ESO204A, Fluid Mechanics and Rate Processes

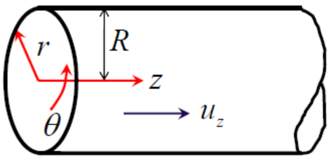
Incompressible flows through pipes and ducts (Internal Flow)

Engineering applications of Fluid Mechanics

Chapter 6 of F M White
Chapter 8 of Fox McDonald

Incompressible flows in pipes/ducts

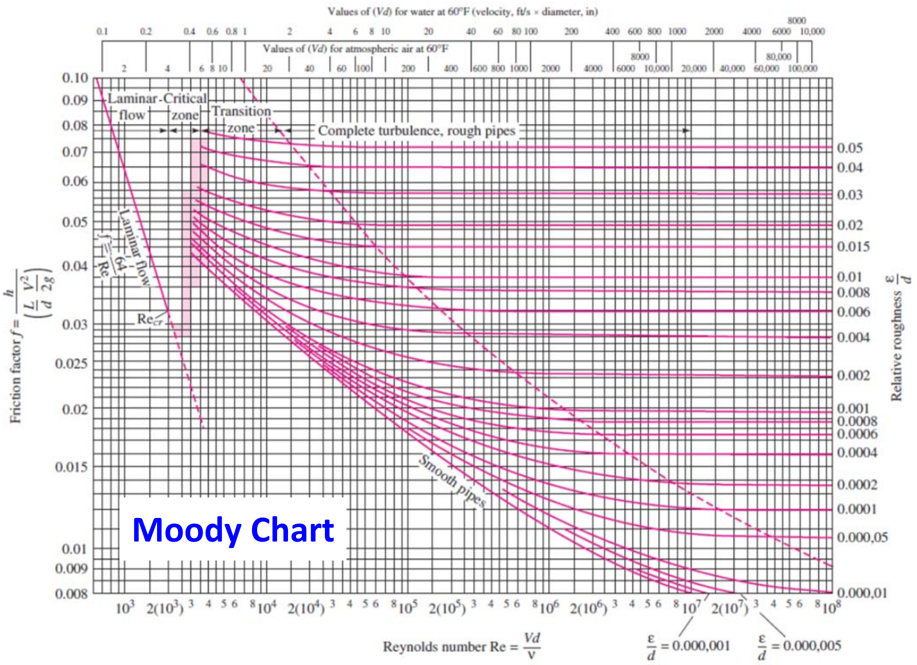
head loss in pipe flow $h_f = f\dfrac{L}{d}\dfrac{u_{\text{av}}^2}{2g}$



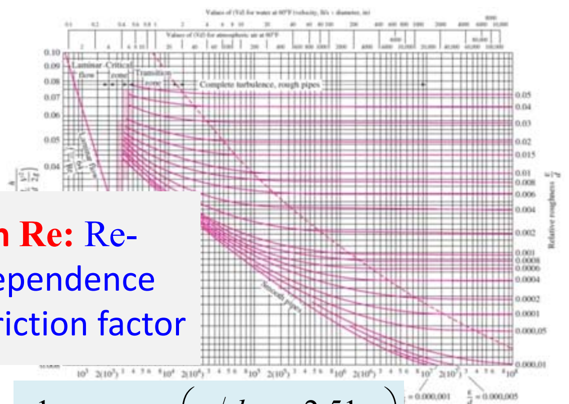
$$f = \frac{8\tau_w}{\rho u_{\text{av}}^2} = 4C_f$$

Laminar: $\mathrm{Re}_d < 1800$ $\quad f = \dfrac{64}{\mathrm{Re_d}}$

Turbulent: $\mathrm{Re}_d > 2000$ $\quad f = f\left(\mathrm{Re_d}, \dfrac{\varepsilon}{d}\right)$



Moody Chart



Low Re: follows exact solution (H-P solution)

High Re: Re-independence of friction factor

Moody chart is a plot of Colebrook formula:

$$\frac{1}{\sqrt{f}} = -2\log\left(\frac{\varepsilon/d}{3.7} + \frac{2.51}{Re_d\sqrt{f}}\right)$$

