ESO204A, Fluid Mechanics and rate Processes

Dimensional Analysis: application to model testing

Chapter 5 of F M White
Chapter 7 of Fox McDonald

A running car experiences fluid resistance known as 'drag force' $F = f\left(L, u, \rho, \mu\right)$

We are interested to measure the drag on a similar to estimate the drag on the prototype

$$\frac{F}{\rho u^2 L^2} = \psi\left(\frac{\mu}{\rho u L}\right) \qquad C_D = \psi\left(\frac{1}{\mathrm{Re}}\right)$$

To conduct useful model test, we need to match **Re**, which may need model testing at high-speed

Our crude experiment indicated C_D = constant!!

We can explain this result from the dimensional analysis

$$\frac{F}{\rho u^2 L^2} = \psi\left(\frac{\mu}{\rho u L}\right) \qquad C_D = \psi\left(\frac{1}{\text{Re}}\right)$$

$$C_D = \frac{\text{drag}}{\text{inertia}} \qquad \frac{1}{\text{Re}} = \frac{\text{viscous}}{\text{inertia}}$$

- Dim. analysis indicates three forces
- Dim. analysis scales other forces w. r. t. inertia

Drag force has two sources: viscous + pressure (or 'form' drag)

$$\frac{F}{\rho u^2 L^2} = \psi\left(\frac{\mu}{\rho u L}\right)$$

$$C_D = \psi\left(\frac{1}{\text{Re}}\right)$$

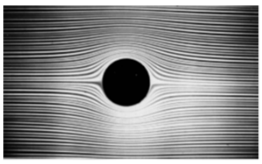
$$C_D = \frac{\text{drag}}{\text{inertia}}$$

$$\frac{1}{\text{Re}} = \frac{\text{viscous}}{\text{inertia}}$$

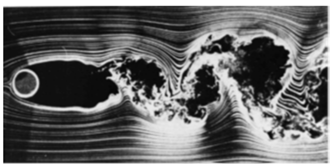
Form drag becomes more important at high velocity

High Re $\equiv$ viscous << inertia

Form drag dominates



Re =0.1



Re =10,000

Flow over cylinders at varying Re

High Re case $F = f(L, u, \rho)$ Dropping μ

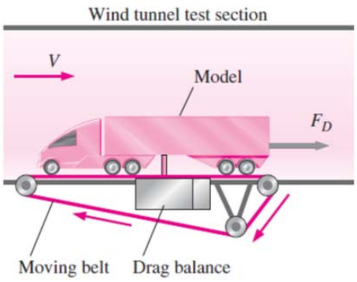
Conduct dimensional analysis $\pi_1 = \frac{F}{\rho u^2 L^2}$

$$\Rightarrow \psi\left(\frac{F}{\rho u^2 L^2}\right) = 0 \quad \Rightarrow \frac{F}{\rho u^2 L^2} = C_D = \text{constant}$$

Above relation usually holds for $\mathrm{Re} \sim 10^3$ or more

The model study can be conducted up to the point where C_D reaches the Re-independent value

Example: model testing of a truck



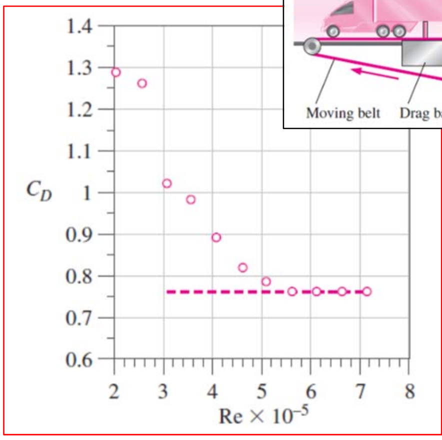
	Model	Prototype
L	.991m	15.9m
u	70m/s max	26.8 m/s (100km/hr)

