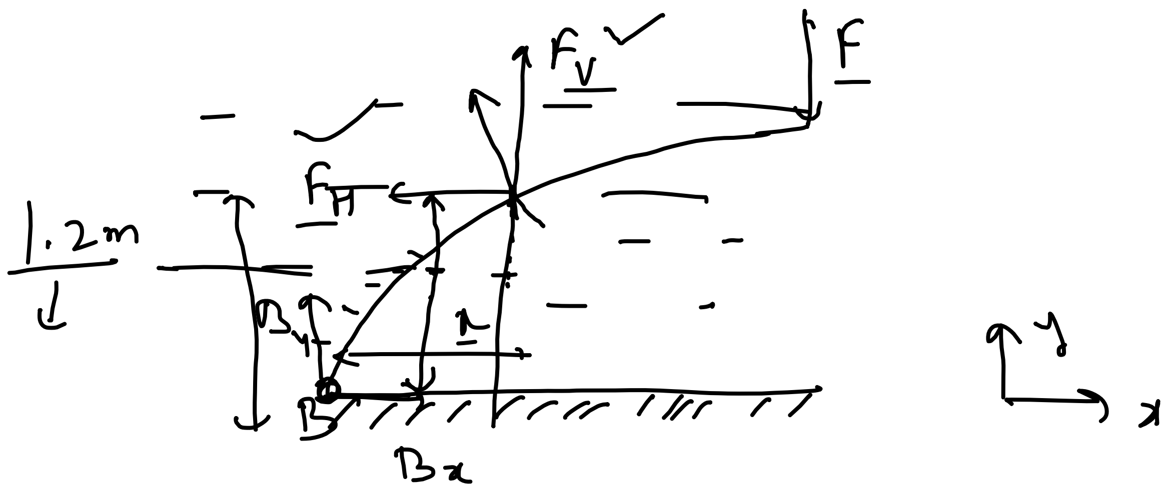
Quiz 1 - Solution

Que 1

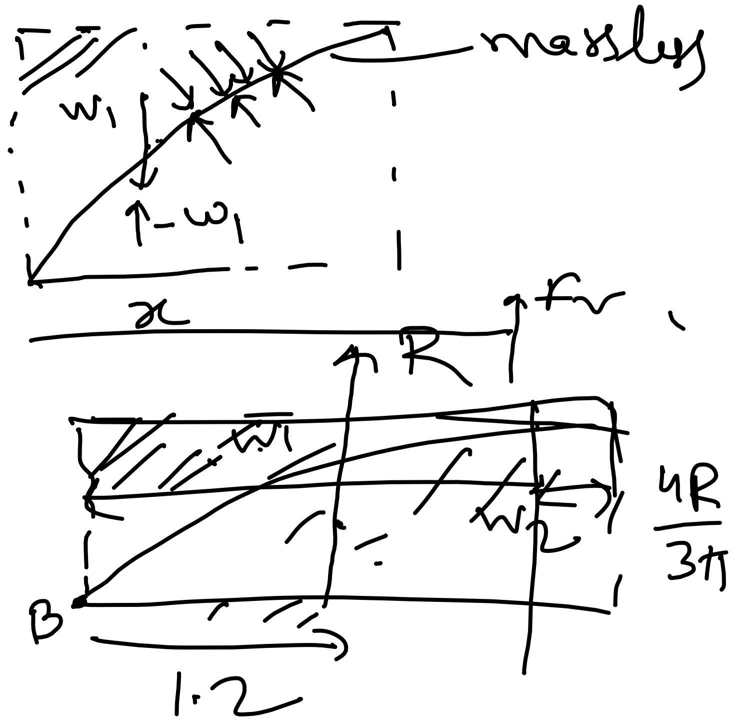


Method 1 → more detail

$F_H = \rho g h_{cg} A_{Project}$

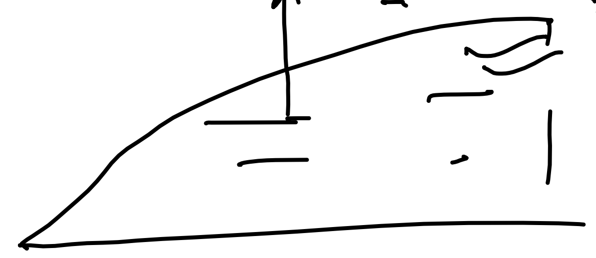
y_{cp} →

F_V How to find F_V and its line of action



⇒

$-W_1 = F_V$



$\vec{R} = \vec{W_1} + \vec{W_2}$

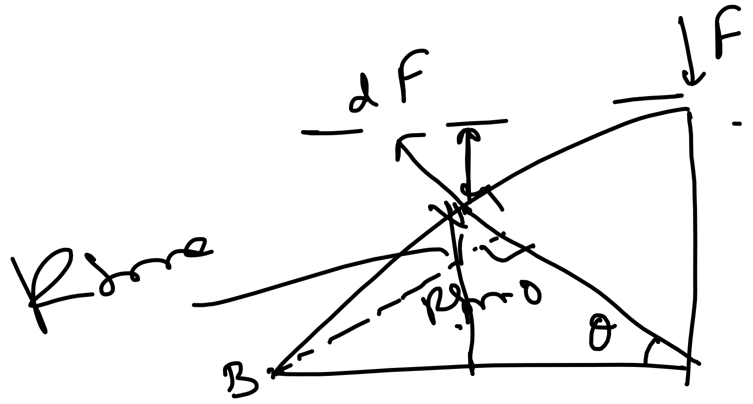
$\vec{W_1} = \vec{R} - \vec{W_2}$

$-\vec{F_V} = \vec{R} - \vec{W_2} = (\vec{W_1} + \vec{W_2}) - \vec{W_2}$ — (i)

