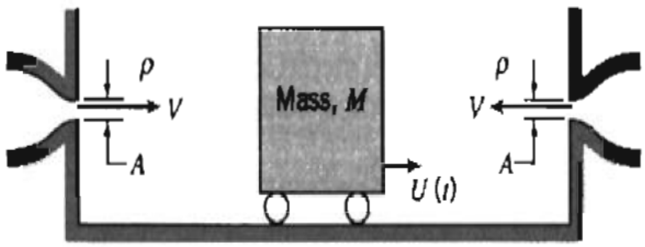
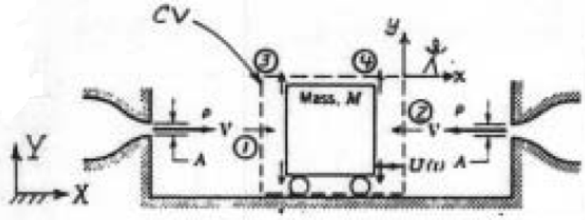
Fluid Mechanics and Rate Processes: Integral Formulation Tutorial; September 01, 2016

P1. A rectangular block of mass, M with vertical faces, rolls on a horizontal surface between two opposite jets as shown in Fig. P1. Assume that at t=0, when the block is at x=0, it is set into motion at speed U_0=10m/s, to the right. Calculate the time required to reduce the block speed to U=0.5m/s, and the block position at that instant.



P1



Solution: Apply x momentum equation to linearly accelerating CV.

Basic equation:

$$\overset{=0(1)}{F_{Sx}} + \overset{=0(2)}{F_{Bx}} - \int_{CV} a_{rf_x}\rho\, d\forall = \overset{\approx 0(3)}{\frac{\partial}{\partial t}\int_{CV} u_{xyz}\rho\, d\forall} + \int_{CS} u_{xyz}\rho \vec{V}_{xyz}\cdot d\vec{A}$$

Assumptions: (1) No pressure or friction forces, so $F_{Sx}=0$
(2) Horizontal, so $F_{Bx}=0$
(3) Neglect mass of liquid in CV; $u \approx 0$ in CV
(4) Uniform flow at each section
(5) Measure velocities relative to CV

Then

$$-a_{rf_x}M = -M\frac{dU}{dt} = u_1\{-|\rho(V-U)A|\} + u_2\{-|\rho(V+U)A|\} + u_3\{\dot{m}_3\} + u_4\{\dot{m}_4\}$$

$$u_1 = V-U \qquad u_2 = -(V+U) \qquad u_3 = 0 \qquad u_4 = 0$$

or

$$-M\frac{dU}{dt} = \rho A\left[-(V-U)^2 + (V+U)^2\right] = \rho A\left[4UV\right] = 4\rho VAU$$

Thus

$$\frac{dU}{U} = -\frac{4\rho VA}{M}\,dt$$

Integrating, $$\int_{U_0}^{U}\frac{dU}{U} = \ln U\Big]_{U_0}^{U} = \ln\frac{U}{U_0} = -\frac{4\rho VA}{M}\,t \qquad (1)$$

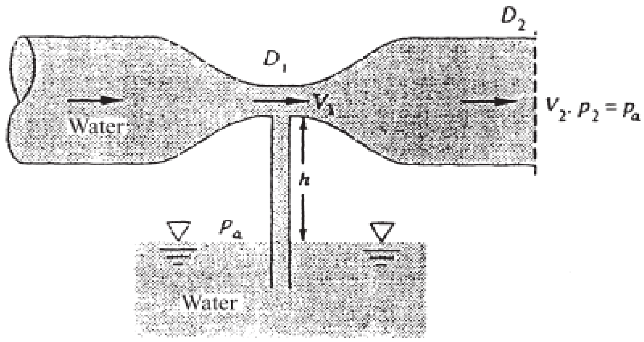
Thus $$t = -\frac{M}{4\rho VA}\ln\frac{U}{U_0} = -\frac{1}{4}\cdot\frac{M}{\rho VA}\ln\frac{0.5}{10} = 0.750\,\frac{M}{\rho VA}$$ t

From Eq. 1, $$U(t) = \frac{dX}{dt} = U_0\, e^{-\frac{4\rho VA}{M}t}$$

Integrating, $$X = \int_0^X dX = \int_0^t U_0\, e^{-\frac{4\rho VA}{M}t}\,dt = -\frac{MU_0}{4\rho VA}\, e^{-\frac{4\rho VA}{M}t}\Big]_0^t$$

$$X = \frac{MU_0}{4\rho VA}\left[1 - e^{-\frac{4\rho VA}{M}t}\right] = \frac{0.95}{4}\,\frac{MU_0}{\rho VA} = 0.238\,\frac{MU_0}{\rho VA}$$ X

P2. A necked-down section in a pipe flow, called a venturi, develops a low throat pressure which can aspirate fluid upward from a reservoir, as in Fig. P2. Using Bernoulli's equation with no losses, derive an expression for the velocity V_1 which is just sufficient to bring reservoir fluid into the throat.



