ESO204A, Fluid Mechanics and rate Processes

Energy conservation: integral formulation

Useful for calculation of
power, heat transfer, frictional losses

Chapter 3 of F M White
Chapter 4 of Fox McDonald

Energy conservation: $\frac{dE}{dt} = \dot{Q} - \dot{W}$

Reynolds Transport Theorem:

$$\frac{dE}{dt} = \frac{\partial}{\partial t}\int_{CV} \rho e d\forall + \int_{CS} \rho e (\vec{u}.\vec{n}) dA \qquad e = \frac{E}{m}$$

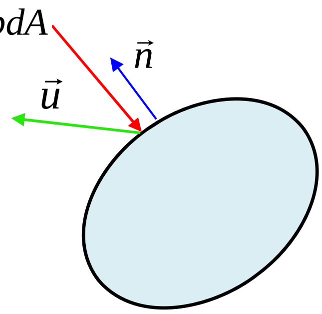
Rate of work: $\dot{W} = \dot{W}_{\text{shaft}} + \dot{W}_{\text{shear}} + \dot{W}_{\text{pressure}} + \dot{W}_{\text{others}}$

Combining:

$$\dot{Q} - \dot{W}_{\text{shaft}} - \dot{W}_{\text{shear}} - \dot{W}_{\text{pressure}} - \dot{W}_{\text{others}} = \frac{\partial}{\partial t}\int_{CV} \rho e d\forall + \int_{CS} \rho e (\vec{u}.\vec{n}) dA$$

Elemental pressure force: $-p\vec{n}dA$

Displacement rate: $-(\vec{u}.\vec{n})\vec{n}$



$$\dot{W}_{\text{pressure}} = \int_{\text{CS}} (-p\vec{n}dA).\left[-(\vec{u}.\vec{n})\vec{n}\right]$$
$$= \int_{\text{CS}} p(\vec{u}.\vec{n})\,dA$$

Energy Equation:

$$\dot{Q} - \dot{W}_{\text{shaft}} - \dot{W}_{\text{shear}} - \dot{W}_{\text{pressure}} - \dot{W}_{\text{others}} = \frac{\partial}{\partial t}\int_{\text{CV}} \rho e d\mathcal{V} + \int_{\text{CS}} \rho e(\vec{u}.\vec{n})\,dA$$

$$\dot{Q} - \dot{W}_{\text{shaft}} - \dot{W}_{\text{shear}} - \dot{W}_{\text{others}} = \frac{\partial}{\partial t}\int_{\text{CV}} \rho e d\mathcal{V} + \int_{\text{CS}} \rho\left(e + \frac{p}{\rho}\right)(\vec{u}.\vec{n})\,dA$$

Energy Equation:

$$\dot{Q} - \dot{W}_{\text{shaft}} - \dot{W}_{\text{shear}} - \dot{W}_{\text{others}} = \frac{\partial}{\partial t}\int_{\text{CV}} \rho e d\mathcal{V} + \int_{\text{CS}} \rho\left(e + \frac{p}{\rho}\right)(\vec{u}.\vec{n})\,dA$$

Steady flow, non deformable CV:

$$\int_{\text{CS}} \rho\left(e + \frac{p}{\rho}\right)(\vec{u}.\vec{n})\,dA = \dot{Q} - \dot{W}_{\text{shaft}} - \dot{W}_{\text{shear}} - \dot{W}_{\text{others}}$$

Energy: $e = \text{internal} + \text{kinetic} + \text{potential} \; = ie + \dfrac{u^2}{2} + gz$

$$e + \frac{p}{\rho} = h + \frac{u^2}{2} + gz \qquad h:\ \text{specific enthalpy}$$

$$\int_{CS} \rho\left(h+\frac{u^2}{2}+gz\right)(\vec{u}.\vec{n})\,dA=\dot{Q}-\dot{W}_{\text{shaft}}-\dot{W}_{\text{shear}}-\dot{W}_{\text{others}}$$

Steady, non-deformable CV, uniform flow at inlets, exits

$$\sum\left(h+\frac{u^2}{2}+gz\right)\rho\vec{u}.\vec{A}=\dot{Q}-\dot{W}_{\text{shaft}}-\dot{W}_{\text{shear}}-\cancelto{=0\ (\text{assume})}{\dot{W}_{\text{others}}}$$

$\dot{W}_{\text{shaft}} \neq 0$ for fluid machines (pump, turbine) within CV

$\dot{W}_{\text{shear}}$ indicates losses due to frictional (viscous) effects

$$\sum\left(h+\frac{u^2}{2}+gz\right)\rho\vec{u}.\vec{A}=\dot{Q}-\dot{W}_{\text{shaft}}-\text{Losses}$$

Known as: Steady Flow Energy Eqn.

