$

$$= -\frac{dp}{2}A - \frac{dp}{2}(A + dA)$$

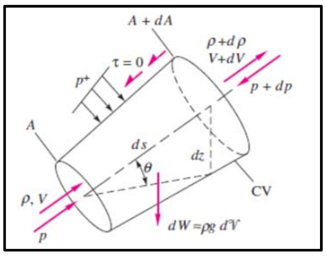
$$= -Adp$$

$$F_{bs} = -\left(A + \frac{dA}{2}\right)ds\rho g \sin\theta \quad = -Adz\rho g$$

Momentum conservation: $-Adp - Adz\rho g = \rho VAdV$

$$\rho VdV + dp + \rho gz = 0$$

Momentum conservation:



$$\rho V A dV + A dp + A \rho g dz = 0$$

$$\Rightarrow d\left(\frac{V^2}{2} + \frac{p}{\rho} + gz \right) = 0$$

$$\frac{V^2}{2} + \frac{p}{\rho} + gz = B \ \ (\text{constant along a streamline})$$

Known as **Bernoulli Equation,** applicable for steady, incompressible, frictionless flow

Bernoulli Equation, may also be derived for more general situation such as, unsteady, compressible, viscous flows

Bernoulli Equation: $\frac{V^2}{2} + \frac{p}{\rho} + gz = B$

No real flow is strictly ‘frictionless’, Bernoulli Eq. is still used in real flow to have an idea about the pressure change

Frictionless flow: flows away from solid wall may ‘sometimes’ be considered frictionless

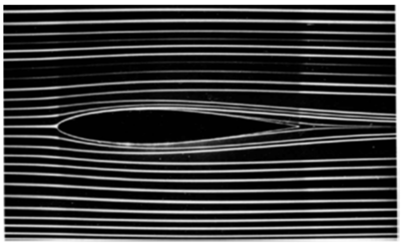
Uniform flows away from solid wall often falls under a special class of flow, known as ‘irrotational flow’. In such cases B is constant everywhere (not just along a streamline)

Bernoulli Equation: $\frac{V^2}{2} + \frac{p}{\rho} + gz = B$

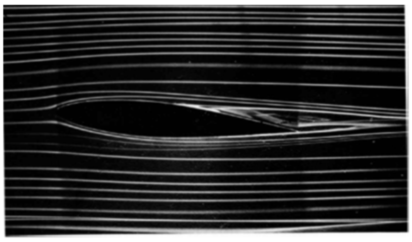
Generally, frictional effects are significant in

- long/narrow flow passages
- diverging sections
- wake region, downstream of an object
- near wall flow
- separated flow

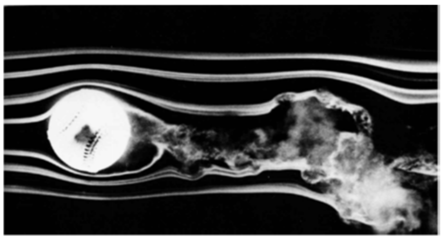
Bernoulli Equation **should not be used** in such cases



1. Flow past an airfoil, streaklines are 'attached' to the solid surface



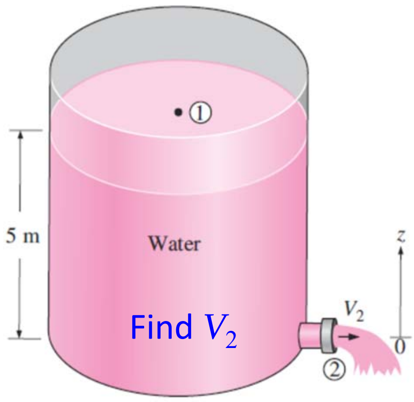
2. Streaklines are 'separated' as we change the angle of attack



3. Wake behind a fast-moving baseball

Bernoulli Eq. is applicable to case 1 when we are just a little away from the solid surface; in cases 2 and 3, we have to be far far away from the solid surfaces

We assume steady (slow enough to consider quasi-steady), incompressible, frictionless flow



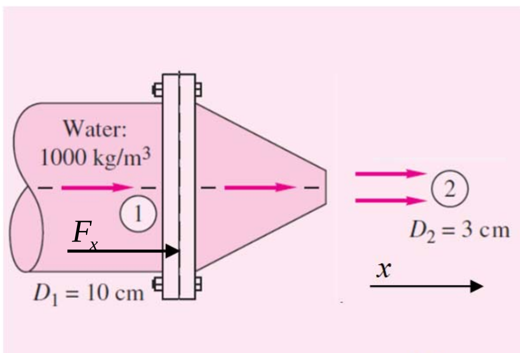
Mass conservation:

$$A_1V_1 = A_2V_2 \quad \Rightarrow V_1 = \frac{A_2}{A_1}V_2$$

Bernoulli: $$\frac{p_1}{\rho}+\frac{V_1^2}{2}+gz_1 = \frac{p_2}{\rho}+\frac{V_2^2}{2}+gz_2$$

$$V_2^2 - V_1^2 = 2g\left(z_1 - z_2\right) \quad \Rightarrow V_2 = \sqrt{2gh}\left[1-\left(\frac{A_2}{A_1}\right)^2\right]^{-\frac{1}{2}} \approx \sqrt{2gh}$$

Find F_x at the flange for $Q_{\text{water}} = 1.5\text{m}^3/\text{min}$



Fire hose with a nozzle

We assume steady, incompressible, frictionless flow

$p_2 = p_a$

Mass conservation: $$A_1u_1 = A_2u_2 = Q$$

Bernoulli: $$p_1 - p_2 = \frac{1}{2}\rho\left(u_2^2 - u_1^2\right)$$

Momentum: $$F_x + \left(p_1 - p_a\right)A_1 = \rho Q\left(u_2 - u_1\right)$$