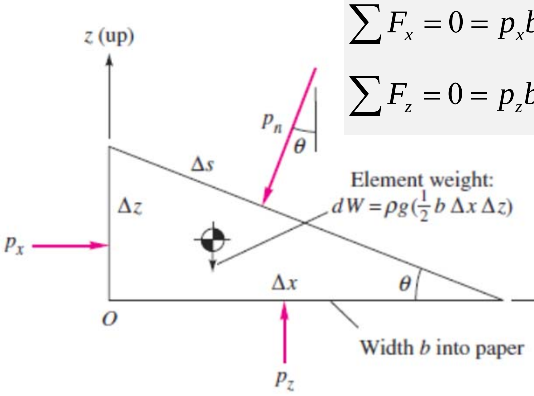
## Static equilibrium of a fluid element




$$\sum F_x = 0 = p_x b\Delta z - p_n b\Delta s \sin\theta$$

$$\sum F_z = 0 = p_z b\Delta x - p_n b\Delta s \cos\theta - \frac{1}{2}\rho g b\Delta x\Delta z$$

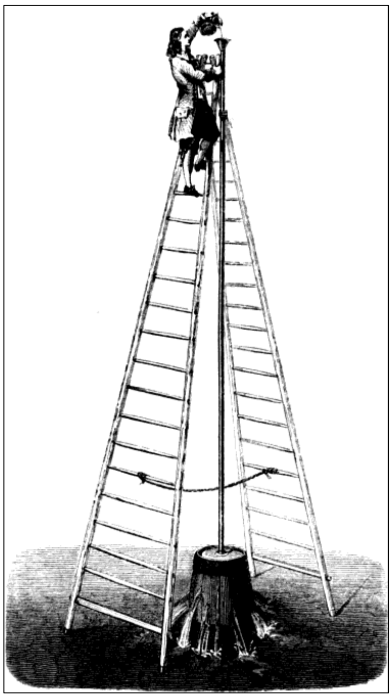
$$\sin\theta = \frac{\Delta z}{\Delta s}; \cos\theta = \frac{\Delta x}{\Delta s}$$

$$p_n = p_x = p_z - \frac{1}{2}\rho g\Delta z$$

For an infinitesimal fluid element $\Delta z \to 0 \Rightarrow p_n = p_x = p_z$

**Pressure at a point is a scalar**

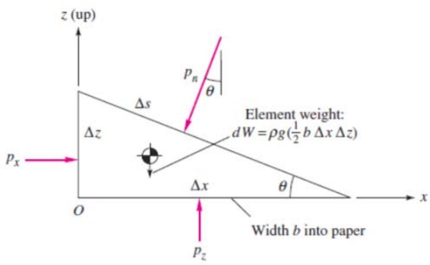
**(direction-independent)**



Blaise Pascal (1623-1662)

Pascal's 'Barrel-Buster' experiment (~1646)

## Static equilibrium of a fluid element




Body force

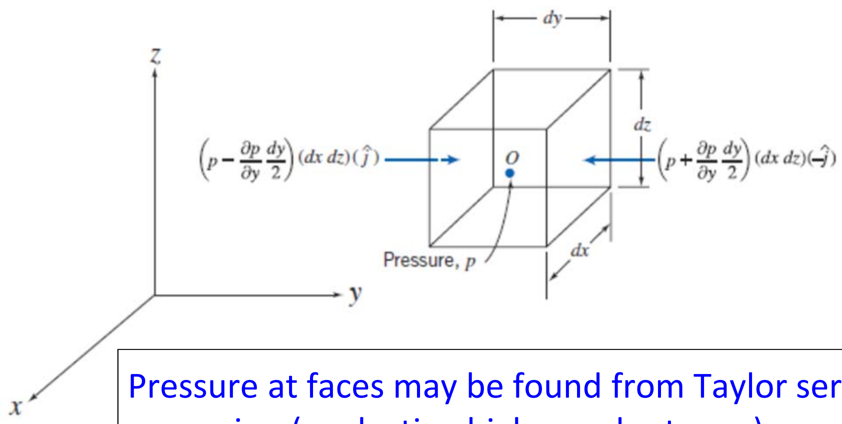
Surface force

$$p_x(b\Delta z) - p_n(b\Delta s\sin\theta) = 0 = p_z(b\Delta x) - p_n(b\Delta s\cos\theta) - \rho g\left(\frac{1}{2}b\Delta x\Delta z\right)$$

