Laminar boundary layer... Continued

$[\,U_0 \equiv U_\infty\,]$

U_∞, U_0

Drag force D

$$\frac{dD}{dx} = b\tau_w$$

$$D(x) = b\int_0^x \tau_w \, dx$$

$$c_f = \frac{\tau_w}{\frac{1}{2}\rho U_\infty^2} = \frac{0.664}{\left(\frac{U_\infty x \rho}{\mu}\right)^{1/2}} \Rightarrow \tau_w = 0.332 \frac{\rho^{1/2}\mu^{1/2}U_0^{1.5}}{x^{1/2}}$$

$$D(x) = b\int 0.332 \frac{\rho^{1/2}\mu^{1/2}U_0^{1.5}}{x^{1/2}}\,dx \, ; \quad \text{(Drag on one-side of the plate)}$$

$$= 0.664\, b\rho^{1/2}\mu^{1/2}U_0^{1.5}x^{1/2}$$

$$C_D = \frac{D(L)}{\frac{1}{2}\rho U_0^2 (bL)} = \frac{(0.664)(b)\rho^{1/2}\mu^{1/2}U_0^{1.5}L^{0.5}}{\frac{1}{2}\rho U_0^2 bL} = \frac{1.328}{Re_L^{0.5}} \, ; \quad Re_L = \frac{\rho U_0 L}{\mu}$$

$$= 2c_f(L)$$

$$\frac{\theta(x)}{x}$$

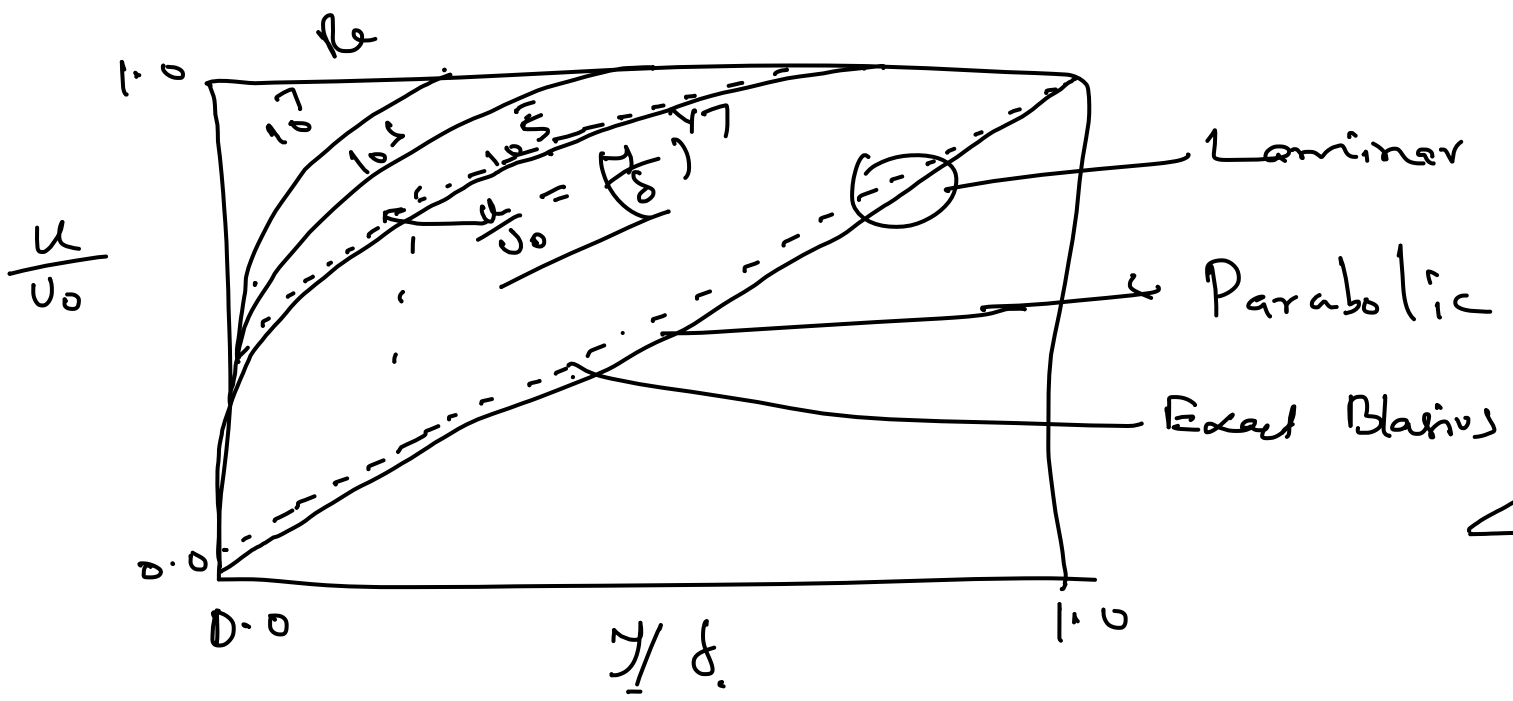
$$C_D = \frac{D(L)}{\frac{1}{2}\rho U_0^2 bL} \, ; \quad C_D(x) = \frac{D(x)}{\frac{1}{2}\rho U_0^2 bx}$$

