ESO204A, Fluid Mechanics and rate Processes

Momentum conservation: integral formulation

Very useful for calculation of forces

Chapter 3 of F M White
Chapter 4 of Fox McDonald (uploaded)

Reynolds Transport Theorem:

$$\frac{dB_{sys}}{dt} = \frac{\partial}{\partial t}\int_{CV} \rho\beta d\forall + \int_{CS} \rho\beta\left(\vec{u}.\vec{n}\right)dS$$

For Momentum conservation: $B_{sys} = m\vec{u} \Rightarrow \beta = \vec{u}$

$$\frac{d\left(m\vec{u}\right)}{dt} = \frac{\partial}{\partial t}\int_{CV} \rho\vec{u} d\forall + \int_{CS} \rho\vec{u}\left(\vec{u}.\vec{n}\right)dS$$

Momentum conservation principle: $\frac{d\left(m\vec{u}\right)}{dt} = \vec{F}$

Momentum conservation (intgral formulation)

$$\vec{F} = \frac{\partial}{\partial t}\int_{CV} \rho \vec{u} d\forall + \int_{CS} \rho \vec{u}\left(\vec{u}.\vec{n}\right) dS$$

$$\vec{F} = \vec{F}_S + \vec{F}_B$$

$\vec{F}_S$: Surface force, all forces acting at the control surface

$\vec{F}_B$: Body forces (gravity, electromagnetic)

Surface forces usually come from pressure, shear and interaction with solid objects/surfaces

$$\vec{F}_S + \vec{F}_B = \frac{\partial}{\partial t}\int_{CV} \rho \vec{u} d\forall + \int_{CS} \rho \vec{u}\left(\vec{u}.\vec{n}\right) dS$$

Incompressible flow: $\vec{F}_S + \vec{F}_B = \rho\frac{\partial}{\partial t}\int_{CV} \vec{u} d\forall + \rho\int_{CS} \vec{u}\left(\vec{u}.\vec{n}\right) dS$

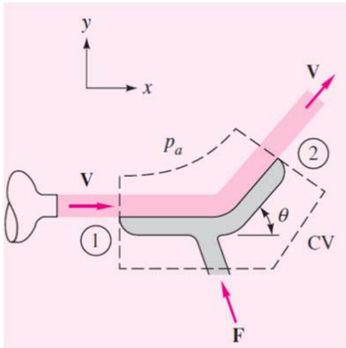
In a non-deformable CV

Steady flow: $\vec{F}_S + \vec{F}_B = \int_{CS} \rho \vec{u}\left(\vec{u}.\vec{n}\right) dS$

Steady, incompressible flow: $\vec{F}_S + \vec{F}_B = \rho\int_{CS} \vec{u}\left(\vec{u}.\vec{n}\right) dS$

Water jet (area A) hits a fixed vane, flow direction changes

Find the force F necessary to hold the vane fixed



Assumption: 1. steady, incompressible flow, 2. flow is in horizontal plane, 3. uniform flow at inlet/exit, 4. $p = p_a$

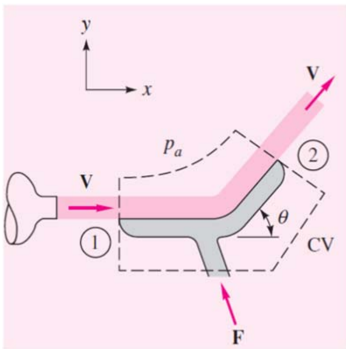
Analysis: $\int_{CS} (\vec{u}.\vec{n})\,dS = 0$ $\quad \vec{F}_S = \rho \int_{CS} \vec{u}\,(\vec{u}.\vec{n})\,dS$

$$\int_{CS} (\vec{u}.\vec{n})\,dS = 0$$

$$u_1 = u_2 = V \quad (\text{since } A = \text{constant})$$

$$\vec{F}_S = \rho \int_{CS} \vec{u}\,(\vec{u}.\vec{n})\,dS = \rho \sum \vec{u}\left(\vec{u}.\vec{A}\right)$$

$$\left(\vec{u}.\vec{A}\right)_1 = -VA = -\left(\vec{u}.\vec{A}\right)_2$$

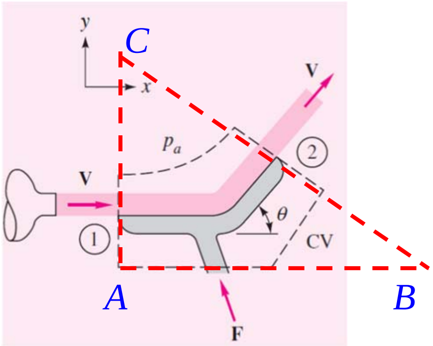


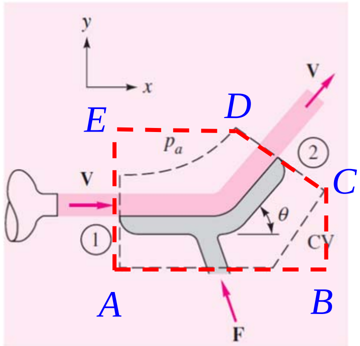
In x direction:

