ESO204A, Fluid Mechanics and rate Processes

Heat Transfer Fundamentals

Chapters 1, 2 of Cengel

RECALL

Energy conservation: $\frac{dE}{dt} = \dot{Q}_{in} - \dot{W}_{out}$

Reynolds Transport Theorem:

$$\frac{dE}{dt} = \frac{\partial}{\partial t}\int_{CV} \rho e d\forall + \int_{CS} \rho e (\vec{u}.\vec{n}) dA \qquad e = \frac{E}{m}$$

Rate of work: $\dot{W} = \dot{W}_{\text{shaft}} + \dot{W}_{\text{shear}} + \dot{W}_{\text{pressure}} + \dot{W}_{\text{others}}$

Combining:

$$\dot{Q}_{in} - \dot{W}_{\text{shaft}} - \dot{W}_{\text{shear}} - \dot{W}_{\text{others}} = \frac{\partial}{\partial t}\int_{CV} \rho e d\forall + \int_{CS} \rho \left(e + \frac{p}{\rho} \right)(\vec{u}.\vec{n}) dA$$

$$\dot{Q}_{in} - \dot{W}_{\text{shaft}} - \dot{W}_{\text{shear}} - \dot{W}_{\text{others}} = \frac{\partial}{\partial t}\int_{CV} \rho e\, d\mathcal{V} + \int_{CS} \rho\left(e+\frac{p}{\rho}\right)\cancel{(\vec{u}.\vec{n})}\, dA$$

$\dot{E}_{\text{gen}}$ assume $u = 0$

$$-\dot{E}_{\text{gen}} = \cancel{\dot{W}_{\text{shaft}}} + \cancel{\dot{W}_{\text{shear}}} + \dot{W}_{\text{others}} = \dot{W}_{\text{elec}} + \dot{W}_{\text{chem}} + \ldots$$

$$\frac{\partial}{\partial t}\int_{CV} \rho e\, d\mathcal{V} = \dot{Q}_{in} + \dot{E}_{\text{gen}} \qquad \dot{E}_{\text{gen}} = \int_{CV} \dot{e}_{\text{gen}}\, d\mathcal{V}$$

$\dot{e}_{\text{gen}}$: volumetric energy (heat) generation rate (W/m^3)

Energy conservation

$$\frac{\partial}{\partial t}\int_{CV} \rho e\, d\mathcal{V} = \dot{Q}_{in} + \int_{CV} \dot{e}_{\text{gen}}\, d\mathcal{V}$$

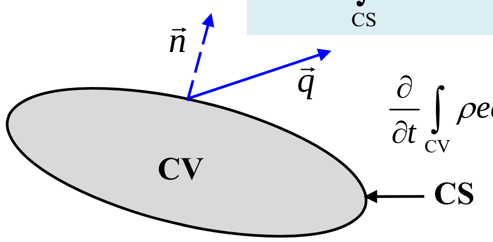
$$\frac{\partial}{\partial t}\int_{CV} \rho e\, d\mathcal{V} = \dot{Q}_{in} + \int_{CV} \dot{e}_{\text{gen}}\, d\mathcal{V}$$

Heat flow in the direction of the area (scalar)

= heat flux (vector) . area (vector) $\Rightarrow \dot{Q} = \vec{q}.\vec{A} = \vec{q}.\vec{n}A$

Heat flow in to a CV through the CS

$$\dot{Q}_{in} = -\int_{CS} \vec{q}.d\vec{A} = -\int_{CS} \vec{q}.\vec{n}dA$$



$$\frac{\partial}{\partial t}\int_{CV} \rho e\, d\mathcal{V} = -\int_{CS} \vec{q}.\vec{n}dA + \int_{CV} \dot{e}_{\text{gen}}\, d\mathcal{V}$$

$$\frac{\partial}{\partial t}\int_{CV}\rho e\,d\cancel{V} = -\int_{CS}\vec{q}.\vec{n}dA + \int_{CV}\dot{e}_{gen}d\cancel{V} \;= -\int_{CV}\nabla.\vec{q}d\cancel{V} + \int_{CV}\dot{e}_{gen}d\cancel{V}$$

$$\int_{CV}\frac{\partial(\rho e)}{\partial t}d\cancel{V} = \int_{CV}\left(-\nabla.\vec{q}+\dot{e}_{gen}\right)d\cancel{V} \qquad \boxed{\frac{\partial(\rho e)}{\partial t} = -\nabla.\vec{q}+\dot{e}_{gen}}$$

Considering internal energy only

$$e = cT$$

Specific heat Temperature

Fourier's law of heat conduction

$$\vec{q} = -k\nabla T$$

Thermal conductivity (material property)

$$\frac{\partial(\rho e)}{\partial t} = -\nabla.\vec{q}+\dot{e}_{gen} \qquad e = cT \qquad \vec{q} = -k\nabla T$$

$$\boxed{\frac{\partial(\rho cT)}{\partial t} = \nabla.\left(k\nabla T\right)+\dot{e}_{gen}}$$

