ESO204A, Fluid Mechanics and rate Processes

Announcements

- Quiz tomorrow- in respective tutorials
- 20mins, closed book/notes: statics, kinematics (RTT not included)
- Bring your calculator
- One problem will be solved after the quiz, problem is uploaded as usual
- Office hours tomorrow: 1700-1800hrs, SL-210

ESO204A, Fluid Mechanics and rate Processes

Conservation of mass: integral formulation

We develop mass/momentum/energy conservation Equations; integral form provides quick and easy (albeit approximate) estimates of useful quantities, cannot derive detail 'field' information

Chapter 3 of F M White
Chapter 4 of Fox McDonald (uploaded)

Reynolds Transport Theorem (RTT)

Rate of change of B of the system

$$\frac{dB_{sys}}{dt} = \frac{\partial}{\partial t}\int_{CV} \rho\beta d\forall + \int_{CS} \rho\beta\left(\vec{u}.\vec{n}\right)dS$$

Rate of change of B in the CV

Rate of flow of B through the CS

$$\beta = B/m$$

Conservation of mass: integral form

$$\frac{dB_{sys}}{dt} = \frac{\partial}{\partial t}\int_{CV} \rho\beta d\forall + \int_{CS} \rho\beta\left(\vec{u}.\vec{n}\right)dS \qquad \text{here } B = m \Rightarrow \beta = 1$$

$$\frac{dm_{sys}}{dt} = \frac{\partial}{\partial t}\int_{CV} \rho d\forall + \int_{CS} \rho\left(\vec{u}.\vec{n}\right)dS \qquad \frac{dm_{sys}}{dt} = 0$$

$$\frac{\partial}{\partial t}\int_{CV} \rho d\forall + \int_{CS} \rho\left(\vec{u}.\vec{n}\right)dS = 0$$

Equation of mass conservation
(integral form)

Equation of mass conservation: integral form

$$\frac{\partial}{\partial t}\int_{CV}\rho d\forall+\int_{CS}\rho\left(\vec{u}.\vec{n}\right)dS=0$$

Constant density (incompressible) flow:

$$\frac{\partial \forall_{CV}}{\partial t}+\int_{CS}\left(\vec{u}.\vec{n}\right)dS=0$$

Incompressible flow, non-deformable CV:

$$\Rightarrow \int_{CS}\left(\vec{u}.\vec{n}\right)dS=0$$

Equation of mass conservation: integral form

$$\frac{\partial}{\partial t}\int_{CV}\rho d\forall+\int_{CS}\rho\left(\vec{u}.\vec{n}\right)dS=0$$

Steady flow:

$$\frac{\partial}{\partial t}\int_{CV}\rho d\forall+\int_{CS}\rho\left(\vec{u}.\vec{n}\right)dS=0$$

Steady flow, non-deformable CV:

$$\Rightarrow \int_{CS}\rho\left(\vec{u}.\vec{n}\right)dS=0$$

Steady flow, non-deformable CV:

$$\Rightarrow \int_{CS} \rho\left(\vec{u}.\vec{n}\right) dS = 0$$

Incompressible flow, non-deformable CV:

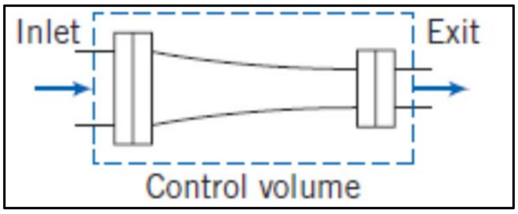
$$\Rightarrow \int_{CS} \left(\vec{u}.\vec{n}\right) dS = 0$$

True for both steady and unsteady cases

Uniform flow (over an area)

$$\int_{CS} \left(\vec{u}.\vec{n}\right) dS = 0 \Rightarrow \sum \vec{u}.\vec{A} = 0$$

