Fluid Mechanics and Rate Processes: Tutorial 11

P1. Consider a 150-W incandescent lamp. The filament of the lamp is 5 cm long and has a diameter of 0.5 mm. The diameter of the glass bulb of the lamp is 8 cm. Determine the heat flux, in W/m^2, (a) on the surface of the filament and (b) on the surface of the glass bulb, and (c) calculate how much it will cost per year to keep that lamp on for eight hours a day every day if the unit cost of electricity is 5.34Rs./kWh.

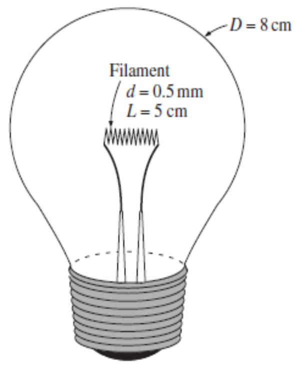


Fig.1 P1

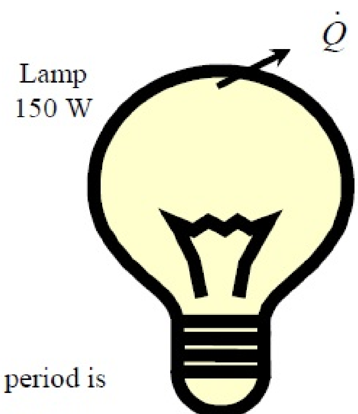
Solution:

Assumptions Heat transfer from the surface of the filament and the bulb of the lamp is uniform.

Analysis (a) The heat transfer surface area and the heat flux on the surface of the filament are

$$A_s = \pi DL = \pi(0.05\text{ cm})(5\text{ cm}) = 0.785\text{ cm}^2$$

$$\dot{q}_s = \frac{\dot{Q}}{A_s} = \frac{150\text{ W}}{0.785\text{ cm}^2} = 191\text{ W/cm}^2 = \mathbf{1.91\times10^6\ W/m^2}$$



(b) The heat flux on the surface of glass bulb is

$$A_s = \pi D^2 = \pi(8\text{ cm})^2 = 201.1\text{ cm}^2$$

$$\dot{q}_s = \frac{\dot{Q}}{A_s} = \frac{150\text{ W}}{201.1\text{ cm}^2} = 0.75\text{ W/cm}^2 = \mathbf{7500\ W/m^2}$$

(c) The amount and cost of electrical energy consumed during a one-year period is

$$\text{Electricity Consumption} = \dot{Q}\Delta t = (0.15\text{ kW})(365\times 8\text{ h/yr}) = 438\text{ kWh/yr}$$

$$\text{Annual Cost} = (438\text{ kWh/yr})(5.34 Rs/kWh) = 2338.92/yr$$

P2. Starting with an energy balance on a cylindrical shell volume element, derive the steady one-dimensional heat conduction equation for a long cylinder with constant thermal conductivity in which heat is generated at a rate of $\dot{g}$

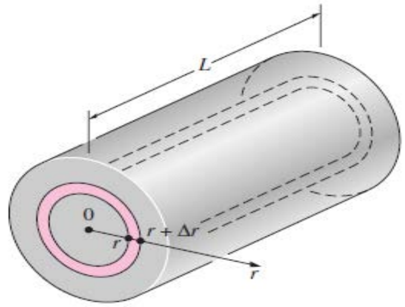


Fig.2 P2

Solution:

We consider a thin cylindrical shell element of thickness Δr in a long cylinder (see Fig. 2-15 in the text). The density of the cylinder is ρ, the specific heat is c, and the length is L. The area of the cylinder normal to the direction of heat transfer at any location is $A = 2\pi rL$ where r is the value of the radius at that location. Note that the heat transfer area A depends on r in this case, and thus it varies with location. An *energy balance* on this thin cylindrical shell element of thickness Δr during a small time interval Δt can be expressed as

$$\dot{Q}_r - \dot{Q}_{r+\Delta r} + \dot{E}_{\text{element}} = \frac{\Delta E_{\text{element}}}{\Delta t}$$

where

$$\Delta E_{\text{element}} = E_{t+\Delta t} - E_t = mc(T_{t+\Delta t} - T_t) = \rho cA\Delta r(T_{t+\Delta t} - T_t)$$

$$\dot{E}_{\text{element}} = \dot{e}_{\text{gen}} V_{\text{element}} = \dot{e}_{\text{gen}} A\Delta r$$

