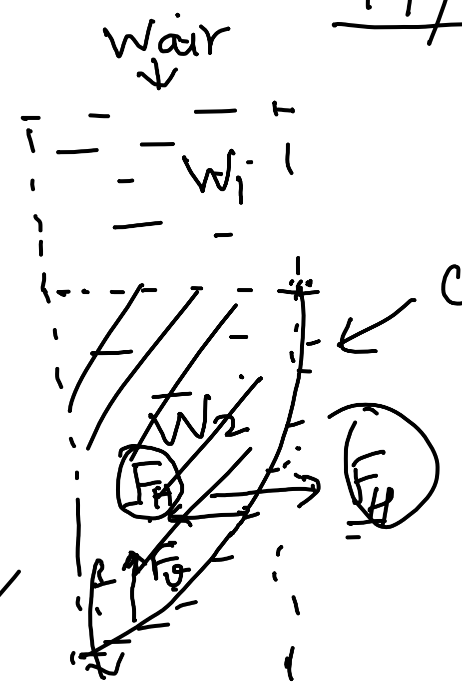
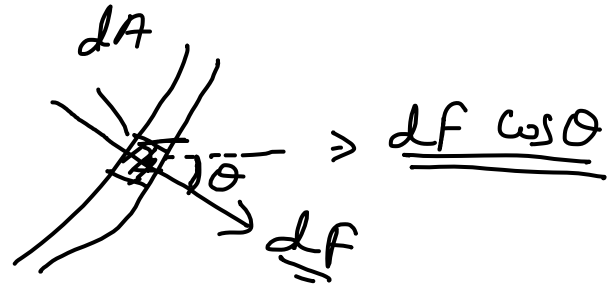
Hydrostatic force on a curved Surface



F = net force acting on the curved surface

F_H = Horizontal component of the force

$$F_H = \int \overbrace{p\,dA}^{dF} \cos\theta = \int p \underbrace{dA\cos\theta}_{\text{Projected Area}}$$



dA_p = Projected Area

$$= \int \underline{p}\, \underline{dA_p}$$

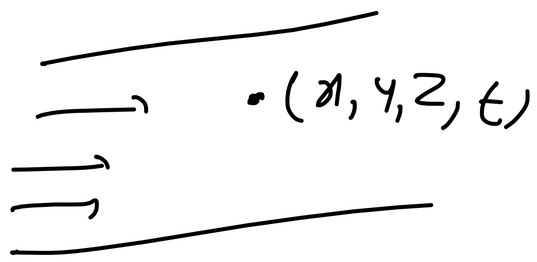
" The horizontal component of the hydrostatic force is the force on a force on a area projected onto the vertical plane "

Vertical component

$$F_V = W_1 + W_2 + W_{air}$$

" The vertical component is the net weight of fluid column above the curved surface ".

Eulerian description vs Lagrangian description



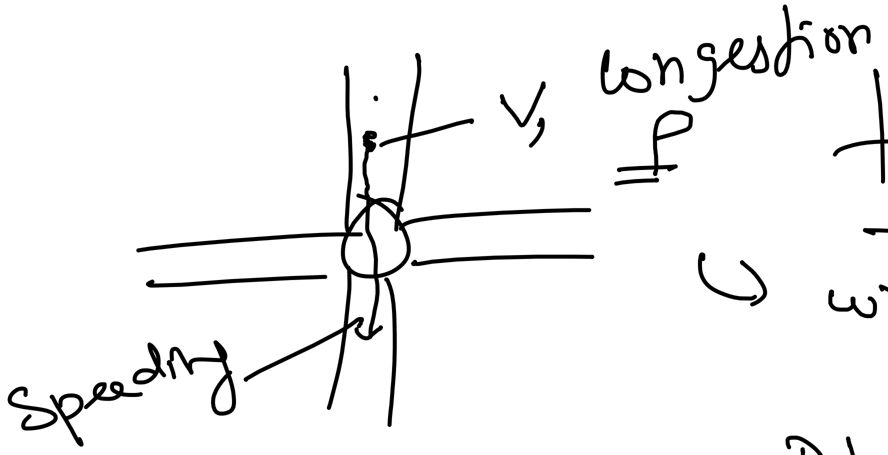
Eulerian → field description e.g. Electric field
Magnetic field