moment of $(-F_V)$ = moment $(\vec{W_1} + \vec{W_2})$ − moment $\vec{W_2}$

$|F_V| . x =$

Method 2



$dF = p\,dA$ $\qquad p = \rho g h$

$dF_H = p\,dA\cos\theta$ $\qquad h = R(1 - \sin\theta)$

$dF_V = p\,dA\sin\theta$

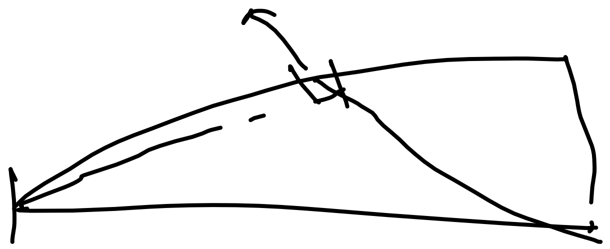
$F_H = \int p\,dA\cos\theta$

$F_V = \int p\,dA\sin\theta$

$\underline{F_R = \sqrt{F_H^2 + F_V^2}}$

$\tan\theta = \dfrac{F_V}{F_H}$

Method 3



$dM_B = \int p\,dA\,R\sin\theta = \underline{F} \cdot 2 \cdot 4$

Non-dimensional analysis of equation of motion

of-the-order

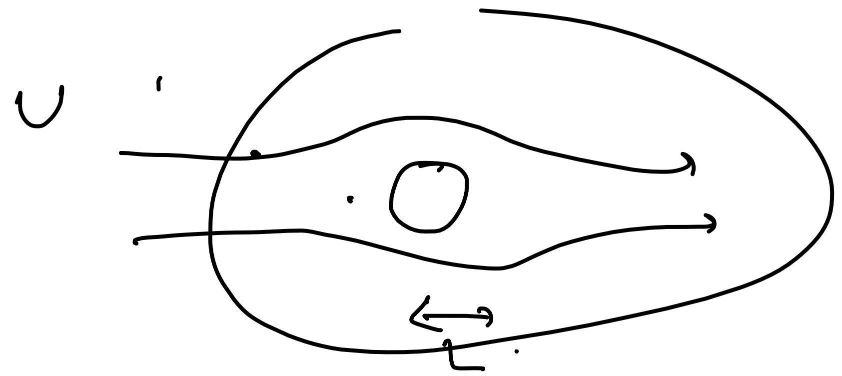
- Incompressible flow

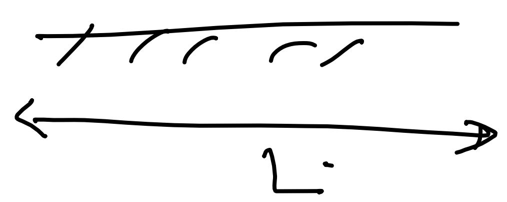
Continuity $\quad \nabla \cdot V = 0$

eqn motion (NS) $\quad \left[\rho \frac{dV}{dt}\right] = \left[\rho g\right] + \left[-\nabla p\right] + \left[\mu \nabla^2 V\right]$

→ Inviscid flow → Euler eqn →

Creeping flow → $v \to 0$





$x = (0, L)$

$\frac{x}{L} = (0, 1)$

scale $V = U$ = velocity far away from surface

scale $x, y, z = L \equiv$ some dimension of surface

$V^* = \frac{V}{U}, \quad u^* = \frac{u}{U}, \quad v^* = \frac{v}{U}, \quad w^* = \frac{w}{U}, \quad x^* = \frac{x}{L}, \quad y^* = \frac{y}{L}, \quad z^* = \frac{z}{L}$

$p^* = \frac{p - \rho g z}{\rho U^2}$

ρU^2 - inertial stress

$\rho U U$ = momentum flux $= \frac{1}{A}\frac{dp_x}{dt} = \tau_{yx}$

$$\nabla = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right) = \frac{1}{L}\left(\frac{\partial}{\partial x^*}\hat{i} + \frac{\partial}{\partial y^*}\hat{j} + \frac{\partial}{\partial z^*}\hat{k}\right) = \frac{1}{L}\nabla^*$$

