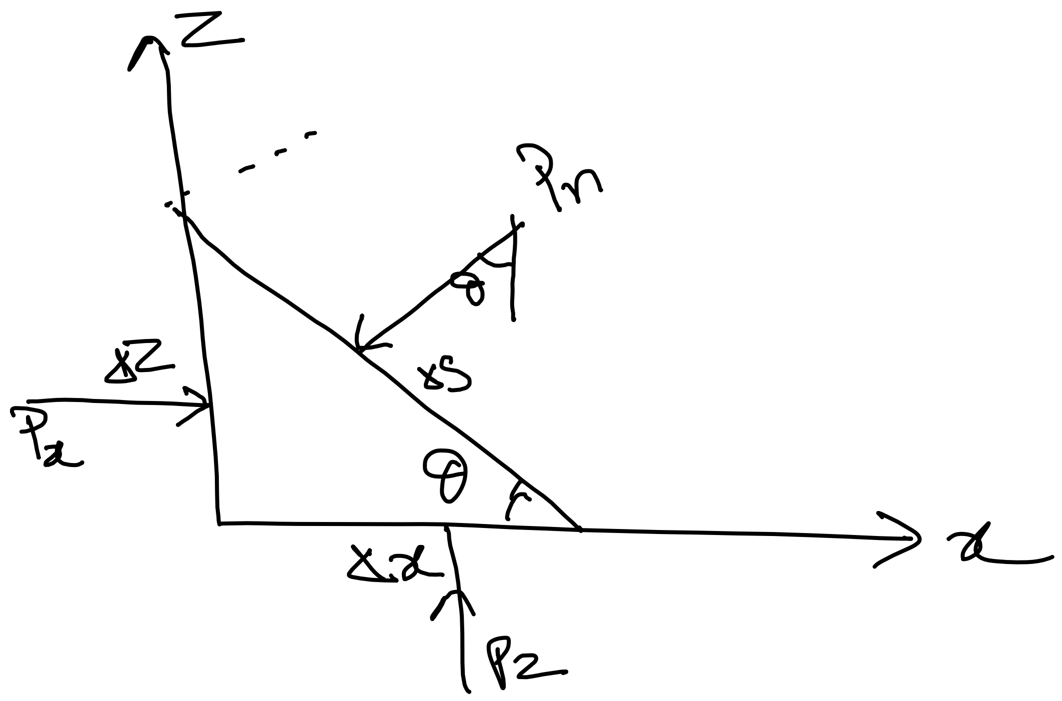
- Pressure is a scalar. When it acts on a surface, it acts like a tensor whose direction is inward normal to the surface

- A wedge of a fluid element



Force balance

$$\sum F_x = P_x\, b\, \Delta z - P_n \sin\theta\, \underline{b\, \Delta s} = 0$$

$$\Delta s \sin\theta = \Delta z$$

$$\Rightarrow P_x = P_n$$

$$\sum F_z = P_z\, \Delta x\, b - P_n\, \Delta s\, b \cos\theta - \frac{1}{2}\underline{\Delta x\, \Delta z\, b\, \rho\, g} = 0$$

