ESO204A, Fluid Mechanics and Rate Processes

Incompressible flows through pipes and ducts (Internal Flow)

Engineering applications of Fluid Mechanics

Chapter 6 of F M White
Chapter 8 of Fox McDonald

Incompressible flows in pipes/ducts

major loss in pipe flow $h_f = f\dfrac{L}{d}\dfrac{u_{\text{av}}^2}{2g}$

$$f = \frac{8\tau_w}{\rho u_{\text{av}}^2} = 4C_f = f\left(\mathrm{Re}_{\mathrm{d}}, \frac{\varepsilon}{d}\right),\ \text{obtained from Moody chart}$$

minor loss in pipe flow $h_{\mathrm{m}} = K\dfrac{u_{\text{av}}^2}{2g}$,

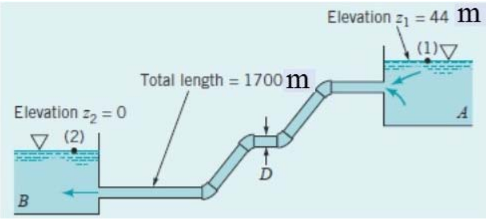
K : minor loss coefficnet, usually obtained from experiments

Pipe Flow: Problem Solving

- Given the pipe geometry, and either flow rate or power/loss, find the other
- Given the power/loss, flow rate and partial information about pipe geometry, find the rest of the geometric parameters

Some of the above problems require iterative solution

Example: Finding diameter



$$Q = 5\,\mathrm{m}^3/\mathrm{s};\ \varepsilon = 150\,\mu\mathrm{m};\ D = ?$$

$$\cancel{\frac{p_1}{\rho g}} + \cancel{\frac{V_1^2}{2g}} + z_1 = \cancel{\frac{p_2}{\rho g}} + \cancel{\frac{V_2^2}{2g}} + z_2 + h_f + h_m$$

$$h_f + h_m = z_1 - z_2$$

$$\boxed{\left(\frac{fL}{D} + \sum K\right)\frac{u^2}{2g} = z_1 - z_2}$$

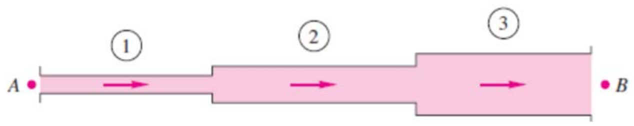
$$u = Q\Big/\left(\frac{\pi}{4}D^2\right) \qquad f = f(D)$$

$$\sum K = K_{\mathrm{SC}} + 4K_{\mathrm{elbow}} + K_{\mathrm{SE}} = 2.2$$

Solve the Eq iteratively to evaluate D

Piping in Series and Parallel

Piping network problem is usually quite complex, we will see few elements of piping network

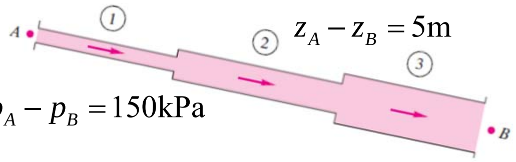


$$Q_1 = Q_2 = Q_3 \qquad h_{L,AB} = h_{L,1} + h_{L,2} + h_{L,3} \qquad h_L = h_f + h_m$$

$$u_1 d_1^2 = u_2 d_2^2 = u_3 d_3^2$$

$$h_{L,AB} = \sum_{i=1}^{3}\left[\frac{u_i^2}{2g}\left(\frac{f_i L_i}{d_i} + \sum_j K_{ij}\right)\right]$$

