

ESO204A, Fluid Mechanics and rate Processes

Dimensional Analysis: application to model testing

Chapter 5 of F M White
Chapter 7 of Fox McDonald

Model studies (similitude)

Certain fluid mechanical phenomenon is governed by

$$f(\pi_1, \pi_2, \dots, \pi_n) = 0 \quad \text{where } \pi_i \text{ are non-dimensional}$$

When the model is **similar** to the prototype

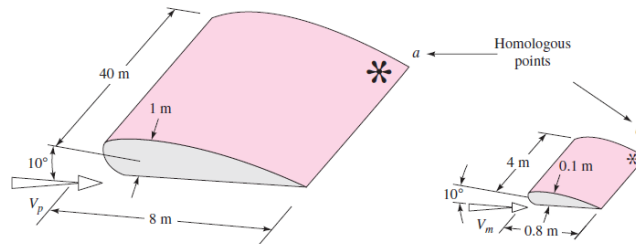
$$(\pi_i)_{\text{model}} = (\pi_i)_{\text{prototype}}, \quad i = 1, 2, \dots, n$$

Complete **similarity** requires

Geometric similarity + **Kinematic** similarity + **Dynamic** similarity

Geometric similarity: length-scale matching

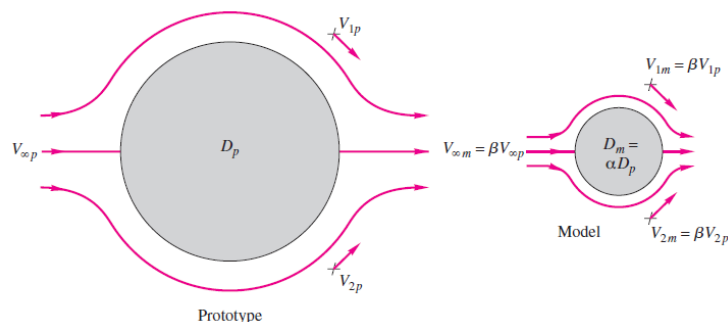
A model and prototype are geometrically similar if and only if all body dimension in all three coordinates have the **same linear scale ratio**



All angles, flow direction, orientation with the surroundings must be preserved

Kinematic similarity (velocity-scale matching)

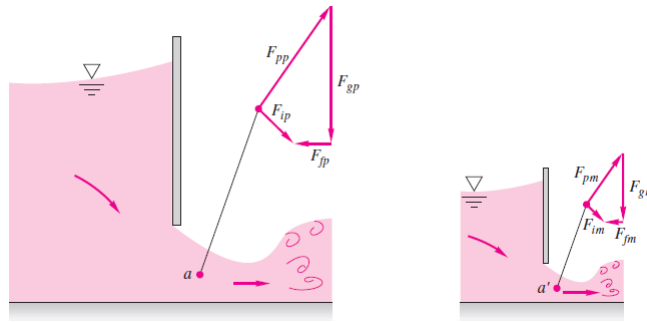
A model and prototype are kinematically similar if homologous particles lie at homologous points at homologous time



Kinematic similarity requires geometric similarity

Dynamic similarity (force-scale matching)

A model and prototype are dynamically similar if ratio of any two forces are same for model and prototype



Dynamic similarity requires geometric, kinematic similarities

When fluid flows over an object, the object experiences fluid resistance known as 'drag force'

For incompressible flow with 'smooth' objects the drag force is given by

$$F = f(L, u, \rho, \mu)$$

Conduct dimensional analysis to identify the dimensionless numbers associated with the above phenomenon