Everywhere $(U_0 \equiv U_\infty)$

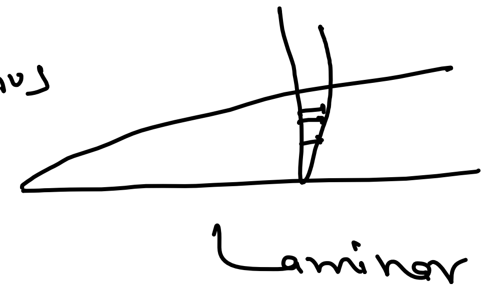
$$D(x) = \rho b \int_0^{\delta(x)} u(U_0 - u)\,dy$$

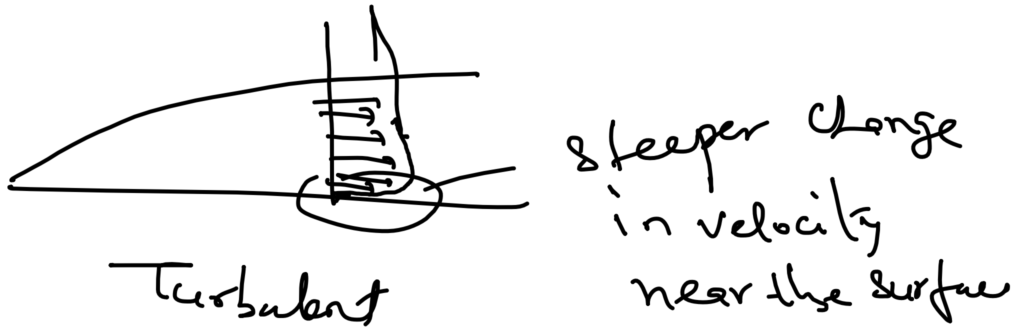
$$\Rightarrow C_D = \frac{2}{x}\int \frac{u}{U_0}\left(1 - \frac{u}{U_0}\right)dy = \frac{2}{x}\theta(x) = \frac{1.328}{Re_x^{1/2}}$$

$$\boxed{\frac{\theta(x)}{x} = \frac{0.664}{Re_x^{1/2}}} \, ; \quad Re_x = \left(\frac{\rho U_0 x}{\mu}\right)$$



$$\underline{f(\eta)} = \frac{u}{U_0}$$





one-seventh law profile for turbulent flow → (empirical) $\frac{u}{U_0} = \left(\frac{y}{\delta}\right)^{1/7}$; ($Re \lesssim 10^5$)

Turbulent boundary layer theory

Logarithmic velocity profile (valid all Re)

$$\frac{u}{u^*} = \frac{1}{k} \ln \frac{y u^*}{\nu} + B$$

$\underline{k = 0.41 \;,\; B = 5.0}$

δ ?
c_f ?
c_D ?

at $y = \delta$ $\quad u = U_0$

$$\left(\frac{U_0}{u^*}\right) = \frac{1}{k} \ln\left(\frac{\delta u^*}{\nu}\right) + B$$

$$c_f = \frac{\tau_w}{\frac{1}{2}\rho U_0^2} = \frac{2 u^{*2}}{U_0^2} \Rightarrow \quad \frac{U_0}{u^*} = \left(\frac{2}{c_f}\right)^{1/2} \Rightarrow u^* = \left(\frac{c_f}{2}\right)^{1/2} U_0$$

$$\frac{\delta u^*}{\nu} = \frac{\delta}{\nu}\left(\frac{c_f}{2}\right)^{1/2} U_0 = \left(\frac{c_f}{2}\right)^{1/2} (Re_\delta)$$

$$\boxed{\left(\frac{2}{c_f}\right)^{1/2} \approx 2.44 \ln \left(Re_\delta\right)\left(\frac{c_f}{2}\right)^{1/2} + 5.0}$$ — eq (i)

cumbersome

$\Rightarrow c_f = c_f(\underline{\delta})$, we need $\underline{c_f} = f(Re_x)$

find $\underline{\delta = f(Re_x)}$

numerical values from eq (i)

