Reynolds transport theorem

$$\frac{dB_{sys}}{dt} = \frac{d}{dt}\int_{CV} \beta \rho \, d\forall + \int_{CS} \beta \rho (V.n) \, dA$$

Linear momentum balance

$$B = \underline{mV} \Rightarrow \beta = \frac{d(mV)}{dm} = V$$

$$\Rightarrow \frac{d(mV)_{sys}}{dt} = \underline{\Sigma F} = \frac{d}{dt}\int_{CV} V \rho \, d\forall + \int_{CS} \underline{V \rho (V.n) \, dA} \quad \leftarrow \text{vector equation}$$

$V =$ velocity with respect to an inertial frame of reference

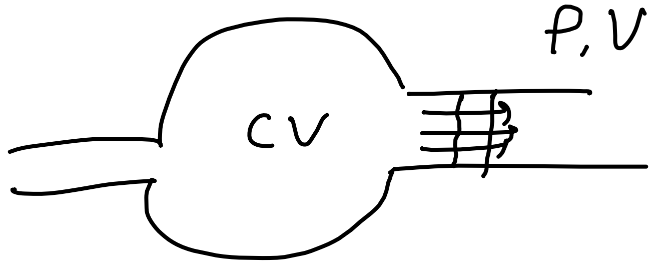
x-momentum balance

$$V = u\hat{i} + v\hat{j} + w\hat{k}$$

$$\Sigma F_x = \frac{d}{dt}\int u \rho \, d\forall + \int \underline{u} \, \underline{\rho (V.n) \, dA}$$

$V \equiv V_r = V - V_s$
$u = u - u_s$

$\rho, V =$ constant

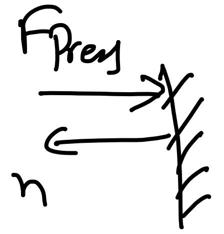


if ρ, V do not depend on the cross-section area

$$\Sigma F_x = \frac{d}{dt}\int u \rho \, d\forall + \Sigma(\dot{m}_i V_i)_{out} - \Sigma(\dot{m}_i v_i)_{in}$$

How to calculate pressure force?

