

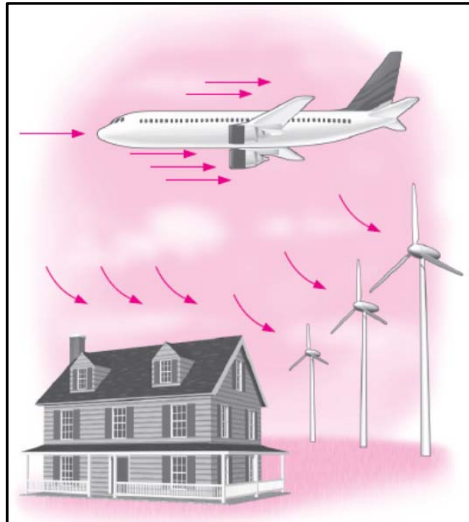
ESO204A, Fluid Mechanics and Rate Processes

Incompressible flows over immersed bodies (External Flow)

Chapter 7 of F M White
Chapter 9 of Fox McDonald

So far, in this course

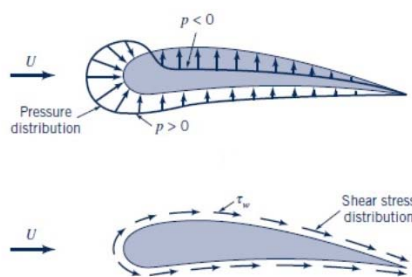
- **Fluid statics**
- **Fluid Kinematics**
- **Integral Formulation**
- **Differential Formulation**
- **Internal (pipe and duct) Flow**
- **External Flow**



External flows are quite common in practice

Study of external flows led to the concept of **boundary layer**, one of the crucial developments in Fluid Mechanics

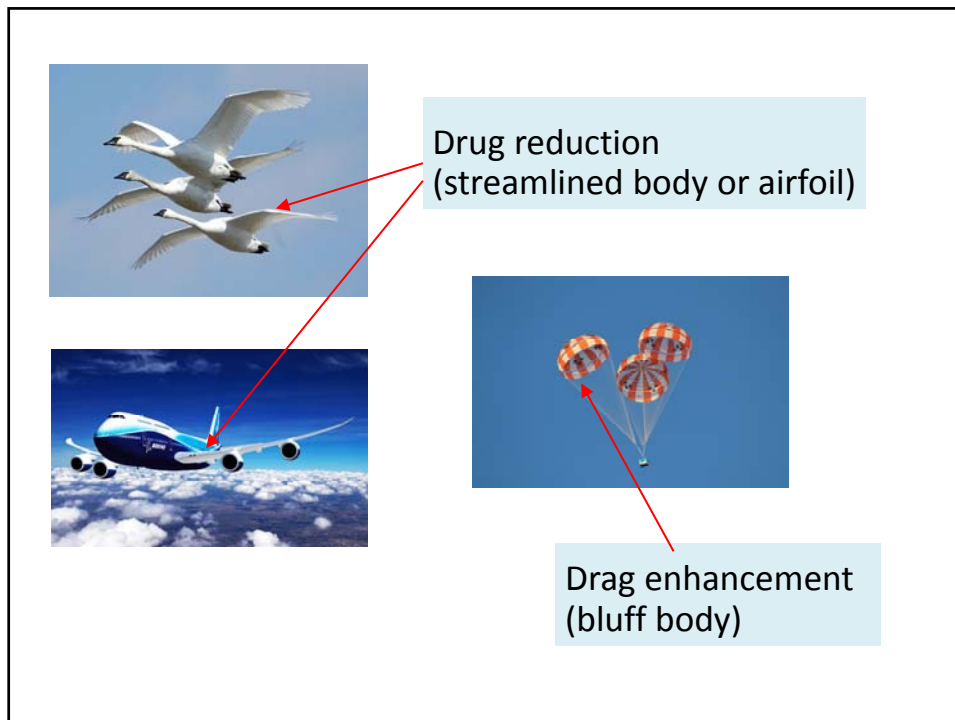
Drag and Lift



Lift: resultant Force normal to flow direction

Drag: resultant force in flow direction

Flow direction: direction of the Free Stream (far away from the body)



$$F_D = F_R \cos \phi$$

$$F_L = F_R \sin \phi$$

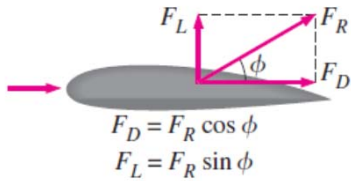
Drag: $F_D = \int_A (-p \cos \theta + \tau_w \sin \theta) dA$

