Fluid Mechanics and Rate Processes: Tutorial 12

P1. Stainless steel ball bearings (ρ=8085 kg/m^3, k =15.1 W/m · °C, C_p= 0.480 kJ/kg · °C, and α=3.91x10^{-6} m^2/s) having a diameter of 1.2 cm are to be quenched in water. The balls leave the oven at a uniform temperature of 900°C and are exposed to air at 30°C for a while before they are dropped into the water. If the temperature of the balls is not to fall below 850°C prior to quenching and the heat transfer coefficient in the air is 125 W/m^2 · °C, determine how long they can stand in the air before being dropped into the water.

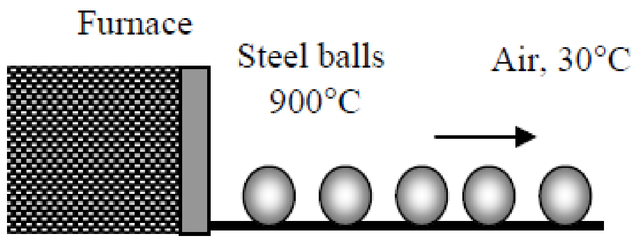

Fig.P1

Solution:

Assumptions **1** The bearings are spherical in shape with a radius of r_o = 0.6 cm. **2** The thermal properties of the bearings are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** The Biot number is Bi < 0.1 so that the lumped system analysis is applicable (this assumption will be verified).

Properties The thermal conductivity, density, and specific heat of the bearings are given to be k = 15.1 W/m.°C, ρ = 8085 kg/m^3, and c_p = 0.480 kJ/kg.°F.

Analysis The characteristic length of the steel ball bearings and Biot number are

$$L_c = \frac{V}{A_s} = \frac{\pi D^3/6}{\pi D^2} = \frac{D}{6} = \frac{0.012\text{ m}}{6} = 0.002\text{ m}$$

$$Bi = \frac{hL_c}{k} = \frac{(125\text{ W/m}^2.°\text{C})(0.002\text{ m})}{(15.1\text{ W/m.°C})} = 0.0166 < 0.1$$

Therefore, the lumped system analysis is applicable.
Then the allowable time is determined to be

$$b = \frac{hA_s}{\rho c_p V} = \frac{h}{\rho c_p L_c} = \frac{125\text{ W/m}^2.°\text{C}}{(8085\text{ kg/m}^3)(480\text{ J/kg.°C})(0.002\text{ m})} = 0.01610\text{ s}^{-1}$$

$$\frac{T(t)-T_\infty}{T_i - T_\infty} = e^{-bt} \longrightarrow \frac{850-30}{900-30} = e^{-(0.0161\text{ s}^{-1})t} \longrightarrow t = \mathbf{3.68\text{ s}}$$

The result indicates that the ball bearing can stay in the air about 4 s before being dropped into the water.