ESO204A: Fluid Mechanics and Rate Processes

Kinematics

We will try to describe the fluid motion without being concerned with the forces necessary to cause the motion

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Announcements

1. Tutorial: Thursday, 1100-1200, T203-T212
2. Please check the Section list
3. Try to solve the problems; bring a copy of the problems in the tutorial
4. **My office hours: Thursday, 1800-1900, SL210;** you are welcome with any questions/doubts

Kinematics: how to describe fluid motion?

Lagrangian vs Eulerian descriptions

Material (total) vs partial (local) derivatives

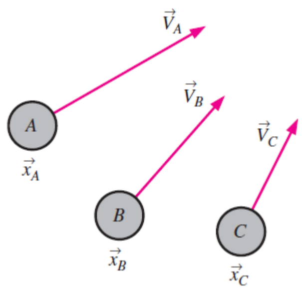
Reference: Chapter 1 of F. M. White

We will now start applying vector calculus concepts, take some time to revisit your Maths book

Lagrangian descriptions

o Lagrangian: Attention is fixed on a particular mass of fluid as it flows

o Track position and velocity vectors of each fluid particles (or any fixed mass system) as a function of time

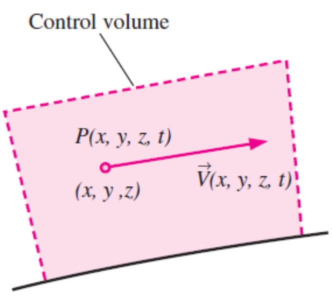


Independent variables: initial position, time

Eulerian description

o We focus on an imaginary control volume (or a point), usually fixed in space; fluid flows in and out of the control volume

o We do not track each particle, instead rely on 'field' variables (usually velocity and pressure fields)



o Lagrangian description helps in deriving the governing Equations, Eulerian frame is used for the solution

Eulerian (Field) representation

o Important flow variables (velocity and pressure fields) are expressed as a function of space of time, such as

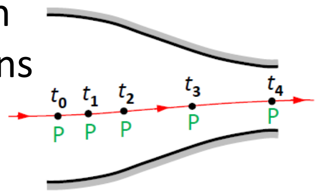
$$\vec{u} = \vec{u}(t, x, y, z),\ p = p(t, x, y, z)$$

o A fluid particle (contains large number of molecules), at a particular point, attains the values (of the field variables) assigned at that point

o Steady field: time derivative to be zero; unsteady otherwise; transient fields evolve into steady or periodic (unsteady) state

o Dimensionality: dependence on spatial coordinate; maximum three (03)

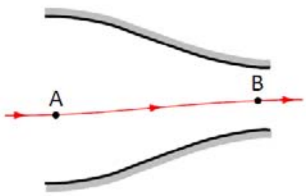
Lagrangian descriptions



Time	Particle P's speed [m/s]
t_0	5
t_1	8
t_2	10
t_3	15
t_4	20

Is this a steady or unsteady flow?

Eulerian description



Time	Speed at A	Speed at B
t_0	5	20
t_1	5	20
t_2	5	20
t_3	5	20
t_4	5	20

Interpret physically $\dfrac{\partial B}{\partial t}, \dfrac{dB}{dt} \left(\dfrac{dB}{dt} \text{ is also written as } \dfrac{DB}{Dt} \right)$

How do we relate rate of change of B of a fluid particle located at point P (x, y, z) at time instant t to the rate of change of B at the point P?

Total (material) derivative

Partial (local) derivative

$$\frac{dB}{dt} = \frac{\partial B}{\partial t} + (\vec{u}.\nabla) B$$

convective derivative

Steadiness requires partial derivatives to be zero for all flow variables

$$\frac{dB(t,x)}{dt} = \lim_{\Delta t \to 0}\left[\frac{B(t+\Delta t, x+\Delta x) - B(t,x)}{\Delta t}\right]$$

$$= \lim_{\Delta t \to 0}\left[\frac{B(t+\Delta t, x) - B(t,x)}{\Delta t}\right] + \lim_{\Delta t \to 0}\left[\frac{B(t+\Delta t, x+\Delta x) - B(t+\Delta t,x)}{\Delta t}\right]$$

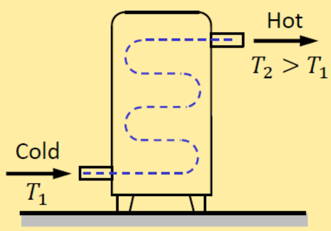
$$= \frac{\partial B}{\partial t} + \lim_{\Delta t \to 0}\left[\frac{B(t+\Delta t, x+\Delta x) - B(t+\Delta t,x)}{\Delta x}\frac{\Delta x}{\Delta t}\right]$$

$$= \frac{\partial B}{\partial t} + u\frac{\partial B}{\partial x} \quad (\text{assuming } \Delta x \to 0)$$

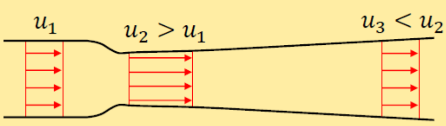
Can be extended for 3-D case; in general

$$\frac{dB}{dt} = \frac{\partial B}{\partial t} + (\vec{u}.\nabla)B \qquad \text{where } \vec{u}.\nabla = u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}$$

Examples

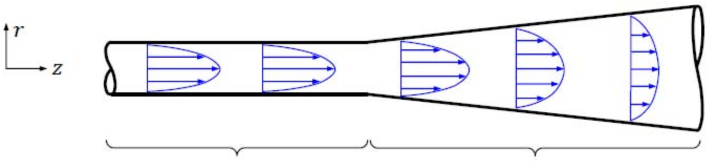


- Steady state operation of a water heater.
- Fluid heats up in the heater.
- $\partial T/\partial t$ of any fluid particle is zero, but dT/dt is not zero.
- Convective derivative of T is not zero.



