ESO204A: Fluid Mechanics and Rate Processes
TUTORIAL 8 PROBLEMS

August-November 2017

1. Review of mid-semester exam (10 minutes).

2. The wall shear stress τ_w in a boundary layer is assumed to be a function of stream velocity U, boundary layer thickness δ, local turbulence velocity u', density ρ, and the local pressure gradient dp/dx. Using (ρ, U, δ) as repeating variables, rewrite this relationship as a dimensionless function.

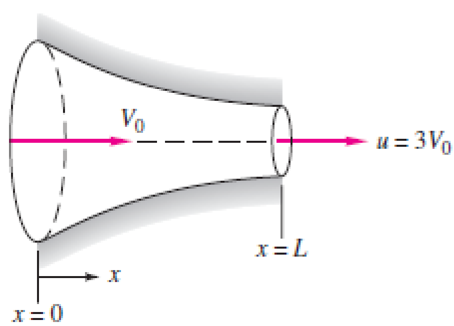
3. A fixed cylinder of diameter D and length L, immersed in a stream flowing normal to its axis at velocity U, will experience zero average lift. However, if the cylinder is rotating at angular velocity Ω, a lift force F will arise. The fluid density ρ is important, but viscosity is found to be secondary and can be neglected in the present analysis. Formulate lift behavior as a dimensionless function.

4. The time t_d to drain a liquid from a hole in the bottom of a tank is a function of the hole diameter d, initial fluid volume v_0, initial liquid depth h_0, and density ρ and viscosity μ of the liquid. Rewrite this relation as a dimensionless function.

5. Flow through the converging nozzle in Fig. P4.2 can be approximated by the one-dimensional velocity distribution

$$u \approx V_0\left(1+\frac{2x}{L}\right) \quad v \approx 0 \quad w \approx 0$$

(*a*) Find a general expression for the fluid acceleration in the nozzle. (*b*) For the specific case $V_0 = 3$ m/s and L = 0.15 m/s, compute the acceleration, in units of g's, at x=0 and L.



P4.2

6. A two-dimensional velocity field is given by

$$\mathbf{V} = \left(x^2 - y^2 + x\right)\mathbf{i} - \left(2xy + y\right)\mathbf{j}$$

in applicable units. At (x, y) = (1, 2), compute (*a*) the accelerations a_x and a_y, (*b*) the velocity component in the direction $\theta = 40^\circ$, (*c*) the direction of maximum velocity, and (*d*) the direction of maximum acceleration.

7. The velocity field near a stagnation point may be written in the form

$$u = \frac{U_0 x}{L} \quad v = -\frac{U_0 y}{L} \quad U_0 \text{ and } L \text{ are constants}$$

(*a*) Show that the acceleration vector is purely radial. (*b*) For the particular case L = 1.5 m, if the acceleration at (x, y) = (1 m, 1 m) is 25 m/s^2, what is the value of U_0?

8. For an incompressible plane flow in polar coordinates, we are given

$$v_r = r^3 \cos\theta + r^2 \sin\theta$$

Find the appropriate form of circumferential velocity for which continuity is satisfied.

9. An idealized incompressible flow has the proposed three-dimensional velocity distribution

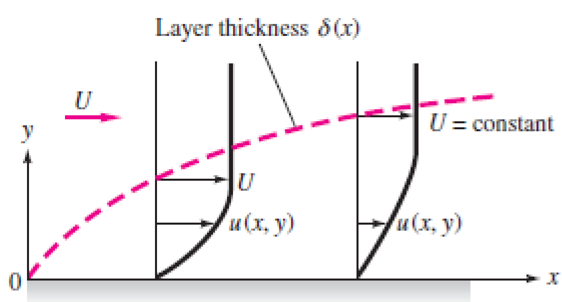
$$\mathbf{V} = 4xy^2\mathbf{i} + f(y)\mathbf{j} + zy^2\mathbf{k}$$

Find the appropriate form of the function $f(y)$ that satisfies the continuity equation $\nabla \cdot \mathbf{u} = 0$.

10. (*For discussion*) An excellent approximation for the two-dimensional incompressible laminar boundary layer on the flat surface in Fig. P4.17 is

$$u \approx U\left(2\frac{y}{\delta} - 2\frac{y^3}{\delta^3} + \frac{y^4}{\delta^4}\right) \quad \text{for } y \le \delta \quad \text{where } \delta = Cx^{1/2},\ C = \text{const}$$

(*a*) Assuming a no-slip condition at the wall, find an expression for the velocity component $v(x, y)$ for $y \le \delta$. (*b*) Then find the maximum value of v at the station x = 1 m, for the particular case of airflow, when U = 3 m/s and $\delta = 1.1$ cm.



P4.17

11. (*For discussion*) In turbulent flow near a flat wall, the local velocity u varies only with distance y from the wall, wall shear stress τ_w, and fluid properties ρ and μ. The following data were taken in a wind tunnel where $\rho = 1.185$ kg/m^3, $\mu = 1.82\text{E-5}$ kg/m·s, and $\tau_w = 1.39$ Pa.

y, mm	0.533	0.889	1.397	2.032	3.048	4.064
u, m/s	15.42	16.52	17.56	18.20	19.35	20.09

(*a*) Plot these data in the form of dimensionless u versus dimensionless y, and suggest a suitable power-law curve fit.
(*b*) Suppose that the tunnel speed is increased until u = 27.5 m/s at y = 3 mm. Estimate the new wall shear stress, in units of Pa.