ESO204A, Fluid Mechanics and rate Processes

Conduction Heat Transfer

Chapter 2 of Cengel

General Eq. of heat conduction $\frac{\partial T}{\partial t} = \alpha \nabla^2 T + \frac{\dot{e}_{\mathrm{gen}}}{\rho c}$

Unsteady, no generation $\frac{\partial T}{\partial t} = \alpha \nabla^2 T$

Steady: $\nabla^2 T + \frac{\dot{e}_{\mathrm{gen}}}{k} = 0$

Steady, no generation: $\nabla^2 T = 0$

k, α : thermal conductivity and diffusivity (both being material properties) respectively

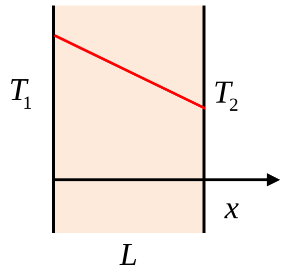
Fourier: 1768-1830

Conduction occurs in all media (solid, liquid gas) through diffusion and lattice vibration

Steady, 1-D Heat Transfer in a Slab $\frac{d^2T}{dx^2}=0 \Rightarrow T=c_1x+c_2$

$T(x=0)=T_1 \quad \Rightarrow c_2=T_1$

$T(x=L)=T_2 \Rightarrow c_1L+c_2=T_2$



$$T=T_1+(T_2-T_1)\frac{x}{L}$$

linear temperature variation

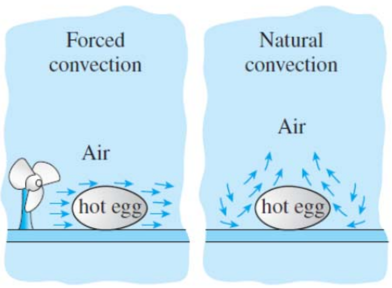
Heat flux $q''_x=-k\frac{dT}{dx}=-k\frac{d}{dx}\left[T_1+(T_2-T_1)\frac{x}{L}\right]=\frac{k(T_1-T_2)}{L}$

Heat transfer rate $\dot{Q}=q''_x\times\text{Area}=\frac{kA(T_1-T_2)}{L}$

Other Heat Transfer Mechanisms

- **Convection**

Heat transfer due to fluid flow

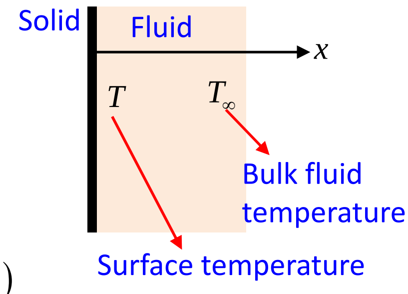


- **Radiation**

Heat transfer due to electromagnetic radiation

Example: solar radiation on earth

Convective heat transfer at a solid fluid interface



Newton's law of cooling

$$q''_x = h\left(T - T_\infty\right) \qquad \dot{Q} = hA\left(T - T_\infty\right)$$

h : convective heat transfer coefficient

Unlike thermal conductivity, **convective heat transfer coefficient is NOT a material property**; it is a function of thermophysical properties and flow variables (velocity, shear)

Radiative Heat Transfer

Every object emits, absorbs, transmits, reflects radiation (at various amounts) all the time; part of radiation manifests as heat

Maximum emission: Blackbody emission,

Stefan-Boltzmann law $\dot{Q} = A\sigma T^4$

Stefan-Boltzmann constant

$5.67\times10^{-8}\,\mathrm{Wm^{-2}K^{-4}}$

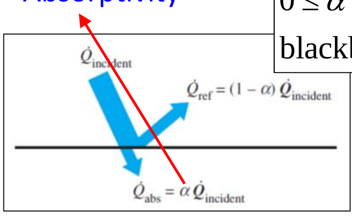
Emissivity

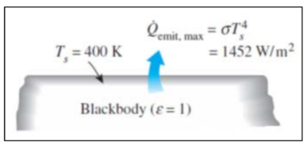
Non-blackbody emission $\dot{Q} = A\varepsilon\sigma T^4 \qquad 0 < \varepsilon \le 1$

Absorptivity

$0 \le \alpha \le 1$

blackbody: $\alpha = 1$





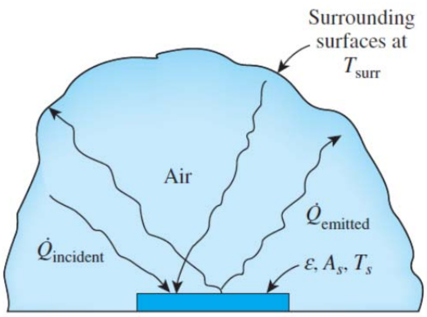
Radiative Heat Transfer

Kirchhoff's law $\varepsilon = \alpha$

$$\dot{Q}_{\text{rad}} = A_s \varepsilon \sigma T_s^4 - A_s \alpha \sigma T_{\text{surr}}^4$$

