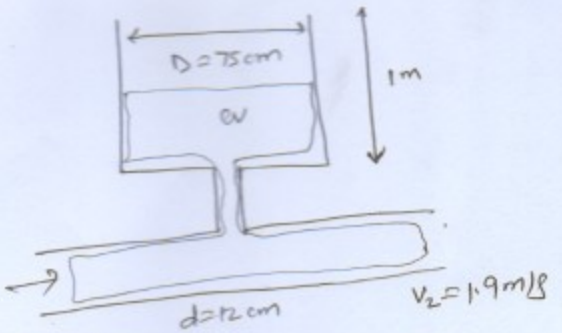
~~4443~~ Tutorial 3

P3.14



$$0 = \frac{\partial}{\partial t}\int \rho \, d\forall + \int \rho (V.n) dA$$

$$0 = \frac{\partial}{\partial t}(\rho A h) + \rho V_2 \frac{\pi}{4} d^2 - \rho V_1 \frac{\pi}{4} d^2$$

$$\frac{\pi}{4} D^2 \frac{dh}{dt} = d^2 (V_1 - V_2)$$

$$\frac{dh}{dt} = \frac{d^2}{D^2}(V_1 - V_2) = \left(\frac{12}{75}\right)^2 (2.5 - 1.9) = 0.01536 \text{ m/s}$$

$$t = \frac{1 - 0.30}{0.01536} = 45.57 \text{ s}$$

P3.18

$$0 = \frac{d}{dt}\int \rho \, d\forall^{\text{S.S.}} + \int \rho (V.n) dA$$

$$0 = 0 + \rho \int u \, dy \, b - \rho \int U_0 \, dy \, b + Q$$

$$Q = \rho U_0 \delta b - \rho b \int_0^{\delta} U_0 \left(\frac{3\eta - \eta^3}{2}\right) dy$$

$$= \rho U_0 \delta b \left[1 - \int_0^1 \left(\frac{3\eta - \eta^3}{2}\right) d\eta\right]$$

$$= \rho U_0 \delta b \left[1 - \left[\frac{3\eta^2}{4} - \frac{\eta^4}{8}\right]_0^1\right] = \rho U_0 \delta b \left[1 - \frac{3}{4} + \frac{1}{8}\right]$$

$$= 0.375 \rho U_0 \delta b$$

P3.19

$\rho = 891 \text{ kg/m}^3$

$$0 = \frac{\partial}{\partial t}\int_{CS} \rho \, dV + \int \rho (v \cdot n) \, dA$$

$$0 = 0 + \rho \int u \, dz \, b - \rho \int U_0 b \, dz$$

$$0 = (891) b \int a z (z_0 - z) \, dz - 891 (0.08) b \, 0.04$$

$$0 = a \left[\frac{z_0 z^2}{2} - \frac{z^3}{3} \right]_0^4 - (0.08)(0.04)$$

$$= a \left[\frac{4 \cdot 4^2}{2} - \frac{4^3}{3} \right] - 0.08 \times 0.04$$

$$10^{-6} (a)(4)^3 \left(\frac{1}{6}\right) = 0.08 \times 0.04$$

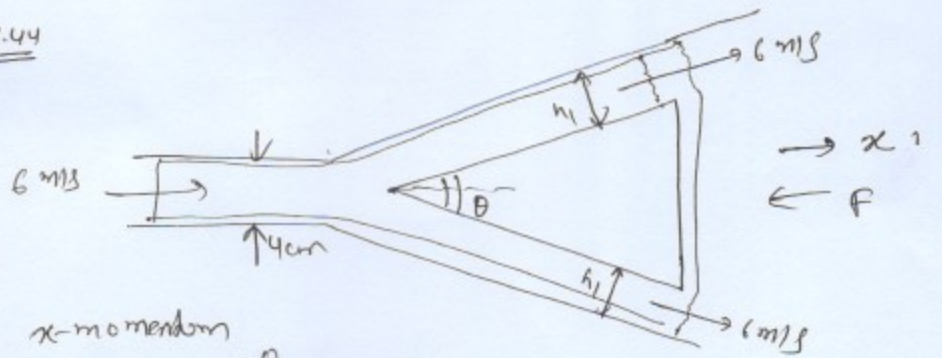
$$a = \frac{3 \times 10^{-4}}{10^{-6}} = 300$$

U_{max} @ $z = 2$ cm

$$u_{max} = \frac{3 \times 10^{-4}}{10^{-6}} \times 0.02 (2) \times 10^{-2}$$

$$= 0.12 \text{ m/s} = 12 \text{ cm/s}$$

P3.44



x-momentum

$$-F = \frac{d}{dt}\int u \rho \, dV + \int u \rho (v \cdot n) \, dA$$

mass balance

$$0 = 6 h_1 \rho b + 6 h_1 \rho b - 6 \times 0.04 \times \rho b$$

$$h_1 = 0.02 \text{ m}$$

x-momentum

$$-F = (6\cos\theta_2)\,\rho\; 6\times h_1 b + (6\cos\theta_2)\,\rho\,(6\times h_1 b) - 6\,\rho(6\times 0.04 b)$$

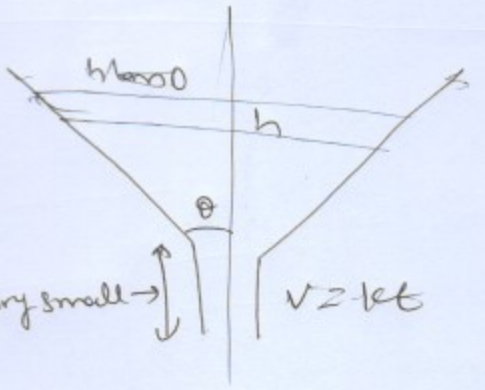
$$-124 = 6\;12\times 1000\times 6\times 0.02\cos\theta_2 - 6\times 1000\times 6\times 0.04$$

$$\cos\theta_2 = 0.914$$

$$\theta_2 = 23.95$$

$$\theta = 47.9^\circ$$

P 3.28



$$dV = \pi (h \tan\theta)^2 dh$$

mass balance

$$0 = \frac{d}{dt}\int \rho\, dV + \int \rho (V \cdot n)\, dA$$

$$= \frac{d}{dt}\int \rho \pi h^2 \tan^2\theta\, dh - \left[\rho (kt) \frac{\pi}{4} d^2\right]$$

$$0 = \frac{d}{dt}\left(\pi \rho \frac{h^3}{3} \tan^2\theta\right) - \rho k t \frac{\pi}{4} d^2$$

$$\int_0 d\left(\frac{h^3}{3}\tan^2\theta\right) = \frac{k}{4} d^2 \int_0 t\, dt$$

$$\frac{h^3}{3}\tan^2\theta = \frac{k}{8} d^2 t^2$$

$$h = \left[\frac{3}{8} k t^2 d^2 \cot^2\theta\right]^{1/3}$$