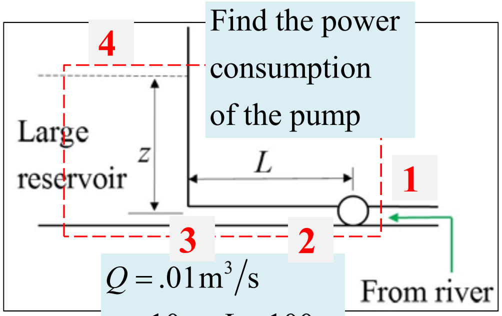
An approximation of the above is Haaland formula:

$$\frac{1}{\sqrt{f}} \approx -1.8\log\left[\left(\frac{\varepsilon/d}{3.7}\right)^{1.11} + \frac{6.9}{Re_d}\right]$$

Example

Pumping of water to a large reservoir

$\dfrac{\varepsilon}{d} = .0001$



$Q = .01\,\mathrm{m}^3/\mathrm{s}$
$z = 10\mathrm{m},\ L = 100\mathrm{m}$
$\mathrm{dia} = 75\mathrm{mm}$

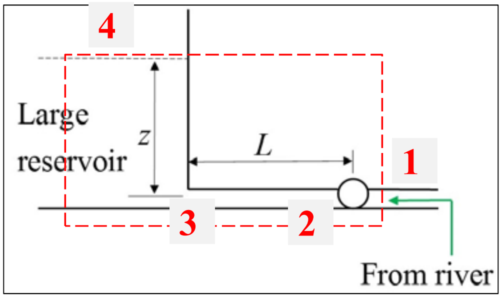
Applying energy Equation between 1-2:

$$\dot{m}\left(\frac{p_1}{\rho} + \cancel{\frac{u_1^2}{2}} + \cancel{gz_1}\right) + \dot{W} = \dot{m}\left(\frac{p_2}{\rho} + \cancel{\frac{u_2^2}{2}} + \cancel{gz_2}\right) + \cancel{\text{friction in pump}}$$

$$\frac{\dot{W}}{\dot{m}g} = \frac{p_2}{\rho g} - \frac{p_1}{\rho g}$$

$$\boxed{\frac{\dot{W}}{\dot{m}g} = \frac{p_2}{\rho g} - \frac{p_1}{\rho g}}$$

Similarly, applying energy Equation between 2-3:



$$\frac{p_2}{\rho g} + \cancel{\frac{u_2^2}{2g}} + \cancel{z_2} = \frac{p_3}{\rho g} + \cancel{\frac{u_3^2}{2g}} + \cancel{z_3} + h_f$$

Please note, mass conservation gives

$$\dot{m} = \dot{m}_1 = \dot{m}_2 = \dot{m}_3$$
$$u_1 = u_2 = u_3$$

$$\boxed{\frac{p_2}{\rho g} - \frac{p_3}{\rho g} = h_f = f\frac{L}{d}\frac{u_1^2}{2g}}$$

combining $$\boxed{\frac{\dot{W}}{\dot{m}g} - \frac{p_3}{\rho g} = -\frac{p_1}{\rho g} + h_f}$$

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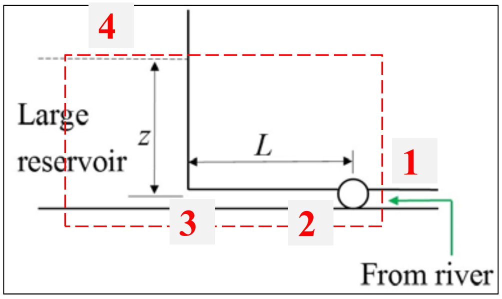


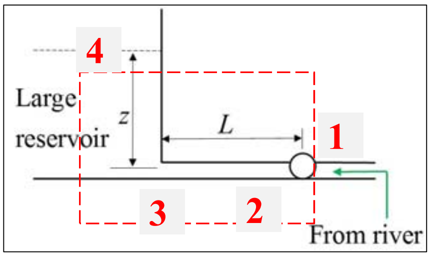
Similarly, applying energy Equation between 3-4:

$$\frac{p_3}{\rho g} + \frac{u_3^2}{2g} + z_3 = \frac{p_4}{\rho g} + \frac{u_4^2}{2g} + z_4 + \cancel{\text{losses}}$$

combining $$\frac{\dot{W}}{\dot{m}g} + \frac{u_3^2}{2g} + z_3 = \frac{p_4}{\rho g} - \frac{p_1}{\rho g} + \frac{u_4^2}{2g} + z_4 + h_f$$