Consider water flow through pipe where



Given:

inlet: $D_i = 50\text{mm}, u_i = 2.5\,\text{m/s}$

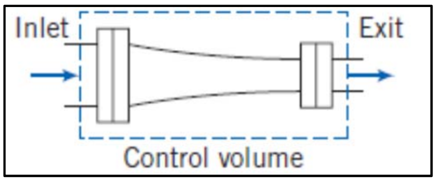
exit: $D_e = 25\text{mm}, u_e = ?$

To find: $u_e = ?$

Step 1: select an appropriate CV

In this case, we have selected a stationary, non-deformable CV

inlet: $D_\text{i} = 50\text{mm}, u_\text{i} = 2.5\,\text{m/s}$
exit: $D_\text{e} = 25\text{mm}, u_\text{e} = ?$



Assumptions:

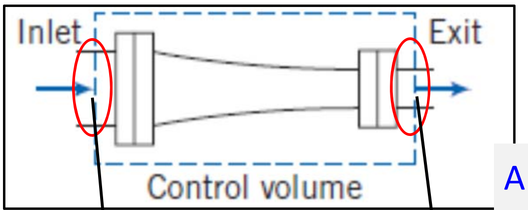
1. Incompressible flow
2. Uniform flows over inlet, exit areas

Analysis:

Integral mass conservation: $\dfrac{\partial}{\partial t}\displaystyle\int_{\text{CV}} \rho d\forall + \int_{\text{CS}} \rho(\vec{u}.\vec{n})\,dS = 0$

Incompressible mass conservation, non-deformable CV: $\displaystyle\int_{\text{CS}} (\vec{u}.\vec{n})\,dS = 0$

Uniform flow over inlet, exit: $\displaystyle\sum_j (\vec{u}.\vec{n})\,A_j = 0$



Area vector points outward from the CV

Typical inlet

$\vec{u} \longrightarrow$
$\vec{n} \longleftarrow$

$\vec{u}.\vec{n}A = -u_\text{i}A_\text{i}$

Typical exit

$\vec{u} \longrightarrow$
$\vec{n} \longrightarrow$

$\vec{u}.\vec{n}A = u_\text{e}A_\text{e}$

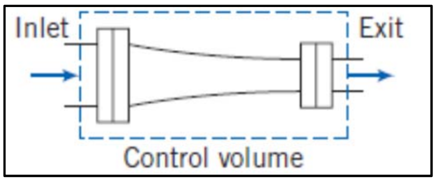
$$\sum_j (\vec{u}.\vec{n})\,A_j = -u_\text{i}A_\text{i} + u_\text{e}A_\text{e}$$

Proper choice of CV could be of great help

Putting everything together

inlet: $D_i = 50\text{mm}, u_i = 2.5\,\text{m/s}$

exit: $D_e = 25\text{mm}, u_e = ?$



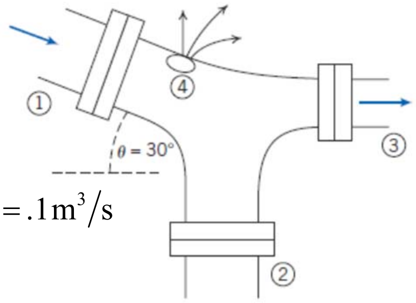
Incompressible mass conservation, non-deformable CV, Uniform flow over inlet, exit:

$$\sum_j (\vec{u}.\vec{n}) A_j = 0$$

$$\Rightarrow -u_i A_i + u_e A_e = 0 \quad \Rightarrow u_e = \frac{A_i}{A_e} u_i \quad = 10\,\text{m/s}$$ (Ans.)

Try to take a CV, such that the velocity and surface normal are either 0° or 180° apart (either same or in opposite directions)

Consider water flow through pipe where



$$A_1 = A_2 = .2\,\mathrm{m}^2, A_3 = .15\,\mathrm{m}^2$$

$$u_1 = 5\,\mathrm{m/s}, u_2 = ?, u_3 = 12\,\mathrm{m/s}, Q_4 = .1\,\mathrm{m}^3/\mathrm{s}$$

$$\text{now} \int_{\mathrm{CS}} (\vec{u}.\vec{n})\,dS = 0 \Rightarrow$$

$$-u_1 A_1 + u_2 A_2 + u_3 A_3 + u_4 A_4 = 0$$

$$\Rightarrow -u_1 A_1 + u_2 A_2 + u_3 A_3 + Q_4 = 0$$

$$\Rightarrow u_2 = -4.5\,\mathrm{m/s}$$