Fluid Mechanics and Rate Processes: Tutorial 10

P1. A small low-speed wind tunnel (Fig. P1) is being designed for calibration of hot wires. The air is at 19°C. The test section of the wind tunnel is 30cm×30cm×30cm. The flow through the test section must be as uniform as possible. The wind tunnel speed ranges from 1 to 8 m/s, and the design is to be optimized for an air speed of V = 4.0 m/s through the test section. (*a*) For the case of nearly uniform flow at 4.0 m/s at the test section inlet, by how much will the centerline air speed accelerate by the end of the test section? (*b*) Recommend a design that will lead to a more uniform test section flow.

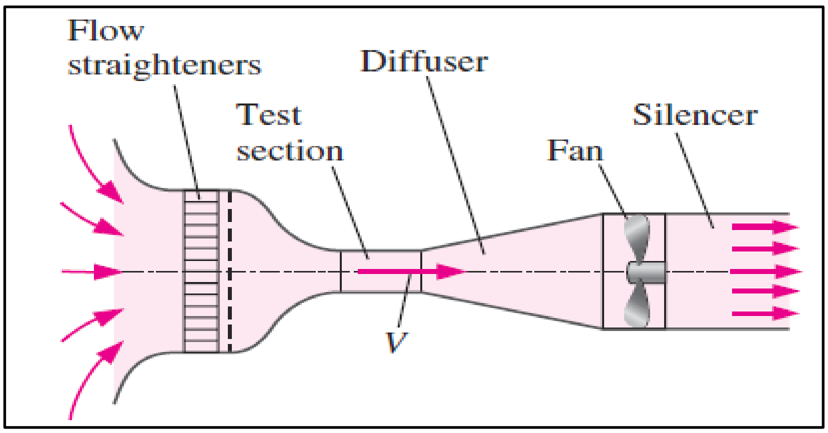


Fig.P1

SOLUTION The acceleration of air through the square test section of a wind tunnel is to be calculated, and a redesign of the test section is to be recommended.

Assumptions **1** The flow is steady and incompressible. **2** The walls are smooth, and disturbances and vibrations are kept to a minimum. **3** The boundary layer is laminar.

Properties The kinematic viscosity of air at 19°C is $\nu = 1.507\times 10^{-5}$ m^2/s.

Analysis **(*a*)** The Reynolds number at the end of the test section is approximately

V=4.0 m/s, then $$\mathrm{Re}_x = \frac{V\times 0.30\mathrm{m}}{\nu} = 7.96\times 10^4$$

Since Re_x is lower than the engineering critical Reynolds number, $Re_{x,cr} = 5 \times 10^5$, and is even lower than $Re_{x,\text{ critical}} = 1 \times 10^5$, and since the walls are smooth and the flow is clean, we may assume that the boundary layer on the wall remains laminar throughout the length of the test section. As the boundary layer grows along the wall of the wind tunnel test section, air in the region of irrotational flow in the central portion of the test section accelerates in order to satisfy conservation of mass.

The boundary-layer and displacement thickness at the end of the test section,

$$\frac{\delta}{x} = 4.91\,\text{Re}_x^{-\frac{1}{2}} \qquad \delta = 5.22\text{mm} \quad (\text{for } x = 30\text{cm})$$

$$\frac{\delta^*}{x} = 1.72\,\text{Re}_x^{-\frac{1}{2}} \qquad \delta^* = 1.83\text{mm} \quad (\text{for } x = 30\text{cm})$$

We apply conservation of mass to calculate the average air speed at the end of the test section,

$$V_{end} A_{end} = V_{begining} A_{begining}$$

$$\rho V \times (30\text{cm})^2 = \rho V_{\text{exit}} \times \left(30\text{cm} - 2\delta^*\right)^2$$

$$V_{\text{exit}} = V \times \left(\frac{30\text{cm}}{30\text{cm} - 2\delta^*}\right)^2 \qquad \Rightarrow V_{\text{exit}} = 4.1\text{m/s}$$

Thus the air speed increases by approximately 2.5 percent through the test section, due to the effect of displacement thickness.

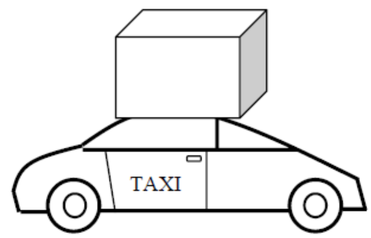
(*b*) Test section may be designed as a diverging duct to avoid flow acceleration.

P2. Advertisement signs are commonly carried by taxicabs for additional income, but they also increase the fuel cost. Consider a sign that consists of a 0.30-m-high, 0.9-m-wide, and 0.9-m-long rectangular block mounted on top of a taxicab such that the sign has a frontal area of 0.3 m by 0.9 m from all four sides. Determine the increase in the annual fuel cost of this taxicab due to this sign. Assume the taxicab is driven 60,000 km a year at an average speed of 50 km/h and the overall efficiency of the engine is 28 percent. Take the density, unit price, and heating value of gasoline to be 0.75 kg/L, Rs65/L, and 42,000 kJ/kg, respectively, and the density of air to be 1.25 kg/m^3.

Fig.P2

Solution An advertisement sign in the form of a rectangular block that has the same frontal area from all four sides is mounted on top of a taxicab. The increase in the annual fuel cost due to this sign is to be determined.

Assumptions **1** The flow of air is steady and incompressible. **2** The car is driven 60,000 km a year at an average speed of 50 km/h. **3** The overall efficiency of the engine is 28%. **4** The effect of the sign and the taxicab on the drag coefficient of each other is negligible (no interference), and the edge effects of the sign are negligible (a crude approximation). **5** The flow is turbulent so that the tabulated value of the drag coefficient can be used.