Overall energy Eqn.

$$\frac{\dot{W}}{\dot{m}g} + \frac{u_3^2}{2g} + z_3 = \frac{p_4}{\rho g} - \frac{p_1}{\rho g} + \frac{u_4^2}{2g} + z_4 + h_f$$



$$\frac{\dot{W}}{\dot{m}g} + \frac{u_1^2}{2g} + z_1 = \frac{p_4}{\rho g} - \frac{p_1}{\rho g} + \frac{u_4^2}{2g} + z_4 + h_f$$

We should start from here

≈ 0, very large reservoir

$$\cancel{\frac{p_1}{\rho g}} + \frac{u_1^2}{2g} + z_1 + \frac{\dot{W}}{\dot{m}g} = \cancel{\frac{p_4}{\rho g}} + \cancel{\frac{u_4^2}{2g}} + z_4 + h_f$$

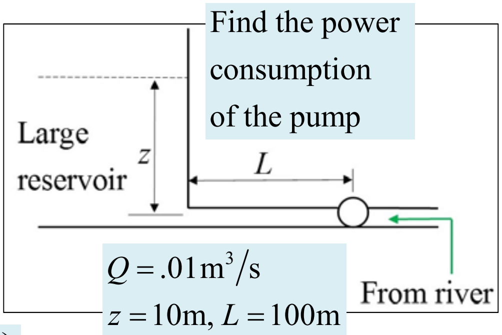
$$\frac{\dot{W}}{\dot{m}g} = z + h_f - \frac{u_1^2}{2g}$$

$$\dot{W} = \dot{m}g\left(z + f\frac{L}{d}\frac{u_1^2}{2g} - \frac{u_1^2}{2g}\right)$$

Example

Pumping of water to a large reservoir

$\frac{\varepsilon}{d} = .0001$

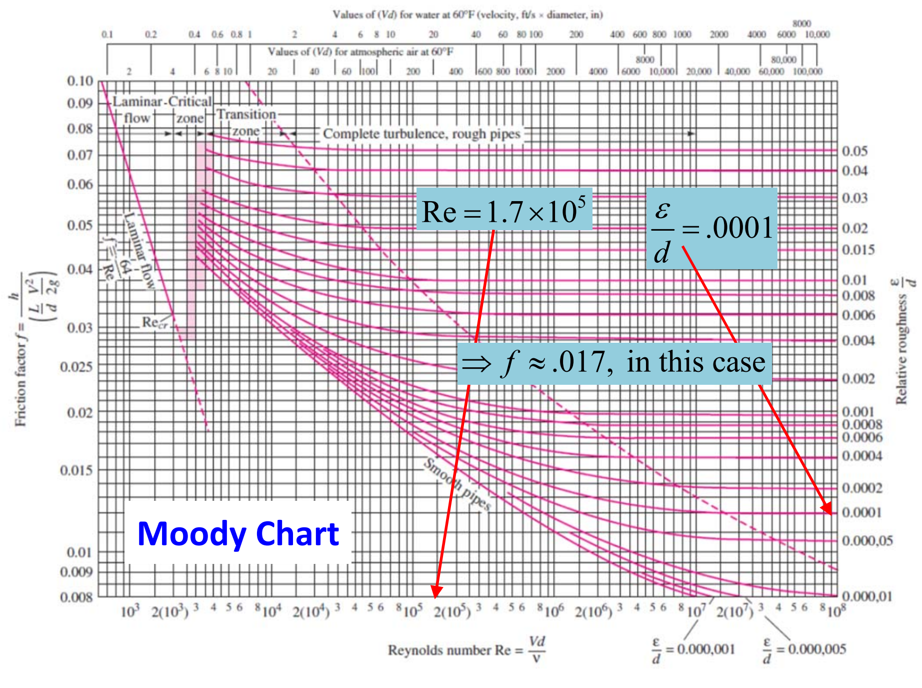


$Q = .01\,\mathrm{m^3/s}$
$z = 10\mathrm{m},\ L = 100\mathrm{m}$
$d = 75\mathrm{mm}$

