Q1) A velocity field is given by:

$$u = y - 1$$
$$v = y - 2.$$

Where, u and v are in m/s and x and y are in meters.

Get an expression for the streamlines.
Plot the streamline that passes through the point (x,y) = (4,3) for the domain $-\infty < x < \infty$ showing the flow direction.
Discuss the flow at x = 0.
Compare this streamline with the pathline through the point (x,y) = (4,3). **[15]**

$u = y - 1$, $v = y - 2$ where the streamlines are obtained from

$$\frac{dy}{dx} = \frac{v}{u} = \frac{y-2}{y-1}$$

or $\displaystyle\int \frac{(y-1)}{(y-2)}\,dy = \int dx$ or $\displaystyle\int \frac{y\,dy}{(y-2)} - \int \frac{dy}{(y-2)} = x + \tilde{C}$, where $\tilde{C}$ is a constant.

From integral tables:

$$\int \frac{y\,dy}{(y-2)} = y - 2 + 2\ln(y-2) \quad \text{and} \quad \int \frac{dy}{(y-2)} = \ln(y-2)$$

Thus, the streamlines are given by

$$y - 2 + 2\ln(y-2) - \ln(y-2) = x + \tilde{C}$$

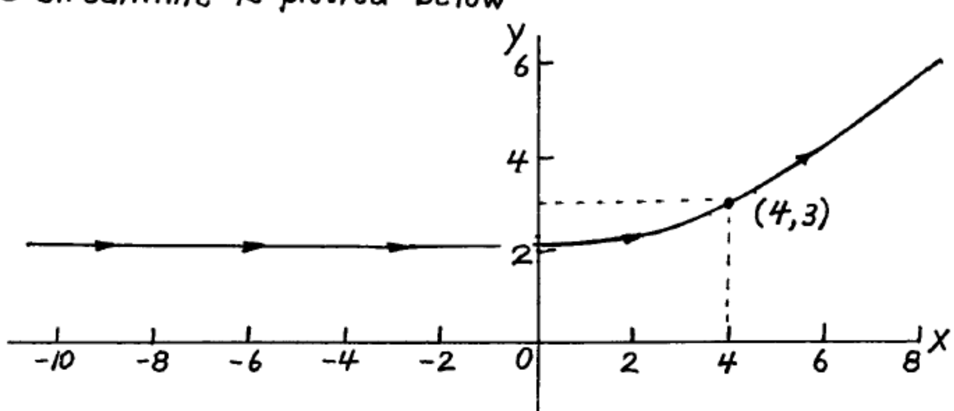
or

$$y + \ln(y-2) = x + C \text{ , where } C \text{ is a constant} \qquad (1)$$

For the streamline that passes through $x = 4$ and $y = 3$, the value of C is found from Eq. (1) as:

$3 + \ln(3-2) = 4 + C$ or $C = -1$ Thus, $\underline{\underline{x = y + \ln(y-2) + 1}}$

This streamline is plotted below:

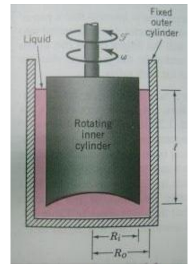


Note: As $x \rightarrow -\infty$, $y \rightarrow 2$. Also, since $u = y - 1$, with $y \geq 2$ anywhere on this streamline, it follows that $u > 0$. The flow is from left to right.

At x = 0, streamlines do not exist for y ≤ 2.0. For y > 2.0, the velocity field will be a function of y as per the velocity components. The value of the constant C will also depend on the specific y and will vary from streamline to streamline.

Since the flow is steady, streamlines = pathlines.