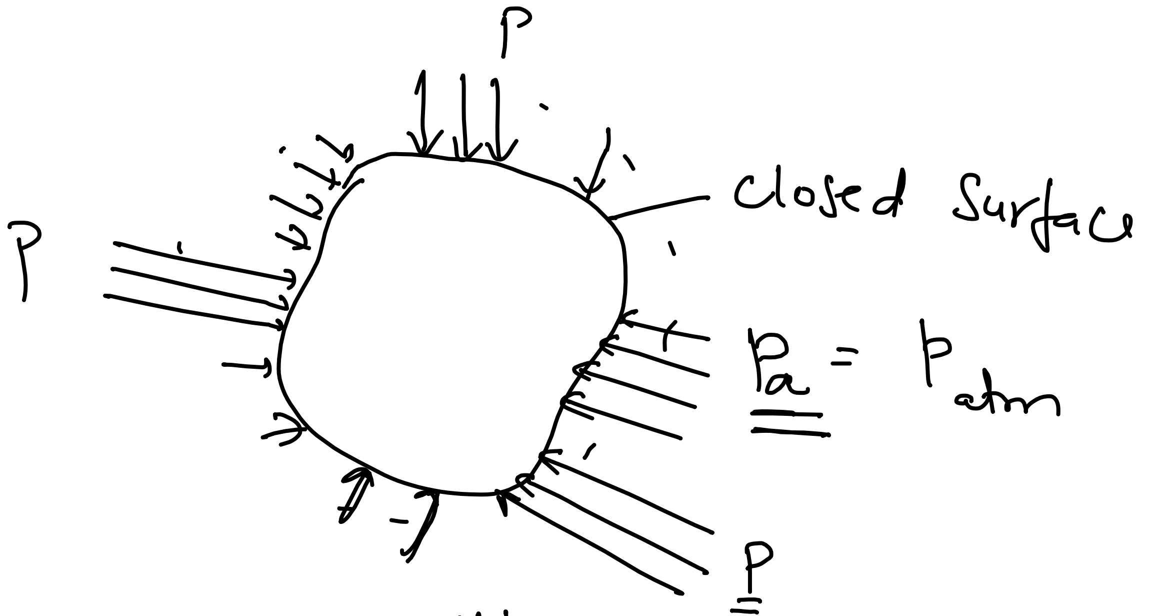
$$F_{Press} = \int_{cs} P\,(-n)\,dA$$

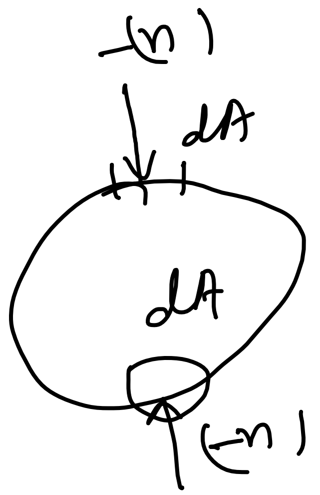


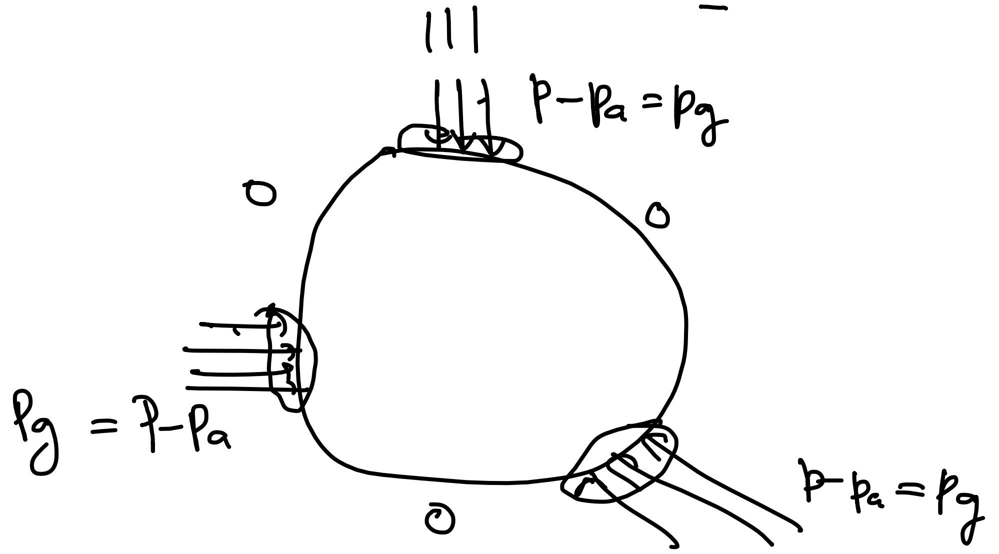
$$F_{uniform} = -P_a \int_{cs} n\,dA = -P_a(0) = 0$$

$$F_{Press} = F_{Press} - F_{uniform}$$

$$= \int (P - P_a)(-n)\,dA = \int (P_{gage})(-n)\,dA$$

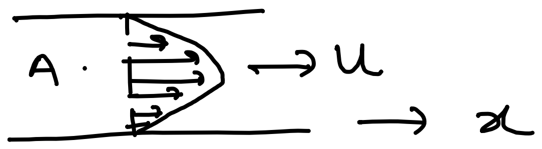






momentum correction factor

$$\text{rate of momentum flow} = \int \rho\, u\, \underbrace{(V \cdot n)}_{u}\, dA$$



$$= \int \rho u^2 dA = \beta\, \rho\, V_{avg}\, A\, V_{avg}$$

$n = \hat{i}$

$V = u\hat{i}$

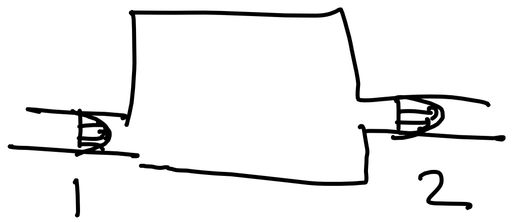
$$\int \rho u^2 dA = \beta \rho V_{avg}^2 A$$

$$V_{avg} = \frac{\int u\,dA}{A}$$

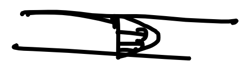
Avg of square $\neq$ square of avg

$$\boxed{\beta = \frac{1}{A}\int \frac{u^2}{V_{avg}^2}\, dA}$$

$$\sum F_x = \frac{d}{dt}\int u\,\rho\, d\forall + \sum\left(\beta_2\, \rho\, V_{avg_2}^2 A\right) - \sum \beta_1\, \rho\, V_{avg_1}^2 A$$



