ESO204A, Fluid Mechanics and Rate Processes

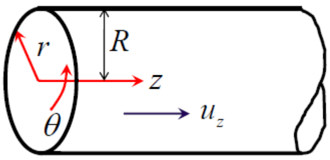
Incompressible flows through pipes and ducts (Internal Flow)

Few applications of the theories we have learned so far

Chapter 6 of F M White
Chapter 8 of Fox McDonald

Laminar pipe flow: Hagen-Poiseuille Flow

Steady, fully-developed, axisymmetric flow in a circular pipe



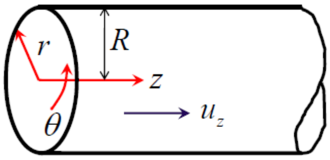
$$u_z = u_{\max}\left(1-\frac{r^2}{R^2}\right); u_r = u_\theta = 0 \qquad u_{\max} = -\frac{R^2}{4\mu}\frac{dp}{dz}$$

$$u_{av} = \frac{1}{\pi R^2}\int_0^R u.2\pi r dr = \frac{1}{2}u_{\max} \qquad \tau_w = -\mu\left[\frac{du}{dr}\right]_{r=R} = \frac{8\mu u_{av}}{d}$$

$$C_f = \frac{\tau_w}{\frac{1}{2}\rho u_{av}^{\ 2}} = \frac{8\mu u_{av}}{\frac{1}{2}\rho u_{av}^{\ 2} d} = \frac{16}{\mathrm{Re_d}}$$

Laminar pipe flow: Hagen-Poiseuille Flow

$$u_{\max} = 2u_{\mathrm{av}} = -\frac{R^2}{4\mu}\frac{dp}{dz}$$



$$\frac{dp}{dz} = \text{constant} = \frac{p_2 - p_1}{L}$$

$$h_f = \frac{p_1 - p_2}{\rho g} = -\frac{L}{\rho g}\frac{dp}{dz} = \frac{L}{\rho g}\frac{8\mu u_{\mathrm{av}}}{R^2} = \frac{L}{\rho g}\frac{32\mu u_{\mathrm{av}}}{d^2} = f\frac{L}{d}\frac{u_{\mathrm{av}}^2}{2g}$$

$$f = \frac{64}{\mathrm{Re_d}}$$

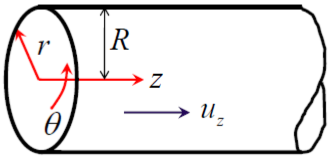
f : Darcy friction factor, $f = 4C_f$

C_f : Fanning friction factor (skin friction coefficient)

Darcy-Weisbach Equation: $h_f = f\dfrac{L}{d}\dfrac{u_{\mathrm{av}}^2}{2g}$

Laminar pipe flow: Hagen-Poiseuille Flow

$$u_{\max} = 2u_{\mathrm{av}} = -\frac{R^2}{4\mu}\frac{dp}{dz} \qquad \tau_w = \frac{8\mu u_{\mathrm{av}}}{d}$$



$$\frac{dp}{dz} = \text{constant} = \frac{p_2 - p_1}{L}$$

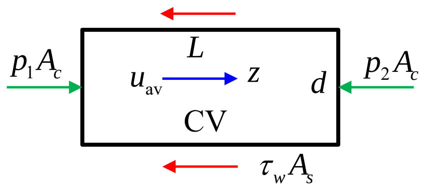
$$p_1 - p_2 = -L\frac{dp}{dz} = L\frac{32\mu u_{\mathrm{av}}}{d^2} = \frac{4L}{d}\frac{8\mu u_{\mathrm{av}}}{d} = \frac{4L}{d}\tau_w$$

$$h_f = \frac{p_1 - p_2}{\rho g} = \frac{4L}{d}\frac{\tau_w}{\rho g} = f\frac{L}{d}\frac{u_{\mathrm{av}}^2}{2g} \qquad f = \frac{8\tau_w}{\rho u_{\mathrm{av}}^2} \qquad f = 4C_f$$