→ velocity field $V(x, y, z, t)$
ρ
p

Pressure gage
Lagrangian → (•) → measure pressure experienced by the particle as it moves
Particle

$p(t)$

e.g. traffic flow



traffic engineer
↪ width ⇒ eulerian

Police !

– Fluid dynamics measurements are suited to Eulerian description

→ acceleration is experienced by a particle

→ we are given with velocity field $V(x, y, z, t)$

→ we want to find out acceleration ⇒ <u>Local acceleration</u> + <u>convective acceleration</u>

fluid flow = steady ⇒ all the time derivatives are zero

$$\frac{\partial V}{\partial t} = 0$$

$$\vec{V} = u(x,y,z,t)\,\hat{i} + v(x,y,z,t)\,\hat{j} + w(x,y,z,t)\,\hat{k}$$

$\frac{d}{dt}$ = Total derivative
Substantial derivative
material derivative

$$\frac{dV}{dt} = \frac{du}{dt}\hat{i} + \frac{dv}{dt}\hat{j} + \frac{dw}{dt}\hat{k}$$

$$\frac{du}{dt} = \frac{\partial u}{\partial t} + \frac{\partial u}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial u}{\partial y}\frac{\partial y}{\partial t} + \frac{\partial u}{\partial z}\frac{\partial z}{\partial t}$$

$$= \frac{\partial u}{\partial t} + \underline{\frac{\partial u}{\partial x}u + \frac{\partial u}{\partial y}v + \frac{\partial u}{\partial z}w}$$

$$= \underbrace{\frac{\partial u}{\partial t}}_{\text{local}} + \underbrace{(V\cdot\nabla)u}_{\text{convective}} \quad \text{— (i)}$$

Similarly

$$\frac{dv}{dt} = \frac{\partial v}{\partial t} + (V\cdot\nabla)v \quad \text{— (ii)}$$

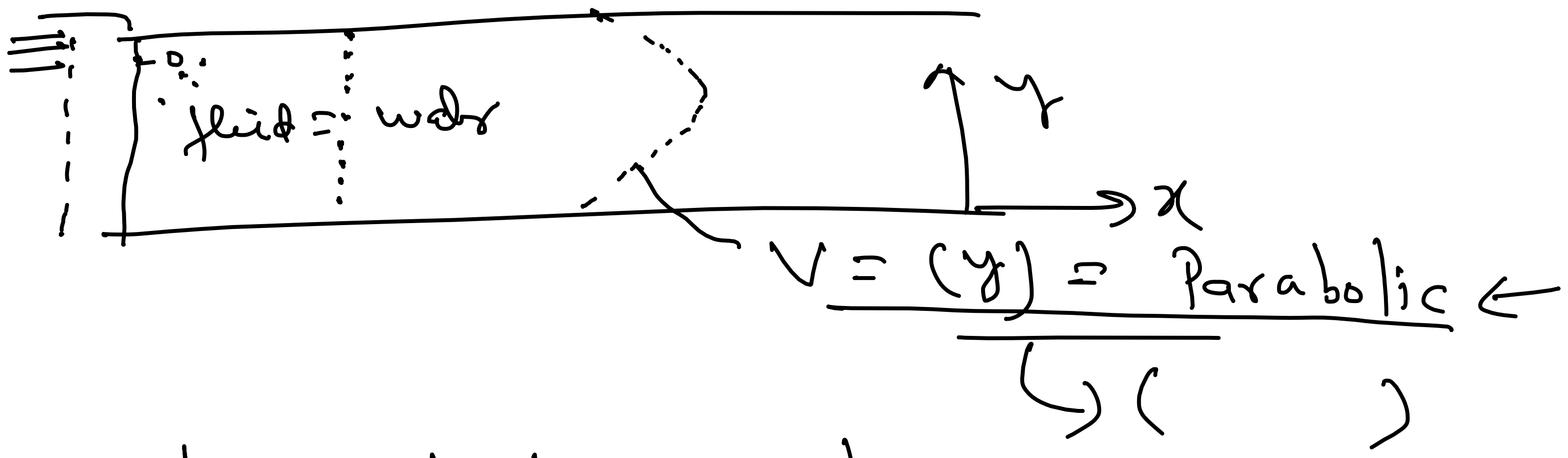
$$\frac{dw}{dt} = \frac{\partial w}{\partial t} + (V\cdot\nabla)w \quad \text{— (iii)}$$