$$\dot{Q}_{\text{rad}} = A_s \varepsilon \sigma \left(T_s^4 - T_{\text{surr}}^4\right)$$

$$q''_{\text{rad}} = \varepsilon \sigma \left(T_s^4 - T_{\text{surr}}^4\right)$$



At high temperature, radiation dominates over other modes

Following form is sometimes useful

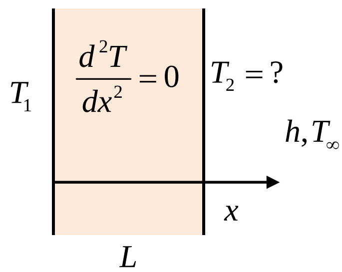
$$\dot{Q}_{\text{rad}} = h_{\text{rad}} A_s \left(T_s - T_{\text{surr}}\right)$$

$$\text{where } h_{\text{rad}} = \frac{\varepsilon \sigma \left(T_s^4 - T_{\text{surr}}^4\right)}{T_s - T_{\text{surr}}} = \varepsilon \sigma \left(T_s + T_{\text{surr}}\right)\left(T_s^2 + T_{\text{surr}}^2\right)$$

In many practical heat transfer applications, all three modes occur simultaneously

Steady, 1-D Heat Transfer in a Slab $T = c_1 x + c_2$

$$T(x=0) = T_1 \quad \Rightarrow c_2 = T_1$$



Net heat flux is zero at $x = L$

$$x = L: -k\frac{dT}{dx} + h\left(T_\infty - T\right) = 0$$

$$\Rightarrow -kc_1 + h\left(T_\infty - c_1 L - c_2\right) = 0$$

$$\Rightarrow c_1 = \frac{h\left(T_\infty - T_1\right)}{k + hL}$$

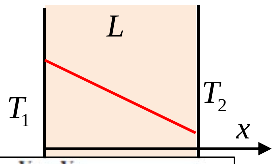
$$T = T_1 + \frac{h\left(T_\infty - T_1\right)}{k + hL} x$$

$$T_2 = T(x = L) = T_1 + \frac{hL\left(T_\infty - T_1\right)}{k + hL}$$

$$\dot{Q} = q''_x \times \text{Area} = \frac{kAh\left(T_1 - T_\infty\right)}{k + hL}$$

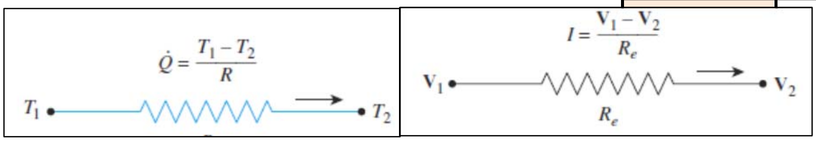
Steady, 1-D Heat Transfer in a Slab: Electrical Analogy

$$\dot{Q} = \frac{kA(T_1 - T_2)}{L} \qquad T_1 - T_2 = \Delta T$$



$$\boxed{\Delta T = \dot{Q}\left(\frac{L}{kA}\right)} \equiv \boxed{\Delta V = IR} \qquad \boxed{R_{\text{cond}} = \frac{L}{kA}}$$

voltage current resistance



Similarly $\dot{Q}_{\text{conv}} = hA\Delta T$ $\qquad \boxed{R_{\text{conv}} = \frac{1}{hA}} \qquad \boxed{R_{\text{rad}} = \frac{1}{h_{\text{rad}}A}}$

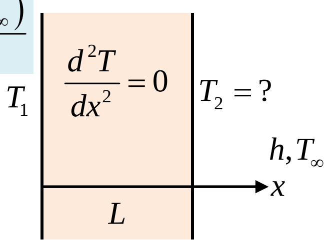
$\dot{Q}_{\text{rad}} = h_{\text{rad}} A \Delta T$

We can now have series and parallel combination as in electrical circuit

Steady, 1-D Heat Transfer in a Slab

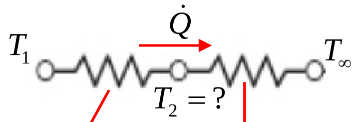
$$\boxed{T = T_1 + \frac{h(T_\infty - T_1)}{k + hL}x}$$

$$\boxed{T_2 = T_1 + \frac{hL(T_\infty - T_1)}{k + hL}} \qquad \boxed{\dot{Q} = \frac{kAh(T_1 - T_\infty)}{k + hL}}$$



Electrical analogy $\qquad \dot{Q} = \frac{T_1 - T_\infty}{L/(kA) + 1/(hA)}$

To find T_2



$$\dot{Q} = \frac{T_1 - T_\infty}{L/(kA) + 1/(hA)} = \frac{T_1 - T_2}{L/(kA)}$$

$$\frac{T_2 - T_1}{T_\infty - T_1} = \frac{L/(kA)}{L/(kA) + 1/(hA)} = \frac{hL}{k + hL}$$

$$R_{\text{cond}} = \frac{L}{kA} \qquad R_{\text{conv}} = \frac{1}{hA}$$