Tutorial 6

5.2

$V = 150$ mi/h $\quad$ $V = ?$

$T = 20°C$ $\quad$ $T = -10°C$

$P = 1$ atm $\quad$ $P = 83$ kPa

L_m $\quad$ L_p

model

$$\rho_m = \frac{PM}{RT} = \frac{1.0134\times10^5 \times 0.029}{8.314 \times 293} = 1.206 \text{ kg/m}^3$$

$$\mu_m = 1.8\times10^{-5} \text{ Pa.s. (Table 4.4)}$$

Prototype

$$\rho_p = \frac{83000 \times 0.029}{8.314 \times 263} = 1.1 \text{ kg/m}^3$$

$$\mu_m = 1.8\times10^{-5} \text{ Pa.s.}$$

$$L_p = 7 L_m$$

$$\frac{F}{\rho V^2 L^2} = g\left(\frac{\rho V L}{\mu}\right)$$

for dynamic similarity $Re_m = Re_p$

$$V_m = 150 \text{ mi/h} = 150 \times 4.4704\times10^{-1} \text{ m/s} = 67.056 \text{ m/s}$$

$$\frac{\rho_p V_p L_p}{\mu_p} = \frac{\rho_m V_m L_m}{\mu_m}$$

$$\frac{1.1 \times V_p \times 7}{1.8\times10^{-5}} = \frac{1.206 \times 67.056 \times 1}{1.8\times10^{-5}}$$

$$V_p = 10.50 \text{ m/s} = \frac{10.50}{0.44704} = 23.49 \text{ mi/h}$$

5.4

m

water 

$V_m = 2$ m/s

$D_m = 0.08$ m

$F_m = 5$

$T = 20^\circ C$

water

$\rho = 1000$ kg/m3

$\mu = 1$ cp $= 0.001$ Pa.s

p

$V_p = ?$

$D_p = 1.5$ m

$F_p = ?$

$T = 25^\circ C$

$P = 1$ atm

Air

$\rho = 1.2$ kg/m3

$\mu = 1.8 \times 10^{-5}$ Pa.s.

~~Dyna~~ $$\frac{F}{\rho V^2 D^2} = g\left(\frac{\rho V D}{\mu}\right)$$

Dynamic similarity $$\left(\frac{\rho V D}{\mu}\right)_m = \left(\frac{\rho V D}{\mu}\right)_p$$

$$\frac{1000 \times 2 \times 0.08}{0.001} = \frac{1.2 \times V_p \times 1.5}{1.8 \times 10^{-5}} \Rightarrow V_p = 1.6 \text{ m/s}$$

$$\left(\frac{F}{\rho V^2 D^2}\right)_m = \left(\frac{F}{\rho V^2 D^2}\right)_p$$

$$\frac{5}{1000 \times (2)^2 \times (0.08)^2} = \frac{F_p}{1.2 \times (1.6)^2 \times (1.5)^2}$$

$F_p = 1.35$ N

5.39

$Q = f(g, b, H)$

$L^3T^{-1} \quad LT^{-2} \quad L \quad L$

$n = 4, \quad J = 2 \qquad n - J = 4 - 2 = 2$ Pi groups

$$\frac{Q}{g^{1/2} H^{5/2}} = g\left(\frac{b}{H}\right)$$

$Q \propto b$

$$\Rightarrow \frac{Q}{g^{1/2} H^{5/2}} = K \frac{b}{H} \qquad \text{where } K = \text{constant}$$

$$\frac{Q}{b\, g^{1/2} H^{3/2}} = K$$

5.41

(a) Dimension of $\rho c_p \frac{\sigma}{\mu} \frac{\partial p}{\partial y} \frac{\partial T}{\partial y} \equiv$ dimension of $k \frac{\partial^2 T}{\partial y^2}$

$$\rho \equiv [ML^{-3}], \quad c_p = \frac{J}{kg\,K} = \frac{MLT^{-2}L}{MK} = L^2T^{-2}K^{-1}$$

