→ Aim of Solving Navier-stokes(NS) is to find drag force on a surface (

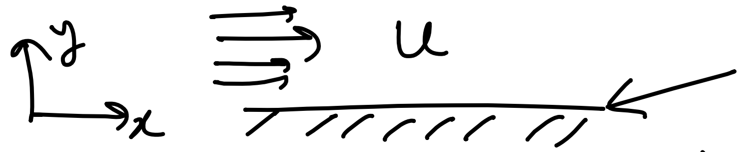
→ Problem geometry → velocity profile → stresses → forces

→ But NS can be solved only for simple problems

→ Example: Finding drag force on a submarine is too complicated for analytical solution of NS.

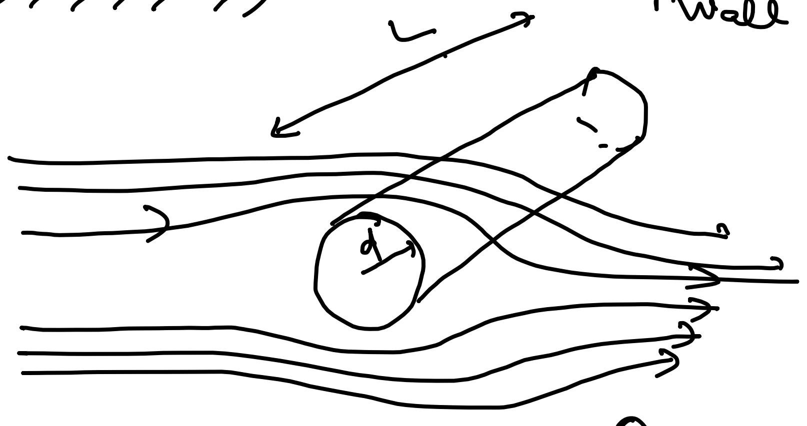
Drag force : Net force on a solid in the direction of fluid flow

Simple problem $\xrightarrow{e.g.}$



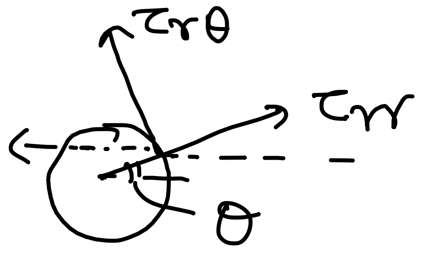
$\tau_{yx}\big|_{wall} = \tau_{yx}\big|_{y=0}$, Drag force $F_d = \int \tau_{yx}\big|_{y=0} \, dA$

A more complicated Problem



$V = v_r \hat{r} + v_\theta \hat{\theta}$

→



$$\text{Drag force} = \int \tau_{rr}\big|_R \cos\theta \, dA - \int \tau_{r\theta}\big|_R \sin\theta \, dA$$

To find τ_{rr} & $\tau_{r\theta}$ we need v_r & v_θ

→ 2 D problem

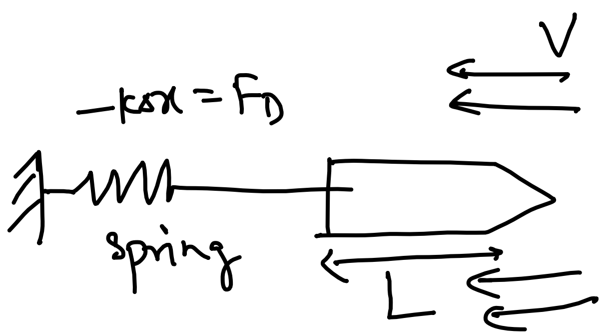
$$\text{Drag Coefficient } C_D = \frac{\text{Drag force}}{\frac{1}{2}\rho U^2 \,(\text{Projected})\,\text{Area}}$$

$$C_{D\,\text{cylinder}} = \frac{F_D}{\frac{1}{2}\rho U^2 L d} \;, \quad C_{D\,\text{Sphere}} = \frac{F_D}{\frac{1}{2}\rho U^2 \frac{1}{4}\pi d^2}$$

