Fluid Mechanics and Rate Processes: Tutorial 5

P1. A viscous liquid of constant density and viscosity falls due to gravity between two parallel plates a distance 2h apart, as in the figure. The flow is fully developed, that is, w = w(x) only. There are no pressure gradients, only gravity. Set up and solve the Navier-Stokes equation for the velocity profile w(x).

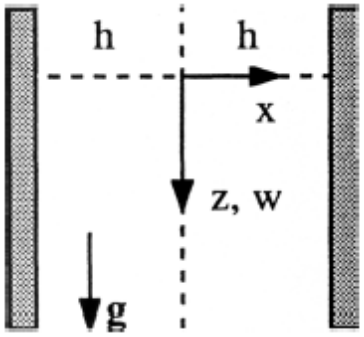

Fig.P1

Solution: Only the z-component of Navier-Stokes is relevant:

$$\rho \frac{dw}{dt} = 0 = \rho g + \mu \frac{d^2 w}{dx^2}$$

$$\text{or} \quad w'' = -\frac{\rho g}{\mu} \qquad (1)$$

With boundary condition,

$$w(-h) = w(+h) = 0 \qquad \text{(no-slip)}$$

After integrating equation (1) and using boundary condition,

$$w = \frac{\rho g}{2\mu}(h^2 - x^2) \qquad \text{Ans.}$$