Energy balance $\frac{p_1}{\rho g} + \frac{1}{2g}\alpha_1 V_1^2 + z_1 = \frac{p_2}{\rho g} + \frac{1}{2g}\alpha_2 V_2^2 + z_2 + h_f - h_p + h_t$

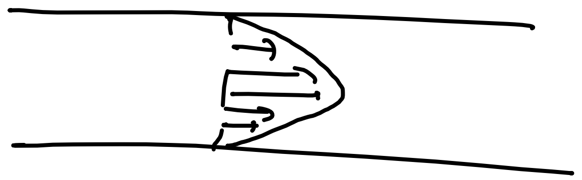
How to find h_f ← Head loss or frictional loss

Empirical method

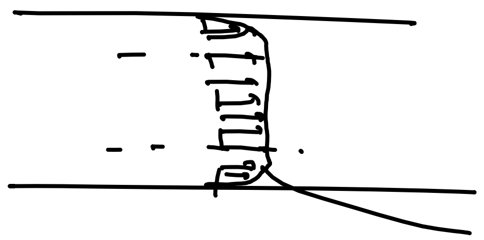
$h_f = f \frac{L}{d} \frac{V^2}{2g}$ for a circular pipe

$f = \frac{64}{Re_d}$ for Laminar flow (by darcy f)
for circular pipe

Turbulent flow → Navier-stokes' is not valid for turbulent flow ⇒ So we can not get the velocity profile ⇒ we use emipiral method to find velocity profile for turbulent flow



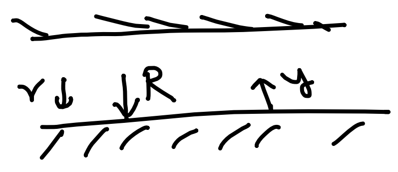
Laminar velocity profile



Turbulent velocity profile

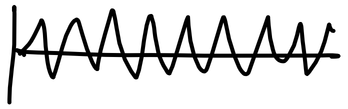
sharp change in velocity near th surface ⇒ can be captured by Logarithmic velocity Profile

$$\boxed{\frac{u}{u^*} = \frac{1}{k} \ln \frac{y u^*}{\nu} + B}$$



$y = R - r$

u = time-averaged velocity $= \frac{\int u\,dt}{\int dt}$

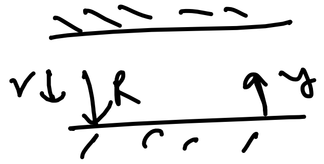


$$u^* = \sqrt{\frac{\tau_w}{\rho}} = \text{dimension of velocity} \Leftarrow \text{frictional velocity}$$

$B, k =$ constant that has to determined for a geometry

find f

$$\frac{u}{u^*} = \frac{1}{k} \ln \frac{y u^*}{\nu} + B$$



$y = R - r$

$$\frac{u(r)}{u^*} = \frac{1}{k} \ln \frac{(R-r) u^*}{\nu} + B$$

$V =$ cross-section averaged velocity $= V = \dfrac{\int u(r) 2\pi r \, dr}{\pi R^2}$ for a circular pipe

$$V = \frac{1}{2} u^* \left(\frac{2}{k} \ln \frac{R u^*}{\nu} + 2B - \frac{3}{k} \right)$$

for circular pipe $k = 0.41$, $B = 5.0$

Smooth pipe ($f \neq 0$)

$$\frac{V}{u^*} \approx 2.44 \ln \left(\frac{R u^*}{\nu} \right) + 1.34$$

$$\left(\frac{V}{u^*} \right) = \sqrt{\frac{\rho V^2}{\tau_w}} \Leftrightarrow f = \frac{\tau_w}{\frac{1}{8} \rho V^2} \Rightarrow \frac{V}{u^*} = \left(\frac{8}{f} \right)^{1/2}$$

$$\frac{R u^*}{\nu} = \frac{1}{2} \frac{d \, u^* V}{\nu V} = \frac{1}{2} Re_d \left(\frac{u^*}{V} \right) = \frac{1}{2} Re_d \left(\frac{f}{8} \right)^{1/2}$$

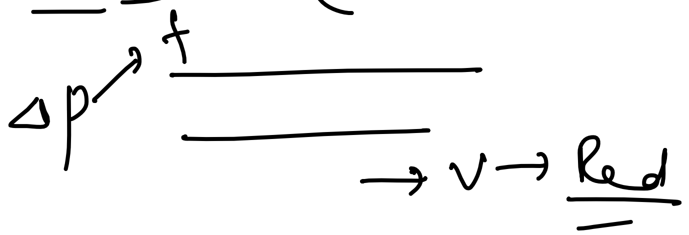
$$\Rightarrow \frac{1}{f^{1/2}} \approx 1.99 \log \left(Re_d f^{1/2} \right) - 1.02$$ (Smooth pipe, turbulent flow)

(implicit in f) $\hookrightarrow$ has been modified by Prandtl

$$\boxed{\frac{1}{f^{1/2}} = 2.0 \log Re_d f^{1/2} - 0.8}$$ (Prandtl)