Re_δ	10^4	10^5	10^6	10^7
c_f	0.00493	0.00315	0.00217	0.00158

$\longrightarrow$ fitted $\Rightarrow$ $c_f = \dfrac{0.02}{Re_\delta^{1/6}}$

$\dfrac{\delta(x)}{x}$

$$\tau_w(x) = \rho U_0^2 \frac{d\theta}{dx}$$

$$\Rightarrow \left(\frac{2\tau_w}{\rho U_0^2}\right) = \boxed{2\frac{d\theta}{dx} = c_f}$$

$$\theta = \int_0^\delta \frac{u}{U_0}\left(1 - \frac{u}{U_0}\right) dy = \theta(\delta)$$

$$\frac{u}{U_0} = \left(\frac{y}{\delta}\right)^{1/7}$$

$$\theta = \int \left(\frac{y}{\delta}\right)^{1/7}\left(1 - \left(\frac{y}{\delta}\right)^{1/7}\right) dy = \frac{7}{72}\delta.$$

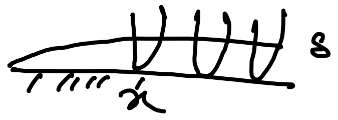
$$\frac{0.02}{Re_\delta^{1/6}} = 2\left(\frac{7}{72}\right)\frac{d\delta}{dx} \; ; \quad Re_\delta = \frac{\rho U_0 \delta}{\mu} \Rightarrow$$

$$\Rightarrow Re_\delta = 0.16\, Re_x^{6/7}$$

$$\frac{Re_\delta}{Re_x} = 0.16\frac{Re_x^{6/7}}{Re_x} \Rightarrow \boxed{\frac{\delta}{x} = \frac{0.16}{Re_x^{1/7}}}$$

$\delta \propto x^{6/7}$ (Turbulent) $\Rightarrow$ more rapid growth boundary

$\delta \propto x^{1/2}$ (Laminar)



c_f

$$c_f = \frac{0.02}{Re_\delta^{1/6}} \; ; \quad \frac{\delta}{x} = \frac{0.16}{Re_x^{1/7}} \Rightarrow \boxed{c_f = \frac{0.027}{Re_x^{1/7}}}.$$

$\underline{\underline{C_D}}$

$$C_D = \frac{D}{\frac{1}{2}\rho U_0^2 b L}$$

$$D = \int_{L} \tau_x b \, dx$$

$$C_D = \frac{\int_0^L |\tau_w| dx}{\frac{1}{2}\rho U_0^2 L} = \frac{\int_0^L c_f \, dx}{L} \quad ; \quad c_f = \frac{0.027}{Re_x^{1/7}}$$

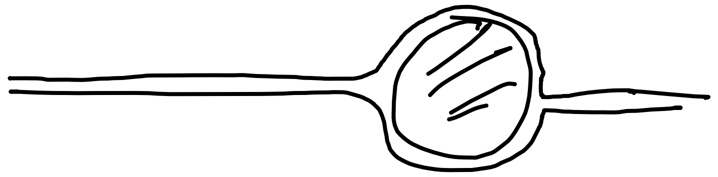
$$\Rightarrow C_D = \frac{0.031}{Re_L^{1/7}} = \frac{7}{6} c_f(L) \quad \text{base on th velocity profile assumed}$$

$$C_D = 2 c_f$$

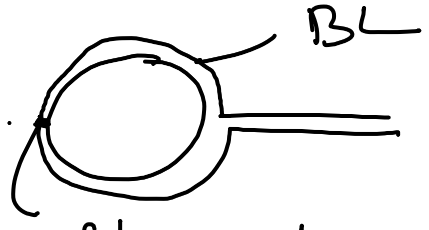
Boundary layer separation

- BL separation is important in aerodynamics
- BL separation happens for flow past blunt bodies (e.g. sphere, cylinder, Air wing etc)

