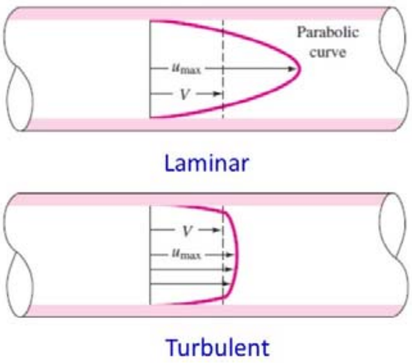
ESO204A, Fluid Mechanics and Rate Processes

Incompressible flows through pipes and ducts (Internal Flow)

Applications of Fluid Mechanics

Chapter 6 of F M White
Chapter 8 of Fox McDonald

Turbulent Pipe Flow



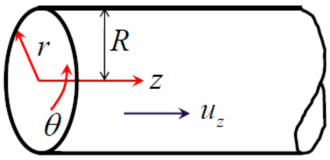
Velocity profile looks more uniform due to increased momentum transport in r -direction

Turbulent flow also includes fluctuation over and above the relatively uniform profile

Engineering applications of pipe flow are in the turbulent regime

Steady, Fully-developed, Pipe Flow

$$h_f = \frac{p_1 - p_2}{\rho g} = f \frac{L}{d} \frac{u_{av}^2}{2g}$$



$$f = \frac{8\tau_w}{\rho u_{av}^2} = 4C_f$$

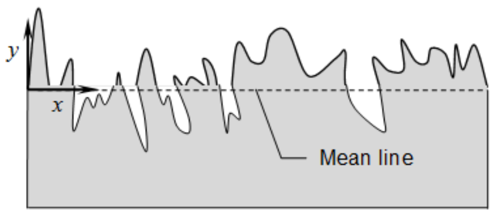
Laminar: $Re_d < 1800$ $\quad f = \dfrac{64}{Re_d}$

Turbulent: $Re_d > 2000$ $\quad f = ?$

Wall shear in turbulent flow is very different than that in laminar flow

Shear Stress in Turbulent Pipe Flow

Shear stress, in turbulent flow, depends on wall roughness, unlike laminar flow



Roughness is measured as rms of deviation from the mean; dimension L

Shear Stress in Turbulent Pipe Flow: Dimensional Analysis

$$\tau_w = f\left(d, u, \rho, \mu, \varepsilon\right)$$

Roughness; dimension L

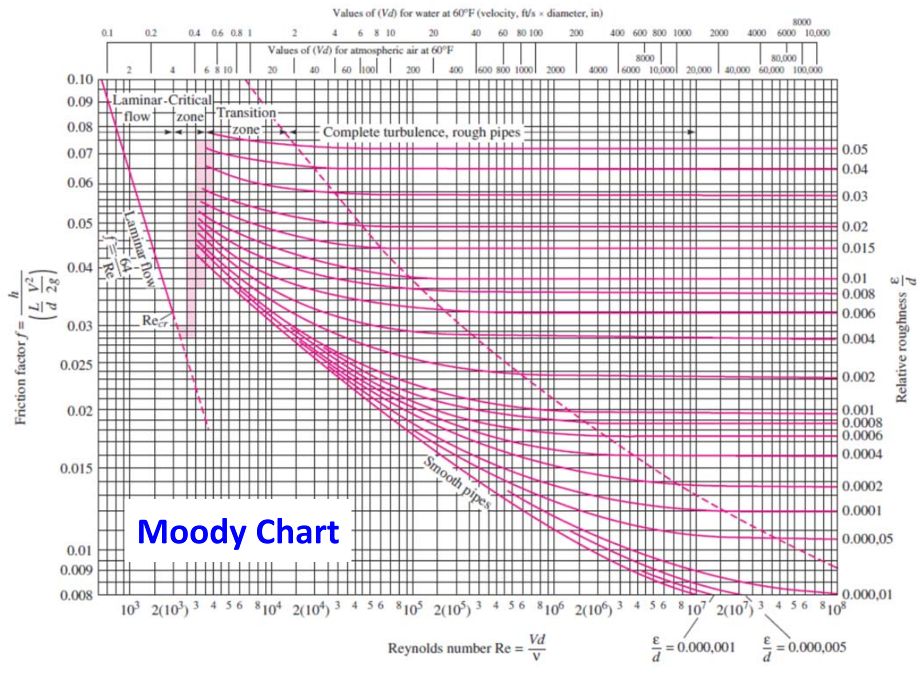
repeating variables : d, u, ρ

$$\pi_1 = \tau_w d^a u^b \rho^c = \frac{\tau_w}{\rho u^2} = \frac{C_f}{2} = 2f \qquad \pi_3 = \varepsilon d^a u^b \rho^c = \frac{\varepsilon}{d}$$