Static equilibrium: $\Sigma$(surface forces + body forces) = 0

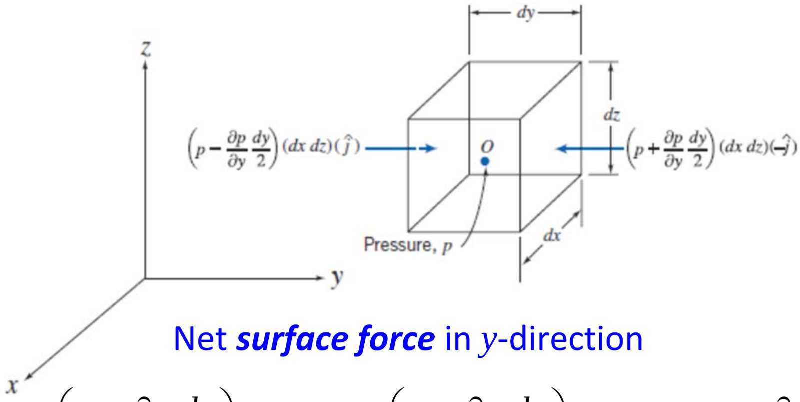
As the size of the fluid element reduces, body force drops faster than surface force

## Governing Equation of fluid statics




Pressure at faces may be found from Taylor series expansion (neglecting higher order terms)

$$p_{y\pm dy/2} = p \pm \frac{\partial p}{\partial y}\frac{dy}{2} + \text{...higher order terms}$$




Net ***surface force*** in $y$-direction

$$d\vec{F}_{sy}=\left(p-\frac{\partial p}{\partial y}\frac{dy}{2}\right)dxdz\left(\hat{j}\right)+\left(p+\frac{\partial p}{\partial y}\frac{dy}{2}\right)dxdz\left(-\hat{j}\right)=-\frac{\partial p}{\partial y}\hat{j}d\forall$$

**Total surface force, considering all three directions:**

$$d\vec{F}_s=-\left(\frac{\partial p}{\partial y}\hat{i}+\frac{\partial p}{\partial y}\hat{j}+\frac{\partial p}{\partial y}\hat{k}\right)d\forall=-\nabla p d\forall$$

***Body force*** (weight) of the fluid element $\quad d\vec{F}_B=\rho\vec{g}d\forall$

Total force on the fluid element

$$d\vec{F}=d\vec{F}_S+d\vec{F}_B=-\nabla p d\forall+\rho\vec{g}d\forall$$

For a static fluid element $\quad d\vec{F}=0\Rightarrow-\nabla p+\rho\vec{g}=0$

**Governing Equation of fluid statics** $\quad \nabla p=\rho\vec{g}$

**Note:** The above concept can be extended for rigid body motion as well, but we will skip that discussion here (interested students may see the texts)

Fluid Statics

# Manometry

Principles of fluid statics used in pressure measurement

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## Governing Equation of fluid statics: $\nabla p = \rho \vec{g}$

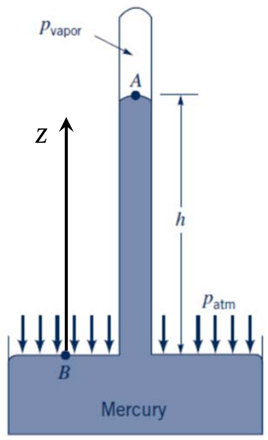
$$x\text{-direction}: \frac{\partial p}{\partial x} = \rho g_x \quad y\text{-direction}: \frac{\partial p}{\partial y} = \rho g_y \quad z\text{-direction}: \frac{\partial p}{\partial z} = \rho g_z$$

