Fluid Mechanics and Rate Processes (ESO204A)

Fluid Statics

1. **Governing Equation**
2. **Manometry**
3. **Hydrostatic force on submerged surfaces**

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Announcements

1. Tutorial starts this Thursday (Aug 04), you will receive necessary info on the course webpage: https://piazza.com/iitk.ac.in/firstsemester2016/eso204a/home
2. Tutorial problems will be uploaded today, please try them before coming to the tutorial
3. Class notes are uploaded after lecture, please go through them regularly, especially before coming to tutorial
4. Your participation in tutorial is essential

Fluid Statics

Hydrostatic Force on Submerged Surfaces

Governing Equation of fluid statics: $\nabla p = \rho \vec{g}$

$$x\text{-direction}: \frac{\partial p}{\partial x} = \rho g_x \quad y\text{-direction}: \frac{\partial p}{\partial y} = \rho g_y \quad z\text{-direction}: \frac{\partial p}{\partial z} = \rho g_z$$

Sometimes we align one axis (let's say z) vertically upward (against gravity), such that $g_x = g_y = 0; g_z = -g$

$$\Rightarrow \frac{\partial p}{\partial x} = \frac{\partial p}{\partial y} = 0 \Rightarrow p = p(z) \qquad \frac{\partial p}{\partial z} = -\rho g = -\gamma$$

Integrating $p(z = h) = p(z = 0) - \gamma h$ for constant ρ

Above Eq. shows that elevation difference leads to hydrostatic pressure difference; briefly, this is the starting point for **hydrostatic force calculation**

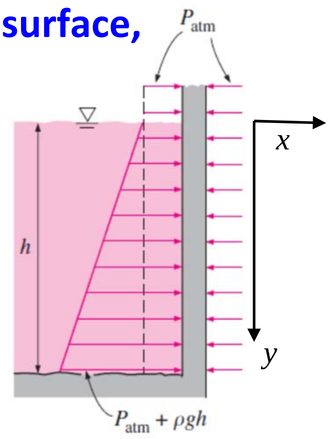
Hydrostatic force on a vertical surface, one side open to atmosphere

$$F = \int p dA \; = \int \left(p_{\text{atm}} + \gamma y \right) dA$$

$$= p_{\text{atm}} A + \gamma \int y b dy \; = p_{\text{atm}} A + \frac{\gamma}{2} b h^2$$

also, by definition $y_{CG} = \frac{1}{A} \int y dA$

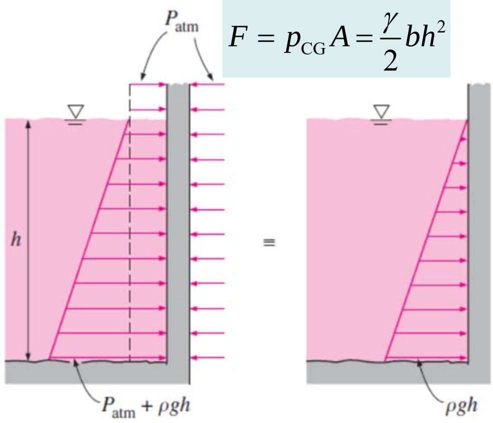
leading to $F = \left(p_{\text{atm}} + \gamma y_{CG} \right) A = p_{CG} A$



Net force (based on gage pressure) in *x*-direction: $F_x = \gamma y_{CG} A = p_{CG} A$

