Fluid Mechanics and Rate Processes: Tutorial 8

P1. In Fig.P1 the pipe entrance is sharp-edged. If the flow rate is 0.004 m^3 /s, what power, in W, is extracted by the turbine?

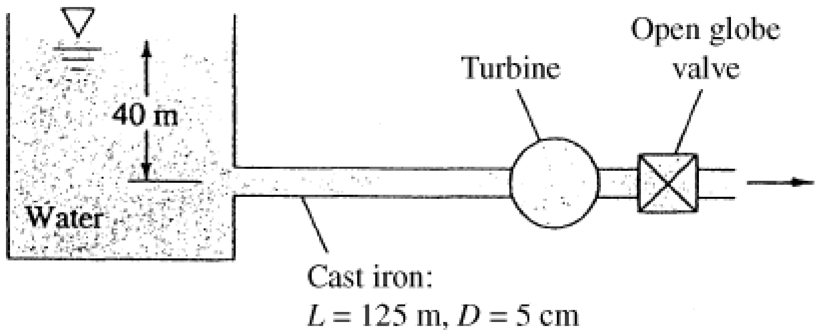

Fig.P1

Solution: For water at 20°C, take ρ = 998 kg/m^3 and $\mu = 0.001$ kg/m·s. For cast iron, $\varepsilon \approx 0.26$ mm, hence $\varepsilon/d = 0.26/50 \approx 0.0052$. The minor loss coefficients are Entrance: $K \approx 0.5$; 5-cm($\approx$2″) open globe valve: $K \approx 6.9$

The flow rate is known, hence we can compute V, Re, and f:

$$V = \frac{Q}{A} = \frac{0.004}{(\pi/4)(0.05)^2} = 2.04\ \frac{m}{s}, \quad Re = \frac{998(2.04)(0.05)}{0.001} \approx 102000, \quad f \approx 0.0316$$

The turbine head equals the elevation difference minus losses and exit velocity head:

$$h_t = \Delta z - h_f - \sum h_m - \frac{V^2}{2g} = 40 - \frac{(2.04)^2}{2(9.81)}\left[(0.0316)\left(\frac{125}{0.05}\right) + 0.5 + 6.9 + 1\right] \approx 21.5\ m$$

$$\text{Power} = \rho g Q h_t = (998)(9.81)(0.004)(21.5) \approx \mathbf{840\ W} \quad \textit{Ans.}$$

P2. The parallel galvanized-iron pipe system of Fig.P2 delivers gasoline at 20°C with a total flow rate of 0.036 m^3 /s. Let the pump be running and delivering 45 kW to the flow in pipe 2. Determine (a) the flow rate in each pipe, and (b) the overall pressure drop.

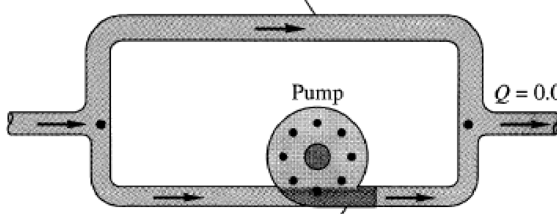

Fig.P2

Solution: For gasoline at 20°C, take ρ = 680 kg/m^3 and μ = 2.92E−4 kg/m·s. For galvanized iron, take ε = 0.15 mm, hence $\varepsilon/d_1 = 0.0030$ and $\varepsilon/d_2 = 0.00375$. The volume-flow relation is the same as in Prob. 6.113, but the head loss in pipe 2 is reduced by the pump head delivered:

$$h_{f1} = f_1 \frac{L_1}{d_1}\frac{V_1^2}{2g} = h_{f2} - h_{pump} = f_2 \frac{L_2}{d_2}\frac{V_2^2}{2g} - \frac{45000\text{ W}}{\rho g Q_2}$$

$$Q_1 + Q_2 = (\pi/4)d_1^2 V_1 + (\pi/4)d_2^2 V_2 = Q_{total} = 0.036\text{ m}^3/\text{s}$$