Sometimes we align one axis (let's say $z$) vertically upward (against gravity), such that $g_x = g_y = 0; g_z = -g$

$$\Rightarrow \frac{\partial p}{\partial x} = \frac{\partial p}{\partial y} = 0 \Rightarrow p = p(z) \qquad \frac{\partial p}{\partial z} = -\rho g = -\gamma$$

Integrating $p(z=h) = p(z=0) - \gamma h$ for constant $\rho$

Above Eq. shows that measurement of length can help us to know the pressure difference; briefly, this is the **principle of manometry**




$$p(z=h)=p(z=0)-\rho gh$$

## Barometer

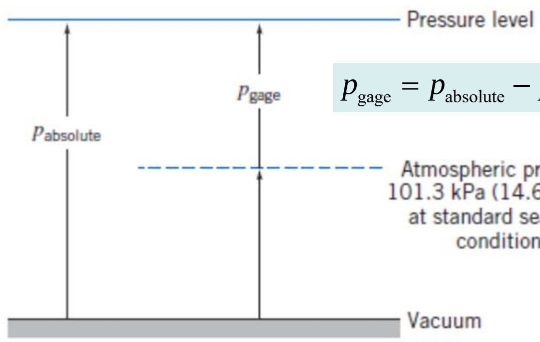
$$p_{vap} = p_{atm} - \rho gh$$

$$p_{vap} \ll p_{atm} \Rightarrow p_{atm} = \rho gh$$

Barometer is the primitive form of manometer, more involved designs are there

Instead of mercury vapor pressure (or vacuum), most gages use ambient pressure as base pressure

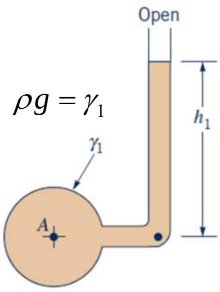
## Absolute and gage pressures




$$p_{gage} = p_{absolute} - p_{atmospheric}$$

Pressure gages usually show gage pressure

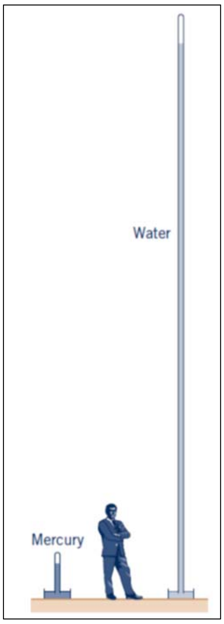
## Simplest manometer (piezometer)




$\rho g = \gamma_1$

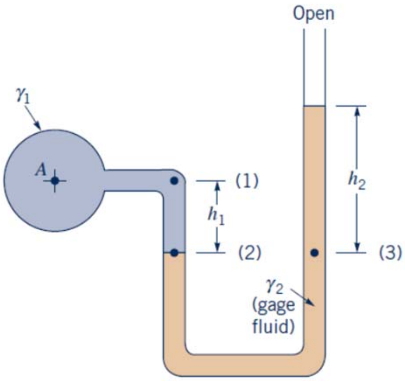
$$p_{\text{A, absolute}} = p_{\text{atm}} + \rho g h_1$$

$$p_{\text{A, gage}} = \rho g h_1$$




Piezometer contains the same fluid as the container A, this could be a serious problem for practical applications

## U-tube manometer




Analysis key: start from one end !!

$$p_{\text{A, absolute}} + \gamma_1 h_1 - \gamma_2 h_2 = p_{\text{atm}}$$

$$p_{\text{A, gage}} = \gamma_2 h_2 - \gamma_1 h_1$$

To obtain gage pressure directly, put atmospheric pressure to be zero

Several other manometer types will be discussed in tutorials/discussions