Substituting,

$$\dot{Q}_r - \dot{Q}_{r+\Delta r} + \dot{e}_{\text{gen}} A\Delta r = \rho cA\Delta r \frac{T_{t+\Delta t} - T_t}{\Delta t}$$

where $A = 2\pi rL$. Dividing the equation above by $A\Delta r$ gives

$$-\frac{1}{A}\frac{\dot{Q}_{r+\Delta r} - \dot{Q}_r}{\Delta r} + \dot{e}_{\text{gen}} = \rho c \frac{T_{t+\Delta t} - T_t}{\Delta t}$$

Taking the limit as $\Delta r \to 0$ and $\Delta t \to 0$ yields

$$\frac{1}{A}\frac{\partial}{\partial r}\left(kA\frac{\partial T}{\partial r}\right) + \dot{e}_{\text{gen}} = \rho c \frac{\partial T}{\partial t}$$

since, from the definition of the derivative and Fourier's law of heat conduction,

$$\lim_{\Delta r \to 0} \frac{\dot{Q}_{r+\Delta r} - \dot{Q}_r}{\Delta r} = \frac{\partial Q}{\partial r} = \frac{\partial}{\partial r}\left(-kA\frac{\partial T}{\partial r}\right)$$

Noting that the heat transfer area in this case is $A = 2\pi rL$ and the thermal conductivity is constant, the one-dimensional transient heat conduction equation in a cylinder becomes

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \dot{e}_{\text{gen}} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

where $\alpha = k / \rho c$ is the thermal diffusivity of the material.

P3. Water flows through a pipe at an average temperature of T_∞ = 50°C. The inner and outer radii of the pipe are r_1=6 cm and r_2 = 6.5 cm, respectively. The outer surface of the pipe is wrapped with a thin electric heater that consumes 300 W per m length of the pipe. The exposed surface of the heater is heavily insulated so that the entire heat generated in the heater is transferred to the pipe. Heat is transferred from the inner surface of the pipe to the water by convection with a heat transfer coefficient of h =55 W/m^2 °C. Assuming constant thermal conductivity and one-dimensional heat transfer, express the mathematical formulation (the differential equation and the boundary conditions) of the heat conduction in the pipe during steady operation. Do not solve.

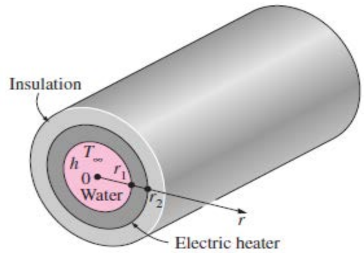


Fig.3 P3

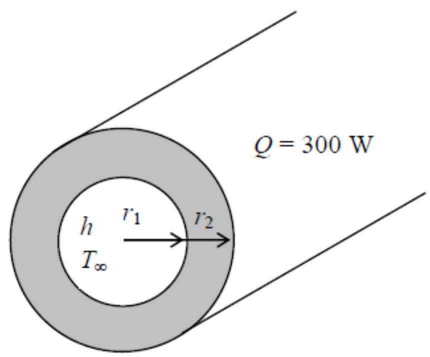
Solution:

Assumptions **1** Heat transfer is given to be steady and one-dimensional. **2** Thermal conductivity is given to be constant. **3** There is no heat generation in the medium. **4** The outer surface at $r = r_2$ is subjected to uniform heat flux and the inner surface at $r = r_1$ is subjected to convection.