P2

Solution: Water will begin to aspirate into the throat when $p_a - p_1 = \rho gh$. Hence:

$$\text{Volume flow:}\quad V_1 = V_2(D_2/D_1)^2;\quad \text{Bernoulli } (\Delta z = 0):\quad p_1 + \frac{1}{2}\rho V_1^2 \approx p_{atm} + \frac{1}{2}\rho V_2^2$$

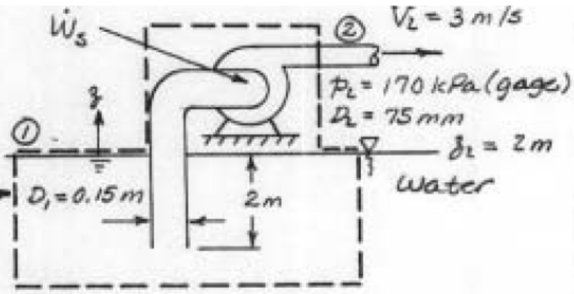
$$\text{Solve for}\quad p_a - p_1 = \frac{\rho}{2}(\alpha^4 - 1)V_2^2 \geq \rho gh,\quad \alpha = \frac{D_2}{D_1},\quad \text{or:}\quad \mathbf{V_2 \geq \sqrt{\frac{2gh}{\alpha^4 - 1}}}\quad \textit{Ans.}$$

$$\textit{Similarly,}\quad \mathbf{V_{1,\,min} = \alpha^2 V_{2,\,min} = \sqrt{\frac{2gh}{1-(D_1/D_2)^4}}}\quad \textit{Ans.}$$

P3. A pump draws water from a reservoir through a 150mm diameter suction pipe and delivers it to a 75 mm diameter discharge pipe. The end of the suction pipe is 2 m below the free surface of the reservoir. The pressure gage on the discharge pipe (2 m above the reservoir surface) reads 170 kPa. The average speed in the discharge pipe is 3m/s. If the pump efficiency is 75 percent, determine the power required to drive it.

Given: Pump system as shown.

$\eta_{pump} = 0.75$



Find: Power required.

Solution: Apply first law to CV shown, noting that flow enters with negligible velocity at section ①.

Basic equation:

$$\dot{Q} - \dot{W}_{shaft} - \underbrace{\dot{W}_{shear}}_{=0(1)} - \underbrace{\dot{W}_{other}}_{=0(1)} = \underbrace{\frac{\partial}{\partial t}\int_{CV} e\rho\, d\forall}_{=0(2)} + \int_{CS}\left(e + \frac{p}{\rho}\right)\rho \vec{V}\cdot d\vec{A}$$

$$e = u + \frac{V^2}{2} + gz$$

Assumptions: (1) $\dot{W}_{shear} = \dot{W}_{other} = 0$
(2) Steady flow
(3) $V_1 \simeq 0$
(4) $z_1 = 0$
(5) $p_1 = 0$ (gage)
(6) Uniform flow at each section
(7) Incompressible flow; $V_1 A_1 = V_2 A_2$

Then

$$\dot{Q} - \dot{W}_s = \Big(u_1 + \underbrace{\frac{V_1^2}{2}}_{\simeq 0(3)} + \underbrace{gz_1}_{=0(4)} + \underbrace{\frac{p_1}{\rho}}_{=0(5)}\Big)\{-\dot{m}\} + \Big(u_2 + \frac{V_2^2}{2} + gz_2 + \frac{p_2}{\rho}\Big)\{\dot{m}\}$$

or

$$-\dot{W}_s = \dot{m}\left[\frac{p_2}{\rho} + \frac{V_2^2}{2} + gz_2 + \left(u_2 - u_1 - \frac{\delta Q}{dm}\right)\right]$$

Obtain the ideal or minimum power input by neglecting thermal effects. Thus

$$-\dot{W}_{s,ideal} = \dot{m}\left[\frac{p_2}{\rho} + \frac{V_2^2}{2} + gz_2\right]$$

For the system,

$$\dot{m} = \rho V_2 A_2 = 999\,\frac{kg}{m^3} \times 3\,\frac{m}{s} \times \frac{\pi}{4}(0.075)^2 m^2 = 13.2\ kg/s$$

and

$$-\dot{W}_{s,ideal} = 13.2\,\frac{kg}{s}\left[1.70\times10^5\,\frac{N}{m^2}\times\frac{m^3}{999\,kg} + \frac{1}{2}(3)^2\frac{m^2}{s^2}\times\frac{N\cdot s^2}{kg\cdot m} + 9.81\,\frac{m}{s^2}\times 2\,m \times \frac{N\cdot s^2}{kg\cdot m}\right]$$

$$\dot{W}_{s,ideal} = -2560\,\frac{N\cdot m}{s} \times \frac{kW\cdot s}{10^3\,N\cdot m} = -2.56\ kW$$

Finally

$$\dot{W}_{s,actual} = \frac{\dot{W}_{s,ideal}}{\eta} = \frac{-2.56\ kW}{0.75} = -3.41\ kW$$

$\dot{W}_{s,actua}$