This Eq. can be proved even without using the velocity profile

Fully developed pipe flow: force balance

$$p_1 A_c - p_2 A_c - \tau_w A_s = 0$$



$$\left(p_1 - p_2\right) A_c = \tau_w A_s$$

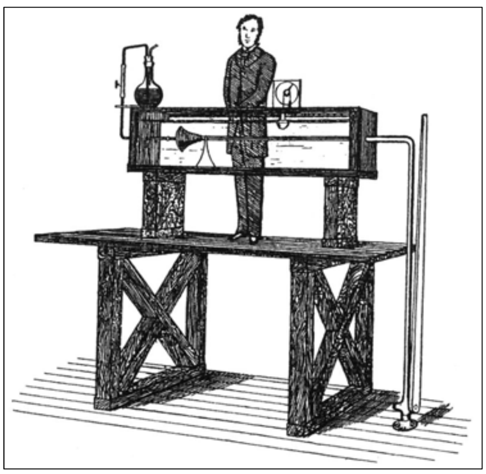
$$p_1 - p_2 = \frac{A_s}{A_c}\tau_w = \frac{\pi d L}{\frac{\pi}{4} d^2}\tau_w = \frac{4L}{d}\tau_w$$

$$h_f == f\frac{L}{d}\frac{u_{av}^2}{2g} \qquad f = \frac{8\tau_w}{\rho u_{av}^2} = 4C_f$$

The applicability of these Equations is not limited to any particular velocity profile;

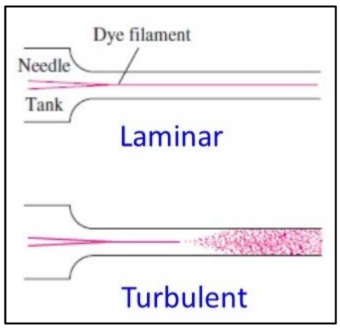
The only assumption: no momentum change (fully-dev)

Reynold’s experiments, 1880

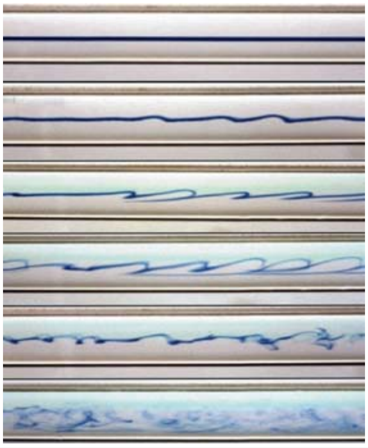


Osborne Reynolds with his experimental setup

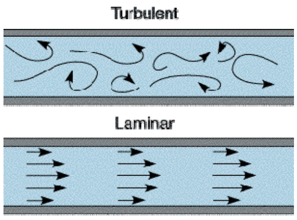
Osborne Reynolds
1842-1912



Laminar vs Turbulent Flow



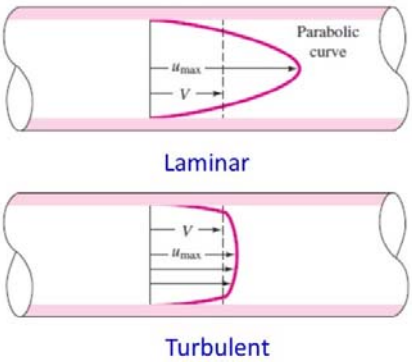
Reynolds experiment reproduced; laminar to transient transition in a pipe flow



o Turbulent flow shows small-scale random fluctuation in flow variables (velocity, pressure); averaged quantities are well-behaved

o Flow is inherently 3-D and unsteady; averaged quantities may be steady, 2-D or fully-developed

Turbulent Pipe Flow



Velocity profile looks more uniform due to increased momentum transport in r -direction

Transition depends on Reynolds number

Laminar: $\mathrm{Re}_d < 1800$

Turbulent: $\mathrm{Re}_d >\sim 2000$

Engineering applications of pipe flow are in the turbulent regime