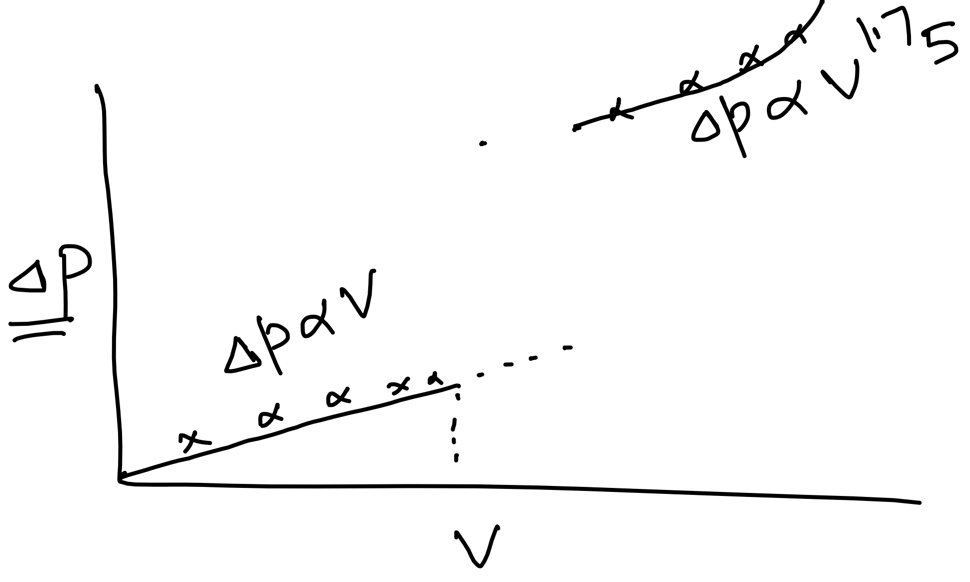
Explicit eqn for f $4000 < Re_d < 10^5$

$$\boxed{f = \frac{0.316}{Re_d^{1/4}}}$$ (Blasius) (Smooth



$$f = f\left(Re_d, \frac{\varepsilon}{d}, \text{shape} \right)$$

Hagen's experiment



$f = \frac{64}{Re_d}$

$\ln f = \ln 64 - \ln(Re_d)$

for Laminar flow

$$h_f = \frac{\Delta p}{\rho g} + \cancel{\Delta z}^{\,0} = f \frac{L}{d} \frac{V^2}{2g} = \frac{64 \mu}{\rho \cancel{V} d} \frac{L}{d} \frac{V^2}{2g} \propto V$$

for turbulent

$$h_f = \frac{\Delta p}{\rho g} + \cancel{\Delta z} = \frac{0.316 \mu^{1/4}}{(\rho V d)^{1/4}} \frac{L}{d} \frac{V^2}{2g} \propto \underline{\underline{V^{1.75}}}$$

$\left(\frac{\varepsilon}{d}\right)$ **The moody chart**

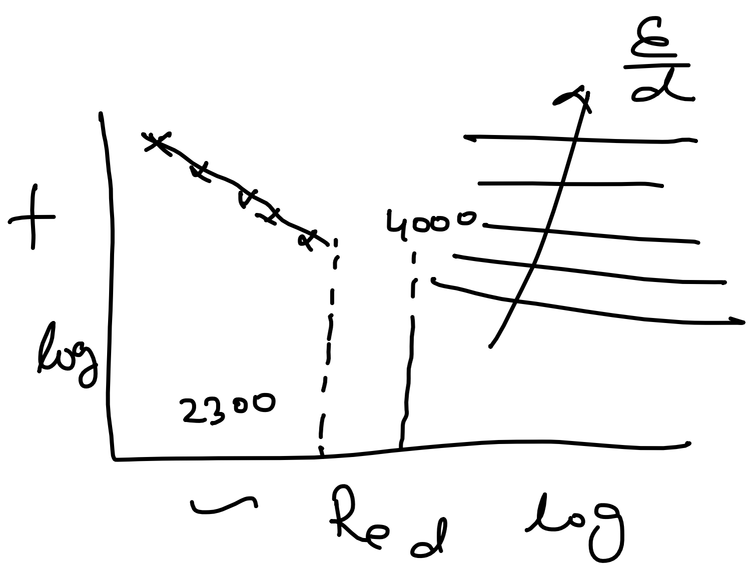
At constant Re $\quad \frac{1}{f^{1/2}} = -2.0 \log\left(\frac{\varepsilon/d}{3.7}\right)$ by experiment (vary $\varepsilon \to \Delta p$)

for smooth pipe $\quad \frac{1}{f^{1/2}} = -2.0 \log Re_d f^{1/2} - 0.8$

Colebrook $\rightarrow$ $\quad \frac{1}{f^{1/2}} = -2.0 \log\left[\frac{\varepsilon/d}{3.7} + \frac{2.51}{Re_d f^{1/2}}\right]$ (Rough pipe)