Lift: $F_L = - \int_A (p \sin \theta + \tau_w \cos \theta) dA$

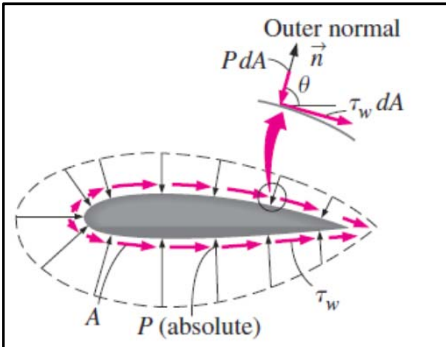
Drag coefficient: $C_D = F_D / \left(\frac{1}{2} \rho u^2 A \right)$

Lift coefficient: $C_L = F_L / \left(\frac{1}{2} \rho u^2 A \right)$

Projected area against the force



Drag:

$$F_D = \int_A (-p \cos \theta + \tau_w \sin \theta) dA$$


Form (pressure) drag

Viscous (friction) drag

Dominates at high Re, Geometry dependent

Dominates at low Re

Drag:

$$F_D = \int_A (\tau_w \sin \theta) dA + \int_A (-p \cos \theta) dA$$

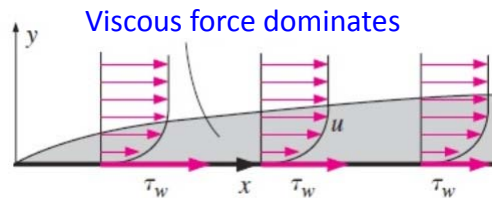
Viscous (friction) drag

Form (pressure) drag

$$\frac{F_D}{\frac{1}{2} \rho u^2 A} = \frac{\int_A (\tau_w \sin \theta) dA}{\frac{1}{2} \rho u^2 A} + \frac{\int_A (-p \cos \theta) dA}{\frac{1}{2} \rho u^2 A}$$

$$C_D = C_{D, \text{viscous}} + C_{D, \text{pressure}}$$

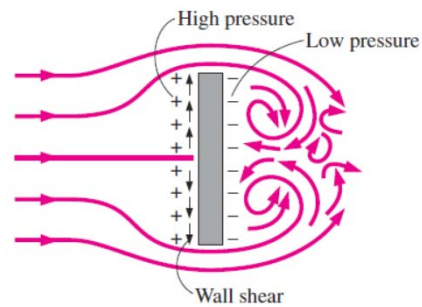
Flow over a flat plate



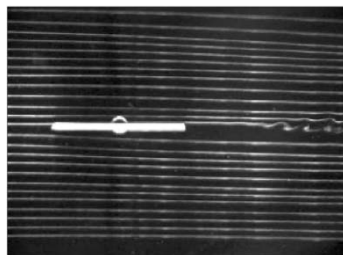
Drag is due to viscous force only

Flow normal to a flat plate

Drag is primarily due to pressure difference across the plate



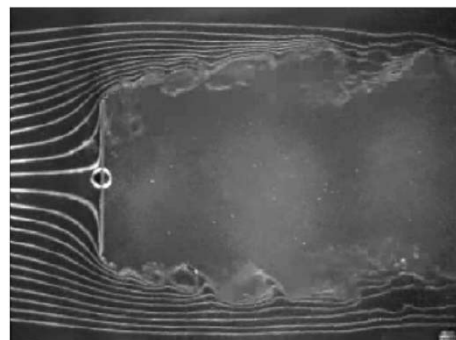
Flow visualization for flow over a flat plate



Drag is due to viscous force only

Flow normal to a flat plate

Drag is primarily due to pressure difference across the plate



Recall dimensional analysis

$$\text{Laminar: } C_D = f(\text{Re}) \quad \text{Turbulent: } C_D = f\left(\text{Re}, \frac{\varepsilon}{d}\right)$$

Low Re $C_D = \frac{\text{constant}}{\text{Re}}$, creeping (Stoke's) flow

High Re $C_D = \text{constant}$ (laminar) or $f\left(\frac{\varepsilon}{d}\right)$ only (turbulent)

In external flow, flows (close to a smooth body) usually remain laminar up to $\text{Re} = \sim 10^5$, unlike internal flows

Recall dimensionless N-S/Continuity Eqns.

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} = -\nabla p + \frac{1}{\text{Re}} \nabla^2 \mathbf{u} \quad \nabla \cdot \mathbf{u} = 0$$

The above Equations may be successfully solved (for many cases) after dropping the viscous term (known as potential flow)

Potential flow solution predicts zero drag/lift for all objects, a phenomenon known as D'Alembert's paradox