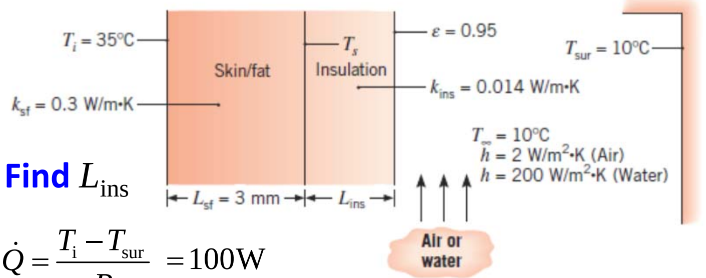
ESO204A, Fluid Mechanics and rate Processes

1-D Heat Conduction

Chapter 2 of Cengel

Designing an winter garment

Heat generation in human body in rest condition is about 100 W

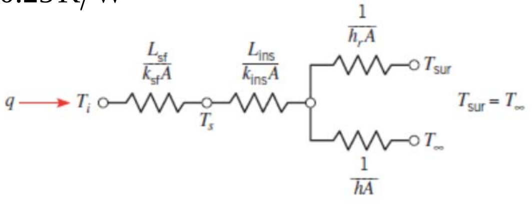


Find L_{ins}

$$\dot{Q} = \frac{T_{\text{i}} - T_{\text{sur}}}{R} = 100\,\text{W}$$

$$\Rightarrow R = 0.25\,\text{K/W}$$

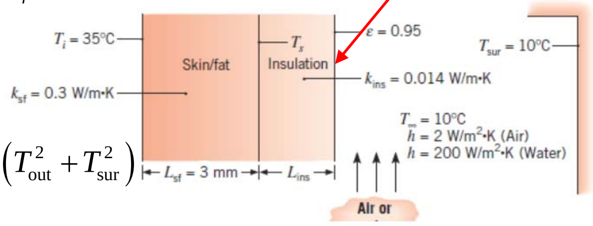
$$R = \frac{L_{\text{sf}}}{k_{\text{sf}}A} + \frac{L_{\text{ins}}}{k_{\text{ins}}A} + \frac{1}{\dfrac{1}{1/hA} + \dfrac{1}{1/h_r A}}$$



$$\frac{L_{\text{sf}}}{k_{\text{sf}}A}+\frac{L_{\text{ins}}}{k_{\text{ins}}A}+\frac{1}{hA+h_rA}=.25\,\text{K/W}$$

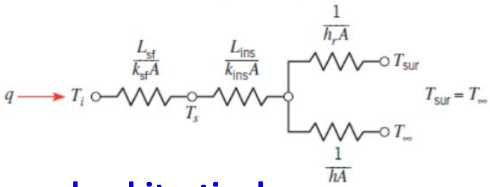
T_{out}

Find L_{ins}



$$h_r=\varepsilon\sigma\left(T_{\text{out}}+T_{\text{sur}}\right)\left(T_{\text{out}}^2+T_{\text{sur}}^2\right)$$

$$\frac{T_{\text{i}}-T_{\text{out}}}{\dfrac{L_{\text{sf}}}{k_{\text{sf}}A}+\dfrac{L_{\text{ins}}}{k_{\text{ins}}A}}=\dot{Q}=100\text{W}$$

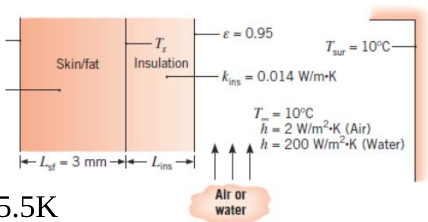


Above three Eqns to be solved itertively to find h_r, T_{out}, L_{ins}

Designing an winter garment

Find L_{ins}

$$\frac{L_{\text{sf}}}{k_{\text{sf}}A}+\frac{L_{\text{ins}}}{k_{\text{ins}}A}+\frac{1}{hA+h_rA}=.25\,\text{K/W}$$



$$\text{assume } T_{\text{out}}\approx 273+\frac{35+10}{2}=295.5\text{K}$$

$$h_r=\varepsilon\sigma\left(T_{\text{out}}+T_{\text{sur}}\right)\left(T_{\text{out}}^2+T_{\text{sur}}^2\right)=5.2\,\text{W/m}^2\text{-K}$$