General Eq. of heat conduction

Assuming constant properties $\boxed{\rho c\frac{\partial T}{\partial t} = k\nabla^2 T+\dot{e}_{gen}}$

Another useful form $\boxed{\frac{\partial T}{\partial t} = \alpha\nabla^2 T+\frac{\dot{e}_{gen}}{\rho c}}$

Thermal diffusivity $= \dfrac{k}{\rho c}$

Thermal diffusivity indicates 'speed' of heat transfer through a material

General Eq. of heat conduction $\frac{\partial T}{\partial t} = \alpha \nabla^2 T + \frac{\dot{e}_{\text{gen}}}{\rho c}$

Unsteady, no generation $\frac{\partial T}{\partial t} = \alpha \nabla^2 T$

Steady: $0 = \alpha \nabla^2 T + \frac{\dot{e}_{\text{gen}}}{\rho c} \Rightarrow \nabla^2 T + \frac{\dot{e}_{\text{gen}}}{k} = 0$

Steady, no generation: $\nabla^2 T = 0$

Steady, no generation, 1-D Cartesian: $\frac{d^2 T}{dx^2} = 0$

Conductivity, diffusivity of important materials

Material	Conductivity ($Wm^{-1}K^{-1}$)	Diffusivity (m^2s^{-1})
Mild steel	50	1.17×10^{-5}
Copper	480	1.11×10^{-4}
Aluminium	300	9.7×10^{-5}
Gold	300	1.27×10^{-4}
Silver	450	1.66×10^{-4}
Concrete	.77	3.5×10^{-7}
Brick	1.3	5.2×10^{-7}
Glass	1	3.4×10^{-7}
Glass wool	.04	1.1×10^{-8}
Air	.02	1.9×10^{-5}
Water	.5	1.4×10^{-7}

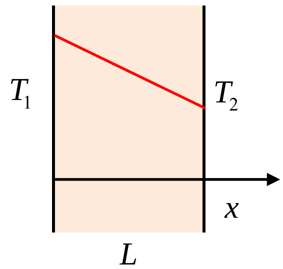
conductor

insulator

Steady, 1-D Heat Transfer in a Slab $\frac{d^2T}{dx^2}=0 \Rightarrow T=c_1x+c_2$

$$T(x=0)=T_1 \quad \Rightarrow c_2=T_1$$

$$T(x=L)=T_2 \quad \Rightarrow c_1L+c_2=T_2$$



$$T=T_1+(T_2-T_1)\frac{x}{L}$$

linear temperature variation

$$\frac{T-T_1}{T_2-T_1}=\frac{x}{L}$$

Nondimensional form

Heat flux $q''_x=-k\frac{dT}{dx}=-k\frac{d}{dx}\left[T_1+(T_2-T_1)\frac{x}{L}\right]=\frac{k(T_1-T_2)}{L}$

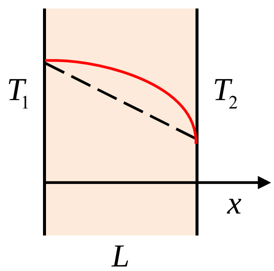
Heat transfer rate $\dot{Q}=q''_x\times\text{Area}=\frac{kA(T_1-T_2)}{L}$

Steady, 1-D heat transfer, with uniform generation $\frac{d^2T}{dx^2}+\frac{\dot{e}_{\text{gen}}}{k}=0 \Rightarrow \frac{dT}{dx}=-\frac{\dot{e}_{\text{gen}}}{k}x+c_1$

$$\Rightarrow T=-\frac{\dot{e}_{\text{gen}}}{2k}x^2+c_1x+c_2 \quad T(x=0)=T_1$$

$$\Rightarrow c_2=T_1$$

$$T(x=L)=T_2 \quad \Rightarrow -\frac{q'''}{2k}L^2+c_1L+c_2=T_2$$



$$T=T_1+(T_2-T_1)\frac{x}{L}+\frac{\dot{e}_{\text{gen}}}{2k}\left(Lx-x^2\right)$$

$$\frac{T-T_1}{T_2-T_1}=\frac{x}{L}+\frac{\dot{e}_{\text{gen}}L^2}{2k(T_2-T_1)}\left(\frac{x}{L}-\frac{x^2}{L^2}\right)$$

Nondimensional form

$$q''_x=-k\frac{dT}{dx}=\frac{k(T_1-T_2)}{L}+\frac{\dot{e}_{\text{gen}}}{2}(2x-L)$$

Contribution from source

$$\dot{Q}=\frac{kA(T_1-T_2)}{L}+\frac{\dot{e}_{\text{gen}}A}{2}(2x-L)$$