Analysis The heat flux at the outer surface of the pipe is

$$\dot{q}_s = \frac{\dot{Q}_s}{A_s} = \frac{\dot{Q}_s}{2\pi r_2 L} = \frac{300 \text{ W}}{2\pi(0.065 \text{ cm})(1 \text{ m})} = 734.6 \text{ W/m}^2$$

Noting that there is thermal symmetry about the center line and there is uniform heat flux at the outer surface, the differential equation and the boundary conditions for this heat conduction problem can be expressed as



$$\frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0$$

$$k\frac{dT(r_1)}{dr} = h[T(r_i) - T_\infty] = 85[T(r_i) - 70]$$

$$k\frac{dT(r_2)}{dr} = \dot{q}_s = 734.6 \text{ W/m}^2$$

P4. Consider the base plate of a 800-W household iron with a thickness of L =0.6 cm, base area of A =160 cm^2, and thermal conductivity of k =20 W/m°C. The inner surface of the base plate is subjected to uniform heat flux generated by the resistance heaters inside. When steady operating conditions are reached, the outer surface temperature of the plate is measured to be 85°C. Disregarding any heat loss through the upper part of the iron, (*a*) express the differential equation and the boundary conditions for steady one-dimensional heat conduction through the plate, (*b*) obtain a relation for the variation of temperature in the base plate by solving the differential equation, and (*c*) evaluate the inner surface temperature.

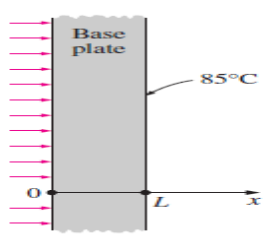


Fig.4 P4

Solution:

Assumptions **1** Heat conduction is steady and one-dimensional since the surface area of the base plate is large relative to its thickness, and the thermal conditions on both sides of the plate are uniform. **2** Thermal conductivity is constant. **3** There is no heat generation in the plate. **4** Heat loss through the upper part of the iron is negligible.

Properties The thermal conductivity is given to be $k = 20$ W/m·°C.

Analysis (*a*) Noting that the upper part of the iron is well insulated and thus the entire heat generated in the resistance wires is transferred to the base plate, the heat flux through the inner surface is determined to be

$$\dot{q}_0 = \frac{\dot{Q}_0}{A_{\text{base}}} = \frac{800\text{ W}}{160\times10^{-4}\text{ m}^2} = 50{,}000\text{ W/m}^2$$

Taking the direction normal to the surface of the wall to be the x direction with $x = 0$ at the left surface, the mathematical formulation of this problem can be expressed as

$$\frac{d^2T}{dx^2} = 0$$

and $$-k\frac{dT(0)}{dx} = \dot{q}_0 = 50{,}000\text{ W/m}^2$$

$$T(L) = T_2 = 85°\text{C}$$

(*b*) Integrating the differential equation twice with respect to x yields

$$\frac{dT}{dx} = C_1$$

$$T(x) = C_1x + C_2$$

where C_1 and C_2 are arbitrary constants. Applying the boundary conditions give

$x = 0$: $$-kC_1 = \dot{q}_0 \;\rightarrow\; C_1 = -\frac{\dot{q}_0}{k}$$

$x = L$: $$T(L) = C_1L + C_2 = T_2 \;\rightarrow\; C_2 = T_2 - C_1L \;\rightarrow\; C_2 = T_2 + \frac{\dot{q}_0 L}{k}$$

Substituting C_1 and C_2 into the general solution, the variation of temperature is determined to be

$$\begin{aligned} T(x) &= -\frac{\dot{q}_0}{k}x + T_2 + \frac{\dot{q}_0 L}{k} = \frac{\dot{q}_0(L-x)}{k} + T_2 \\ &= \frac{(50{,}000\text{ W/m}^2)(0.006-x)\text{m}}{20\text{ W/m}\cdot°\text{C}} + 85°\text{C} \\ &= 2500(0.006-x) + 85 \end{aligned}$$

(*c*) The temperature at $x = 0$ (the inner surface of the plate) is

$$T(0) = 2500(0.006-0) + 85 = \mathbf{100°C}$$

Note that the inner surface temperature is higher than the exposed surface temperature, as expected.