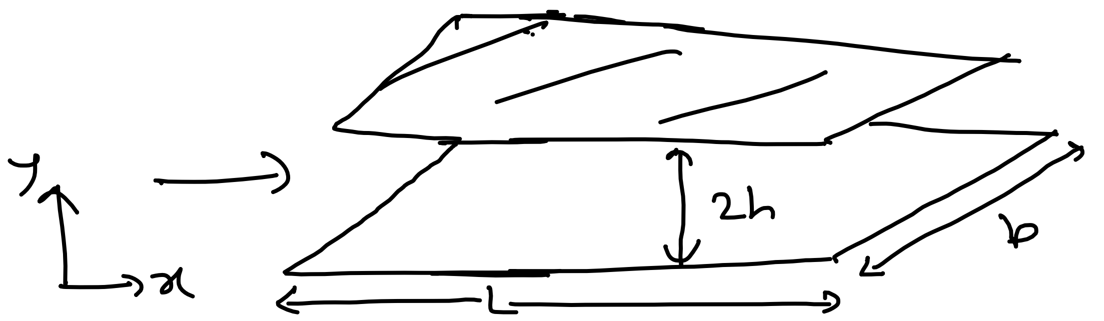
- For laminar flow in a non circular duct, calculating friction factor using $f = \frac{64}{Re_d}$ gives large error. But good thing is that we can analytically derive relationship between f & Re_d for any geometry for laminar flow.

Example: Flow between parallel plates



$b \to \infty$

$$D_h = \frac{4A}{P} = \lim_{b\to\infty} \frac{(4)(2h)b}{2b + 4h} = \frac{4\ 2hb}{2b} = \underline{\underline{4h}}$$

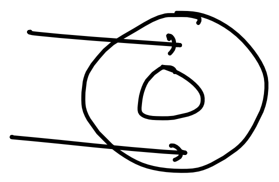
$$u = u_{max}\left(1 - \frac{y^2}{h^2}\right)$$ (by Solving Navier-Stokes) (We have done it in earlier Lecture)

$$u_{max} = \frac{h^2}{2\mu}\frac{\Delta p}{L} \quad ; \quad \left(-\frac{dp}{dx} = \frac{\Delta p}{L}\right)$$

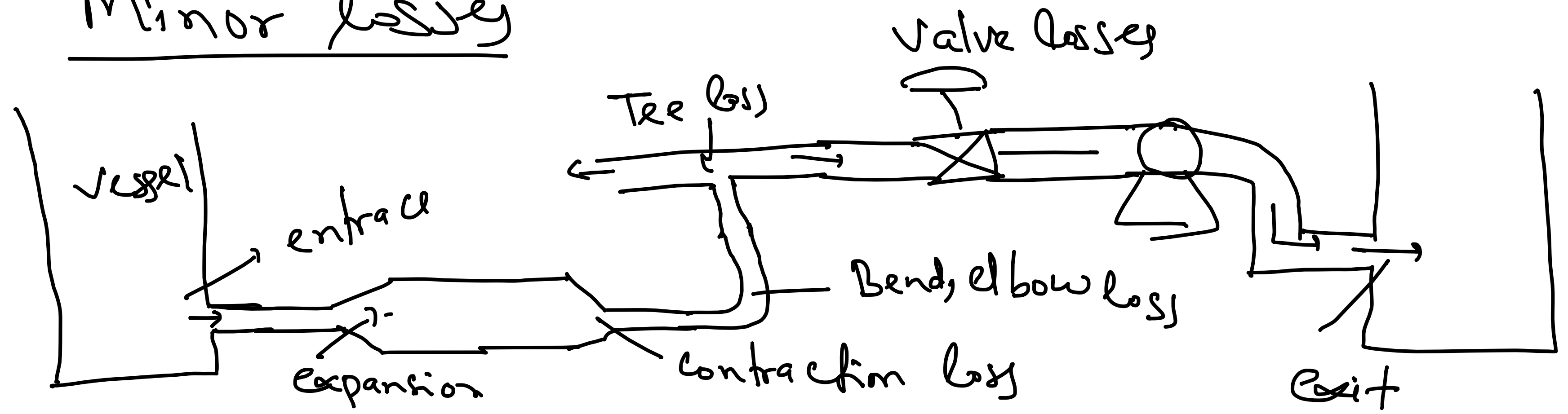
$$V = \frac{\int_{-h}^{h} u\, b\, dy}{2bh} = \frac{2}{3} u_{max}$$

$$\tau_w = \mu \left|\frac{\partial u}{\partial y}\right|_{y=h} = \frac{2\mu\, u_{max}}{h} = \frac{3V\mu}{h}$$

$$f = \frac{\tau_w}{\frac{1}{8}\rho V^2} = \frac{3V\mu}{h\,\frac{1}{8}\rho V^2} = \frac{24\mu}{h\rho V} = \frac{96\mu}{(4h)(\rho V)} = \frac{96\mu}{D_h \rho V} = \left(\underline{\underline{\frac{96}{Re_{D_h}}}}\right) \neq \frac{64}{Re_{D_h}}$$



