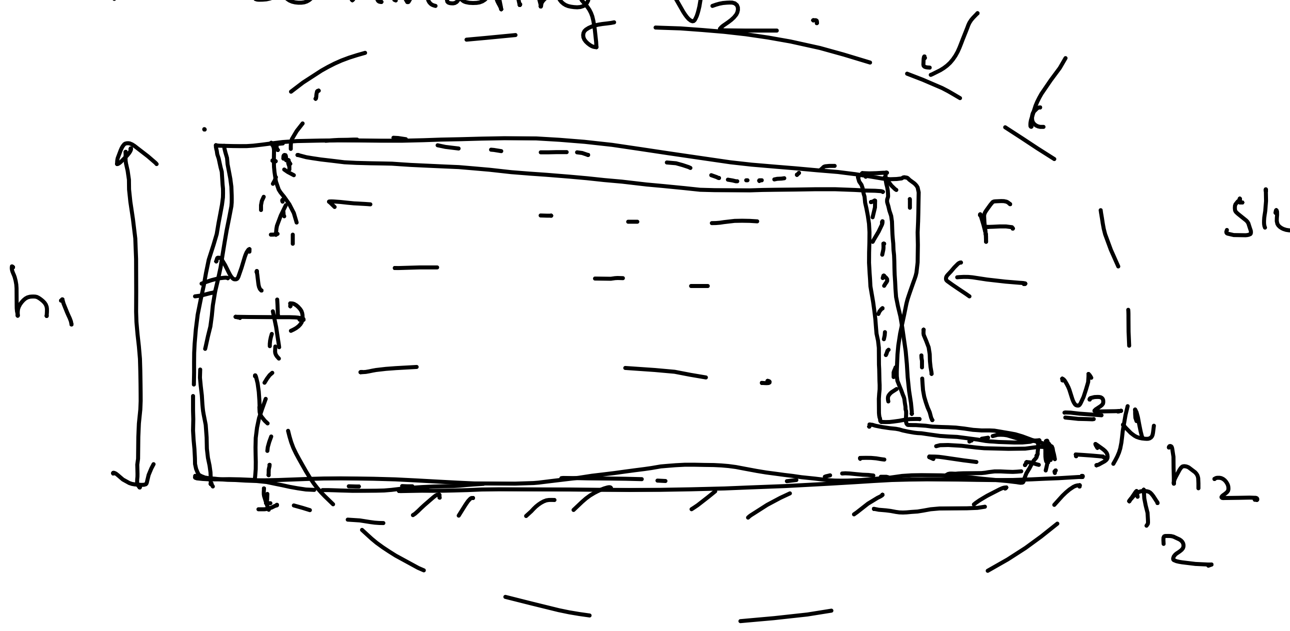
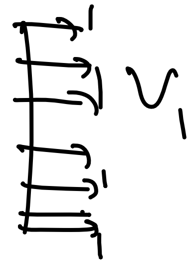
Example: The sluice gate in fig below controls flow in an open channel. At section 1 the flow is uniform and pressure is hydrostatic. Neglecting bottom friction and atmospheric pressure derive a formula for the horizontal force F required to hold the gate. Express your final formula in terms of inlet velocity V_1 eliminating V_2.



sluice gate width b



$\rightarrow x$

Solution:

mass balance

$$0 = \cancel{\frac{d}{dt}\int_{cv} \rho \, d\forall}^{\,0} + \int_{cs} \rho \, \underline{(V \cdot n)\, dA}$$

$$+\rho V_2 A_2 + (-)\,\rho V_1 A_1 = 0$$

$$\rho \underline{\underline{V_2}}\, h_2 b - \rho \underline{\underline{V_1}}\, h_1 b = 0 \quad \text{—(i)}$$