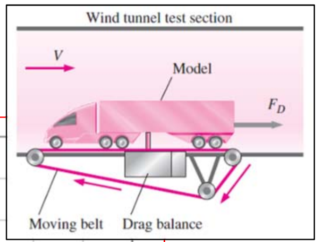
For Re matching $(uL)_m = (uL)_p$ $u_m = 429\,\mathrm{m/s}$

The model speed is in compressible regime and also cannot be attained in the present wind-tunnel

Wind tunnel model test results

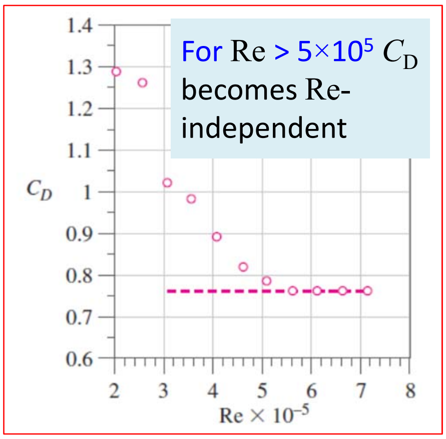
u_m (m/s)	F_m (N)
20	12.4
25	19
30	22.1
35	29
40	34.3
45	39.9
50	47.2
55	55.5
60	66
65	77.6
70	89.9



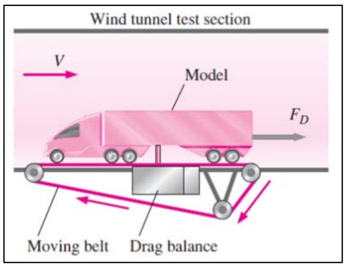


For Re > 5×10^5 C_D becomes Re-independent

Application to prototype



For Re > 5×10^5 C_D becomes Re-independent



$$\mathrm{Re}_{p,26.8\mathrm{m/s}} = 4.37\times10^6$$

At this Re, C_D is Re-independent

$$C_{D,p,26.8\ \mathrm{m/s}} = C_{D,m,\mathrm{Re}>5\times10^5} = .76$$

$$F_p = \rho u_p^2 L_p^2 C_D$$

$$F = f(L,u,\rho,\mu) \qquad \frac{F}{\rho u^2 L^2} = \psi\left(\frac{\mu}{\rho u L}\right) \qquad C_D = \psi\left(\frac{1}{\mathrm{Re}}\right)$$

$$C_D = \frac{\text{drag}}{\text{inertia}} \qquad \frac{1}{\mathrm{Re}} = \frac{\text{viscous}}{\text{inertia}}$$

Low Re case $\Rightarrow$ small inertia $\Rightarrow$ drag ~ viscous

As discussed before, drag force has two components: pressure (form) and viscous

For low Re, viscous part dominates

Low Re case $\quad F = f(L,u,\mu) \quad$ Dropping ρ

Conduct dimensional analysis $\quad \pi_1 = \dfrac{F}{\mu u L}$

$$\Rightarrow \psi\left(\frac{F}{\mu u L}\right) = 0 \quad \Rightarrow \frac{F}{\mu u L} = \text{constant} \quad \Rightarrow \frac{F}{\rho u^2 L^2} = \frac{\text{constant}.\mu u L}{\rho u^2 L^2}$$

$$\Rightarrow C_D = \frac{\text{constant}}{\mathrm{Re}}$$

Above relation holds for $\mathrm{Re} < 1$ (creeping flow or Stoke's flow) and is very useful for viscosity measurement, microflows, biological systems

Low Re

$$\text{Drag} = \text{constant}.\mu u L$$

High Re

$$\text{Drag} = \text{constant}.\rho u^2 L^2$$

At high speed drag is proportional to u^2 while at low speed drag is proportional to u

In a highly viscous (low Re) environment, it is very difficult to start/maintain motion

