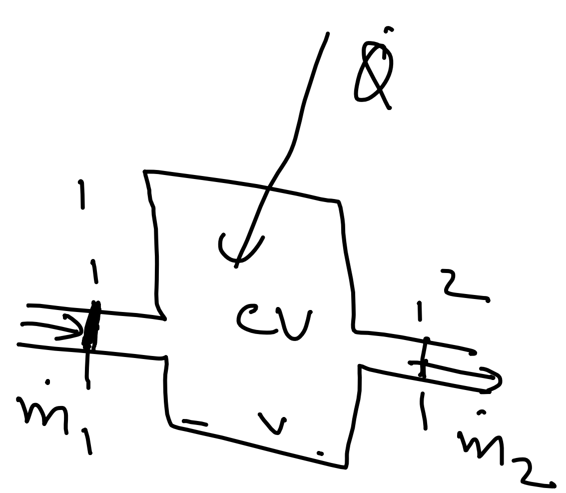
Energy balance.... (continued)



$$\boxed{\dot{Q} - \dot{W}_s - \dot{W}_v = \dot{m}_2\left(\hat{h}_2 + \tfrac{1}{2}\alpha_2 V_{2av}^2 + g z_2\right) - \dot{m}_1\left(\hat{h}_1 + \tfrac{1}{2}\alpha_1 V_{1av}^2 + g z_1\right)}$$

$$\dot{m}_1 = \dot{m}_2 = \dot{m}$$

$$\hat{h}_1 + \tfrac{1}{2}\alpha_1 V_{1av}^2 + g z_1 = \hat{h}_2 + \tfrac{1}{2}\alpha_2 V_{2av}^2 + g z_2 - q + w_s + w_v \quad ; \quad q = \frac{\dot{Q}}{\dot{m}}, \; w_s = \frac{\dot{W}_s}{\dot{m}}, \; w_v = \frac{\dot{W}_v}{\dot{m}}$$

$$\frac{\hat{u}_1}{g} + \frac{p_1}{\rho g} + \frac{1}{2g}\alpha_1 V_{1av}^2 + z_1 = \frac{p_2}{\rho g} + \frac{\hat{u}_2}{g} + \frac{1}{2g}\alpha_2 V_{2av}^2 + z_2 - h_q + h_s + h_v$$

All term unit ≡ Length (m)
Each term is called a "head".

Adiabatic $h_q = 0$; viscous work $h_v \approx$ small ≈ 0

friction

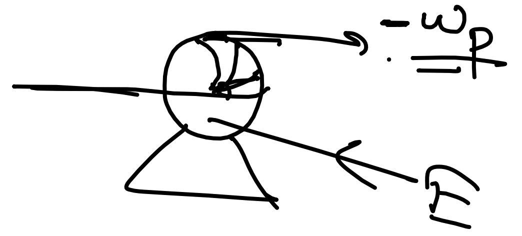
$$\frac{p_1}{\gamma} + \frac{1}{2g}\alpha_1 V_{1av}^2 + z_1 = \frac{p_2}{\gamma} + \frac{1}{2g}\alpha_2 V_{2av}^2 + z_2 + \frac{\hat{u}_2 - \hat{u}_1}{g} + h_s$$

$h_s = -h_p$ for pump, $h_s = +h_t$ (for turbine)

$$\boxed{\frac{p_1}{\gamma} + \frac{1}{2g}\alpha_1 V_{1av}^2 + z_1 = \frac{p_2}{\gamma} + \frac{1}{2g}\alpha_2 V_{2av}^2 + h_f - h_p + h_t}$$

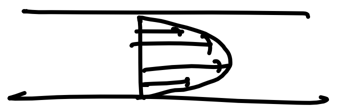
Energy balance

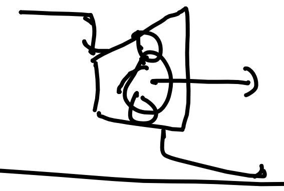
$h_f = \frac{\hat{u}_2 - \hat{u}_1}{g}$ ≡ frictional losses