Note the equivalence of the following systems

$$F = p_{CG} A = \frac{\gamma}{2} b h^2$$

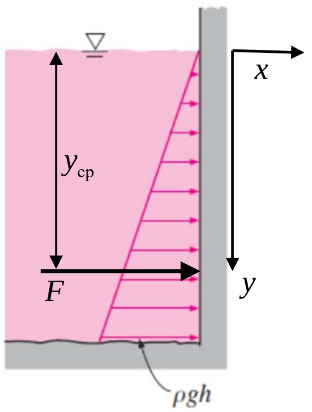


Also, force = the volume of *pressure prism*

$$= \frac{1}{2} (\gamma h) . h . b = \frac{\gamma}{2} b h^2$$

Force can be calculated from direct integration or one of the above shortcuts

Where does this force act : ***center of pressure***



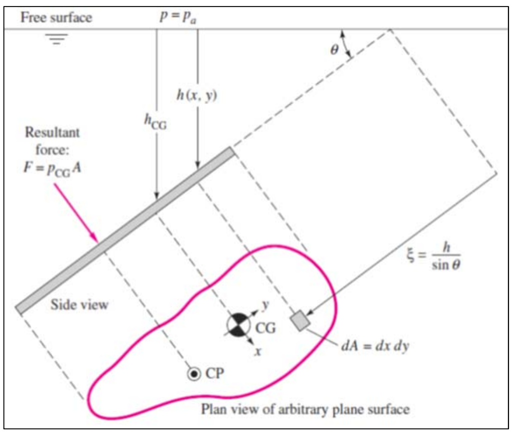
$$Fy_{cp} = \int y dF = \int_0^h y(\gamma y)(bdy)$$

$$\Rightarrow \frac{\gamma}{2} bh^2 y_{cp} = \frac{\gamma}{3} bh^3 \Rightarrow y_{cp} = \frac{2h}{3}$$

Force acts through the *center of gravity* of the *pressure prism*

Direct integration is applicable to all kind of surfaces, shortcuts are always true but usable only for simple cases, such as plane surfaces of constant width

Let's now take an inclined, submerged, plane surface



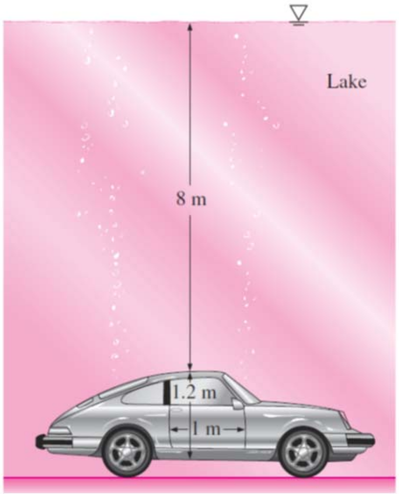
$$F = \int pdA = \int \gamma h dA$$
$$= \int \gamma (h_{CG} - y\sin\theta) dA$$
$$= \gamma h_{CG} A - \gamma \sin\theta \int y dA$$
$$= \gamma h_{CG} A \quad = p_{CG} A$$

$$Fy_{cp} = \int y\gamma h dA$$
$$= \gamma \int y (h_{CG} - y\sin\theta) dA$$
$$= \gamma h_{CG} \int y dA - \gamma \sin\theta \int y^2 dA$$
$$= -\gamma \sin\theta \int y^2 dA$$

$$\Rightarrow y_{cp} = -\frac{\gamma \sin\theta I_{xx}}{F}$$

similarly $x_{cp} = -\frac{\gamma \sin\theta I_{xy}}{F}$ **(prove)**

How to open up the car door from inside?



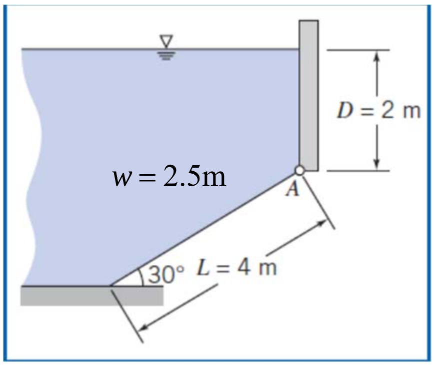
$$F = 10^3 \frac{\text{kg}}{\text{m}^3} \times 9.81 \frac{\text{m}}{\text{s}^2} \times \left(8 + \frac{1.2}{2}\right)\text{m}$$
$$\times (1\text{m} \times 1.2\text{m}) = 101.3\text{kN}$$

Moment needed to open the door

$$F \times .5\text{m} = 50.6\text{kNm}$$

Key: let some water in!!

Find the force on the inclined gate hinged at A



$$F = 10^4 \frac{\text{kg}}{\text{m}^2\text{s}^2} \times \left(2 + \frac{4}{2}\sin 30^\circ\right)\text{m}$$
$$\times (4\text{m} \times 2.5\text{m}) \ = 300\text{kN}$$

Find the CP, using the expressions of MI given in your text (F M White)

Notes for problem solving

Be consistent in gage/abs pressure

Remember useful properties and expressions or remember how to deduce them quickly