Q2) The viscosity of liquids can be measured through the use of a rotating cylinder viscometer (as shown in the figure). In this device the outer cylinder is fixed and the inner cylinder is rotated with an angular velocity, $\boldsymbol{\omega}$. The torque $\boldsymbol{\tau}$ required to develop $\boldsymbol{\omega}$ is measured and the viscosity is calculated from these two measurements. Develop an equation relating $\boldsymbol{\mu}$, $\boldsymbol{\omega}$, $\boldsymbol{\tau}$, $\boldsymbol{l}$, $\boldsymbol{R_o}$ and $\boldsymbol{R_i}$. Neglect the end effects and assume the velocity distribution in the gap is linear. **[10]**



Torque, $d\mathcal{T}$, due to shearing stress on inner cylinder is equal to

$$d\mathcal{T} = R_i\, \tau\, dA$$

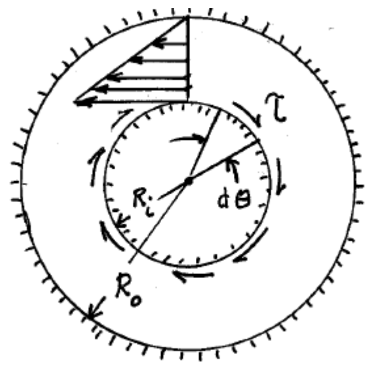
where $dA = (R_i\, d\theta)\,\ell$. Thus,

$$d\mathcal{T} = R_i^2\, \ell\, \tau\, d\theta$$

and torque required to rotate inner cylinder is

$$\mathcal{T} = R_i^2\, \ell\, \tau \int_0^{2\pi} d\theta$$

$$= 2\pi R_i^2\, \ell\, \tau$$



top view

($\ell \sim$ cylinder length)

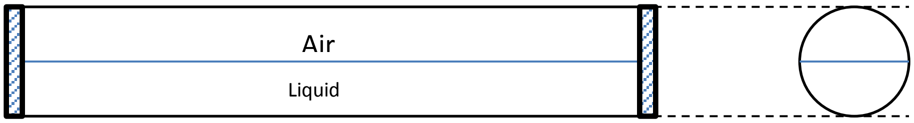
For a linear velocity distribution in the gap

$$\tau = \mu\, \frac{R_i\, \omega}{R_o - R_i}$$

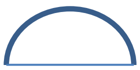
so that

$$\underline{\underline{\mathcal{T} = \frac{2\pi R_i^3\, \ell\, \mu\, \omega}{R_o - R_i}}}$$

Q3) A horizontal 2 m diameter conduit is half-filled with a liquid (SG = 1.6) and is capped at both ends with plane vertical surfaces. The air pressure in the conduit above the liquid surface is 150 kPa. Determine the resultant force of the fluid acting on one of the end caps, and locate this force relative to the bottom of the conduit. Use $g = 9.81\ m/s^2$ and $\rho_{water} = 1000\ kg/m^3$. **[15]**