→ BL separation in case of laminar flow



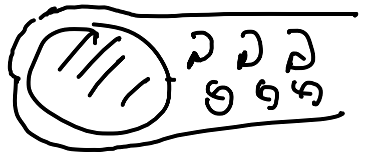
ideal



ideal

separation of BL

Real case



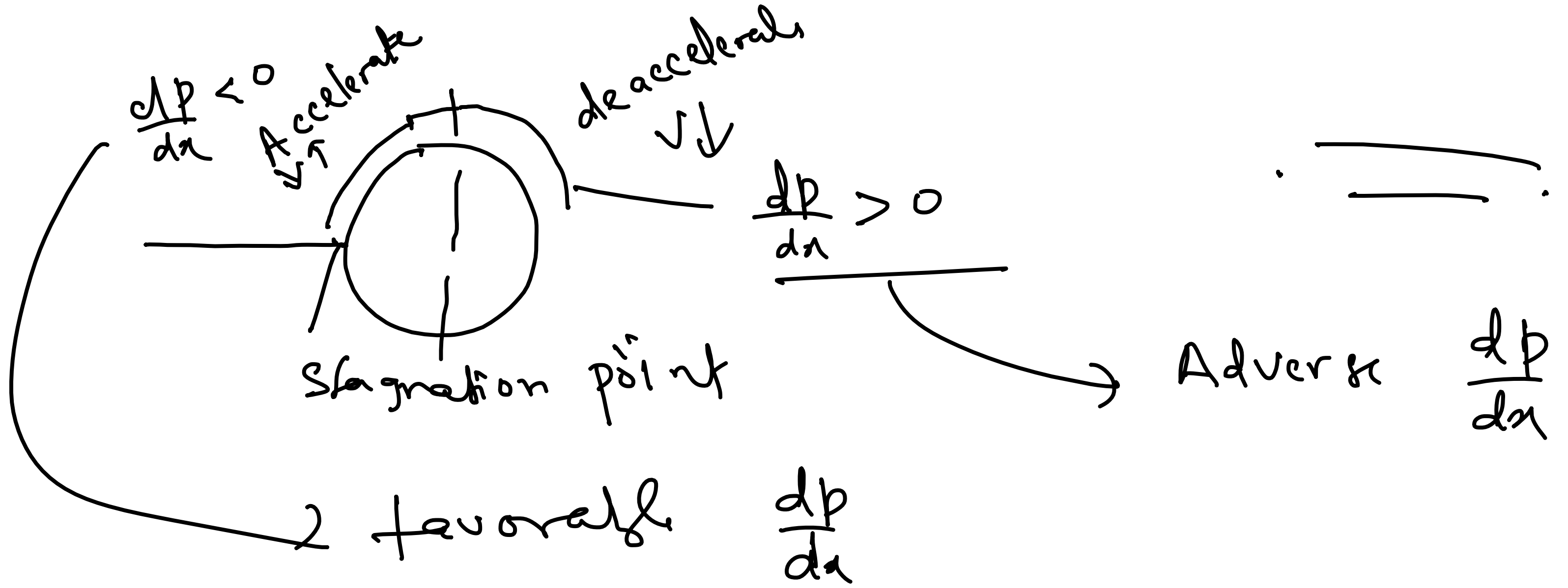
How does BL Separation happen?

→ first, we take inviside around a sphere → Euler eqn

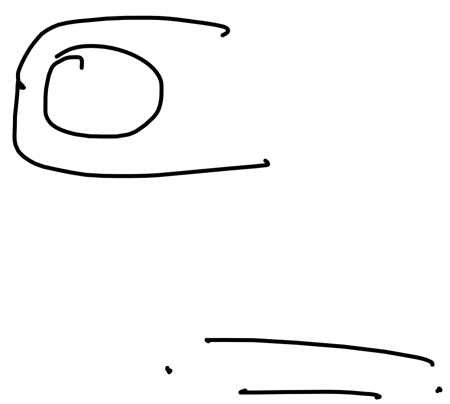
$$\underline{\underline{\rho \frac{dV}{dt}}} = \underline{\underline{\rho g}} - \nabla p$$