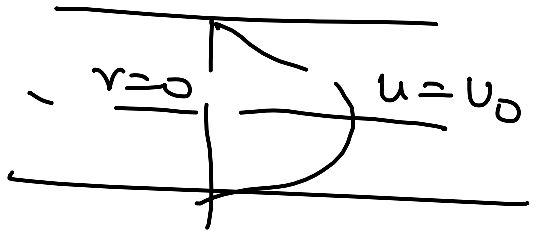
Laminar flow V = low $u = u_0\left[1 - \left(\frac{r}{R}\right)^2\right]$ Pipe



$$\beta = \frac{1}{A}\int \left(\frac{u^2}{u_{avg}^2}\right) 2\pi r\, dr$$

$$= \frac{4}{3}$$

$$u_{avg} = \frac{\int u\, 2\pi r\, dr}{\pi R^2}$$

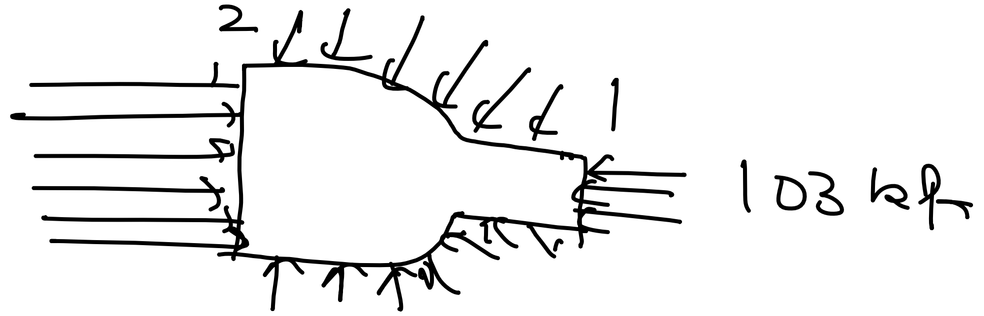


Turbulent flow V = high $u = u_0\left[1 - \frac{r}{R}\right]^m$ $\frac{1}{9} \le m \le \frac{1}{5}$

$$\beta = \frac{(1+m)^2\,(2+m)^2}{2(1+2m)\,(2+2m)}$$

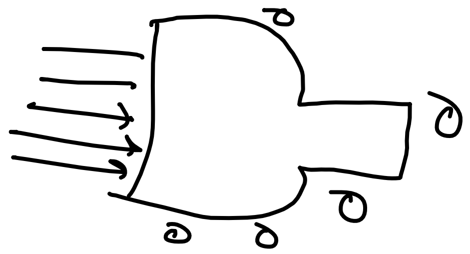
$\beta \approx 1.0$

Example 1: A control volume of a nozzle section has surface pressure of 276 kPa (abs) at Section 1 and atmospheric pressure 103 kPa absolute at Section 2 and the external rounded part of the nozzle. Compute the net pressure force if $D_1 = 75$ mm and $D_2 = 25$ mm