→ Mechanical → Minor losses



fittings → minor loss (

→ How to calculate minor losses → empirical method

$$h_m = (\text{head loss in fitting}) = (\underline{\underline{K}})\left(\frac{V^2}{2g}\right)$$

$$h_{tot} = (\text{fittings} + \text{pipe or duct}) = \underbrace{\left[\frac{V_1^2}{2g}\left(\frac{f_1 L_1}{d_1} + \Sigma K_i\right)\right]}_{\text{for a constant diameter}} + \frac{V_2^2}{2g}\left(\frac{f_2 L_2}{d_2} + \Sigma K_i\right) \cdots$$

↳ energy eqn h_f

External flow ← How to calculate drag force on a Solid

→ Brute force → N-S → $\vec{V}$ → τ_{ij} → $\vec{F}$ ← (solving N-S may be difficult)

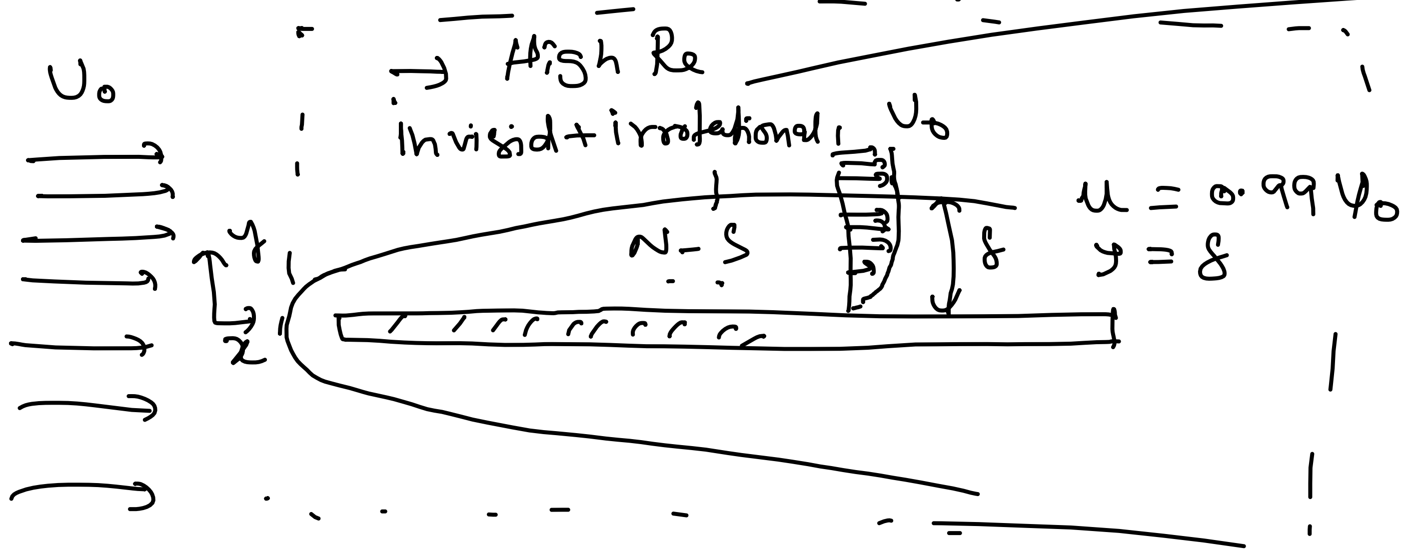
→ make some assumptions → simplify eqn to be solved → may need computer

Boundary layer theory (for laminar)

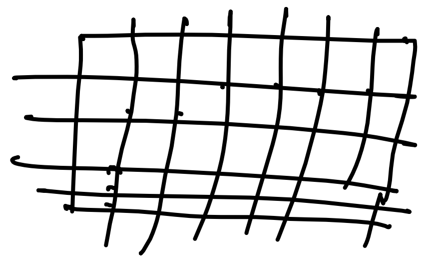
for example ⇒ flow around a flat plate

→ Re → low

→ $\delta \cong L$



$$\left.\begin{aligned} \nabla^2 \phi &= 0 \\ \nabla^2 \psi &= 0 \end{aligned}\right]$$



For the boundary layer (incompressible, laminar, Newtonian)

$u_{xx} = \frac{\partial^2 u}{\partial x^2}$

continuity $\quad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$

momentum $\quad \rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$

$\rho\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = -\frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right)$