$$F_x + F_{px}\,(\text{pressure forces}) = \rho V \left(\vec{u}.\vec{A}\right)_1 + \rho V \cos\theta \left(\vec{u}.\vec{A}\right)_2$$
$$= -\rho V^2 A + \rho V^2 A \cos\theta$$

$$F_x = -\rho V^2 A + \rho V^2 A \cos\theta$$

$F_{px} = 0$ Since gage pressure is zero everywhere



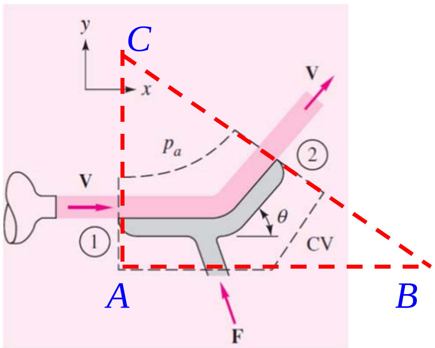


$$F_{px} = p \times AC - p \times BC\cos\theta$$
$$= 0$$

$$F_{px} = p \times AE - p \times BC$$
$$- p \times CD\cos\theta \quad = 0$$

Pressure force always at the CS toward the CV (compressive)

In y direction:



$$F_y + F_{py} = \rho V\sin\theta\left(\vec{u}.\vec{A}\right)_2$$
$$= \rho V^2 A\sin\theta$$

$$F_{py} = p \times AB - p \times BC\sin\theta$$
$$= 0$$

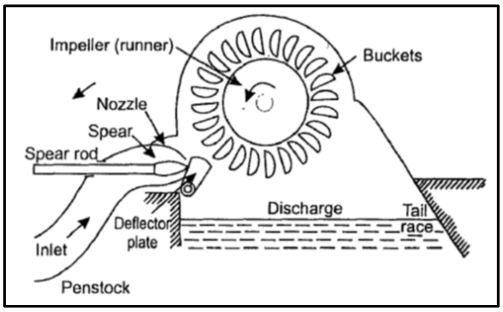
$$F_y = \rho V^2 A\sin\theta$$

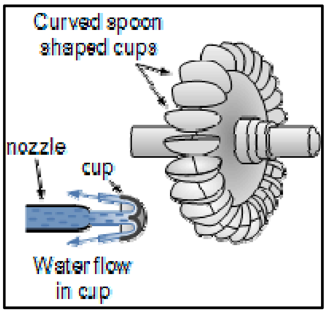
$$F_x = -\rho V^2 A + \rho V^2 A\cos\theta$$

$$F = \sqrt{F_x^2 + F_y^2} = \rho V^2 A\sqrt{\sin^2\theta + \left(\cos\theta - 1\right)^2} = 2\rho V^2 A\sin\frac{\theta}{2}$$

Also find the direction of the above force

- Pressure force always at the CS toward the CV (compressive)
- **Thumb rules for choice of CV:** surface normal (at CS) should be along or opposite to the flow direction or coordinate axes
- Uniform pressure distribution over a closed CS leads to zero pressure force (proof is left as an exercise)
- Wise choice of 'reference pressure' simplifies the force calculation



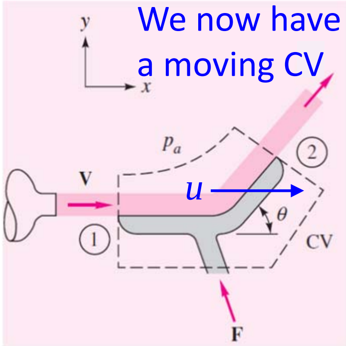


Application: Hydraulic turbine such as Pelton Wheel

Vane moving in x-direction with velocity u (constant)

Nozzle exit velocity V

Observer (and the axes) will move with the CV



$$\left(\vec{u}.\vec{A}\right)_1 = -\left(\vec{u}.\vec{A}\right)_2 = -\left(V-u\right)A = -V_1 A$$

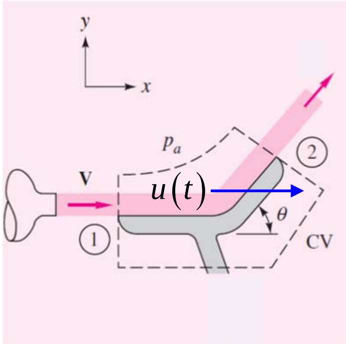
In x direction: $F_x == \rho V_1^2 A\left(\cos\theta - 1\right)$

In y direction: $F_y = \rho V_1^2 A \sin\theta$

Starting of a turbine

If we remove the force F_x the vane will accelerate

We are in a non-inertial frame now



$$\left(\vec{u}.\vec{A}\right)_1 = -\left(\vec{u}.\vec{A}\right)_2 = -\left(V-u\right)A = -V_1\left(t\right)A$$

$$-m\frac{du}{dt} = \rho\left(V-u\right)^2 A\left(\cos\theta - 1\right); u\left(t=0\right) = 0$$

$$\frac{u}{V} = \frac{Vbt}{1+Vbt}; b = \frac{\left(1-\cos\theta\right)\rho A}{m}$$

What happens during shutdown?