$\equiv$

173 kPa 2

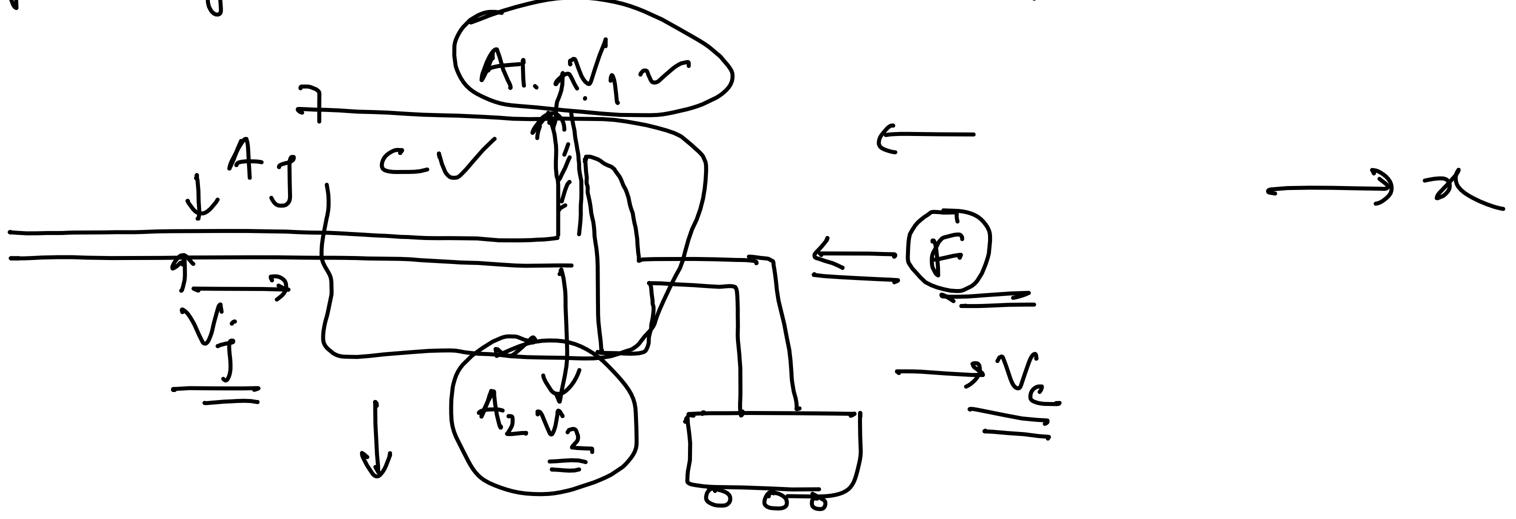


$$F_{Pres} = \int p\,(-n)\, dA$$

$$= (173\times10^3)(-1)(-i)\,\frac{\pi D_2^2}{4}$$

$$= (\quad\quad)\ i$$

Example 2 A water jet of velocity V_j impinges normal to a flat plate that moves to the right at velocity V_c as shown in fig below. Find the force required to keep the plate moving at constant velocity if the jet density is 1000 kg/m³, the jet area is 3 cm² and V_J and V_c are 20 and 15 m/s respectively. Neglect the weight of the jet and plate and assume steady flow with respect to moving plate with the jet splitting into an equal upward and downward half jet.



$A_1 = A_2 = \frac{1}{2} A_j$

Solution

RTT mass balance

$$0 = \cancel{\frac{d}{dt}\int \rho\, d\forall}^{\,0} + \int \rho (V\cdot n)\, dA$$

$\rho =$ constant

$$0 = \rho(+)V_1A_1 + \rho V_2A_2 + \rho(V_j - V_c)(-1)A_j$$

$$V_1A_1 + V_2A_2 = A_j(V_j - V_c)$$

$$(V_1 + V_2) = 2(V_j - V_c)$$

$$V_1 + V_2 = 2 \times 5 \text{ m/s} \quad \text{--- (1)}$$

y-momentum balance

$$\sum F_y = 0 = \cancel{\frac{d}{dt}\int v\rho\, d\forall}^{\,0} + \int v\rho (V\cdot n)\, dA$$

$$0 = V_1\cancel{\rho}(V_1)\cancel{A_1} - (V_2\cancel{\rho}\, V_2 \cancel{A_2})$$

$$V_1 = V_2$$

x-momentum

$$-F_x = \cancel{\frac{d}{dt}}^{\,0} + (0) + \rho(V_j - V_c)_x \cdot \left(-(V_j - V_c)A_J\right)$$

$$= -\rho(V_j - V_c)^2 A_J$$