$$\Delta s \cos\theta = \Delta x$$

$$P_z = P_n + \frac{1}{2}\underline{\Delta z}\, \rho\, g$$

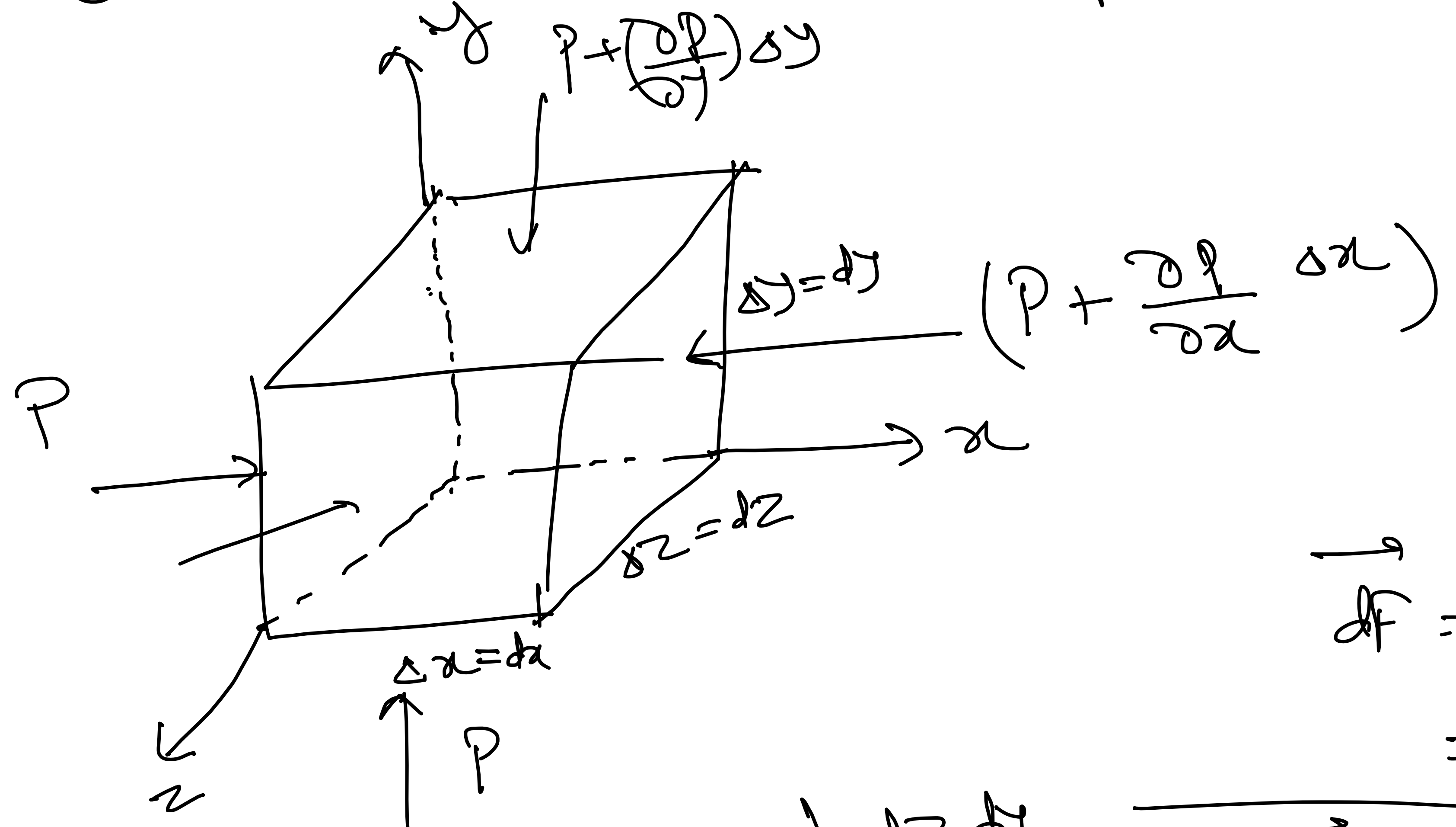
$$\Delta z \to 0$$

$$P_z = P_n$$

$$P_x = \underline{P_n} = P_z$$

- It is a point property

Force on a fluid element due to spatial variation of pressure



$$dV = dx\,dy\,dz$$

$$\vec{dF} = dF_x\,\hat{i} + dF_y\,\hat{j} + dF_z\,\hat{k}$$

$$= -\left(\frac{\partial p}{\partial x}\hat{i} + \frac{\partial p}{\partial y}\hat{j} + \frac{\partial p}{\partial z}\hat{k}\right) dx\,dy\,dz$$

$$dF_x = p\,dz.dy - \left(p + \frac{\partial p}{\partial x} dx\right) dz\,dy$$

$$= -\frac{\partial p}{\partial x} dx\,dy\,dz$$

$$\boxed{\frac{\vec{dF}}{dV} = -\nabla p}$$

similarly

$$dF_y = -\frac{\partial p}{\partial y} dx\,dy\,dz$$

$$dF_z = -\frac{\partial p}{\partial z} dx\,dy\,dz$$

Other forces acting on element

$$\sum \text{Body forces} + \sum \text{Surface force} = 0$$

Hydrostatics → no viscous stress

- electromagnetic force
- gravity

$$-\nabla p \, dV + \rho \vec{g} \, dV = 0$$

$$\nabla p = \rho \vec{g}$$

$$\left(\frac{\partial p}{\partial x} \hat{i} + \frac{\partial p}{\partial y} \hat{j} + \frac{\partial p}{\partial z} \hat{k} \right) = \rho \left(0 \hat{i} + 0 \hat{j} - g \hat{k} \right)$$