$$\underline{\frac{d\vec{V}}{dt}} = \frac{\partial \vec{V}}{\partial t} + \underline{(V\cdot\nabla)\vec{V}}$$

$$\frac{dP}{dt} = \frac{\partial P}{\partial t} + (V\cdot\nabla)P \quad \text{---(ii)}$$

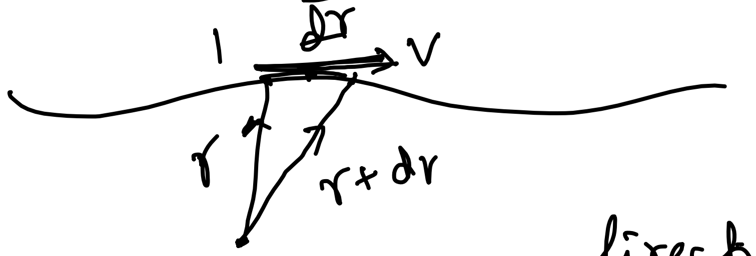
Flow visualization

↳ with the help of 4 lines → lines can be created with the help of H_2 bubbles using electrolysis of water

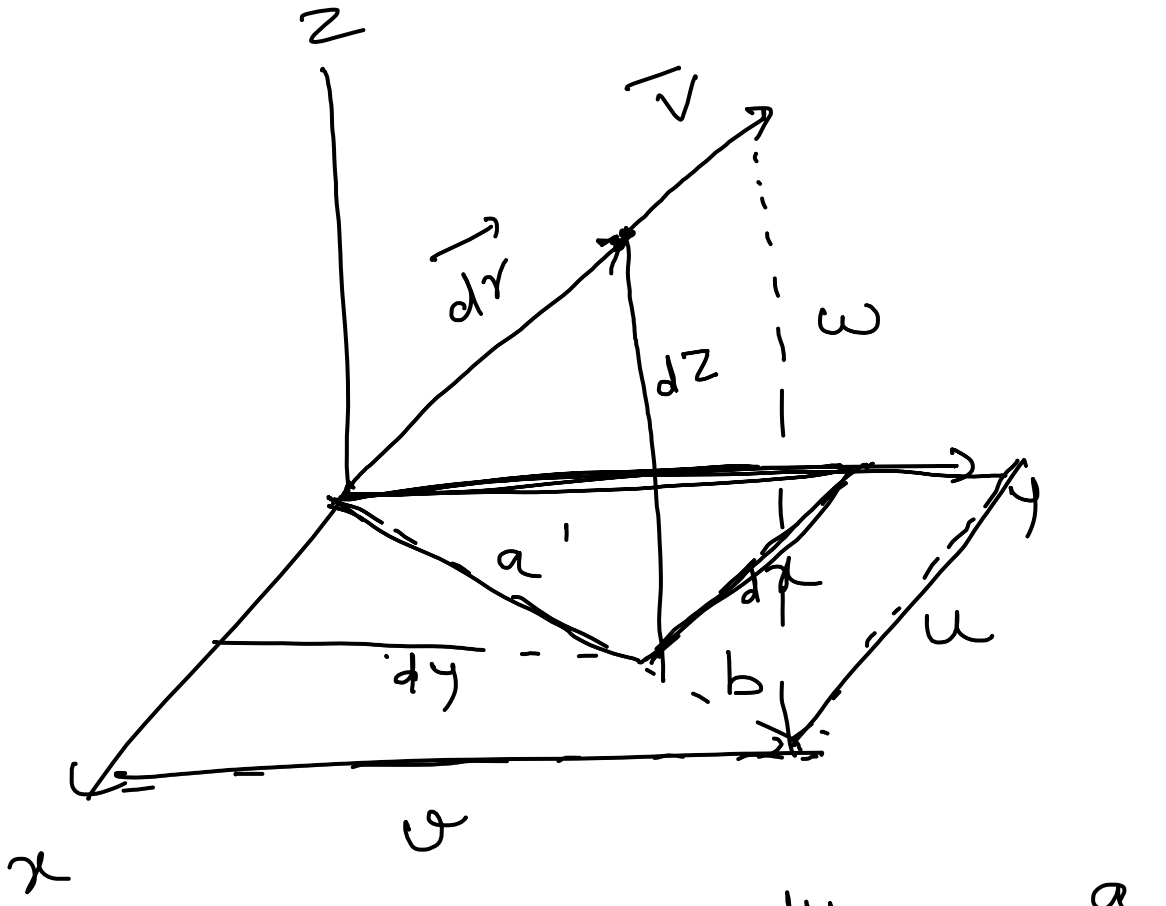


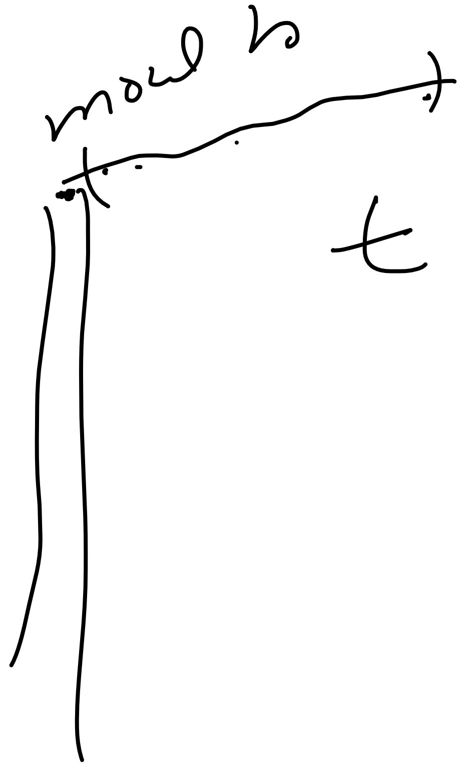
—(i) Stream line

→ " A curve which is everywhere tangent to velocity vector".



