ESO204A: Fluid Mechanics and Rate Processes
TUTORIAL 11 PROBLEMS

August-November 2017

1. Review of Tutorial 10.

2. The velocity profile for pressure-driven laminar flow between parallel plates (see Fig. 4.12*b*) has the form $u = C(h^2 - y^2)$, where C is a constant. (*a*) Determine if a stream function exists. (*b*) If so, find a formula for the stream function.

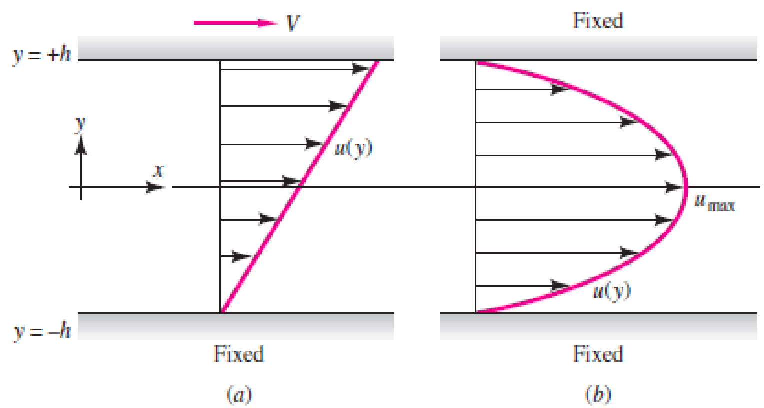


Fig. 4.12 Incompressible viscous flow between parallel plates: (*a*) no pressure gradient, upper plate moving; (*b*) pressure gradient $\partial p/\partial x$ with both plates fixed.

3. An incompressible stream function is defined by

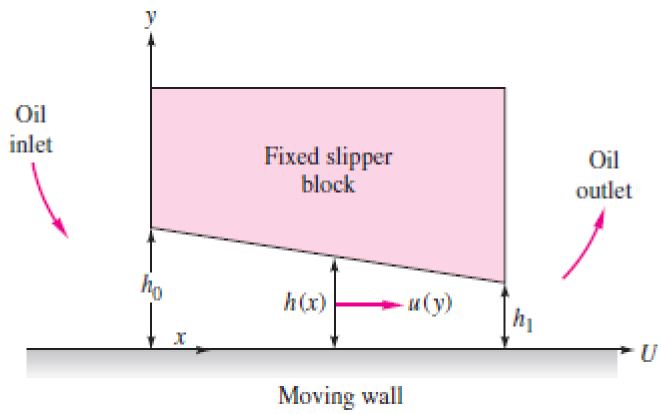
$$\Psi(x, y) = \frac{U}{L^2}\left(3x^2 y - y^3\right)$$

where U and L are (positive) constants. Where in this chapter are the streamlines of this flow plotted? Use this stream function to find the volume flow Q passing through the rectangular surface whose corners are defined by $(x, y, z) = (2L, 0, 0)$, $(2L, 0, b)$, $(0, L, b)$, and $(0, L, 0)$. Show the direction of Q.

4. {*For discussion*} The flow pattern in bearing lubrication can be illustrated by Fig. P4.83, where a viscous oil $(\rho,\ \mu)$ is forced into the gap $h(x)$ between a fixed slipper block and a wall moving at velocity U. If the gap is thin, $h << L$, it can be shown that the pressure and velocity distributions are of the form $p = p(x)$, $u = u(y)$, $u = w = 0$. Neglecting gravity, reduce the Navier-Stokes equations (4.38) to a single differential equation for $u(y)$. What are the proper boundary conditions? Integrate and show that

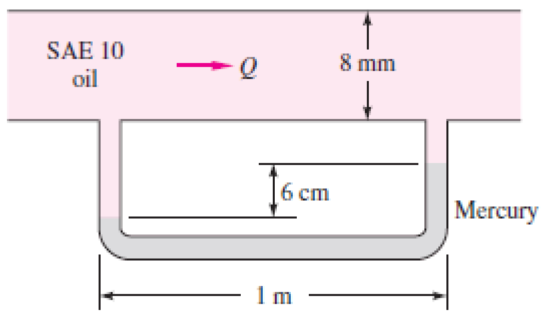
$$u = \frac{1}{2\mu}\frac{dp}{dx}\left(y^2 - yh\right) + U\left(1 - \frac{y}{h}\right)$$

where $h = h(x)$ may be an arbitrary, slowly varying gap width.