$$\nabla \cdot V = 0 \Rightarrow \frac{1}{L}\nabla^* \cdot V^* U = 0 \Rightarrow \boxed{\nabla^* \cdot V^* = 0}$$

eqn of motion

$$t^* = \frac{t}{L/U}$$

$$\rho\left[\frac{\partial V}{\partial t} + u\frac{\partial V}{\partial x} + v\frac{\partial V}{\partial y} + w\frac{\partial V}{\partial z}\right] = \rho g - \nabla p + \mu \nabla^2 V$$

$$\rho\left[\frac{U}{L/U}\frac{\partial V^*}{\partial t^*} + \frac{u^* U U}{L}\frac{\partial V^*}{\partial x^*} + \frac{v^* U U}{L}\frac{\partial V^*}{\partial y^*} + \frac{w^* U U}{L}\frac{\partial V^*}{\partial z^*}\right] = \rho g - \frac{\nabla^*}{L}\left(p^* \rho U^2 + \underline{\rho g z^* L}\right) + \mu \frac{\nabla^{*2}}{L^2} U V^*$$

$\nabla^* =$

$$\nabla \cdot V = \frac{\nabla^*}{L} \cdot \frac{V^*}{L}$$

$$\left(\rho \frac{U^2}{L}\right)\left[\frac{\partial V^*}{\partial t^*} + u^*\frac{\partial V^*}{\partial x^*} + v^*\frac{\partial V^*}{\partial y^*} + w^*\frac{\partial V^*}{\partial z^*}\right] = \cancel{\rho g} - \frac{\rho U^2}{L}\nabla^* p^* - \cancel{\rho g} + \frac{U\mu}{L^2}\nabla^{*2} V^*$$

$$\boxed{\frac{dV^*}{dt^*} = -\nabla^* p^* + \left(\frac{\mu}{\rho L U}\right)\nabla^{*2} V^*}$$

Non-dimensional eqn of motion

$$\boxed{\frac{dV^*}{dt^*} = -\nabla^* p^*} + \frac{1}{Re}\nabla^{*2} V^*$$

$Re \to \infty \longrightarrow$ invisid condition $\Rightarrow$ Euler eqn

$Re \to 0 \longrightarrow Re\left(\frac{dV^*}{dt}\right) \to 0 \Rightarrow 0 = -\nabla^* p^* + \frac{1}{Re}\nabla^{*2} V^*$

$V \to 0$ Creeping