A running car experiences fluid resistance known as 'drag force'

$$F = f(L, u, \rho, \mu)$$

We are interested to conduct a model study in a wind tunnel to know the drag on the prototype

$$n = 5 \quad k = 3 \quad m = 2$$

Repeating: L, u, ρ

$$\pi_1 = F(L)^a (u)^b (\rho)^c$$

$$\pi_2 = \mu(L)^a (u)^b (\rho)^c$$

	M	L	T
u			
L			
ρ			
F			
μ			

Drag on a car

$$F = f(L, u, \rho, \mu)$$

$$\pi_1 = \frac{F}{\rho u^2 L^2}$$

$$\pi_2 = \frac{\mu}{\rho u L}$$

$$\frac{F}{\rho u^2 L^2} = \psi \left(\frac{\mu}{\rho u L} \right)$$

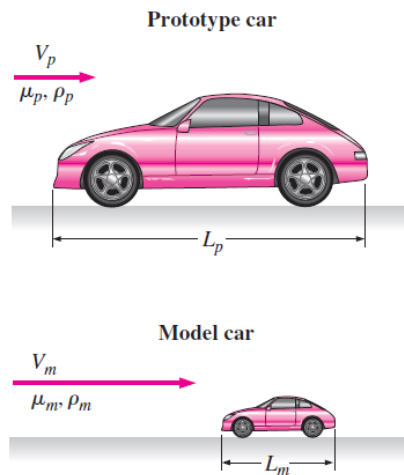


Drag
coefficient

	M	L	T
u	0	1	-1
L	0	1	0
ρ	1	-3	0
F	1	1	-2
μ	1	-1	-1

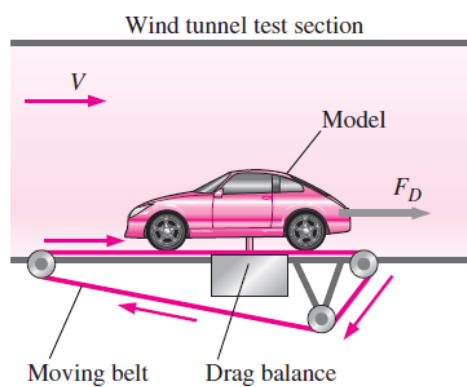
$$C_D = \psi \left(\frac{1}{\text{Re}} \right)$$

Testing a model car in a wind tunnel: geometric similarity



What would be a reasonable scale ratio?

How to achieve kinematic similarity



Kinematic similarity requires moving platform

Model testing: to find the drag on prototype

To conduct useful model test, we need to match **Re**

$$\text{Re}_m = \text{Re}_p \Rightarrow \left(\frac{\rho u L}{\mu} \right)_m = \left(\frac{\rho u L}{\mu} \right)_p$$

Taking same properties of fluid $\Rightarrow (uL)_m = (uL)_p$

Now let's match the **C_D** $\Rightarrow \left(\frac{F}{\rho u^2 L^2} \right)_p = \left(\frac{F}{\rho u^2 L^2} \right)_m$

$$\Rightarrow F_p = F_m \frac{(u^2 L^2)_p}{(u^2 L^2)_m} \Rightarrow F_p = F_m$$

Model testing

$$\frac{L_m}{L_p} = \frac{1}{10}, \text{ say}$$

$$(uL)_m = (uL)_p \Rightarrow \frac{u_m}{u_p} = 10 \quad \begin{array}{l} \text{seems unrealistic for} \\ \text{automobile applications} \end{array}$$

Probable options:

- Use of high-speed (supersonic) wind-tunnel
- Bigger model
- Different fluid

None of the options are quite easy

Summary: For fluid flows over an object, drag force is given by $F = f(L, u, \rho, \mu)$

Using dimensional analysis: $\frac{F}{\rho u^2 L^2} = \psi\left(\frac{\mu}{\rho u L}\right)$ $C_D = \psi\left(\frac{1}{\text{Re}}\right)$

$$C_D = \frac{\text{drag}}{\text{inertia}} \quad \frac{1}{\text{Re}} = \frac{\text{viscous}}{\text{inertia}}$$

- Dim. analysis indicates three forces
- Dim. analysis **scales** other forces w. r. t. inertia
(based on our choice of repeating variables)