Tutorial 1

P1.56 $U = 10.8\ m/s$, $\delta = 3\ cm$

$T = 20^\circ c$, $P = 1\ atm$

(a) $$\tau_w = \mu \frac{\partial u}{\partial y}\bigg|_{y=0} = \frac{\mu U \pi}{2\delta} \cos \frac{\pi y}{2\delta}\bigg|_{y=0}$$

$$= \frac{\mu U \pi}{2\delta}$$

$\mu_{He} = 1.97 \times 10^{-5}$ Pa.s (Table 4.4)

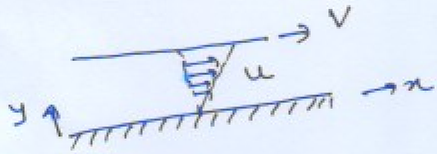
(b) $\tau = \frac{1}{2} \tau_w$

$$\frac{\mu U \cancel{\pi}}{\cancel{2}\delta} \cos \frac{\pi y}{2\delta} = \frac{1}{2} \frac{\mu U \cancel{\pi}}{\cancel{2}\delta}$$

$$\cancel{\frac{\pi y}{2\delta}} = \frac{2\cancel{\pi}}{6}$$

$$y = \frac{2}{3}\delta$$

P1.57



$$u(y) = \frac{V}{h} y$$

$$\tau\big|_{y=h} = \mu \frac{\partial u}{\partial y}\bigg|_{y=h} = \frac{\mu V}{h}$$

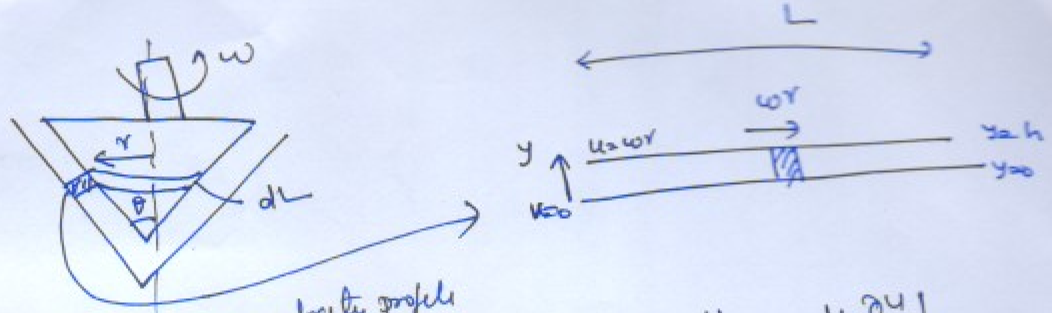
$$F = \tau A = \left(\frac{\mu V}{h}\right) bL$$

$$P = F.V = \frac{\mu V^2}{h} bL$$

$\mu_{SAE\ 30W\ oil} = 0.29$ Pa.s

$$P = \frac{(0.29)(2.5)^2 (0.60)(2)}{0.03} = 73\ W$$

P1. 58



Assuming linear velocity profile

$$u = \left(\frac{\omega r}{h}\right) y \Rightarrow \tau_w = \frac{\omega r}{h}\mu = \mu \left.\frac{\partial u}{\partial y}\right|_{y=h}$$

$$dF = \tau_w \, dA = \frac{\mu \omega r}{h}\left(2\pi r \, dL\right)$$

$$L = \frac{r}{\sin\theta} \Rightarrow dL = \frac{dr}{\sin\theta}$$

$$dF = \frac{\mu \omega r}{h} \frac{2\pi r \, dr}{\sin\theta}$$

$$d(\text{Torque}) = r . dF = r\left(\frac{\mu \omega r}{h}\right)\frac{2\pi r}{\sin\theta} dr$$

$$= \frac{\mu\omega}{h\sin\theta} 2\pi r^3 \, dr$$

$$M = \text{Torque} = \int_0^{r_0} \frac{\mu\omega}{h\sin\theta} 2\pi r^3 dr = \frac{\pi\mu\omega r_0^4}{2h\sin\theta}$$

if there is no applied Torque, the cone will slow down due to viscous torque

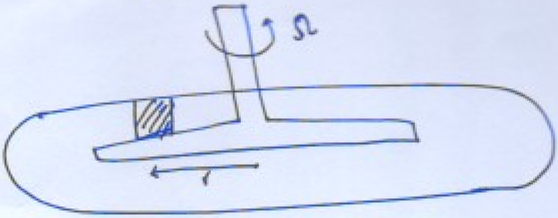
$$M = -I_0 \frac{d\omega}{dt} \qquad I_0 = \frac{3}{10} m r_0^2 \qquad m = \text{cone mass}$$

