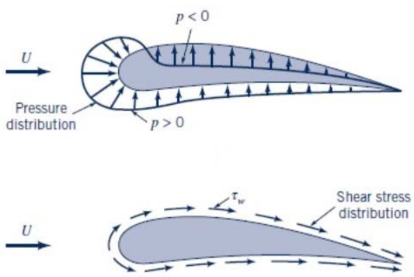
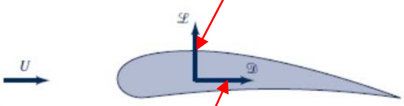
ESO204A, Fluid Mechanics and Rate Processes

Incompressible flows over immersed bodies (External Flow)

Chapter 7 of F M White
Chapter 9 of Fox McDonald

Drag and Lift

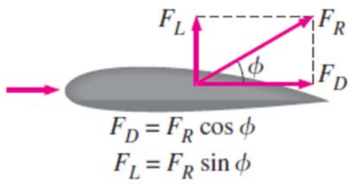


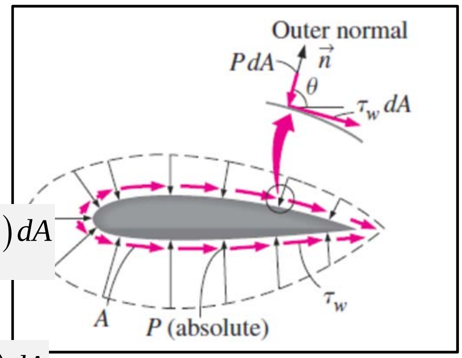


Lift: resultant Force normal to flow direction

Drag: resultant force in flow direction

Flow direction: direction of the Free Stream (far away from the body)





Drag: $F_D = \int_A \left(-p\cos\theta + \tau_w \sin\theta\right) dA$

Lift: $F_L = -\int_A \left(p\sin\theta + \tau_w \cos\theta\right) dA$

Drag coefficient: $C_D = F_D \Big/ \left(\frac{1}{2}\rho u^2 A\right)$

Lift coefficient: $C_L = F_L \Big/ \left(\frac{1}{2}\rho u^2 A\right)$

Projected area against the force

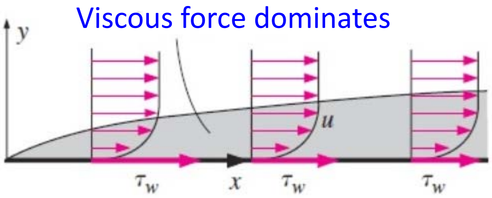
Drag: $F_D = \int_A \left(\tau_w \sin\theta\right) dA + \int_A \left(-p\cos\theta\right) dA$

Viscous (friction) drag **Form (pressure) drag**

$$\frac{F_D}{\frac{1}{2}\rho u^2 A} = \frac{\int_A \left(\tau_w \sin\theta\right) dA}{\frac{1}{2}\rho u^2 A} + \frac{\int_A \left(-p\cos\theta\right) dA}{\frac{1}{2}\rho u^2 A}$$

$$C_D = C_{D,\text{viscous}} + C_{D,\text{pressure}}$$

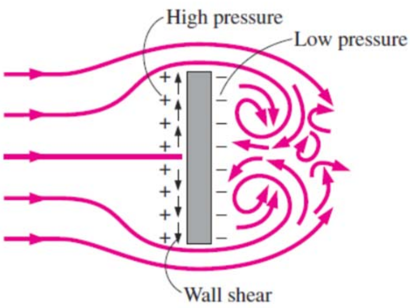
Flow over a flat plate



Drag is due to viscous force only

Flow normal to a flat plate

Drag is primarily due to pressure difference across the plate



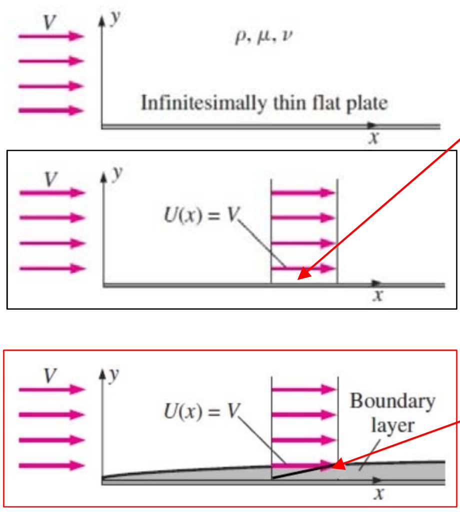
Dimensionless N-S/Continuity Eqns.

$$\frac{\partial \mathbf{u}}{\partial t}+(\mathbf{u}.\nabla)\mathbf{u}=-\nabla p+\frac{1}{\mathrm{Re}}\nabla^2\mathbf{u} \qquad \nabla.\mathbf{u}=0$$

The above Equations may be successfully solved (for many cases) after dropping the viscous term (known as potential flow)

Potential flow solution predicts zero drag/lift for all objects, a phenomenon known as D'Alembert's paradox

Consider flow over a flat plate

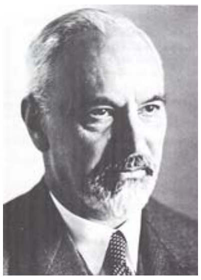


Potential flow solution of the above problem (fluid slips over the plate)

