Fluid Mechanics and Rate Processes: Tutorial 8

P1. The reservoirs in Fig. P1 contain water at 20°C. If the pipe is smooth with L = 4500m and d = 4 cm, what will the flow rate in m^3 /h be for Δz = 100 m?

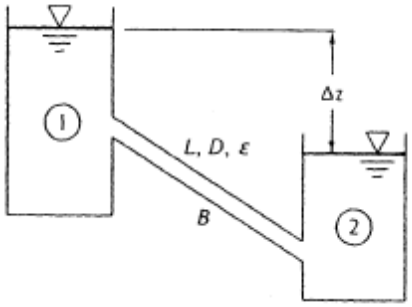

Fig. P1

Solution: For water at 20°C, take ρ = 998 kg/m^3 and $\mu = 0.001$ kg/m·s. The energy equation from surface 1 to surface 2 gives

$$p_1 = p_2 \quad \text{and} \quad V_1 = V_2,$$

$$\text{thus} \quad h_f = z_1 - z_2 = 100 \text{ m}$$

$$\text{Then } 100 \text{ m} = f\left(\frac{4500}{0.04}\right)\frac{V^2}{2(9.81)}, \quad \text{or} \quad fV^2 \approx 0.01744$$

Iterate with an initial guess of $f \approx 0.02$, calculating V and Re and improving the guess:

$$V \approx \left(\frac{0.01744}{0.02}\right)^{1/2} \approx 0.934 \ \frac{m}{s}, \quad Re \approx \frac{998(0.934)(0.04)}{0.001} \approx 37300, \quad f_{smooth} \approx 0.0224$$

$$V_{better} \approx \left(\frac{0.01744}{0.0224}\right)^{1/2} \approx 0.883 \ \frac{m}{s}, \quad Re_{better} \approx 35300, \quad f_{better} \approx 0.0226, \text{ etc.....}$$

This process converges to

$$f = 0.0227, \ Re = 35000, \ V = 0.877 \text{ m/s}, \ Q \approx 0.0011 \text{ m}^3/\text{s} \approx \mathbf{4.0 \ m^3/h}. \quad \textit{Ans.}$$