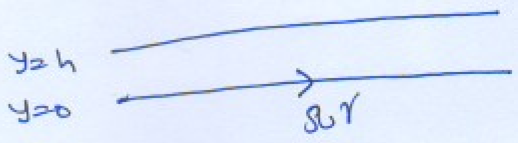
$$\frac{\pi\mu\omega r_0^{\cancel{4}\,2}}{\cancel{2}h\sin\theta} = -\frac{3}{\cancel{10}_5} m \cancel{r_0^2} \frac{d\omega}{dt}$$

$$\int_{\omega_0}^{\omega} \frac{d\omega}{\omega} = -\frac{5}{3m}\frac{\pi\mu r_0^2}{h\sin\theta}\int_0^t dt$$

$$\ln\frac{\omega}{\omega_0} = -\frac{5\pi\mu r_0^2 t}{3mh\sin\theta} \Rightarrow \omega = \omega_0 \exp\left[\frac{-5\pi\mu r_0^2 t}{3mh\sin\theta}\right]$$

P1.59





$u = \left(\Omega r\right)$ at $y=0$

$u = 0$ at $y=h$

$u = \left(\frac{0-\Omega r}{h-0}\right) y + c \Rightarrow u = -\left(\frac{\Omega r}{h}\right) y + c$

at $y=h$ $u=0$

$0 = -\Omega r + c \Rightarrow c = \Omega r$

$u = -\left(\frac{\Omega r}{h}\right) y + \Omega r = \Omega r\left(1-\frac{y}{h}\right)$

$\tau_w = \left| \mu \frac{du}{dy}\Big|_{y=0} \right| = \left| \mu \Omega r \left(-\frac{1}{h}\right) \right| = \frac{\mu \Omega r}{h}$

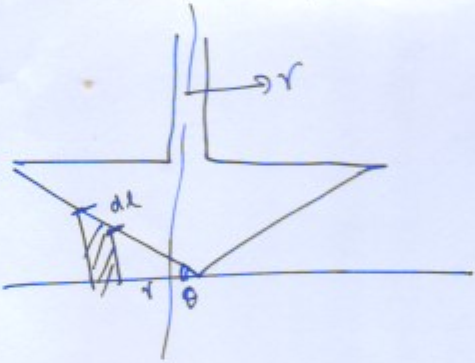
$dF = \tau_w \, dA = \frac{\mu \Omega r}{h} 2\pi r \, dr$

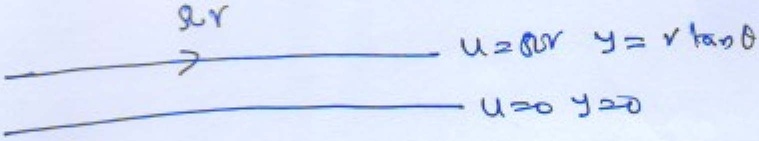
$dM = dF \cdot r = \frac{2\mu\Omega\pi}{h} r^3 \, dr$

On both side $= 2dm = \left(\frac{4\pi\mu\Omega}{h}\right) r^3 \, dr$

$m = \int_0^R r^3 dr \times \frac{4\pi\mu\Omega}{h} = \frac{\pi\mu\Omega}{h} R^4$

P1.61





$$u = \left(\frac{\Omega r}{r\tan\theta}\right) y$$

$$\tau_w = \mu \left.\frac{du}{dy}\right|_{y = r\tan\theta} = \frac{\mu \Omega r}{r\tan\theta}$$

$$dF = \tau_w\, dA = \frac{\mu\Omega r}{r\tan\theta}\, 2\pi r\, dl$$

$$l = \frac{r}{\cos\theta} \qquad dl = \frac{dr}{\cos\theta}$$

$$dF = \frac{\mu\Omega r}{r\tan\theta}\, 2\pi r\, \frac{dr}{\cos\theta}$$

$$dM = dF \cdot r = \frac{\mu\Omega r\, 2\pi r}{r\tan\theta\cos\theta} \cdot r\, dr$$

$$\int dM = \int_0^R \frac{2\pi\mu\Omega r^2}{\sin\theta}\, dr$$

$$M = \frac{2\pi\mu\Omega R^3}{3\sin\theta} \Rightarrow \mu = \frac{3M\sin\theta}{2\pi\Omega R^3}$$