; U = Velocity far from surface

Dimensional Analysis = Theory + experiment → to ~~for~~ example drag

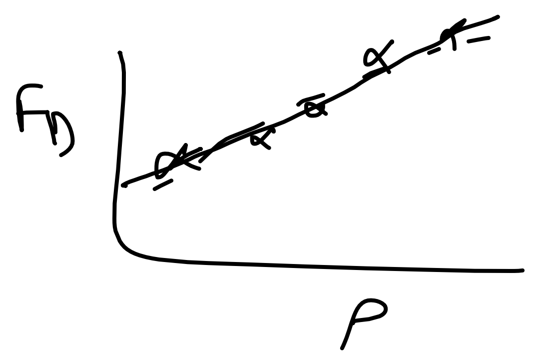
$-kx = F_D$



fluid, (ρ, μ)

$$F_D = f(\rho, \mu, V, L)$$

$10 \times 10 \times 10 \times 10 \rightarrow 10^4$



$\mu, V, L = \text{const}$

(i) Reduction no. of independent variables

$$\left(\frac{F_D}{\rho V^2 L^2}\right) = g\left(\frac{\rho V L}{\mu}\right)$$

$$C_f = g(Re)$$

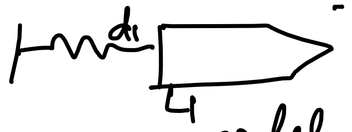
Air ⇐ Incompressible

Scale

(ii) Scaling law

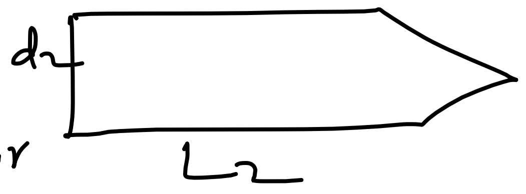
water

Prototype



model
wind tunnel

$C_f = g(Re)$



$$\frac{L_2}{L_1} = \frac{d_2}{d_1}$$

— geometrical similar

— Dynamic similarity ⇒ $Re = \text{constant}$

e $\quad (Re)_m = (Re)_{Prototype} \Rightarrow (C_f)_m = (C_f)_{Prototype}$

$$\frac{F_{D,p}}{F_{Dm}} = \left(\frac{\rho_p}{\rho_m}\right)\left(\frac{V_p}{V_m}\right)^2\left(\frac{L_p}{L_m}\right)^2$$

The Buckingham PI theorem

- for reducing no. of dimensional variables into dimensionless groups.

"if a physical process involves $\underline{n}$ dimensional variables, it can be reduced to a relation between only $\underline{k}$ dimensionless variables or Π_s.
The reduction $\underline{\underline{J}} = n - k$ equals the maximum no. of variables that do not form a Pi among themselves and is always less than or equal to the number of fundamental dimensions describing the variables".

$$\underline{f} = f(\rho, \mu, L, V) \qquad n = 5$$

$$\rho: \underline{M}L^{-3}, \quad \mu: ML^{-1}\underline{T}^{-1}, \quad L, \quad LT^{-1} \qquad J_{max} = 3$$

$$\underline{\underline{k}} = n - J \geq 5 - 3 \geq \underline{\underline{2}}$$

- if we $J = J_{max} = 3$ $\qquad k = 2$ (exactly)

  if we $J < J_{max}$ $\qquad$ reduce J by 1 $\rightarrow$ look $\underline{\underline{J_{max} - 1}}$ variables that do not make a Pi group among themselves

Steps for dimensional Analysis

(i) List the n- variables

(ii) List dimension of each variables according to $\{M L T \theta\}$

(iii) Find J. initially guess J equal to the no. of different dimensions present and look for J variables that do not form a Pi product.

if No Luck → reduce J by 1 and look again

iv) Select $\underline{\underline{J}}$ <u>scaling variable</u> that do not form a Pi product

v) Add one additional variable to your J <u>repeating variable</u> and form a power product.
Algebraically find the exponents that make the product dimensionless

vi) write the final dimensionless function and check to make sure all Pi are dimensionless.

$a+b-3c=0$

$-b=0$

$c=0$

<u>Example</u>: $F = f(\underbrace{L, V, \rho, \mu}_{MLT})$

Step i: $n=5$

Step 2: $J_{max}=3$ take $\underline{\underline{J}} = \underline{\underline{J_{max}}} = \underline{\underline{3}}$

$L^a V^b \rho^c \Rightarrow M^0 L^0 T^0$ only and only if

$L^a (LT^{-1})^b (ML^{-3})^c = M^0 L^0 T^0$ either $a=0$, or $b=0$, or $c=0$

repeating variables = L, V, ρ

$L, V, \mu \longrightarrow$

$\Pi_1 = (L^{a_1} V^{b_1} \rho^{c_1}) F = M^0 L^0 T^0 \longrightarrow$ $a_1 = -2$, $b_1 = -2$, $c_1 = -1$

$\Pi_2 = (L^{a_2} V^{b_2} \rho^{c_2}) \mu = M^0 L^0 T^0$

$$\Pi_1 = \frac{F}{L^2 V^2 \rho}$$

$a_2 = b_2 = c_2 = 1$ $\qquad$ $a_2 = b_2 = c_2 = -1$

$$\Pi_2 = \frac{L V \rho}{\mu} = Re \qquad \Pi_2 = \frac{\mu}{L V \rho}$$