$g = 9.81 \ m/s^2$

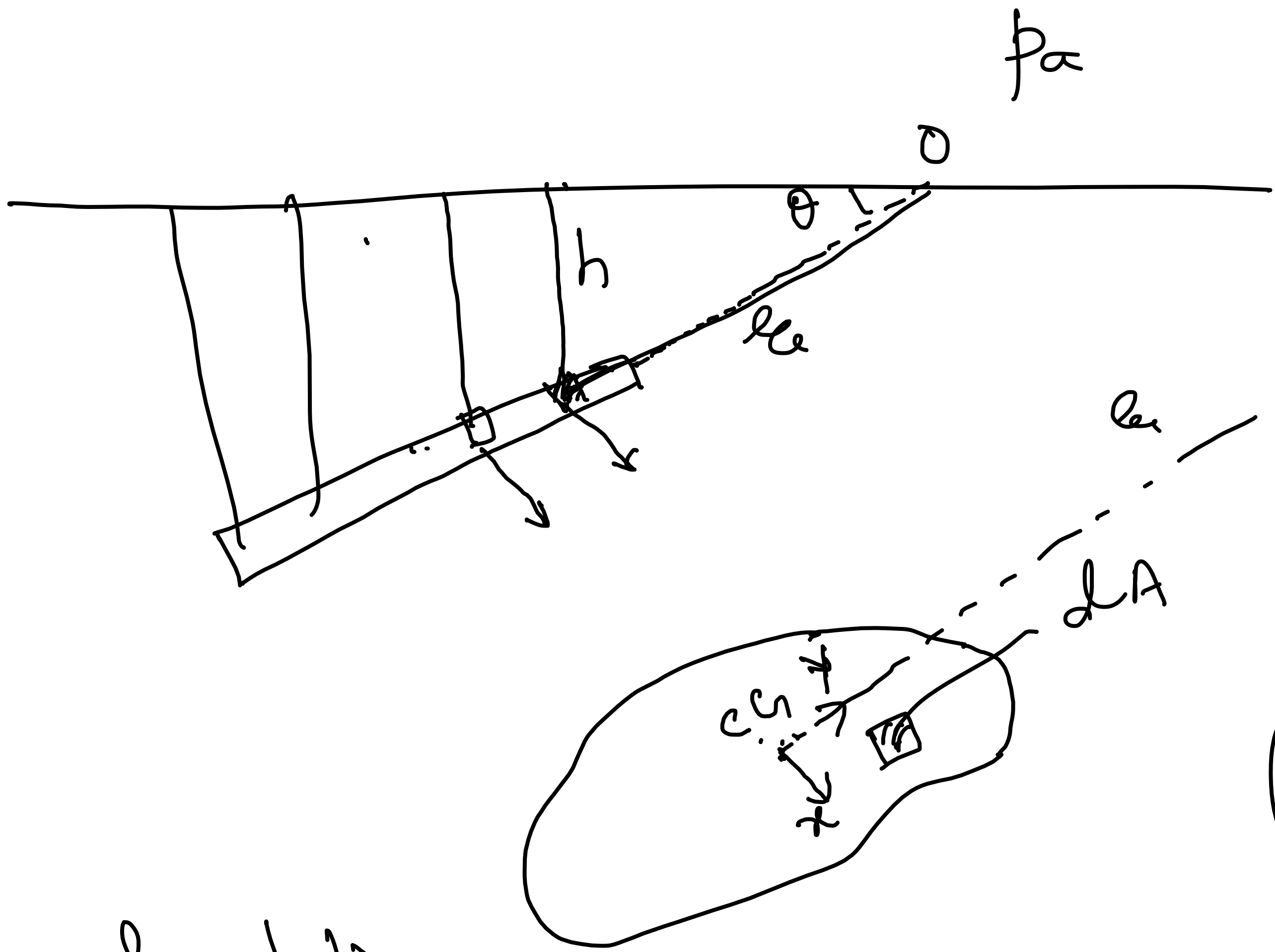
$$\boxed{\frac{\partial p}{\partial z} = -\rho g}$$ ← Hydrostatic condition

$\int \rightarrow \rho g$

$P_{gage} = P - P_a > 0$

$P_{vacuum} = P_a - P > 0$

Hydrostatic force on a surface (one side)