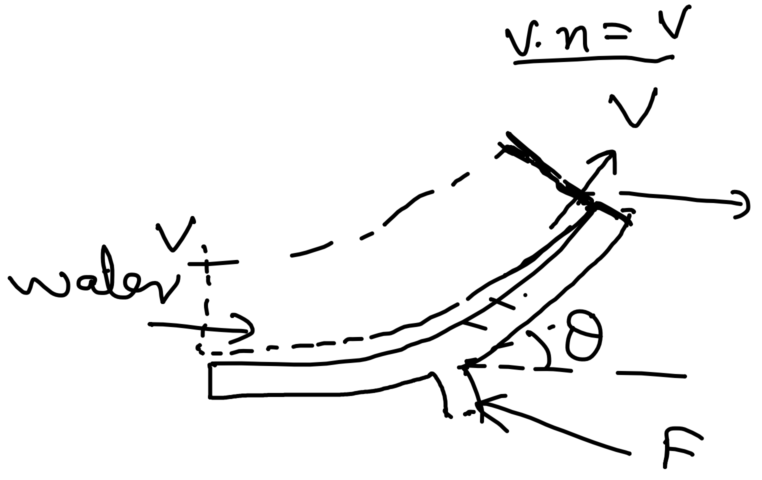
x-momentum balance

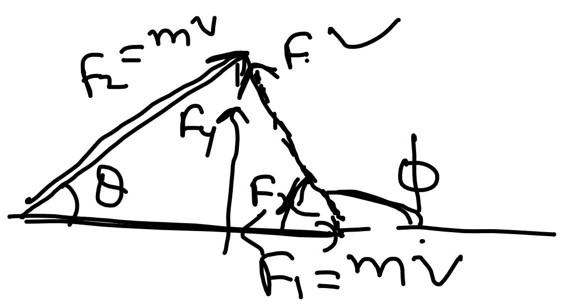
$$\underline{\underline{\Sigma F_x}} = \cancel{\frac{d}{dt}\int u \rho \, d\forall}^{\,0} + \underline{\int u \rho (V \cdot n)\, dA}$$

$$-F + \gamma\, \underline{\underline{h_{cg_1}}} A_1 - \gamma\, \underline{\underline{h_{cg_2}}} A_2 = V_2 \rho (+V_2) A_2 + (V_1)\rho(-V_1) A_1 \quad \text{—(ii)}$$

$$-\underline{\underline{F}} + \rho g\left(\frac{h_1}{2}\right)(h_1 b) - \rho g\left(\frac{h_2}{2}\right)(h_2 b) = \_\_\_\_\_\_ \quad \text{(ii)}$$

Example As shown in fig below, a fixed vane turns water jet of area A through an angle θ without changing its velocity magnitude. The flow is steady. Pressure is p_a everywhere and friction on the vane is negligible. (a) find the components F_x and F_y of the applied force on the vane (b) find expression for the force magnitude F and the angle ϕ between F and horizontal, plot them versus θ.





$$\vec{F_2} - \vec{F_1} = \vec{F}$$

$$\vec{F_1} + \vec{F} = \vec{F_2}$$

(a) x-momentum balance

$$F_x = \frac{d}{dt}\left(\cancel{\quad}\right) + \int_{cs} u\rho(V.n)\,dA$$

$$= V\cos\theta\,\rho(V)A - V\rho VA$$

$$F_x = \underline{\rho A V^2(\cos\theta - 1)}$$

$$F_y = \cancel{(\quad)}^{0} + \int v\rho(V.n)\,dA = \rho V\sin\theta\, VA - 0 = \underline{\rho V^2 A\sin\theta}$$

(b) $F = \sqrt{F_x^2 + F_y^2} = \rho V^2 A\, 2\sin\theta/2$

$\phi = 180 - \tan^{-1}\frac{F_y}{F_x} = 180 - \tan^{-1}\left(\frac{\sin\theta}{\cos\theta - 1}\right) = \underline{90 + \theta/2}$

if θ is small

$F \approx \rho V^2 A\,\theta$

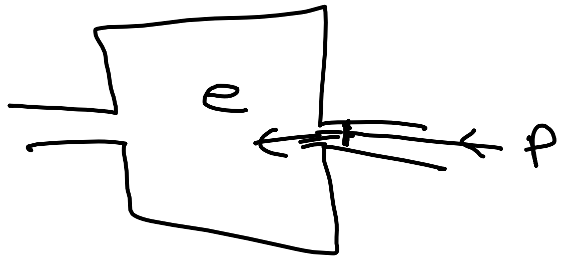
$\phi = 90^\circ$

lift force

→ Principle of airfoil

The energy balance

E = Total energy of the fluid, $\beta = e = \frac{dE}{dm}$



R.T.T. $\dot{Q} - \dot{W} = \frac{dE_{syst}}{dt} = \frac{d}{dt}\int_{CV} e\,\rho\, d\forall + \int_{CS} e\,\rho\,(V\cdot n)\,dA$