direction of $\vec{dr}$ ≡ direction of velocity vector





$$\frac{dx}{u} = \frac{dy}{v} = \frac{a}{b}$$

$$\frac{a}{b} = \frac{dz}{\omega}$$

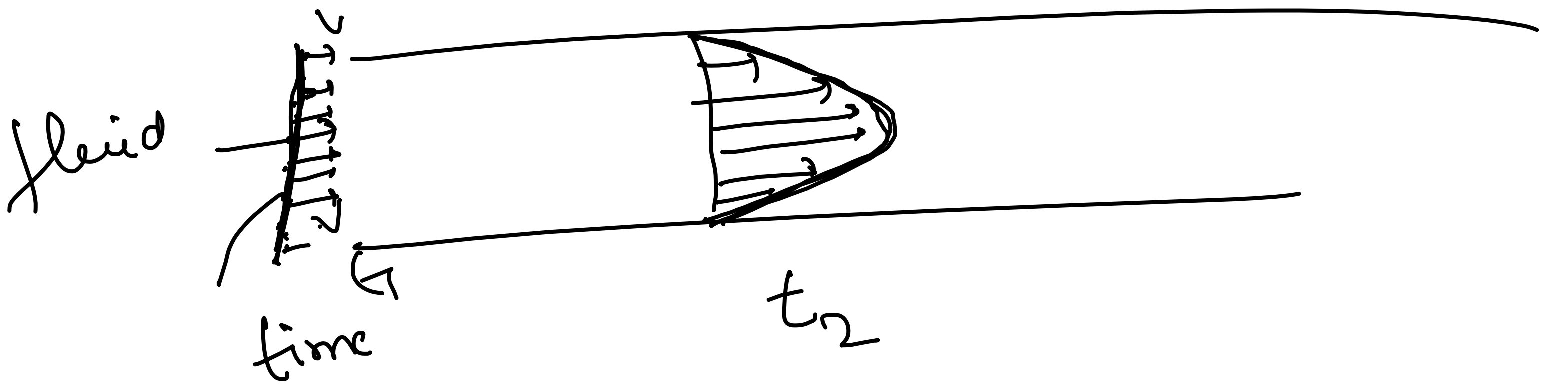
$$\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{\omega}$$ ← eqn for streamline

- Path line = Actual path traversed by a fluid particle

→ streakline = locus of particles which have been to prescribed point at some earlier time

↳ Chimney

- **time line:** set of particles that make a line at some specified time

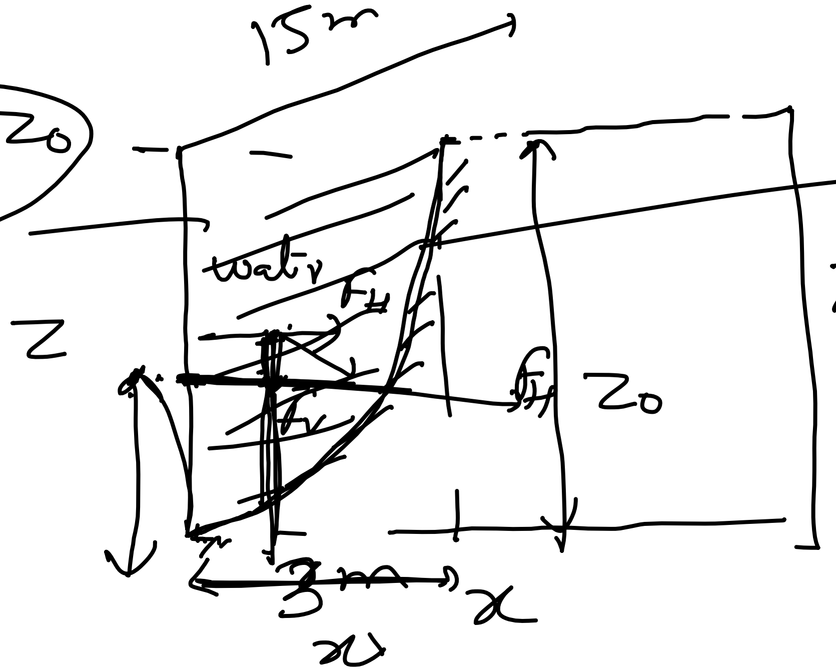


Example A dam has a parabolic shape $\frac{z}{z_0} = \left(\frac{x}{x_0}\right)^2$ as shown in fig with $x_0 = 3m$ and $z_0 = 7.5m$. The fluid is water $\gamma = 9.8\ kN/m^3$ and atmospheric pressure may be omitted. Compute the forces F_H and F_V on the dam and their line of action. The width of the dam is 15 m

$F_H = \gamma\, h_{cg} A$

$h_{cg} = \frac{7.5}{2}$

$A = \frac{2}{3} x_0 z_0$



$\frac{z}{z_0} = \left(\frac{x}{x_0}\right)^2$

$F_V \Rightarrow \rho g A b$

$b = 15m$

$z_{cp} = -\frac{\gamma \sin\theta\, I_{xx}}{h_{cg} A} = -1.25\ m$