$$h=2\,\text{W/m}^2\text{-K}$$

$$L_{\text{ins}}=4.1\text{mm}$$

We may now recalculate T_{out} and improve the result itertively

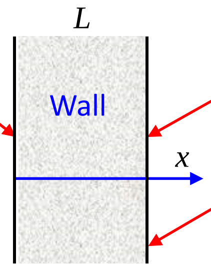
$$h=200\,\text{W/m}^2\text{-K}$$

$$L_{\text{ins}}=6.1\text{mm}$$

$$\frac{T_{\text{i}}-T_{\text{out}}}{\dfrac{L_{\text{sf}}}{k_{\text{sf}}A}+\dfrac{L_{\text{ins}}}{k_{\text{ins}}A}}=100\text{W}$$

Example: calculating Air-conditioning load in a constant temperature room

constant temperature T_{in}



solar radiant flux q_{rad}

convective flux

$$q_{conv} = h_{out}\left(T_\infty - T\right)$$

$$\frac{d^2T}{dx^2} = 0$$

$$\text{BC:} T\left(x=0\right) = T_{in}$$

at $x = L$,

$$-k\frac{dT}{dx} + h_{out}\left(T_\infty - T\right) + q_{rad} = 0$$

$$\frac{dT}{dx} = B$$

$$T = Bx + C$$

Air-conditioning load in a room

$$\frac{dT}{dx} = B \Rightarrow T = Bx + C$$

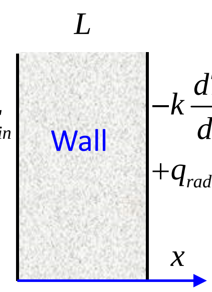
$$C = T_{in}$$

$$-kA + h_{out}\left(T_\infty - BL - C\right)$$
$$+q_{rad} = 0$$

$$-\left(k + h_{out}L\right)B + h_{out}\left(T_\infty - C\right)$$
$$+q_{rad} = 0$$

$$B = \frac{h_{out}\left(T_\infty - C\right) + q_{rad}}{k + h_{out}L}$$

$T = T_{in}$



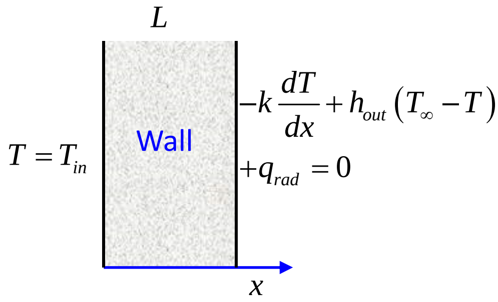
$$-k\frac{dT}{dx} + h_{out}\left(T_\infty - T\right)$$
$$+q_{rad} = 0$$

$$T = T_{in} + \frac{h_{out}\left(T_\infty - C\right) + q_{rad}}{k + h_{out}L}x$$

Air-conditioning load in a room

$$T = Bx + C \qquad C = T_{in}$$

$$B = \frac{h_{out}\left(T_\infty - C\right) + q_{rad}}{k + h_{out}L} = \frac{h_{out}\left(T_\infty - T_{in}\right) + q_{rad}}{k + h_{out}L}$$



Heat load at $x = 0$

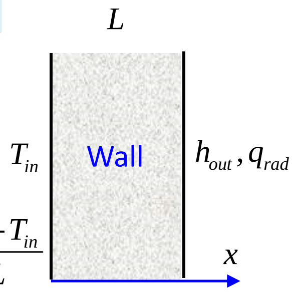
$$= A_{wall}k\frac{dT}{dx} \quad = A_{wall}kB \quad = A_{wall}k\frac{h_{out}\left(T_\infty - T_{in}\right) + q_{rad}}{k + h_{out}L}$$

Air-conditioning load in a room

$$Q = A_{wall}k\frac{h_{out}\left(T_\infty - T_{in}\right) + q_{rad}}{k + h_{out}L}$$

For $h_{out} \to \infty$

$$Q = A_{wall}k\frac{T_\infty - T_{in} + q_{rad}/h_{out}}{k/h_{out} + L} \approx A_{wall}k\frac{T_\infty - T_{in}}{L}$$