$A u_{xx} + B u_{xy} + C u_{yy} = D$

$B^2 - 4AC < 0 \Rightarrow$ Elliptic

$= 0 \Rightarrow$ Parabolic

$> 0 \Rightarrow$ Hyperbolic

$A > 0,\ B = 0,\ C > 0$

$B^2 - 4AC \le 0 \Rightarrow$ Elliptic

⇒ must be solved simulteneosly for entir flow field

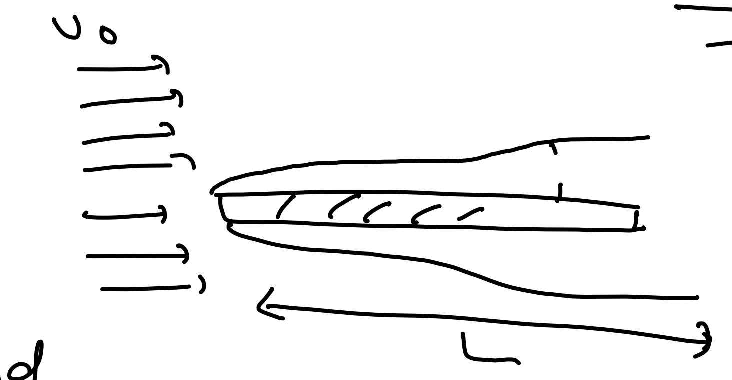
Difficult to solve

$Re \rightarrow$ High (But laminar flow)

↳ (i) $\delta << L$

(ii) eq^{n}s are simplified

(iii) Assumption outer sol^n to be inviscid & irrotation $\Rightarrow$ more valid



$\delta << L$

eq^n order of magnitude analysis

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad u \approx U_0,\quad x \approx L,\quad y \approx \delta$$

$$O\left(\frac{U_0}{L}\right) \approx O\left(\frac{v}{\delta}\right) \qquad O \equiv \text{means order}$$

$$O(v) = O\left(\frac{U_0 \delta}{L}\right)$$

x- momentum

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} + \mu\left(\cancel{\frac{\partial^2 u}{\partial x^2}} + \frac{\partial^2 u}{\partial y^2}\right)$$

smaller Large

order $\longrightarrow$

$$\rho\left(\frac{U_0 U_0}{L} + \frac{U_0\delta}{L}\frac{U_0}{\delta}\right) = O\left(\frac{\partial p}{\partial x}\right) + O\left[\mu\frac{U_0}{L^2} + \mu\frac{U_0}{\delta^2}\right]$$

$$O\left(\frac{\rho U_0^2}{L}\right) = O\left(\mu\frac{U_0}{\delta^2}\right)$$

$$O\left(\frac{\partial p}{\partial x}\right) = O\left(\frac{U_0^2}{L}\right)$$

$$\delta^2 = \frac{\mu L}{\rho U_0}$$

$$\frac{\delta^2}{L^2} = \left(\frac{\mu}{\rho U_0 L}\right) \Rightarrow \frac{\delta}{L} = O\sqrt{\frac{1}{Re}} \leftarrow \text{High } Re \rightarrow \delta \rightarrow \text{Small}$$

y-momentum $\rho\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)$

order of magnitude $\rho\left(U_0\frac{U_0\delta}{L}+\frac{U_0\delta}{L}\frac{U_0\delta}{L\delta}\right)=O\left(\frac{\partial p}{\partial y}\right)+\mu\left(\cancel{\frac{U_0\delta}{L^2 L}}+\frac{U_0\delta}{L\delta^2}\right)$

$$\frac{U_0^2\delta}{L}\;\overset{=}{\sim}\;\frac{U_0}{(L\delta)}$$

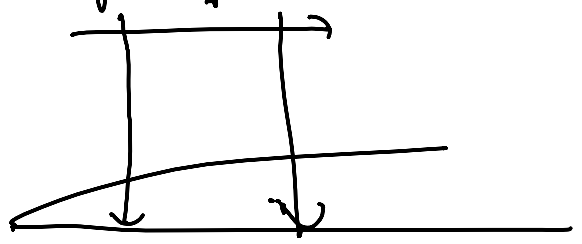
$$\delta=O\sqrt{\frac{1}{Re_L}}$$

$O\left(\frac{U_0^2}{L}\right)>>O\left(\frac{U_0^2\delta}{L}\right)\Rightarrow$ y-momentum eqn is negligible

$O\left(\frac{\partial p}{\partial y}\right)=\frac{U_0^2\delta}{L}\longrightarrow\quad\frac{\partial p}{\partial y}<<\frac{\partial p}{\partial x}$

$O\left(\frac{\partial p}{\partial x}\right)=\frac{U_0^2}{L}\qquad p\neq p(y)$

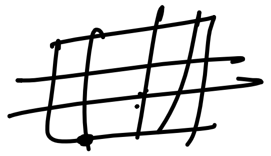
$p=p(x)$



Every $\quad p+\frac{1}{2}\rho U_0^2=0\,($

$$\frac{dp}{dx}=-\rho U_0\frac{dU_0}{dx}$$

eqn
$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$
$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=+\rho U_0\frac{dU_0}{dx}+\mu\left(\frac{d^2 u}{\partial y^2}\right)$$



$A\,u_{xx}+B\,u_{xy}+C\,u_{yy}=D$

$A=0,\ B=0\quad C>0$

$B^2-4AC=0-0=0\Rightarrow$ Parabolic

$\Rightarrow$ Parabolic are solved numerically by beggining at the leading edge and marching downstream