$\frac{dV}{dt} = +ve$ for front end

$\nabla p = -ve$ $\quad$ $p \downarrow$



invisid flow



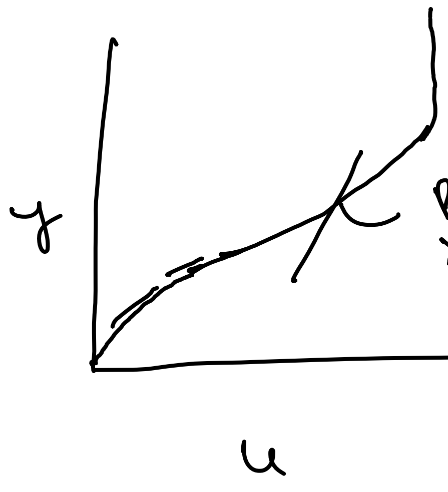
— Prandtl has shown that BL separation happens due to excessive loss of momentum due to skin friction and adverse $\frac{dp}{dx}$

Why the separation happens?

BL eqn $\quad \rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{dp}{dx} + \mu\frac{\partial^2 u}{\partial y^2}$

At the surface $\quad u = v = 0$

$$\mu \frac{\partial^2 u}{\partial y^2}\bigg|_{y=0} = \left(\frac{dp}{dx}\right) \leftarrow \text{base on sign of } \frac{dp}{dx}$$



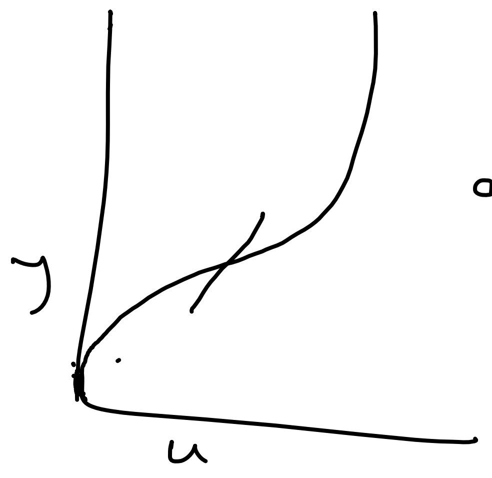
point of inflection

$\frac{du}{dy} < 0$

$\frac{dp}{dx} > 0$ ✓ $= a_1$

> 0

$\frac{\partial^2 u}{\partial y^2}\Big|_{y=0} > 0$



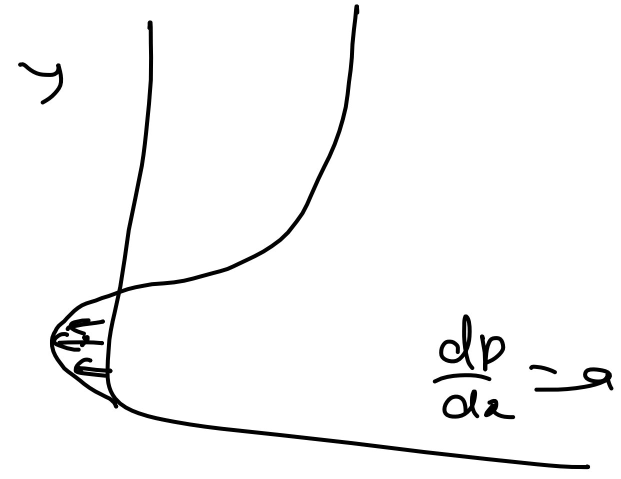
$\frac{dp}{dx} = a_2 > 0$

$a_2 > a_1$

$\frac{\partial^2 u}{\partial y^2} \;\; a_2 > a_1$

Critical Separation

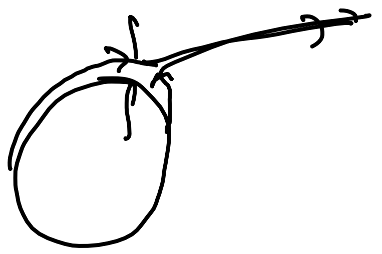
$\frac{du}{dy}\Big|_{y=0} = 0$



$\frac{dp}{dx} = a_3$

$a_3 > a_2 > a_1$

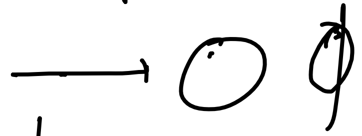
full Separation



Drag Coefficient $\quad C_D = \frac{\text{drag}}{\frac{1}{2}\rho U_0^2 A}$

A = characteristic Area

= frontal Area for thick, blunt bodies, (e.g. sphere or cylinder



= Planform Area → the body area as seen from Above

suitable wide flat bodies (e.g. flat Plate)

$A = bL$