Usually h increases with Re

Above approximation is possible in case of rain/thunderstorm; note that outside wall temperature reaches ambient temperature

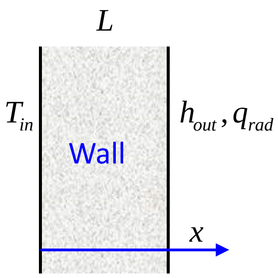
convective flux

$$q_{conv} = h_{out}\left(T_\infty - T\right) \qquad \Rightarrow T_\infty - T = q_{conv}/h_{out} \approx 0$$

Air-conditioning load in a room

$$Q = A_{wall} k \frac{h_{out}\left(T_\infty - T_{in}\right) + q_{rad}}{k + h_{out} L}$$

For $\quad h_{out} \to 0 \qquad Q = A_{wall} q_{rad}$



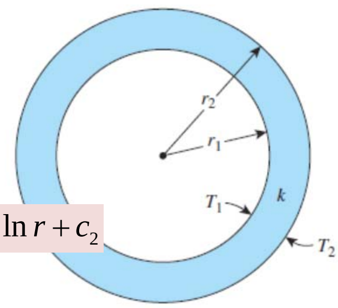
Above approximation is possible in case of a clear-weather, sunny day with low wind speed; note that the thermal properties of wall material does not play any role here

Steady, 1-D heat transfer, cylindrical coordinate

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \cancel{\frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2}} + \cancel{\frac{\partial^2 T}{\partial z^2}} = 0$$

$$T\left(r = r_1\right) = T_1 \qquad T\left(r = r_2\right) = T_2$$

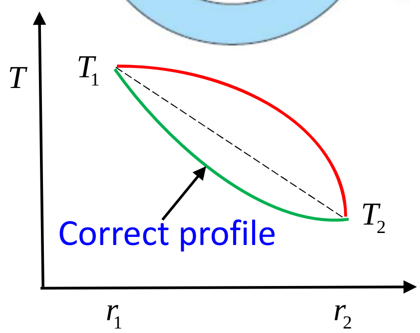
$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0 \quad \Rightarrow r\frac{dT}{dr} = c_1 \quad \boxed{T = c_1 \ln r + c_2}$$



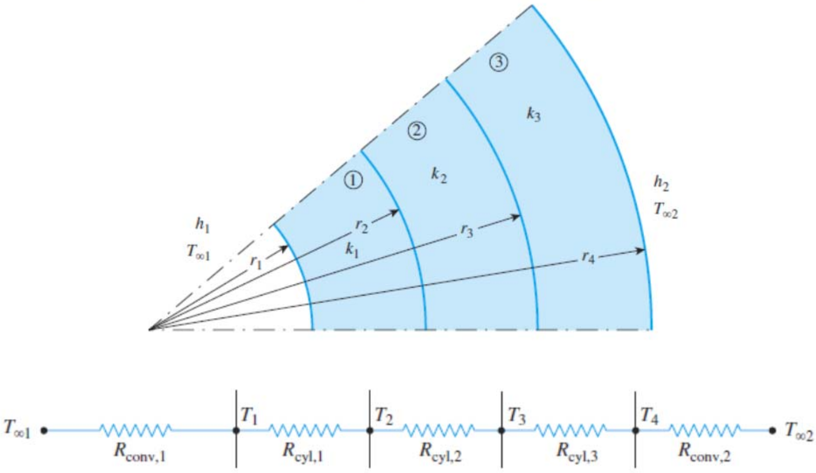
$$\boxed{T = T_1 + \left(T_1 - T_2\right)\frac{\ln\left(r/r_1\right)}{\ln\left(r_1/r_2\right)}}$$