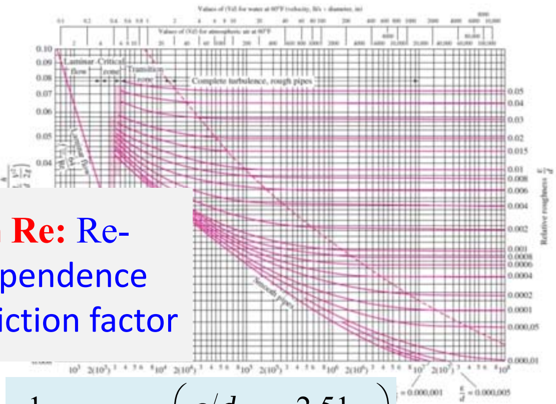
$$\pi_2 = \mu d^a u^b \rho^c = \frac{\mu}{\rho u d} = \frac{1}{\mathrm{Re}}$$

Relative roughness

$$f = \psi\left(\mathrm{Re}, \frac{\varepsilon}{d}\right)$$



Moody Chart



Low Re: follows exact solution (H-P solution)

High Re: Re-independence of friction factor

Moody chart is a plot of Colebrook formula:

$$\frac{1}{\sqrt{f}} = -2\log\left(\frac{\varepsilon/d}{3.7} + \frac{2.51}{\mathrm{Re}_d\sqrt{f}}\right)$$

An approximation of the above is Haaland formula:

$$\frac{1}{\sqrt{f}} \approx -1.8\log\left[\left(\frac{\varepsilon/d}{3.7}\right)^{1.11} + \frac{6.9}{\mathrm{Re}_d}\right]$$

Pipe Flow: Problem Solving

Moody Chart is used in almost all pipe flow problems; fluid properties are also generally known

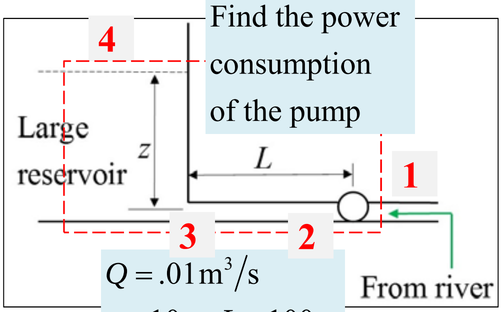
- Given the pipe geometry, and either flow rate or power/loss, find the other
- Given the power/loss, flow rate and partial information about pipe geometry, find the rest of the geometric parameters

Some of the above problems require iterative solution

Example

Pumping of water to a large reservoir

$\frac{\varepsilon}{d} = .0001$



Find the power consumption of the pump

$Q = .01\,\mathrm{m}^3/\mathrm{s}$

$z = 10\mathrm{m},\ L = 100\mathrm{m}$

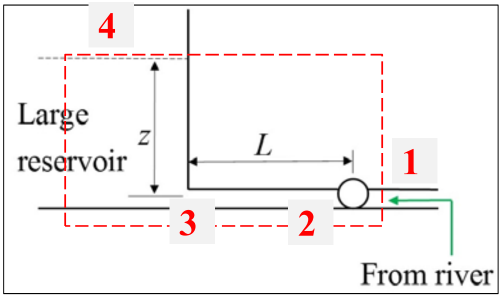
$\mathrm{dia} = 75\mathrm{mm}$

Applying energy Equation between 1-2:

$$\dot{m}\left(\frac{p_1}{\rho} + \cancel{\frac{u_1^2}{2}} + \cancel{gz_1}\right) + \dot{W} = \dot{m}\left(\frac{p_2}{\rho} + \cancel{\frac{u_2^2}{2}} + \cancel{gz_2}\right) + \cancel{\text{friction in pump}}$$

$$\frac{\dot{W}}{\dot{m}g} = \frac{p_2}{\rho g} - \frac{p_1}{\rho g}$$

$$\frac{\dot{W}}{\dot{m}g} = \frac{p_2}{\rho g} - \frac{p_1}{\rho g}$$



Similarly, applying energy Equation between 2-3:

$$\frac{p_2}{\rho g} + \cancel{\frac{u_2^2}{2g}} + \cancel{z_2} = \frac{p_3}{\rho g} + \cancel{\frac{u_3^2}{2g}} + \cancel{z_3} + h_f$$

$$\frac{p_2}{\rho g} - \frac{p_3}{\rho g} = h_f = f\frac{L}{d}\frac{u_1^2}{2g}$$

Please note, mass conservation gives

$$\dot{m} = \dot{m}_1 = \dot{m}_2 = \dot{m}_3$$

$$u_1 = u_2 = u_3$$

combining $\frac{\dot{W}}{\dot{m}g} - \frac{p_3}{\rho g} = -\frac{p_1}{\rho g} + h_f$