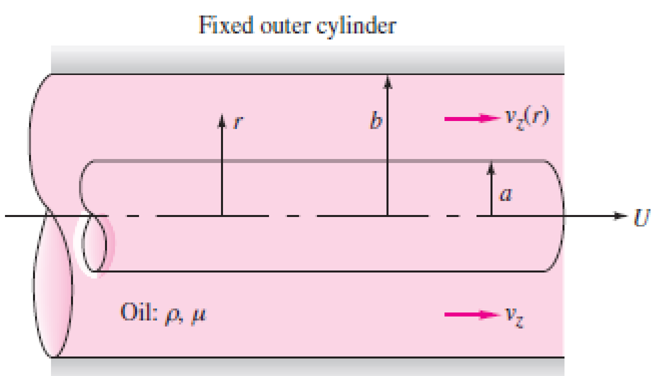
P4.83

5. SAE 10 oil at 20°C flows between parallel plates 8 mm apart, as in Fig. P4.86. A mercury manometer, with wall pressure taps 1 m apart, registers a 6-cm height, as shown. Estimate the flow rate of oil for this condition.



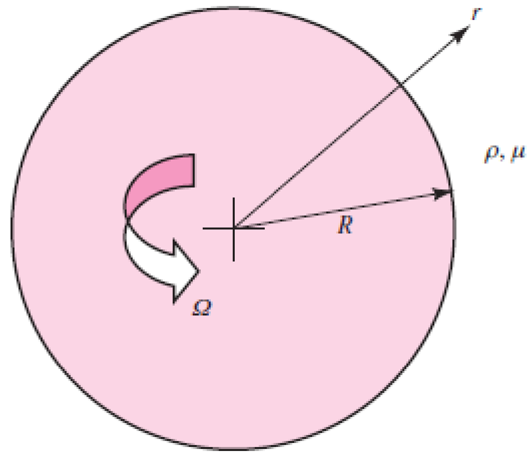
P4.86

6. The viscous oil in Fig. P4.88 is set into steady motion by a concentric inner cylinder moving axially at velocity U inside a fixed outer cylinder. Assuming constant pressure and density and a purely axial fluid motion, solve the Navier-Stokes equations for the fluid velocity distribution $v_z(r)$. What are the proper boundary conditions?



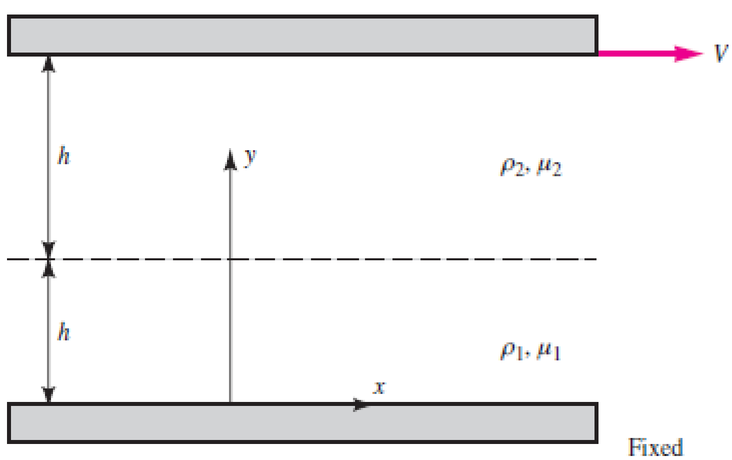
P4.88

7. A long solid cylinder rotates steadily in a very viscous fluid, as in Fig. P4.94. Assuming steady, incompressible laminar flow, solve the Navier-Stokes equation in polar coordinates to determine the resulting velocity distribution. The fluid is at rest far from the cylinder. [*Hint:* the cylinder does not induce any radial motion.]



P4.94

8. Two immiscible liquids of equal thickness h are being sheared between a fixed and a moving plate, as in Fig. P4.95. Gravity is neglected, and there is no variation with x. Find an expression for (a) the velocity at the interface and (b) the shear stress in each fluid. Assume steady incompressible laminar flow.



P4.95