$$\dot{Q} = -kA\frac{dT}{dr} = -k\left(2\pi r L\right)\frac{dT}{dr}$$

$$= \frac{T_1 - T_2}{R} \qquad R = \frac{\ln\left(r_2/r_1\right)}{2\pi k L}$$



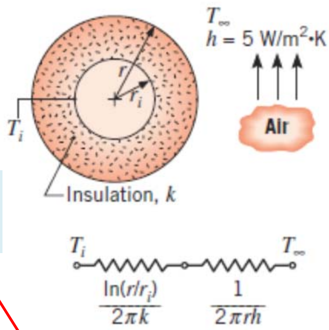
Electrical analogy: multilayer cylinder

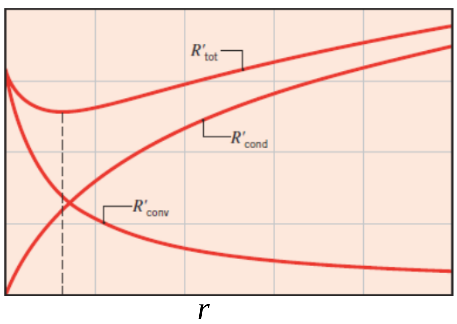


Critical thickness of insulation

$$\dot{Q} = \frac{T_1 - T_2}{R} \qquad R = \frac{\ln(r/r_i)}{2\pi k} + \frac{1}{2\pi r h}$$

$$\frac{dR}{dr} = 0 \qquad \frac{1}{2\pi r k} - \frac{1}{2\pi r^2 h} = 0 \qquad r = \frac{k}{h}$$





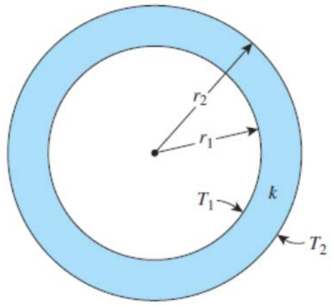
Critical thickness of insulation that maximizes rate of heat transfer

Steady, 1-D heat transfer, spherical coordinate

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right)=0 \qquad T=-2c_1/r+c_2$$

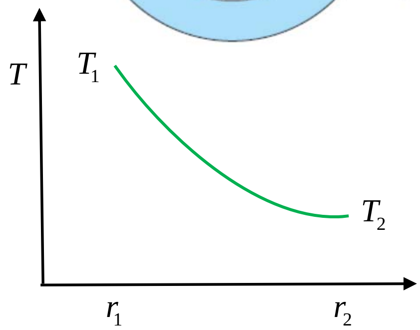
$$T(r=r_1)=T_1 \qquad T(r=r_2)=T_2$$

$$T=T_1+(T_2-T_1)\frac{1-r_1/r}{1-r_1/r_2}$$



$$\dot{Q}=-kA\frac{dT}{dr}=-k\left(4\pi r^2\right)\frac{dT}{dr}$$

$$=\frac{T_1-T_2}{R} \qquad R=\frac{1-r_1/r_2}{4\pi r_1 k}$$



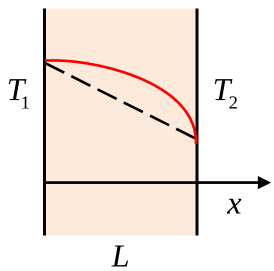
Steady, 1-D heat transfer, with uniform generation

$$\frac{d^2T}{dx^2}+\frac{\dot{e}_{\rm gen}}{k}=0\Rightarrow\frac{dT}{dx}=-\frac{\dot{e}_{\rm gen}}{k}x+c_1$$

$$\Rightarrow T=-\frac{\dot{e}_{\rm gen}}{2k}x^2+c_1x+c_2 \qquad T(x=0)=T_1$$

$$\Rightarrow c_2=T_1$$

$$T(x=L)=T_2\Rightarrow-\frac{\dot{e}_{\rm gen}}{2k}L^2+c_1L+c_2=T_2$$



$$T=T_1+(T_2-T_1)\frac{x}{L}+\frac{\dot{e}_{\rm gen}}{2k}\left(Lx-x^2\right)$$

$$\frac{T-T_1}{T_2-T_1}=\frac{x}{L}+\frac{\dot{e}_{\rm gen}L^2}{2k(T_2-T_1)}\left(\frac{x}{L}-\frac{x^2}{L^2}\right)$$

Nondimensional form

$$q''_x=-k\frac{dT}{dx}=\frac{k(T_1-T_2)}{L}+\frac{\dot{e}_{\rm gen}}{2}(2x-L)$$

Contribution from source

$$\dot{Q}=\frac{kA(T_1-T_2)}{L}+\frac{\dot{e}_{\rm gen}A}{2}(2x-L)$$