ESO 204A: Fluid Mechanics and Rate Processes

Revisiting Kinematics

Lagrangian and Eulerian Descriptions

Anything else (on kinematics) you want to discuss

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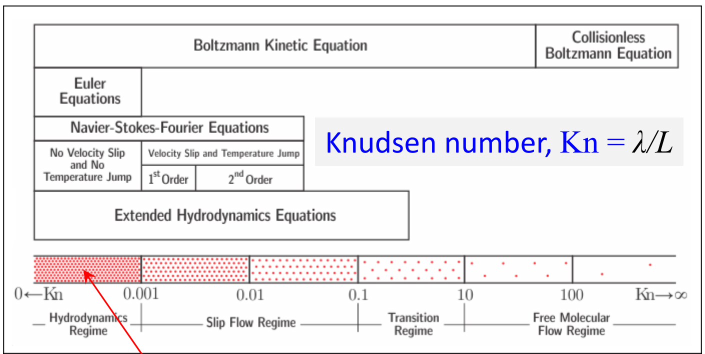
Knudsen number and continuum limit

Continuum hypotheses is applicable when length-scale of interest $(L) >>$ molecular length-scale (mean free path, λ)



Knudsen number, $\mathrm{Kn} = \lambda / L$

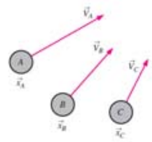
We are interested in this regime

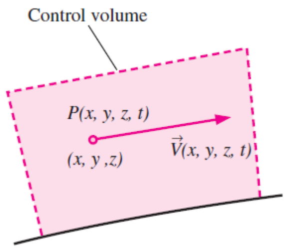
Lagrangian (particle)

- We model the fluid as a bunch of particles; many many particles in our case (due to continuum assumption)
- Describe fluid motion by stating the instantaneous **position and velocity** of each particle

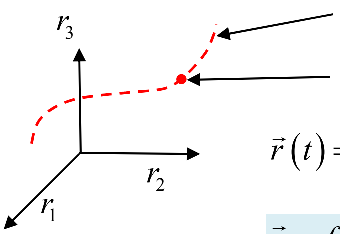
Eulerian (field)

- We define (many many) points in space
- Fluid particles arrives at a point occupies the **position and velocity of that point**
- Eulerian concept is applicable as long as continuum concept is valid, Lagrangian is far more general





Lagrangian description



Pathline of a particle

Particle position: $\vec{r} \equiv (r_1, r_2, r_3)$

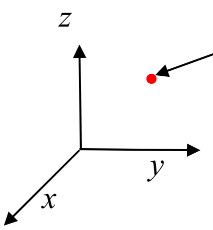
$$\vec{r}(t) = \vec{r}_0 + \int_{t_0}^{t} \vec{u}dt; \text{ initial condition : } \vec{r}(t = t_0) = \vec{r}_0$$

$$\boxed{\vec{r} = f(\vec{r}_0, t_0, t)} \qquad \vec{u} = \frac{d\vec{r}}{dt}, \vec{a} = \frac{d^2\vec{r}}{dt^2}$$

in general, any property $f = f(\vec{r}, t)$

In Lagrangian description, position of a particle is **NOT an independent variable** and is a function of the independent variable: **time.** Particles are identified based on the **initial condition**

Eulerian description



Point in space

At continuum scale, the point is always occupied by some (not same) fluid particle

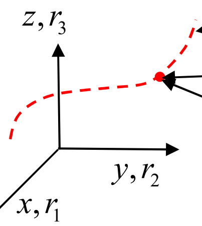
Point coordinate: $\vec{x} \equiv (x, y, z)$

in general, any property $f = f(\vec{x}, t)$

Physically, the above property is the property of the fluid particle occupying the point

In Eulerian description, point coordinate and time (x, y, z, t) are the independent variables

Combining everything



Pathline of a particle

Particle position: $\vec{r} \equiv (r_1, r_2, r_3)$

Point coordinate: $\vec{x} \equiv (x, y, z)$

$\vec{r} \equiv \vec{x}$

therefore, $f(\vec{r}, t) = f(\vec{x}, t)$ Using same ‘clock’

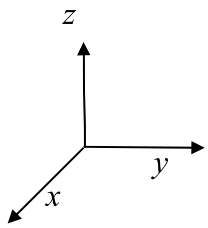
$$\frac{df(\vec{x},t)}{dt} = \frac{df(\vec{r},t)}{dt} = \frac{\partial f}{\partial t} + \frac{\partial f}{\partial r_1}\frac{dr_1}{dt} + \frac{\partial f}{\partial r_2}\frac{dr_2}{dt} + \frac{\partial f}{\partial r_3}\frac{dr_3}{dt}$$

$$= \frac{\partial f}{\partial t} + (\vec{u}.\nabla) f$$

Material derivative in Eulerian sense

Total (not partial) time derivative in Lagrangian sense

Quiz:



Given $u = \frac{dx}{dt}; v = \frac{dy}{dt};$ (x, y) indicates

1. Position of a fluid particle (Lagrangian)
2. Location of a point in space (Eulerian)

Whenever you see ***x*, *y*, *z* as a function of *t*,** we mean **Lagrangian** frame (particle position)

(u, v) indicates:

1. Velocity of a fluid particle (Lagrangian)
2. Velocity at a point in space (Eulerian)
3. Both of the above

Example:

Find the velocity **field** from the Lagrangian description:

$$x = x_0 \exp(-kt) + y_0 \left[1 - \exp(-2kt)\right]; y = y_0 \exp(-kt)$$

$$u = \frac{dx}{dt}; v = \frac{dy}{dt} \qquad u = -kx_0 e^{-kt} + 2ky_0 e^{-2kt}; v = ky_0 e^{kt}$$

Velocity components of a specific particle at time t

Initial condition of the particle: $\vec{x}(t=0) = (x_0, y_0)$

To get the velocity in Eulerian sense $\quad u, v$ should be free of x_0, y_0

$$u = -kx + ky\left(e^{-kt} + e^{-3kt}\right); v = ky$$

Velocity both in Eulerian and Lagrangian frames

Closure

1. Keywords: **Particle, Field**
2. Think about the independent variables
3. All the lines (stream, streak etc.) are simply geometric entity
4. Streaklines, pathlines are drawn by taking particle information from Lagrangian descriptions
5. Streamline is described using velocity field data from Eulerian frame