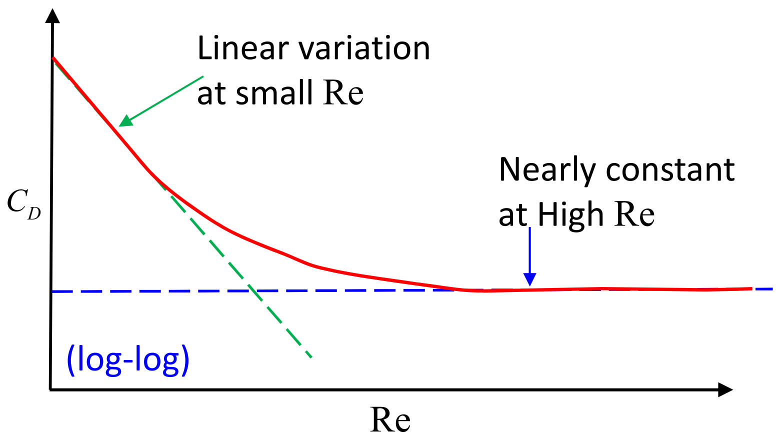
Summary

Low Re

$$C_D = \frac{\text{constant}}{\text{Re}}$$

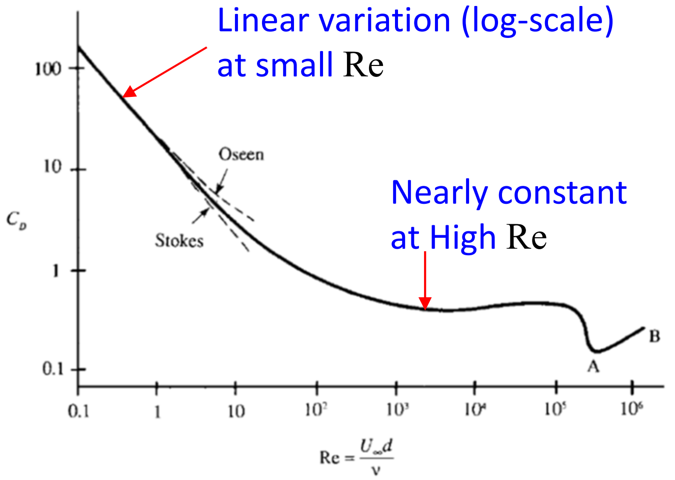
High Re

$$C_D = \text{constant}$$



Drag coefficient for flow over an object (conceptual)

Drag coefficient for flow over a sphere (experimental)

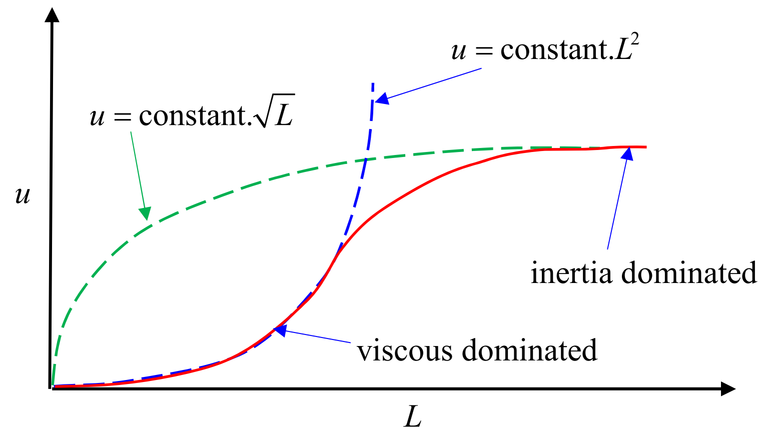


Example: terminal speed of a falling object

Terminal speed: steady speed of the falling object when drag = weight $\text{Drag} = \text{constant}.L^3$

Low Re	High Re
$\text{Drag} = \text{constant}.\mu u L$	$\text{Drag} = \text{constant}.\rho u^2 L^2$
$u \propto L^2$	$u \propto \sqrt{L}$

Terminal speed varies differently with length-scale for smaller and larger objects



Terminal speed of falling objects of varying length-scales (conceptual plot)