SFEE for isothermal, adiabatic, one-inlet one-exit system:

isothermal: $ie=\text{constant} \quad \Rightarrow h=\dfrac{p}{\rho}$

adiabatic: $\dot{Q}=0$

mass flow rate: $\dot{m}=-\left(\rho\vec{u}.\vec{A}\right)_{\text{in}}=\left(\rho\vec{u}.\vec{A}\right)_{\text{out}}$

SFEE: $$\sum\left(h+\frac{u^2}{2}+gz\right)\rho\vec{u}.\vec{A}=\dot{Q}-\dot{W}_{\text{shaft}}-\text{Losses}$$

$$\dot{m}\left(\frac{p}{\rho}+\frac{u^2}{2}+gz\right)_{\text{out}}-\dot{m}\left(\frac{p}{\rho}+\frac{u^2}{2}+gz\right)_{\text{in}}=-\dot{W}_{\text{shaft}}-\text{Losses}$$

$$\dot{m}\left(\frac{p}{\rho}+\frac{u^2}{2}+gz\right)_{\text{out}}-\dot{m}\left(\frac{p}{\rho}+\frac{u^2}{2}+gz\right)_{\text{in}}=-\dot{W}_{\text{shaft}}-\text{Losses}$$

$$\left(\frac{p}{\rho g}+\frac{u^2}{2g}+z\right)_{\text{out}}-\left(\frac{p}{\rho g}+\frac{u^2}{2g}+z\right)_{\text{in}}=-\frac{\dot{W}_{\text{shaft}}}{\dot{m}g}-\frac{\text{Losses}}{\dot{m}g}$$

Pressure head

Velocity head

Elevation head

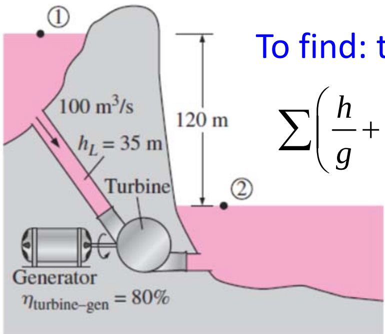
Total Head (indicates total mechanical energy)

shaft work head $\neq 0$ if CV contains pump/turbine

$= -h_L$

Head loss

Power is expressed in terms of length (we call 'head')



To find: turbogenerator power output

$$\sum\left(\frac{h}{g}+\frac{u^2}{2g}+z\right)\rho\vec{u}.\vec{A}=\frac{\dot{Q}-\dot{W}_{\text{shaft}}}{g}-\frac{\text{Losses}}{g}$$

Assumptions: isothermal, negligible change in ambient pressure, kinetic energy

$$\left(\frac{p}{\rho g}+\frac{u^2}{2g}+z\right)_2-\left(\frac{p}{\rho g}+\frac{u^2}{2g}+z\right)_1=-\frac{\dot{W}_{\text{shaft}}}{\dot{m}g}-h_L$$

$$p_1 \approx p_2 \quad u_1 \approx u_2 \quad \Rightarrow \dot{W}_{\text{shaft}}=\dot{m}g\left(z_1-z_2-h_L\right)=\dot{m}g\times 85\text{m}$$

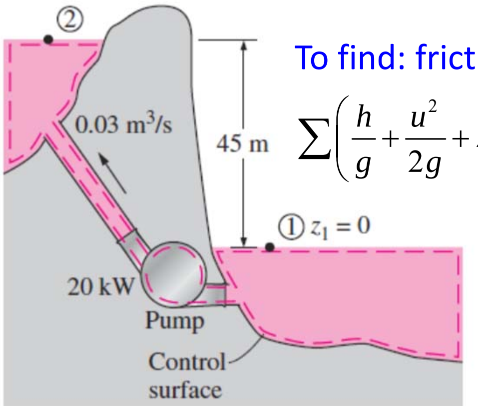
$$\dot{W}_{\text{elec}}=.8\times\dot{W}_{\text{shaft}}$$

$$=\dot{m}g\times 68\text{m}=68\text{MW}$$

Available: 120m

Fluid friction: 35m

Turbogenerator loss: 17m



To find: frictional loss of power

$$\sum\left(\frac{h}{g}+\frac{u^2}{2g}+z\right)\rho\vec{u}.\vec{A}=\frac{\dot{Q}-\dot{W}_{\text{shaft}}}{g}-\frac{\text{Losses}}{g}$$

Assumptions: isothermal, negligible change in kinetic energy

$$\dot{W}_{\text{shaft}} = -20\text{kW}$$

$$h_L = \left(\frac{p}{\rho g}+\frac{u^2}{2g}+z\right)_{\text{in}} - \left(\frac{p}{\rho g}+\frac{u^2}{2g}+z\right)_{\text{out}} - \frac{\dot{W}_{\text{shaft}}}{\dot{m}g} = 23\text{m}$$

$$\dot{m}gh_L = 6.9\text{kW}$$

Input power: 68m   Loss: 23m

Bernoulli Equation and Energy Equation

- Bernoulli Equation is applied along a streamline in a frictionless flow
- In energy Equation velocity and pressure are averaged over control surfaces
- They look similar for adiabatic, isothermal, frictionless flow with no work interaction; in some of these cases any one of them can be used. In such cases energy equation is known as extended Bernoulli Eqn.

Quiz:

A pump is supplying 100kg/s water from Ganga barrage to an open-air reservoir at IITK through a horizontal pipeline of 10km length. The head loss in the pipeline is estimated as 1km. Assuming no loss at pump and motor, find the electrical power requirement