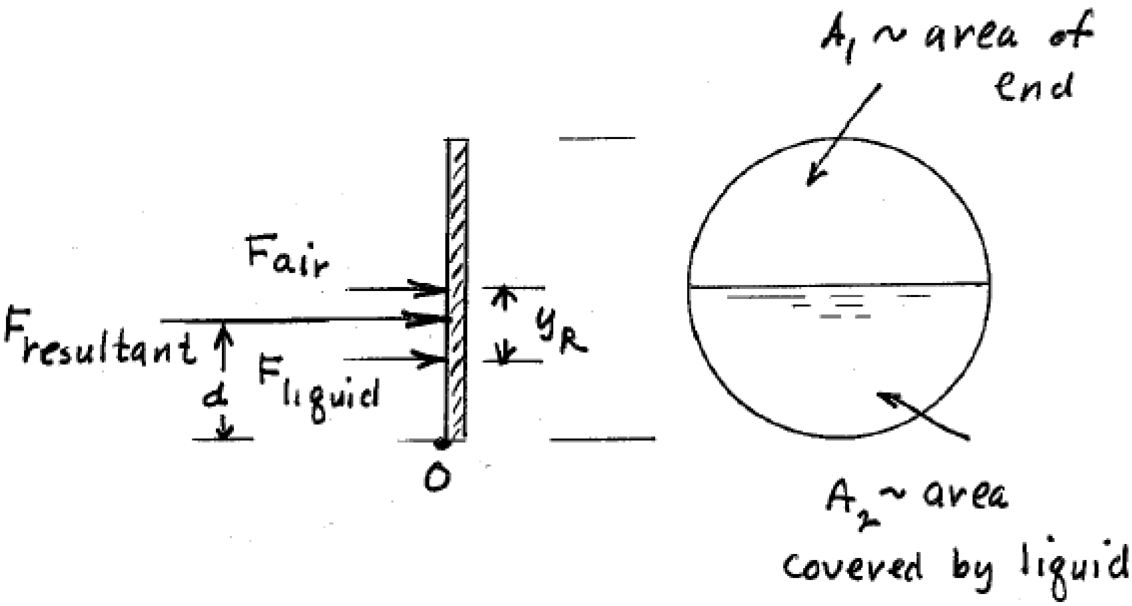


Note the following for a semi-circular plate of radius R (as shown in figure below):



$y_c = \dfrac{4R}{3\pi}$ from the base diameter and

$$I_{xc} = 0.1098R^4$$
$$I_{yc} = 0.3927R^4$$
$$I_{xyc} = 0$$



$F_{air} = p A_1$ where p is air pressure

Thus,

$$F_{air} = \left(150 \times 10^3 \frac{N}{m^2}\right)\left(\frac{\pi}{4}\right)(2m)^2 = 150\pi \times 10^3 \ N$$

$F_{liquid} = \gamma h_c A_2$ where $h_c = \frac{4R}{3\pi}$ (see Fig. 2.18c)

Thus,

$$F_{liquid} = (1.6)\left(9.81 \times 10^3 \frac{N}{m^3}\right)\left[\frac{4(1m)}{3\pi}\right]\left(\frac{1}{2}\right)\left(\frac{\pi}{4}\right)(2m)^2 = 10.5 \times 10^3 \ N$$

For F_{liquid},

$$y_R = \frac{I_{xc}}{y_c A_2} + y_c \qquad \text{where } I_{xc} = 0.1098 \ R^4 \quad \text{(see Fig. 2.18c)}$$

$$\text{and } y_c = h_c = \frac{4R}{3\pi}$$

Thus,

$$y_R = \frac{0.1098\,(1m)^4}{\left[\frac{4(1m)}{3\pi}\right]\left(\frac{1}{2}\right)\left(\frac{\pi}{4}\right)(2m)^2} + \frac{4(1m)}{3\pi} = 0.5891 m$$

Since $F_{resultant} = F_{air} + F_{liquid} = (150\pi + 10.5) \times 10^3 N = \underline{\underline{482 \ kN}}$,

we can sum moments about O to locate resultant to obtain

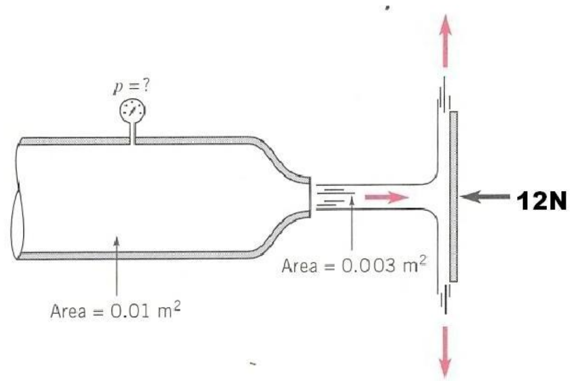
$$F_{resultant}(d) = F_{air}(1m) + F_{liquid}(1m - 0.5891 m)$$