$(\xi_i = \xi_{CG} - y)$

$\rho g = \gamma$ = specific weight of fluid

force due to hydrostatics

$$F = \int dF = \int (p\, dA) = \int (p_a + \rho g h)\, dA$$

$$= p_a A + \gamma \int \xi \sin\theta \, dA = p_a A + \gamma \underline{\sin\theta\, \xi_{CG}}\, A$$

$$\xi_{CG} = \frac{\int \xi_i \, dA}{A}$$

$$= p_a A + \gamma h_{CG} A$$

$$F = (p_a + \gamma h_{CG}) A = P_{CG} A$$

Point of action = (x_{cp}, y_{cp})

= center of Pressure

y_{cp} moment balance

Σ moments due to differential forces = moment due to net force

$$F\, y_{cp} = \int p\, dA\, y = \int (p_a + \gamma h)\, y\, dA$$

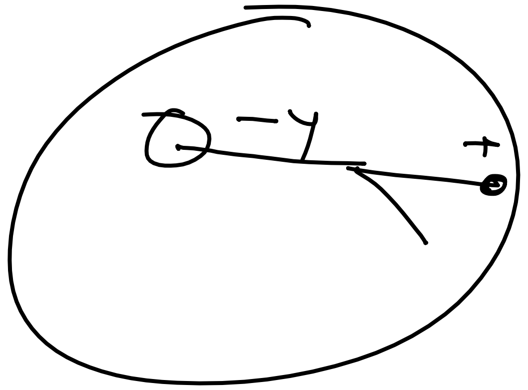
$$= \cancel{\int p_a\, y\, dA}^{\,0} + \gamma \int \xi \sin\theta\, y\, dA$$

$$= \gamma \sin\theta \int (\xi_{cg} - y)\, y\, dA$$

$$= \gamma \sin\theta \left[\cancel{\int \xi_{cg}\, y\, dA}^{\,0} - \int y^2\, dA \right]$$

$$= -\gamma \sin\theta\, I_{xx}$$

$$\boxed{\begin{array}{c} P_{cg} A\, y_{cp} = -\gamma \sin\theta\, I_{xx} \\ y_{cp} = -\dfrac{\gamma \sin\theta\, I_{xx}}{P_{cg} A} \\ \text{no atmospheric pressure} \\ F = \gamma h_{cg} A \\ y_{cp} = -\dfrac{\sin\theta\, I_{xx}}{h_{cg} A} \end{array}}$$