Piping in Series: example



$z_A - z_B = 5\text{m}$

$p_A - p_B = 150\text{kPa}$

Pipe	L (m)	d (cm)	ε/d
1	100	8	0.003
2	150	6	0.002
3	80	4	0.005

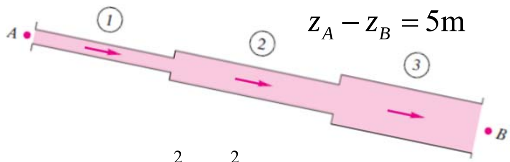
Find $\dot{m}_{\text{water}}$

$$\frac{p_A}{\rho g} + \frac{u_1^2}{2g} + z_A = \frac{p_B}{\rho g} + \frac{u_3^2}{2g} + + z_B + h_{L,AB} \qquad u_1 d_1^2 = u_2 d_2^2 = u_3 d_3^2$$

$$h_{L,AB} = \sum_{i=1}^{3}\left[\frac{u_i^2}{2g}\left(\frac{f_i L_i}{d_i} + \sum_j K_{ij}\right)\right]$$

Four Eqns, four unknowns

$$p_A - p_B = 150\text{kPa}$$



$$z_A - z_B = 5\text{m}$$

$$u_1 d_1^2 = u_2 d_2^2 = u_3 d_3^2$$

$$\sum_{i=1}^{3}\left[\frac{u_i^2}{2g}\left(\frac{f_i L_i}{d_i} + \sum_j K_{ij}\right)\right] = \frac{p_A - p_B}{\rho g} + \frac{u_1^2 - u_3^2}{2g} + z_A - z_B$$

$$\frac{u_1^2}{2g}\left(\frac{f_1 L_1}{d_1} + \sum K_1 - 1\right) + \frac{u_2^2}{2g}\left(\frac{f_2 L_2}{d_2} + \sum K_2\right) + \frac{u_3^2}{2g}\left(\frac{f_3 L_3}{d_3} + \sum K_2 + 1\right) = 20\text{m}$$

Since major losses usually dominate

$$\frac{u_1^2}{2g}\left(\frac{f_1 L_1}{d_1}\right) + \frac{u_2^2}{2g}\left(\frac{f_2 L_2}{d_2}\right) + \frac{u_3^2}{2g}\left(\frac{f_3 L_3}{d_3}\right) = 20\text{m}$$

$$\frac{u_1^2}{2g}\left(\frac{f_1 L_1}{d_1} + \frac{d_1^4}{d_2^4}\frac{f_2 L_2}{d_2} + \frac{d_1^4}{d_3^4}\frac{f_3 L_3}{d_3}\right) = 20\text{m}$$

Pipe	L (m)	d (cm)	ε/d
1	100	8	0.003
2	150	6	0.002
3	80	4	0.005

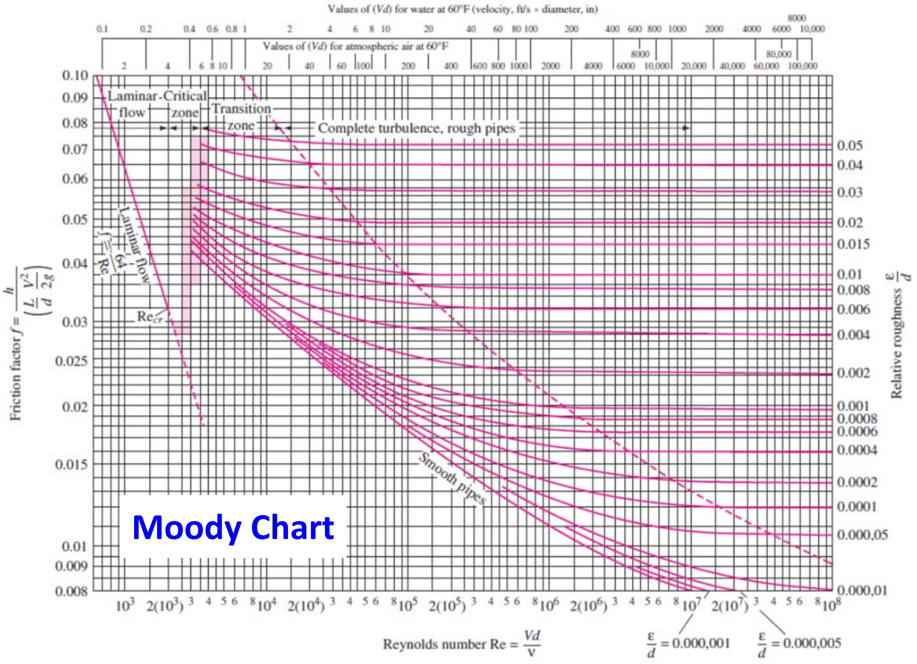
Find

$\dot{m}_{\text{water}}$

$$\frac{u_1^2}{2g}\left(\frac{f_1 L_1}{d_1} + \frac{d_1^4}{d_2^4}\frac{f_2 L_2}{d_2} + \frac{d_1^4}{d_3^4}\frac{f_3 L_3}{d_3}\right) = 20\text{m}$$