$e = e_{internal} + e_{KE} + e_{PE} + e_{other}$

$e_{other} = e_{chem\ rxn} + e_{EM} + e_{nuclear}$

$e = \hat{u} + \frac{1}{2}V^2 + gz$

$\dot{W} = \dot{W}_s + \dot{W}_p + \dot{W}_{viscous}$

work done/time by the CV

$= -f_{viscous}\cdot V$

$= -\int \tau(n)\cdot V\, dA$

$= -\int (\tau_n \cdot V)\, dA$

Pressure work

$F_{Press} = \int p\,(-n)\,dA$ ≡ Force applied by surrounding on the CV

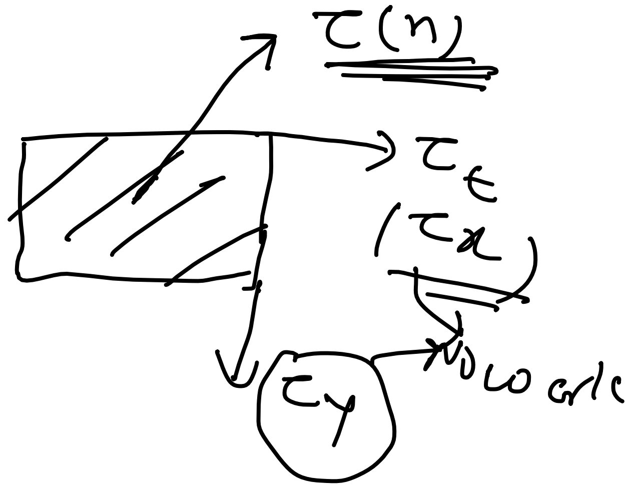
Force applied by the CV $= -F_{Press}$

work done by CV/time $= (-f_{Press})\cdot V = \int (p\,(n)\cdot V)\,dA$

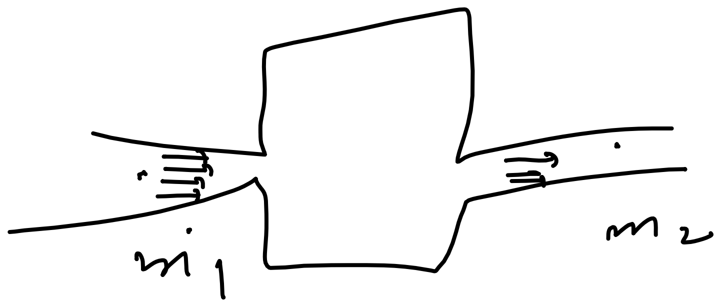
Viscous work

$F_{viscous} = \int \tau(n)\,dA$ = force by the surrounding on CV

Force by the CV on the surrounding $= -f_{viscous}$



$\underline{\underline{\tau}}_t \cdot V = 0 \quad \Leftrightarrow V = 0$ at the surface $\Leftarrow$ No slip condition



$$W_{viscous} = -\int (\tau_n \cdot V)\, dA + (-) \int \overset{0}{(\tau_t \cdot V)}\, dA$$

$$= -\int (\underline{\tau} \cdot V)\, dA$$

$$\dot{Q} - \underline{\underline{W_p}} - W_v - W_s = \frac{d}{dt}\int e \rho\, d\forall + \int e \rho (V \cdot n)\, dA$$

$$\dot{Q} - \dot{W}_v - \dot{W}_s = \frac{d}{dt}\int (\hat{u} + \tfrac{1}{2}V^2 + gz)\rho\, d\forall + \int (\hat{u} + \tfrac{1}{2}V^2 + gz)\, \rho\, (V \cdot n)\, dA + \int p (n \cdot V)\, dA$$

$$= \frac{d}{dt}\int (\hat{u} + \tfrac{1}{2}V^2 + gz)\rho\, d\forall + \int \left[(\hat{u} + \frac{p}{\rho}) + \tfrac{1}{2}V^2 + gz \right] \rho\, (\underline{\underline{V \cdot n}})\, dA$$

$$\frac{1}{\rho} = \underline{\underline{v}}$$

$$\dot{Q} - \dot{W}_v - \dot{W}_s = \frac{d}{dt}\int (\hat{u} + \tfrac{1}{2}V^2 + gz)\rho\, d\forall + \int (\underline{\underline{\hat{h}}} + \tfrac{1}{2}V^2 + gz)\, \underline{\underline{\rho\, (V \cdot n)\, dA}}$$

Assume = Steady flow, $\hat{h} \equiv$ Cross-section Avg, $gz =$ Cross-section Avg

$$\boxed{\dot{Q} - \dot{W}_v - \dot{W}_s = \dot{m}_2 (\hat{h}_2 + \tfrac{1}{2}\underline{\underline{\alpha_2}} V^2_{\underline{\underline{2avg}}} + gz_2) - \dot{m}_1 (\hat{h}_1 + \tfrac{1}{2}\alpha_1 V^2_{1avg} + gz_1)} \leftarrow$$