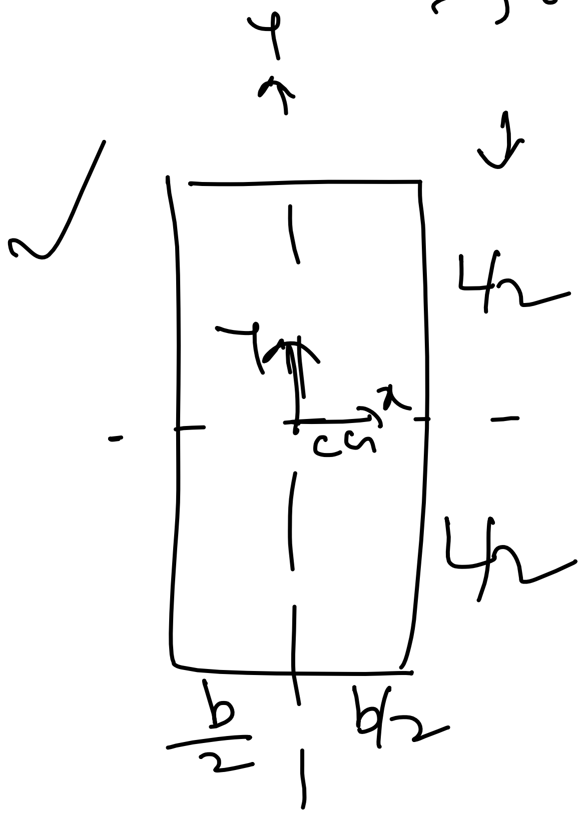
I_{xx} = Area moment of inertia

$$\int^{x} \cancel{y}\, \theta\, dA$$

$$x_{cp} = \frac{-\gamma \sin\theta \, I_{xy}}{P_{cg} A}$$

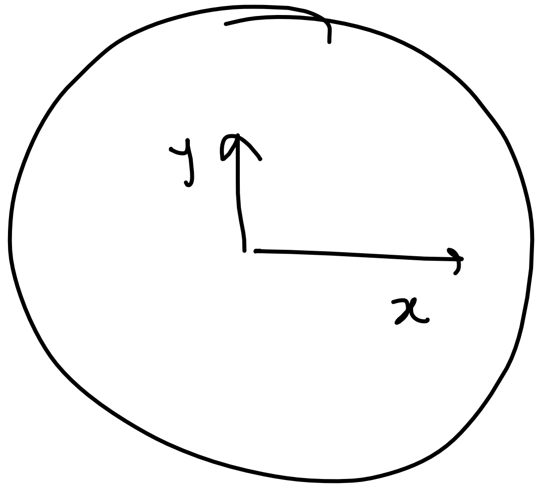
$$I_{xy} = \int xy \, dA$$

for symmetric surfaces $I_{xy} = 0$



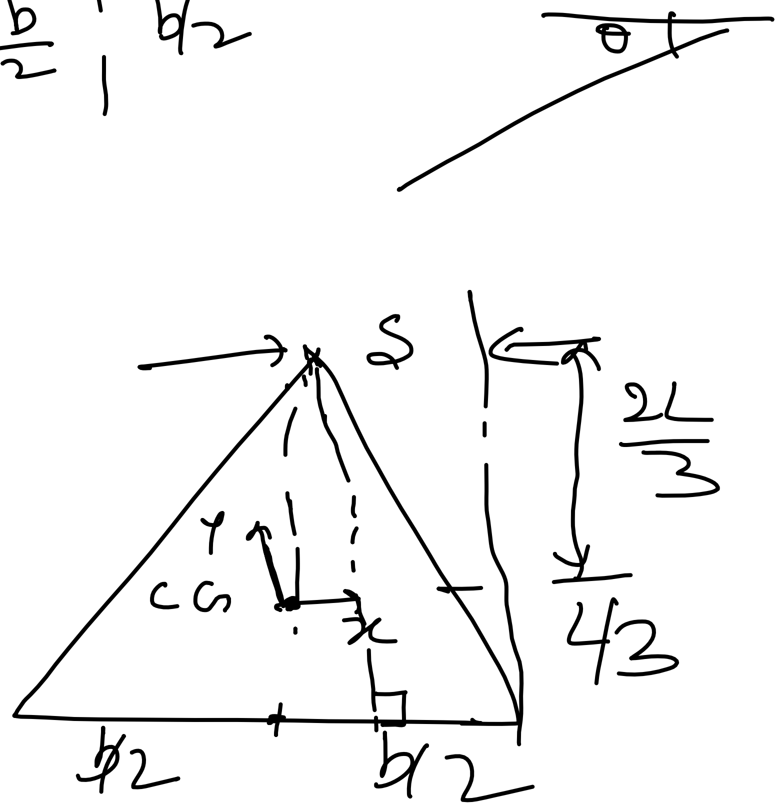
$$I_{xx} = \frac{bL^3}{12}$$

$$I_{xy} = 0$$



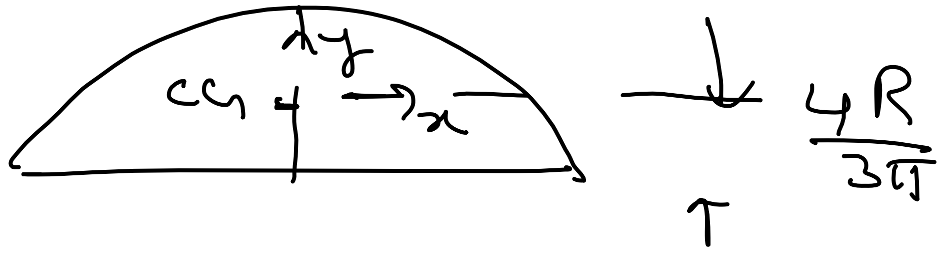
$$I_{xx} = \frac{\pi R^4}{4}$$

$$I_{xy} = 0$$



$$I_{xx} = \frac{bL^3}{36}$$

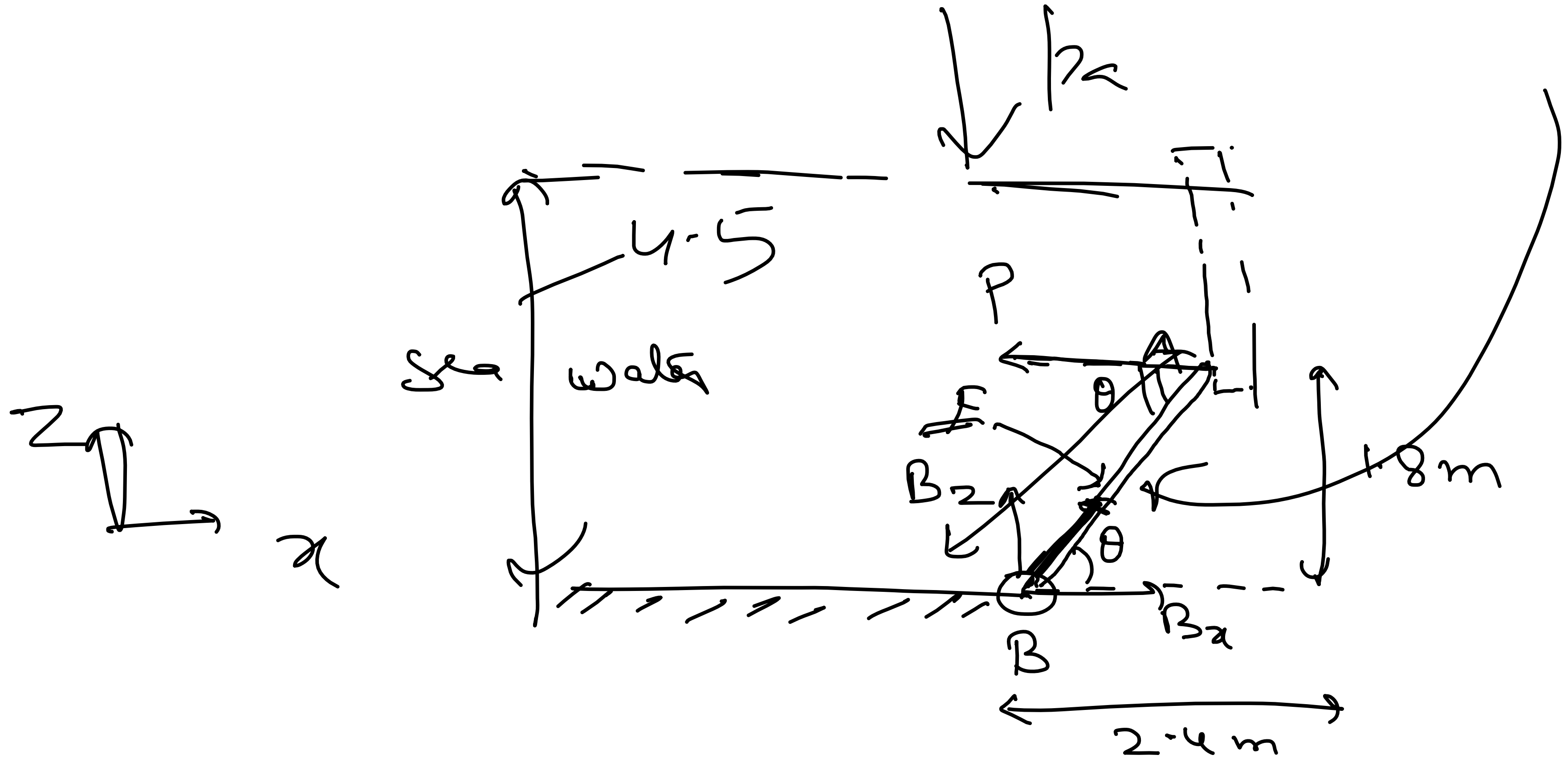
$$I_{xy} = \frac{b(b-2S)L^2}{72}$$



$$I_{xx} = 0.10976 R^4$$

$$I_{xy} = 0$$

__Example__ The gate in fig is 1.5 wide, is hinged at point B and rests against a smooth wall at point A. compute (a) the force on the gate due to sea water (b) the horizontal force P exerted by wall at point A and (c) the reaction at the hinge B.



$\gamma = 10{,}000 \ N/m^3$

$F = \gamma h_{cg} A = 162 \ kN$ $\quad \underline{h_{cg}} = 4.5 - 0.9$

Point of action of $F \rightarrow x_{cp} = 0$

$= y_{cp} = -\dfrac{\sin\theta \ I_{xx}}{h_{cg} A}$

$l = |y_{cp}|$

moment balance about point B

$F(\ ) - \underline{P}x = 0$

force balances in x and z direction