$$\frac{u_1^2}{2g}\left(1250 f_1 + 7900 f_2 + 32000 f_3\right) = 20\text{m}$$

Now we go to Moody chart and take a guess of friction factors



Pipe	L (m)	d (cm)	ε/d
1	100	8	0.003
2	150	6	0.002
3	80	4	0.005

Find $\dot{m}_{water}$

Assuming **Re** to be very high in all sections

$$f_1 = 0.026;\ f_2 = 0.023;\ f_3 = 0.03$$

$$\frac{u_1^2}{2g}\left(1250 f_1 + 7900 f_2 + 32000 f_3\right) = 20\text{m} \qquad \Rightarrow u_1 = .58\,\text{m/s}$$

$$\text{Re}_1 = 45400 \qquad \text{Re}_2 = 60500 \qquad \text{Re}_3 = 90800$$

Update friction factors $f_1 = 0.029;\ f_2 = 0.026;\ f_3 = 0.03$

$$\frac{u_1^2}{2g}(1250 f_1 + 7900 f_2 + 32000 f_3) = 20\text{m}$$

Using friction factors $f_1 = 0.029;\ f_2 = 0.026;\ f_3 = 0.03$

$$\Rightarrow u_1 = .57\ \text{m/s}$$

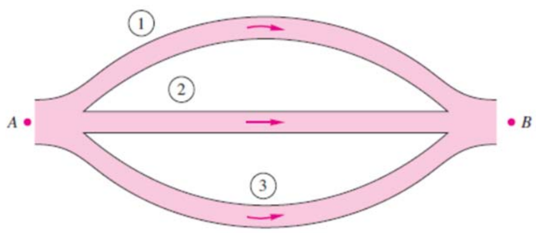
Calculate **Re** and update *f* Find u_1

Continue until changes in velocities are small

$$\dot{m}_{\text{water}} = 10000\ \text{kg/h} \qquad (\text{after two iterations})$$

Solutions may be improved by considering the minor losses

Piping in Parallel



$$Q = Q_1 + Q_2 + Q_3$$

$$h_{AB} = h_1 = h_2 = h_3$$

Example

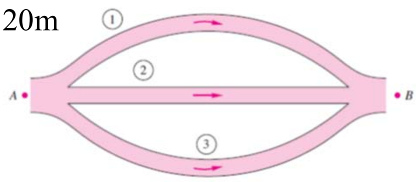
$h_{AB} = 20\text{m}$

Find Q

Pipe	L (m)	d (cm)	ε/d
1	100	8	0.003
2	150	6	0.002
3	80	4	0.005

$$\frac{f_1 L_1}{d_1}\frac{u_1^2}{2g} = \frac{f_2 L_2}{d_2}\frac{u_2^2}{2g} = \frac{f_3 L_3}{d_3}\frac{u_3^2}{2g} = 20\text{m}$$

$$\frac{f_1 L_1}{d_1}\frac{u_1^2}{2g}=\frac{f_2 L_2}{d_2}\frac{u_2^2}{2g}=\frac{f_3 L_3}{d_3}\frac{u_3^2}{2g}=20\text{m}$$



Iterative procedure

Find f_1, f_2, f_3 from fully turbulent zone

Calculate u_1, u_2, u_3 Calculate $\text{Re}_1, \text{Re}_2, \text{Re}_3$ Update f_1, f_2, f_3

Continue till convergence

Present solution, after two iterations:

$$Q = 62.5 + 25.9 + 11.4$$
$$= 99.8\,\text{m}^3/\text{h}$$