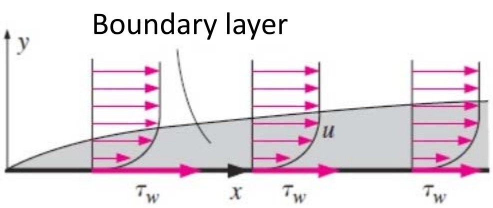
Experiments and viscous flow solution (no-slip condition satisfied)

Boundary Layer

Very thin layer of fluid near solid surface where viscous effects are dominant



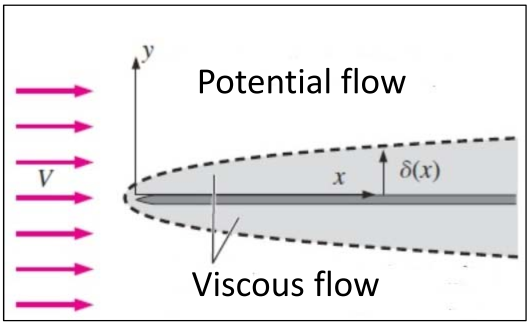
The idea was introduced by Ludwig Prandtl (1875-1953) and his coworkers



The BL concept is the most important development in Fluid Mechanics after N-S Eqn.

Boundary Layer Very thin layer of fluid near solid surface where viscous effects are dominant

Outside boundary layer flow remains largely inviscid (amenable to potential flow solutions)

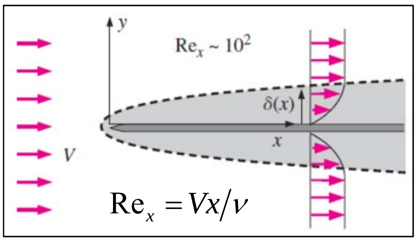


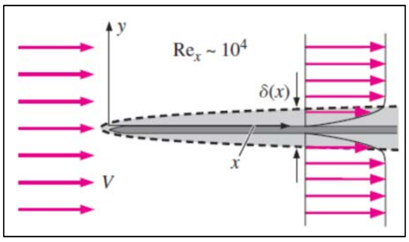
Prandtl showed

$\delta(x) \ll x$

Leads to meaningful approximation of N-S Eq.

Laminar Boundary Layer over a flat plate

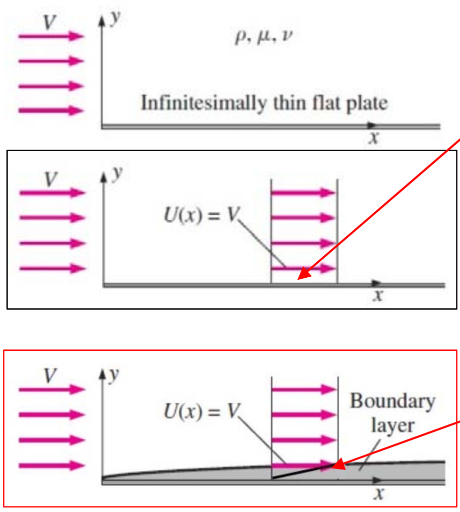




At a given x-location, higher Re_x leads to thinner boundary layer thickness

Boundary layer thickness $u_{y=\delta} = .99 u_{\text{free stream}}$

Boundary layer over a flat plate



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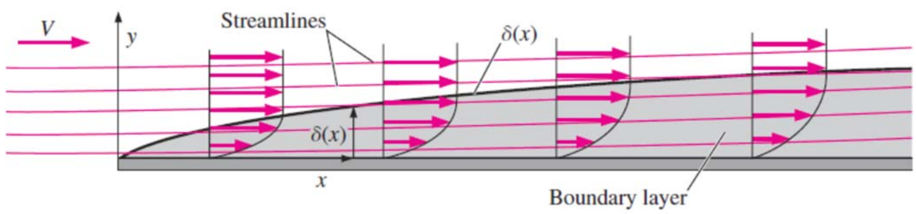
Boundary Layer over a flat plate



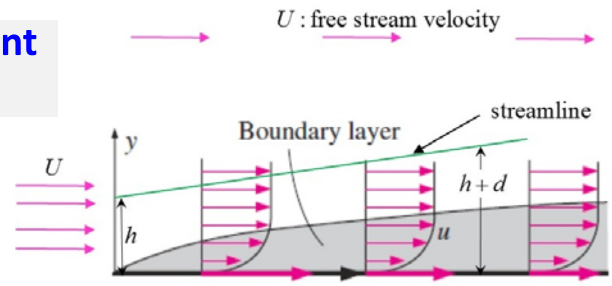
Boundary layer shifts the streamlines upward

Displacement thickness $\delta^* = \int_0^\infty \left(1 - \frac{u}{U}\right) dy$

U : free stream velocity

Displacement thickness

$h \gg \delta$



$$\left(\int_0^h \rho u dy\right)_{x=0} = \int_0^{h+d} \rho u dy \qquad \int_0^h U dy = \int_0^h u dy + \int_0^d u dy$$

$$\int_0^h U dy = \int_0^h u dy + \int_0^d U dy \qquad \int_0^h (U-u)\,dy = Ud$$

$$d = \int_0^h \left(1-\frac{u}{U}\right) dy \qquad \delta^* = \int_0^\infty \left(1-\frac{u}{U}\right) dy \approx \int_0^\delta \left(1-\frac{u}{U}\right) dy$$