$$\mu = \frac{MLT^{-2}/L^2}{LT^{-1}/L} = ML^{-1}T^{-1}; \quad \frac{\partial p}{\partial y} = \frac{MLT^{-2}/L^2}{L} = ML^{-2}T^{-2}$$

$$\frac{\partial T}{\partial y} = KL^{-1}; \quad k = \frac{J/T/L^2}{K/L} = \frac{LMLT^{-2}/TL^2}{K/L} = MLT^{-3}K^{-1}$$

$$\frac{\partial^2 T}{\partial y^2} = KL^{-2}$$

$$\Rightarrow \cancel{M}\cancel{L^{-3}}\cancel{L^2}\cancel{T^{-2}}\cancel{K^{-1}} \frac{\sigma}{\cancel{M}\cancel{L^{-1}}\cancel{T^{-1}}} \cancel{M}\cancel{L^{-2}}\cancel{T^{-2}}\cancel{K}L^{-1} = \cancel{M}L\cancel{T^{-3}}\cancel{K^{-1}}\cancel{K}\cancel{L^{-2}}$$

$$\sigma = L^2$$

(b) $x^* = x/L$, $y^* = y/L$, $p^* = \frac{p}{\rho U^2}$, $T^* = \frac{T}{T_0}$

$$\rho c_p \frac{\sigma}{\mu} \frac{\rho U^2}{\cancel{L}} \frac{\partial p^*}{\partial x^*} \frac{\cancel{T_0}}{\cancel{L}} \frac{\partial T^*}{\partial x^*} + \rho c_p \frac{\sigma}{\mu} \frac{\rho U^2}{\cancel{L}} \frac{\partial p^*}{\partial y^*} \frac{\cancel{T_0}}{\cancel{L}} \frac{\partial T^*}{\partial y^*} + \frac{k\cancel{T_0}}{\cancel{L^2}} \frac{\partial^2 T^*}{\partial y^{*2}} = 0$$

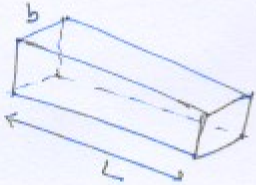
$$\frac{\rho^2 c_p U^2 \sigma}{\mu} \frac{\partial p^*}{\partial x^*} \frac{\partial T^*}{\partial x^*} + \frac{\rho^2 c_p U^2 \sigma}{\mu} \frac{\partial p^*}{\partial y^*} \frac{\partial T^*}{\partial y^*} + k \frac{\partial^2 T^*}{\partial y^{*2}} = 0$$

$$\frac{\rho^2 c_p U^2 \sigma}{\mu k} \left[\frac{\partial p^*}{\partial x^*} \frac{\partial T^*}{\partial x^*} + \frac{\partial p^*}{\partial y^*} \frac{\partial T^*}{\partial y^*} \right] + \frac{\partial^2 T^*}{\partial y^{*2}} = 0$$

5.21

(a) $F = f(\rho, \mu, V, L, b)$

Dimensional Analysis



$$\frac{F}{\rho V^2 bL} = g\left(\frac{\rho V b}{\mu}\right) \qquad L = 1\,m$$

$\rho_{water} = 1000\ kg/m^3$
$\mu_{water} = 1\ cp = 0.001\ Pa.s$
$b = 0.02\,m$

V	1.0	2.0	3.0	4.
F/L	21	85	191	335
$F/\rho V^2 bL$	1.05	1.06	1.06	1.05
$Re = \frac{\rho V b}{\mu}$	20,000	40000	60000	80000

$$\frac{F}{\rho V^2 bL} = \text{Constant} = 1.055$$

now $V = 6\ m/s$, air $\rho = 1.2\ kg/m^3$
$\mu = 1.8\times10^{-5}\ Pa.s$
$b = 0.55\,m$

$$F = 1.055 \times 1.2 \times 6^2 \times 0.55 = 25.1\ N/m$$

(b) $Re_{chimney} = \frac{\rho V b}{\mu} = \frac{1.2\times 6\times 0.55}{1.8\times10^{-5}} = 220\,000$

Data is available in range $20{,}000 < Re < 80{,}000$

But $Re_{chimney}$ is out of range $\Rightarrow$ uncertainty.