Properties The densities of air and gasoline are given to be 1.25 kg/m^3 and 0.75 kg/L, respectively. The heating value of gasoline is given to be 42,000 kJ/kg. The drag coefficient for a square rod for normal flow is C_D = 2.2 (Table 11-1).

Analysis Noting that 1 m/s = 3.6 km/h, the drag force acting on the sign is

$$F_D = C_D A \frac{\rho V^2}{2} = (2.2)(0.9 \times 0.3\ \text{m}^2)\frac{(1.25\ \text{kg/m}^3)(50/3.6\ \text{m/s})^2}{2}\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right) = 71.61\ \text{N}$$

Noting that work is force times distance, the amount of work done to overcome this drag force and the required energy input for a distance of 60,000 km are

$$W_{\text{drag}} = F_D L = (71.61\ \text{N})(60{,}000\ \text{km/year}) = 4.30 \times 10^6\ \text{kJ/year}$$

$$E_{in} = \frac{W_{\text{drag}}}{\eta_{\text{car}}} = \frac{4.30 \times 10^6\ \text{kJ/year}}{0.28} = 1.54 \times 10^7\ \text{kJ/year}$$

Then the amount and cost of the fuel that supplies this much energy are

$$\text{Amont of fuel} = \frac{m_{\text{fuel}}}{\rho_{\text{fuel}}} = \frac{E_{\text{in}}/\text{HV}}{\rho_{\text{fuel}}} = \frac{(1.54 \times 10^7\ \text{kJ/year})/(42{,}000\ \text{kJ/kg})}{0.75\ \text{kg/L}} = \mathbf{489\ L/year}$$

Cost = (Amount of fuel)(Unit cost) = Rs 31785/year

Discussion Note that the advertisement sign increases the fuel cost of the taxicab significantly. The taxicab operator may end up losing money by installing the sign if he/she is not aware of the major increase in the fuel cost, and negotiate accordingly.

P3. Perform a flat-plate momentum analysis on given sinusoidal profile:

$$\frac{u}{U} \approx \sin\left(\frac{\pi y}{2\delta}\right)$$

Compute momentum-integral estimates of C_f, δ/x, δ^*/x, and δ^{**}/x.

Solution: Carry out the same integrations as Section 7.2, but results are more accurate:

$$\delta^{**} = \int_0^\delta \frac{u}{U}\left(1-\frac{u}{U}\right)dy \approx \frac{4-\pi}{2\pi}\delta = 0.1366\delta; \quad \delta^* = \int_0^\delta \left(1-\frac{u}{U}\right)dy \approx \frac{\pi-2}{\pi}\delta = 0.3634\delta$$

$$\tau_w \approx \mu\frac{\pi U}{2\delta} = \rho U^2 \frac{d}{dx}\left[\frac{4-\pi}{2\pi}\delta\right], \quad \text{integrate to:} \quad \frac{\delta}{x} \approx \frac{\pi\sqrt{2}/\sqrt{(4-\pi)}}{\sqrt{Re_x}} \approx \frac{4.80}{\sqrt{Re_x}} \quad (5\%\ \text{low})$$

Substitute these results back to obtain the desired (accurate) dimensionless expressions:

$$\frac{\delta}{x} \approx \frac{\mathbf{4.80}}{\sqrt{\mathbf{Re_x}}}; \quad C_f = \frac{\delta^{**}}{x} \approx \frac{\mathbf{0.655}}{\sqrt{\mathbf{Re_x}}}; \quad \frac{\delta^*}{x} \approx \frac{\mathbf{1.743}}{\sqrt{\mathbf{Re_x}}}; \quad \frac{\delta^{**}}{x} = \frac{0.655}{\sqrt{Re_x}} \qquad \textit{Ans.}\ (a, b, c, d)$$

P4. Consider laminar flow past the square-plate arrangements in the figure below. Compared to the drag of a single plate (1), how much larger is the drag of four plates together as in configurations (a) and (b)? Explain your results.

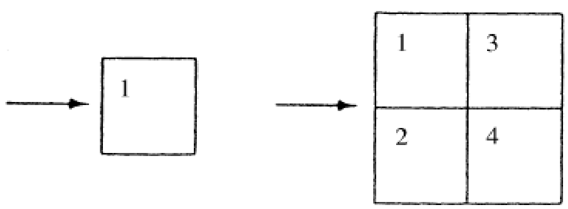


Fig.P4(a)

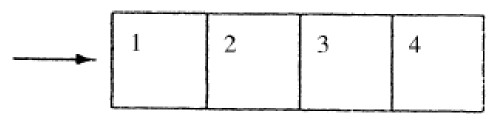


Fig.P4(b)

Solution: The laminar formula $C_D = 1.328/Re_L{}^{1/2}$ means that $C_D \propto L^{-1/2}$. Thus:

$$\text{(a)}\ \ F_a = \frac{const}{\sqrt{2L_1}}(4A_1) = \sqrt{8}F_1 = \mathbf{2.83F_1} \quad \textit{Ans.}\text{ (a)}$$

$$\text{(b)}\ \ F_b = \frac{const}{\sqrt{4L_1}}(4A_1) = \mathbf{2.0F_1} \quad \textit{Ans.}\text{ (b)}$$

The plates near the trailing edge have less drag because their boundary layers are thicker and their wall shear stresses are less. These configurations do ***not*** quadruple the drag.