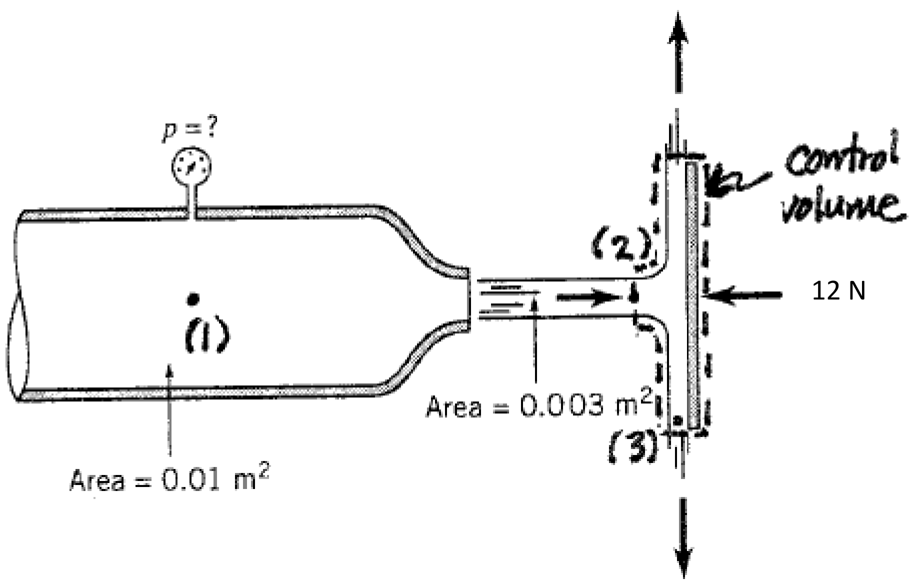
so that

$$d = \frac{(150\pi \times 10^3 N)(1m) + (10.5 \times 10^3 N)(0.4109 m)}{482 \times 10^3 \ N}$$

$$= \underline{\underline{0.987 \ m \text{ above bottom of conduit}}}$$

Q4) Air flows into the atmosphere from a nozzle and strikes a vertical plate as shown in the figure. A horizontal force of 12 N is required to hold the plate in place. Determine the reading on the pressure gage (in kPa). Assume the flow to be steady, incompressible and frictionless. Consider $\rho_{air} = 1.23\ kg/m^3$. **[20]**





To determine the static gage pressure at station (1) we first consider the frictionless and incompressible flow of air from (1) to (2). The Bernoulli equation for this flow is

$$\frac{P_1}{\rho} + \frac{V_1^2}{2} = \frac{\overset{0 \text{ gage}_2}{\cancel{P_2}}}{\rho} + \frac{V_2^2}{2} \qquad (1)$$

We note that V_1 and V_2 are linked by the continuity (conservation of mass) equation

$$Q_1 = Q_2 \quad \text{or} \quad A_1V_1 = A_2V_2 \qquad (2)$$

Combining Eqs. 1 and 2 we obtain

$$\frac{P_1}{\rho} + \frac{\left(\frac{A_2}{A_1} V_2\right)^2}{2} = \frac{V_2^2}{2} \qquad (3)$$

To determine V_2 we use the linear momentum equation for the flow from (2) to (3). For the control volume sketched above the linear momentum principle yields

$$-V_2 \rho V_2 A_2 = -\ 12\text{ N}$$

or

$$V_2 = \sqrt{\frac{(12\text{N})}{\rho A_2}} = \sqrt{\frac{12\text{ N}}{\left(1.23 \frac{\text{kg}}{\text{m3}}\right)\left(1 \frac{\text{N}.\text{s}^2}{\text{kg}.\text{m}}\right)\left(0.003\text{ m}^2\right)}}$$

and

$$V_2 = 57 \frac{\text{m}}{\text{s}}$$

Now, with Eq. 3

$$P_1 = \rho \left[\frac{V_2^2}{2} - \frac{\left(\frac{A_2}{A_1} V_2 \right)^2}{2} \right]$$

or

$$P_1 = \left(1.23 \frac{kg}{m^3}\right)\left(1 \frac{N.s^2}{kg.m}\right)\left\{ \frac{\left(57 \frac{m}{s}\right)^2}{2} - \frac{\left[\left(\frac{0.003\,m^2}{0.01\,m^2}\right)\left(57 \frac{m}{s}\right)\right]^2}{2} \right\}$$

and

$$P_1 = 1820 \frac{N}{m^2} = 1820 \text{ Pa} = \underline{\underline{1.82}} \text{ kPa}$$