ESO204A, Fluid Mechanics and rate Processes

Conservation Equations: integral formulation

Mass conservation
Momentum conservation
Energy conservation

Chapter 3 of F M White
Chapter 4 of Fox McDonald (uploaded)

Reynolds Transport Theorem:

$$\frac{dB_{sys}}{dt} = \frac{\partial}{\partial t}\int_{CV} \rho\beta d\mathcal{V} + \int_{CS} \rho\beta\left(\vec{u}.\vec{n}\right) dS$$

Mass conservation: $$\frac{\partial}{\partial t}\int_{CV} \rho d\mathcal{V} + \int_{CS} \rho\left(\vec{u}.\vec{n}\right) dS = 0$$

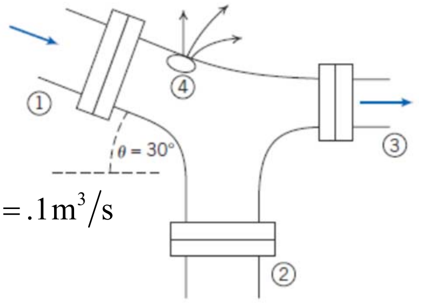
Incompressible flow, non-deformable CV: $$\int_{CS} \left(\vec{u}.\vec{n}\right) dS = 0$$

Steady flow, non-deformable CV: $$\int_{CS} \rho\left(\vec{u}.\vec{n}\right) dS = 0$$

Consider water flow through pipe where

$A_1 = A_2 = .2\,\mathrm{m}^2, A_3 = .15\,\mathrm{m}^2$

$u_1 = 5\,\mathrm{m/s}, u_2 = ?, u_3 = 12\,\mathrm{m/s}, Q_4 = .1\,\mathrm{m}^3/\mathrm{s}$

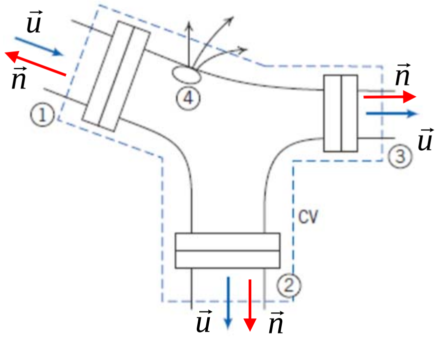


$$\text{now} \int_{CS} (\vec{u}.\vec{n})\, dS = 0 \Rightarrow$$

$$-u_1 A_1 + u_2 A_2 + u_3 A_3 + u_4 A_4 = 0$$

$$\Rightarrow -u_1 A_1 + u_2 A_2 + u_3 A_3 + Q_4 = 0$$

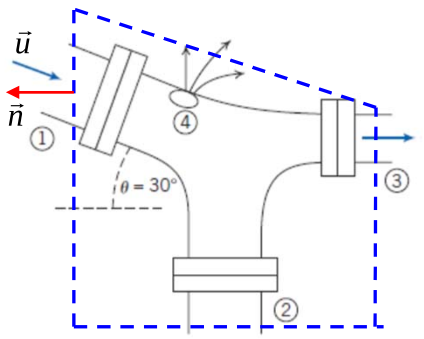
$$\Rightarrow u_2 = -4.5\,\mathrm{m/s}$$



Slightly different CV

$$\dot{m}_1 = \rho u_1 \left(\frac{A_1}{\cos 30^o} \right) .\cos 150^o$$

$$= -\rho u_1 A_1$$



$$\int_{CS} (\vec{u}.\vec{n})\, dS = 0 \Rightarrow$$

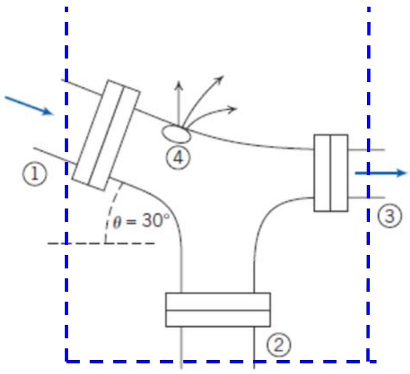
$$-u_1 A_1 + u_2 A_2 + u_3 A_3 + u_4 A_4 = 0$$