- Steady state uniform flow in a converging-diverging nozzle.
- Fluid particles first accelerate and than decelerate.
- $\partial u/\partial t$ of a fluid particle is zero, but du/dt is not zero.
- Convective derivative of u is not zero.

Examples



Axisymmetric flow. Nothing depends on θ

Fully developed flow in a constant diameter pipe
1D flow

Decelerating flow in a pipe of increasing diameter
2D flow

3-D flow field shown by cigarette smoke

Find the expression of $(\vec{u}.\nabla)\vec{u}$ where

$$\vec{u}(t,x,y,z)=u_1(t,x,y,z)\vec{i}-u_2(t,x,y,z)\vec{j}+u_3(t,x,y,z)\vec{k}$$

$$\vec{u}.\nabla=u_1\frac{\partial}{\partial x}+u_2\frac{\partial}{\partial y}+u_3\frac{\partial}{\partial z}$$

$$(\vec{u}.\nabla)\vec{u}=(\vec{u}.\nabla)u_1\vec{i}+(\vec{u}.\nabla)u_2\vec{j}+(\vec{u}.\nabla)u_3\vec{k}$$

$$(\vec{u}.\nabla)u_1=\left(u_1\frac{\partial}{\partial x}+u_2\frac{\partial}{\partial y}+u_3\frac{\partial}{\partial z}\right)u_1=u_1\frac{\partial u_1}{\partial x}+u_2\frac{\partial u_1}{\partial y}+u_3\frac{\partial u_1}{\partial z}$$

$$(\vec{u}.\nabla)u_2=\left(u_1\frac{\partial}{\partial x}+u_2\frac{\partial}{\partial y}+u_3\frac{\partial}{\partial z}\right)u_2=u_1\frac{\partial u_2}{\partial x}+u_2\frac{\partial u_2}{\partial y}+u_3\frac{\partial u_2}{\partial z}$$

$$(\vec{u}.\nabla)u_3=\left(u_1\frac{\partial}{\partial x}+u_2\frac{\partial}{\partial y}+u_3\frac{\partial}{\partial z}\right)u_3=u_1\frac{\partial u_3}{\partial x}+u_2\frac{\partial u_3}{\partial y}+u_3\frac{\partial u_3}{\partial z}$$

$$(\vec{u}.\nabla)\vec{u} = (\vec{u}.\nabla)u_1\vec{i} + (\vec{u}.\nabla)u_2\vec{j} + (\vec{u}.\nabla)u_3\vec{k}$$

$$= \left(u_1\frac{\partial u_1}{\partial x} + u_2\frac{\partial u_1}{\partial y} + u_3\frac{\partial u_1}{\partial z}\right)\vec{i} + \left(u_1\frac{\partial u_2}{\partial x} + u_2\frac{\partial u_2}{\partial y} + u_3\frac{\partial u_2}{\partial z}\right)\vec{j}$$

$$+\left(u_1\frac{\partial u_3}{\partial x} + u_2\frac{\partial u_3}{\partial y} + u_3\frac{\partial u_3}{\partial z}\right)\vec{k}$$

Particular case $u_1 = 4tx, u_2 = -2t^2y, u_3 = 4xz$

$$(\vec{u}.\nabla)\vec{u} = (16t^2x)\vec{i} + (4t^4y)\vec{j} + (16txz + 16x^2z)\vec{k}$$

Find the acceleration of a particle at (-1,1,0) in the following velocity field

$$\vec{u} = u_1\vec{i} + u_2\vec{j} + u_3\vec{k} \quad u_1 = 4tx, u_2 = -2t^2y, u_3 = 4xz$$

$$\vec{a} = \frac{d\vec{u}}{dt} = \frac{\partial\vec{u}}{\partial t} + (\vec{u}.\nabla)\vec{u}$$

$$\frac{\partial\vec{u}}{\partial t} = 4x\vec{i} - 4ty\vec{j} \qquad (\vec{u}.\nabla)\vec{u} = (16t^2x)\vec{i} + (4t^4y)\vec{j} + (16txz + 16x^2z)\vec{k}$$

Local acceleration

Convective acceleration

$$\vec{a} = (4x + 16t^2x)\vec{i} + (-4ty + 4t^4y)\vec{j} + (16txz + 16x^2z)\vec{k}$$

$$\text{at point } (-1,1,0) \quad \vec{a} = -4(1+4t^2)\vec{i} - 4t(1-t^3)\vec{j}$$