$W_v = -\int(\tau \cdot V)\,dA =$ small τ ≡ normal stresses

Kinetic energy correction factor (α)





$$\text{flow rate of K.E.} = \frac{1}{2}\int u^2 \rho (V\cdot n)\,dA = \frac{1}{2}\int u^2 \rho\, u\, dA = \alpha\, \frac{1}{2} V_{av}^2\, \rho V_{av} A$$

$$\boxed{\alpha = \frac{1}{A}\int \frac{u^3}{V_{av}^3}\, dA}$$

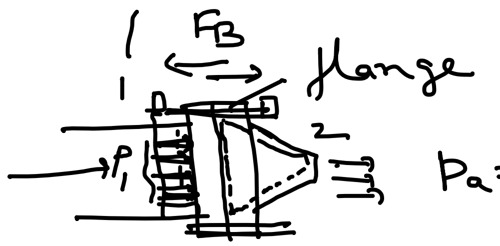
Laminar $\quad u = u_0\left(1-\left(\frac{r}{R}\right)^2\right) \quad u_{av} = \frac{1}{2}u_0 \quad \alpha = 2.0$

Turbulent $\quad u = u_0\left(1-\frac{r}{R}\right)^m \quad u_{av} = \frac{2u_0}{(m+1)(m+2)} \quad \frac{1}{9} \le m \le \frac{1}{5}$

$$\alpha = \frac{(1+m)^2(2+m)^3}{4(1+3m)(2+3m)} \Rightarrow (\alpha \simeq 1.0)$$

Example: A 10 cm fire hose with a 3-cm nozzle discharges $1.5\,m^3/min$ to the atmosphere. Assuming frictionless flow, find the force F_B exerted by the flange bolts to hold the nozzle on the hose.

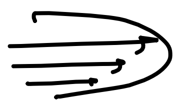
Solution



$$\frac{p_1}{\gamma} + \frac{1}{2g}\alpha_1 V_{1av}^2 + z_1 = \frac{p_2}{\gamma} + \frac{1}{2g}\alpha_2 V_{2av}^2 + z_2 - h_p + h_t + h_f$$

$$\frac{p_{1g}}{\gamma} + \frac{1}{2g}\alpha V_{1avg}^2 + \cancel{z_1} = \left(\frac{p_2}{\gamma}\right)^0 + \frac{1}{2g}\alpha_2 V_{av}^2 + \cancel{z_2} + \cancel{h_t}^0 - \cancel{h_p}^0 + \cancel{h_f}^0$$

$$V_{1avg} = \frac{Q}{A_1}, \quad V_{2avg} = \frac{Q}{A_2}$$



$p_{1g} = ?$

x-momentum balance

$$-F_B + p_{1g}A_1 = \frac{d}{dt}\left(\cancel{\int}\right)^0 + \beta_2\left(\rho V_2^2 A_2\right) - \beta_1 \rho V_1^2 A_1$$

$\beta_2 = \frac{4}{3}$ Laminar

$= 1$ Turbulent ← Assume

$F_B = ?$

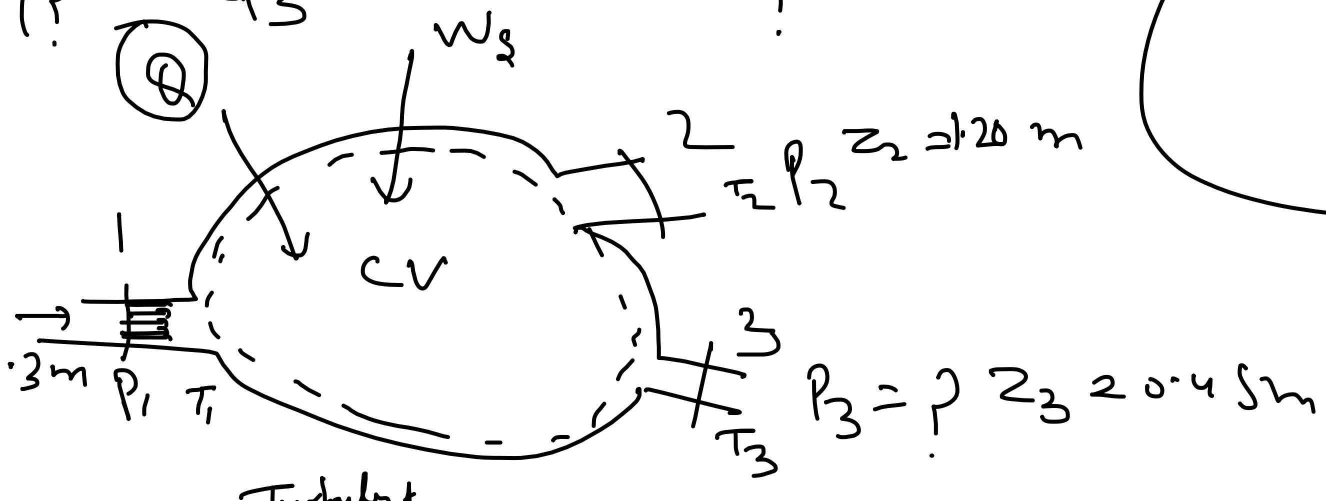
Example A steady flow machine takes in air at section 1 and discharges it at section 2 and 3. The property at each section are as follows:

Section	A, m^2	Q m^3/s		T, °C	P, Pa (abs)
1	0.04	2.8	ρ_1	21	137,900
2	0.1	1.1	ρ	38	206,800
3	0.02	1.4	ρ_3?	93	?

work is provided to the machine at the rate of 120 kW. Find pressure P_3 and heat transfer $\dot{Q}$. Assume air is a perfect gas with $R = 287$ N.m/kg.K and $c_p = 1003$ N.m/kg.K

Solution

$c_p dT$



Turbulent

$$\dot{Q} - \dot{W}_s - \cancel{\dot{\phi}_v}^{\,0} = \dot{m}_3\left(\hat{h}_3 + \tfrac{1}{2}\alpha_3 V_{3av}^2 + z_3\right) + \dot{m}_2\left(\hat{h}_2 + \tfrac{1}{2}\alpha_2 V_{2av}^2 + z_2\right) - \dot{m}_1\left(\hat{h}_1 + \tfrac{1}{2}\alpha_1 V_{1av}^2 + z_1\right)$$

$$\dot{m}_1 = \dot{m}_2 + \dot{m}_3 \qquad \dot{m}_3 = \rho_3 Q_3$$

$$\longrightarrow T \longrightarrow \rho$$

$$PV = \frac{m}{M} RT$$

$$\frac{PM}{R'T} = \rho \qquad \frac{P_3}{R'T} = \rho_3$$

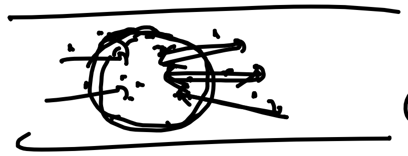
$$0 = \frac{d}{dt}\int \rho \, dV + \int \rho (V \cdot n) dA$$

$$\dot{W}_s = -120000 \text{ W}$$

mass balance, energy balance, momentum over CV ⟹ integral balance

over a point ⟹ differential balance

continuum hypothesis



fluid

differential balance

$$\left\{ \begin{array}{l} dt \rightarrow \\ dx \rightarrow 0 \\ dy \rightarrow 0 \\ dz \rightarrow 0 \end{array} \right.$$

$dV \leftarrow dx \rightarrow$ as small as possible

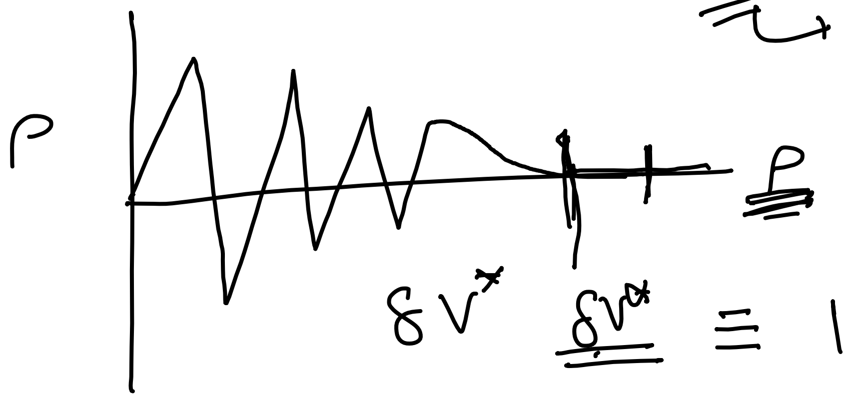
$dx \rightarrow 0$

$$\left. \begin{array}{l} dx \\ dy \\ dz \end{array} \right] \delta V^*$$ all properties will be fluctuating

$\frac{d}{dx}, \frac{d}{dy}, \frac{d}{dz}$ $\quad dV^* \rightarrow$ steady properties

$\hookrightarrow \delta x^*$

$$\rho = \lim_{\delta V \to 0} \frac{\delta m}{\delta V} \qquad \rho = \lim_{\delta V \to \delta V^*} \frac{\delta m}{\delta V} \rightarrow \text{steady}$$



δV^*

$$\delta V \rightarrow \delta V^* \qquad \delta x^a = (\delta V^*)^{1/3}$$

$$\delta V^* \equiv 10^{-9}\ mm^3$$