If we introduce the given data, we obtain two simultaneous algebraic equations:

$$f_1 \frac{60}{0.05}\frac{V_1^2}{2(9.81)} = f_2 \frac{55}{0.04}\frac{V_2^2}{2(9.81)} - \frac{45000}{680(9.81)(\pi/4)(0.04)^2 V_2},$$

$$\text{or:}\quad 61.16\, f_1 V_1^2 = 70.08\, f_2 V_2^2 - 5368/V_2 \quad \text{with V in m/s}$$

$$\text{plus}\quad (\pi/4)(0.05)^2 V_1 + (\pi/4)(0.04)^2 V_2 = 0.036\text{ m}^3/\text{s}$$

The right hand side of the 1st equation should not be negative, hence $V_2 > 15$ m/s. One solution scheme is to guess $V_2 \geq 15$ and then calculate V_1 from each equation. We also guess $f_1 \approx 0.026$ and $f_2 \approx 0.028$ from the solution to Prob. 6.113—but remember, the fluid is *gasoline* now:

$$\text{If } V_2 \approx 15\ \frac{m}{s},\quad \text{head loss gives } V_1 \approx 7.19\ \frac{m}{s},\quad \text{volume flow gives } V_1 \approx 8.73\ \frac{m}{s}$$

$$\text{If } V_2 \approx 16\ \frac{m}{s},\quad \text{head loss gives } V_1 \approx 10.18\ \frac{m}{s},\quad \text{volume flow gives } V_1 \approx 8.09\ \frac{m}{s}$$

Clearly the correct V_2 is somewhere *between* 15 and 16 m/s. The iteration converges to:

$$V_2 = 15.39\text{ m/s},\ Re_2 = 1.43E6,\quad f_2 \approx 0.0280,\quad Q_2 = A_2V_2 = \mathbf{0.0193\ m^3/s}\quad \textit{Ans.}\text{ (a)}$$

$$V_1 = 8.48\text{ m/s},\ Re_1 = 9.94E5,\quad f_1 \approx 0.0263,\quad Q_1 = A_1V_1 = \mathbf{0.0167\ m^3/s}\quad \textit{Ans.}\text{ (a)}$$

The pressure drop is the same in either leg:

$$\Delta p = f_1 \frac{L_1}{d_1}\frac{\rho V_1^2}{2} = f_2 \frac{L_2}{d_2}\frac{\rho V_2^2}{2} - \frac{45000}{Q_2} \approx \mathbf{774,000\ Pa} \quad \textit{Ans.}\ \text{(b)}$$

P3. In Fig.P3 all pipes are 8-cm-diameter cast iron. Determine the flow rate from reservoir (1) if valve C is (a) closed; and (b) open, with $K_{valve} = 0.5$.

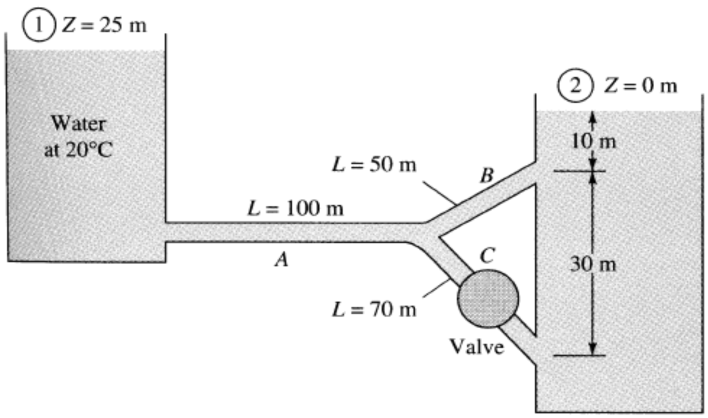

Fig.P3

Solution: For water at 20°C, take $\rho = 998\ kg/m^3$ and $\mu = 0.001$ kg/m·s. For cast iron, $\varepsilon \approx$ 0.26 mm, hence $\varepsilon/d = 0.26/80 \approx 0.00325$ for all three pipes. Note $p_1 = p_2$, $V_1 = V_2 \approx 0$. These are long pipes, but we might wish to account for minor losses anyway:

sharp entrance at A: $K_1 \approx 0.5$; line junction from A to B: $K_2 \approx 0.9$ (Table 6.5)

branch junction from A to C: $K_3 \approx 1.3$; two submerged exits: $K_B = K_C \approx 1.0$

If valve C is closed, we have a straight *series* path through A and B, with the same flow rate Q, velocity V, and friction factor *f* in each. The energy equation yields

$$z_1 - z_2 = h_{fA} + \sum h_{mA} + h_{fB} + \sum h_{mB},$$

$$\text{or:} \quad 25\ \text{m} = \frac{V^2}{2(9.81)}\left[f\frac{100}{0.08} + 0.5 + 0.9 + f\frac{50}{0.08} + 1.0\right], \quad \text{where} \quad f = \text{fcn}\left(\text{Re}, \frac{\varepsilon}{d}\right)$$

Guess $f \approx f_{\text{fully rough}} \approx 0.027$, then $V \approx 3.04$ m/s, $Re \approx 998(3.04)(0.08)/(0.001) \approx 243000$, $\varepsilon/d = 0.00325$, then $f \approx 0.0273$ (converged). Then the velocity through A and B is V = 3.03 m/s, and $Q = (\pi/4)(0.08)^2(3.03) \approx \mathbf{0.0152\ m^3/s}$. *Ans.* (a).

If valve C is open, we have parallel flow through B and C, with $Q_A = Q_B + Q_C$ and, with *d* constant, $V_A = V_B + V_C$. The total head loss is the same for paths A-B and A-C:

$$z_1 - z_2 = h_{fA} + \sum h_{mA\text{-}B} + h_{fB} + \sum h_{mB} = h_{fA} + \sum h_{mA\text{-}C} + h_{fC} + \sum h_{mC},$$

$$\text{or:}\quad 25 = \frac{V_A^2}{2(9.81)}\left[f_A\frac{100}{0.08} + 0.5 + 0.9\right] + \frac{V_B^2}{2(9.81)}\left[f_B\frac{50}{0.08} + 1.0\right]$$

$$= \frac{V_A^2}{2(9.81)}\left[f_A\frac{100}{0.08} + 0.5 + 1.3\right] + \frac{V_C^2}{2(9.81)}\left[f_C\frac{70}{0.08} + 1.0\right]$$

plus the additional relation $V_A = V_B + V_C$. Guess $f \approx f_{\text{fully rough}} \approx 0.027$ for all three pipes and begin. The initial numbers work out to

$$2g(25) = 490.5 = V_A^2(1250f_A + 1.4) + V_B^2(625f_B + 1) = V_A^2(1250f_A + 1.8) + V_C^2(875f_C + 1)$$

$$\text{If } f \approx 0.027,\ \text{solve (laboriously)}\ V_A \approx 3.48\text{ m/s},\ V_B \approx 1.91\text{ m/s},\ V_C \approx 1.57\text{ m/s}.$$

$$\text{Compute}\ Re_A = 278000,\quad f_A \approx 0.0272,\quad Re_B = 153000,\quad f_B = 0.0276,$$
$$Re_C = 125000,\quad f_C = 0.0278$$

Repeat once for convergence: $V_A \approx 3.46$ m/s, $V_B \approx 1.90$ m/s, $V_C \approx 1.56$ m/s. The flow rate from reservoir (1) is $\mathbf{Q_A} = (\pi/4)(0.08)^2(3.46) \approx \mathbf{0.0174\ m^3/s}$. (14% more) *Ans.* (b)