$$\Rightarrow u_2 = -4.5\,\mathrm{m/s}$$

It helps if

$\vec{u}$ and $\vec{n}$ are 0^o or 180^o apart

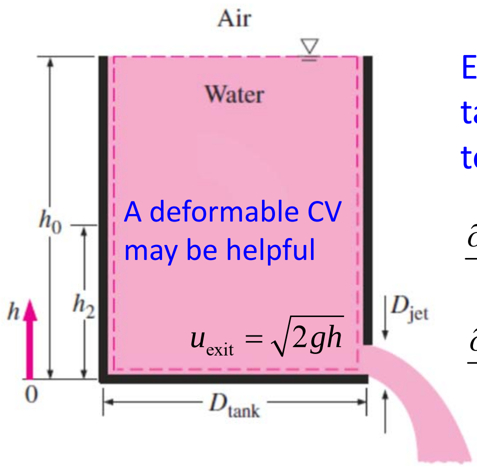
Very large CV (or deformable CV), such that fluid doesn't come out of it



$$\frac{\partial m_{\mathrm{CV}}}{\partial t} + \int_{\mathrm{CS}} \rho(\vec{u}.\vec{n})\,dS = 0$$

$$\frac{\partial m_{\mathrm{CV}}}{\partial t} = \rho Q_4$$

$$Q_4 - u_1 A_1 + u_2 A_2 + u_3 A_3 = 0 \qquad \Rightarrow u_2 = -4.5\,\mathrm{m/s}$$



Evaluate the time it takes for the water level to drop from h_0 to h_2

$$\frac{\partial m_{\mathrm{CV}}}{\partial t} + \int_{\mathrm{CS}} \rho(\vec{u}.\vec{n})\,dS = \frac{dm_{\mathrm{sys}}}{dt} = 0$$

$$\frac{\partial \mathcal{V}_{\mathrm{CV}}}{\partial t} + \int_{\mathrm{CS}} (\vec{u}.\vec{n})\,dS = 0$$

$$\left(\frac{\pi}{4} D_{\mathrm{tank}}^2\right)\frac{dh}{dt} - (uA)_{\mathrm{in}} + (uA)_{\mathrm{out}} = 0 \quad \Rightarrow \left(\frac{\pi}{4} D_{\mathrm{tank}}^2\right)\frac{dh}{dt} = -\sqrt{2gh}\left(\frac{\pi}{4} D_{\mathrm{jet}}^2\right)$$

$$\Rightarrow t = \frac{\sqrt{h_0} - \sqrt{h_2}}{\sqrt{g/2}}\left(\frac{D_{\mathrm{tank}}}{D_{\mathrm{jet}}}\right)^2$$

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Momentum conservation: integral formulation

Very useful for calculation of forces

Chapter 3 of F M White
Chapter 4 of Fox McDonald (uploaded)

Reynolds Transport Theorem:

$$\frac{dB_{\text{sys}}}{dt} = \frac{\partial}{\partial t}\int_{\text{CV}} \rho\beta d\forall + \int_{\text{CS}} \rho\beta\left(\vec{u}.\vec{n}\right)dS$$

For Momentum conservation: $B_{\text{sys}} = m\vec{u} \Rightarrow \beta = \vec{u}$

$$\frac{d\left(m\vec{u}\right)}{dt} = \frac{\partial}{\partial t}\int_{\text{CV}} \rho\vec{u} d\forall + \int_{\text{CS}} \rho\vec{u}\left(\vec{u}.\vec{n}\right)dS$$

Momentum conservation principle: $\dfrac{d\left(m\vec{u}\right)}{dt} = \vec{F}$

Momentum conservation (intgral formulation)

$$\vec{F} = \frac{\partial}{\partial t}\int_{CV} \rho \vec{u}\, d\forall + \int_{CS} \rho \vec{u}\left(\vec{u}.\vec{n}\right) dS$$

$$\vec{F} = \vec{F}_S + \vec{F}_B$$

$\vec{F}_S$: Surface force, all forces acting at the control surface

$\vec{F}_B$: Body forces (gravity, electromagnetic)

Surface forces usually come from pressure, shear and interaction with solid objects/surfaces

$$\vec{F}_S + \vec{F}_B = \frac{\partial}{\partial t}\int_{CV} \rho \vec{u}\, d\forall + \int_{CS} \rho \vec{u}\left(\vec{u}.\vec{n}\right) dS$$