implicit in f

matches $\rightarrow$

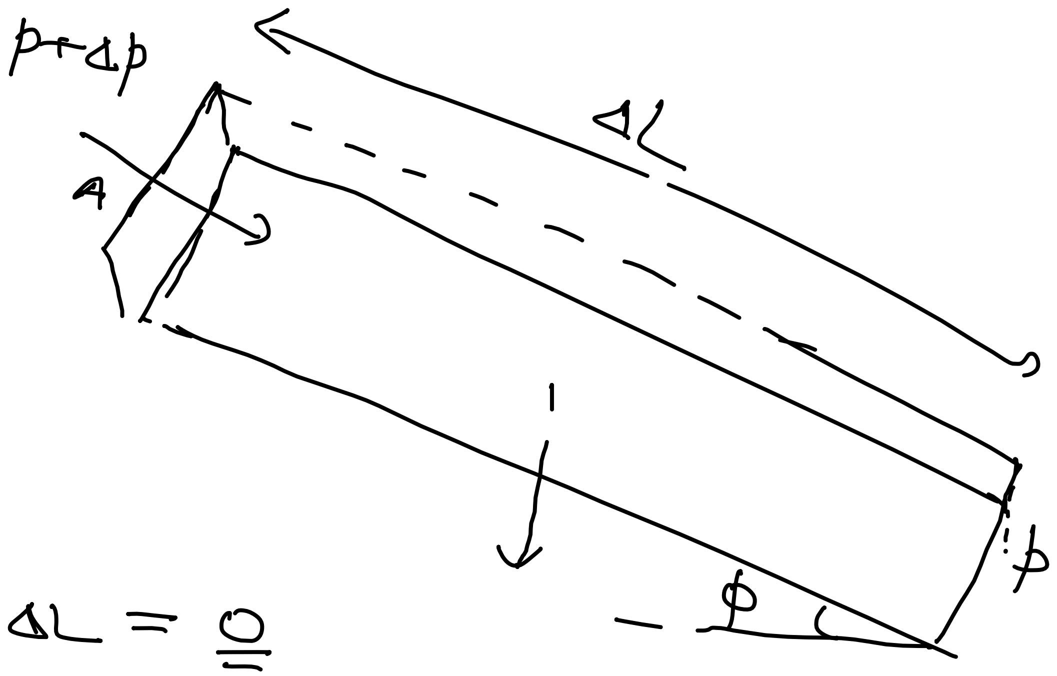


Explicit correlation by Haaland

$$\frac{1}{f^{1/2}} = -1.8 \log\left[\frac{6.9}{Re_d} + \left[\frac{(\varepsilon/d)}{3.7}\right]^{1.11}\right]$$ ← Also matches with 1-6 moody chart

$f = \frac{64}{Re_d}$ → h_f → Δp (so far we have discussed about circular pipe)

Flow in a non-circular Duct



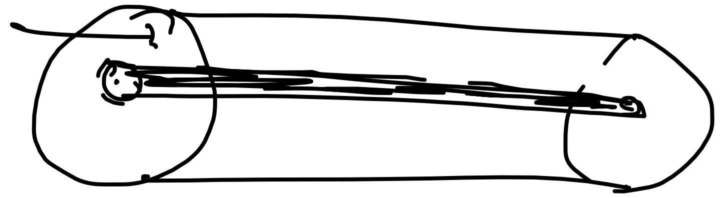
How to define h_f → How to find f

momentum balance

$$(\Delta p) A + \rho g A \Delta L \sin\phi - \tau_w \wp \Delta L = 0$$

$\wp$ = Perimeter

$$h_f = \frac{\Delta p}{\rho g} + \Delta z = \frac{\tau_w}{\rho g} \frac{\Delta L}{(A/\wp)}$$



R_h = hydraulic radius = $\frac{\text{Cross section Area}}{\text{wetted perimeter}}$

$f = \tau_w / (\frac{1}{8} \rho v^2)$

$$h_f = \frac{\frac{1}{8} f \rho v^2}{\rho g} \frac{\Delta L}{A/\wp} = f \frac{\Delta L}{4R_h} \frac{v^2}{2g} = f \frac{\Delta L}{D_h} \frac{v^2}{2g}$$

$$R_h = \frac{\pi (R_2^2 - R_1^2)}{\pi (D_1 + D_2)}$$

Hydraulic diameter $= D_h = 4 R_h$

$$\underline{f} = f\left(\frac{D_h V}{\nu}, \frac{\varepsilon}{D_h}\right) \text{ for any shape}$$

$$f = \begin{cases} \left(\dfrac{64}{Re_{D_h}}\right) & \text{error } \pm 40\% \quad \text{Laminar flow} \\ f_{moody}\left(Re_{D_h}, \dfrac{\varepsilon}{D_h}\right) & \pm \underline{15\%} \quad \text{Turbulent}) \end{cases}$$

→ Although moody chart is for circular pipe, we use it for non circular duct in case of ~~t~~urbulent)

→ for Laminar flow