Incompressible flow: $\vec{F}_S + \vec{F}_B = \rho\frac{\partial}{\partial t}\int_{CV} \vec{u}\, d\forall + \rho\int_{CS} \vec{u}\left(\vec{u}.\vec{n}\right) dS$

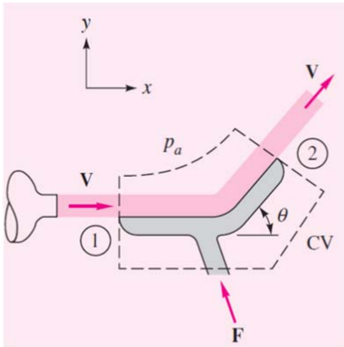
In a non-deformable CV

Steady flow: $\vec{F}_S + \vec{F}_B = \int_{CS} \rho \vec{u}\left(\vec{u}.\vec{n}\right) dS$

Steady, incompressible flow: $\vec{F}_S + \vec{F}_B = \rho\int_{CS} \vec{u}\left(\vec{u}.\vec{n}\right) dS$

Water jet (area A) hits a fixed vane, flow direction changes

Find the force F necessary to hold the vane fixed



Assumption: 1. steady, incompressible flow, 2. flow is in horizontal plane, 3. uniform flow at inlet/exit, 4. $p = p_a$

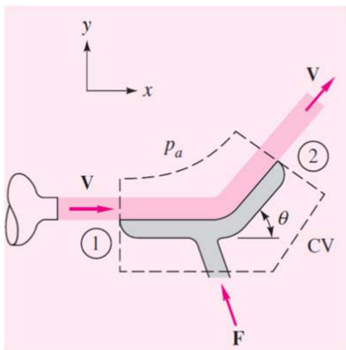
Analysis: $\int_{CS} (\vec{u}.\vec{n})\, dS = 0$ $\quad \vec{F}_S = \rho \int_{CS} \vec{u}\,(\vec{u}.\vec{n})\, dS$

$$\int_{CS} (\vec{u}.\vec{n})\, dS = 0$$

$$u_1 = u_2 = V \quad (\text{since } A = \text{constant})$$

$$\vec{F}_S = \rho \int_{CS} \vec{u}\,(\vec{u}.\vec{n})\, dS = \rho \sum \vec{u}\left(\vec{u}.\vec{A}\right)$$

$$\left(\vec{u}.\vec{A}\right)_1 = -VA = -\left(\vec{u}.\vec{A}\right)_2$$

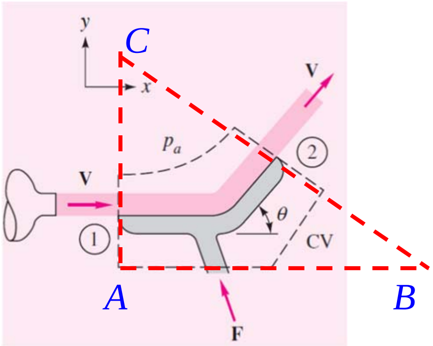


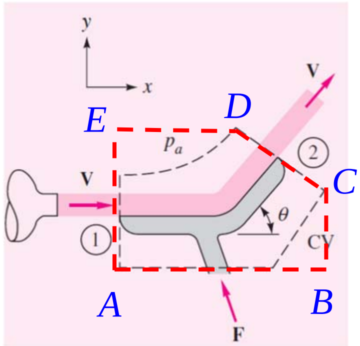
In x direction:

$$F_x + F_{px}\,(\text{pressure forces}) = \rho V \left(\vec{u}.\vec{A}\right)_1 + \rho V \cos\theta \left(\vec{u}.\vec{A}\right)_2$$
$$= -\rho V^2 A + \rho V^2 A \cos\theta$$

$$F_x = -\rho V^2 A + \rho V^2 A \cos\theta$$

$F_{px} = 0$ Since gage pressure is zero everywhere





$$F_{px} = p \times AC - p \times BC \cos\theta$$
$$= 0$$

$$F_{px} = p \times AE - p \times BC$$
$$- p \times CD \cos\theta \ = 0$$

Pressure force always at the CS toward the CV (compressive)