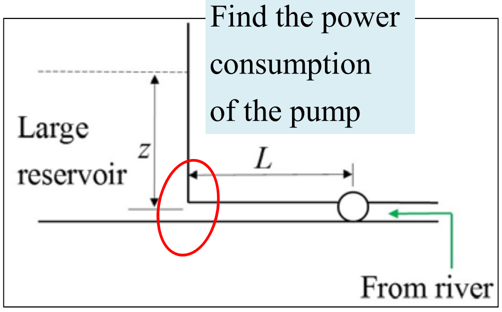
$$\dot{W} = \dot{m}g\left(z + f\frac{L}{d}\frac{u_1^2}{2g} - \frac{u_1^2}{2g}\right)$$

$$u_1 = \frac{Q}{\frac{\pi}{4}d^2} = 2.26\,\mathrm{m/s} \qquad \mathrm{Re} = \frac{\rho u_1 d}{\mu} = \frac{10^3 \times 2.26 \times .075}{10^{-3}} = 1.7 \times 10^5$$

Now, use Moody chart to find f



Example



$$\dot{W} = \dot{m}g\left(z + f\frac{L}{d}\frac{u_1^2}{2g} - \frac{u_1^2}{2g}\right)$$

$f \approx .017$

$Q = .01\,\mathrm{m^3/s}$
$z = 10\mathrm{m},\ L = 100\mathrm{m}$
$\mathrm{dia} = 75\mathrm{mm}$

$$\dot{W} = \dot{m}g\left(10\mathrm{m} + 58.9\mathrm{m} - 2.6\mathrm{m}\right)$$
$$= \rho Q g \times 66.3\mathrm{m} \quad = 663\mathrm{W}$$

There will be additional losses due to sudden expansion; such losses are called 'minor loss'

Minor losses

Minor losses usually occur due to sudden change in flow area or direction

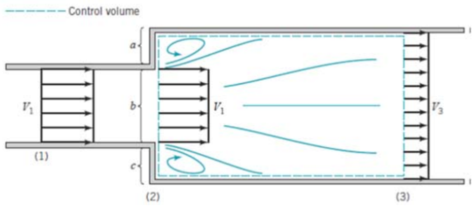
Minor losses may be significant at pipe bends, valves, sudden expansion/contraction, inlet/exit

Minor losses are quantified by minor loss coefficient K

$$h_m = K\frac{u^2}{2g}$$

Experimentally obtained values of K are available in Table/Plots

Minor losses: Example, sudden expansion



Mass: $\rho A_1 V_1 = \rho A_3 V_3$

Momentum: $p_1 A_3 - p_3 A_3 =$

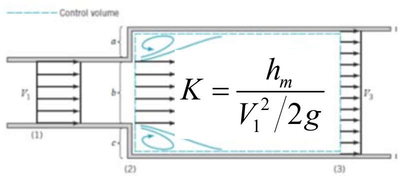
$$\rho A_3 V_3 . V_3 - \rho A_1 V_1 . V_1$$

assume $p_a = p_b = p_c = p_1$

Energy: $\dfrac{p_1}{\rho g} + \dfrac{V_1^2}{2g} = \dfrac{p_3}{\rho g} + \dfrac{V_3^2}{2g} + h_m$

Find, minor loss coefficient $\quad K = \dfrac{h_m}{V_1^2/2g}$

Minor loss coefficient in a sudden expansion



$$K = \frac{h_m}{V_1^2/2g}$$

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} = \frac{p_3}{\rho g} + \frac{V_3^2}{2g} + h_m$$

$$\frac{2p_1}{\rho V_1^2} + 1 = \frac{2p_3}{\rho V_1^2} + \frac{V_3^2}{V_1^2} + K$$

$$K = \frac{2(p_1 - p_3)}{\rho V_1^2} + 1 - \frac{V_3^2}{V_1^2}$$

$$K = 2\frac{V_3^2}{V_1^2} - 2\frac{A_1}{A_3} + 1 - \frac{V_3^2}{V_1^2}$$

$$K = 1 - 2\frac{A_1}{A_3} + \frac{V_3^2}{V_1^2}$$

$$p_1 A_3 - p_3 A_3 = \rho A_3 V_3 . V_3 - \rho A_1 V_1 . V_1$$

$$\frac{(p_1 - p_3)}{\rho V_1^2} = \frac{V_3^2}{V_1^2} - \frac{A_1}{A_3}$$

$$\rho A_1 V_1 = \rho A_3 V_3 \Rightarrow \frac{V_3}{V_1} = \frac{A_1}{A_3}$$

$$K = \left(1 - \frac{A_1}{A_3}\right)^2$$

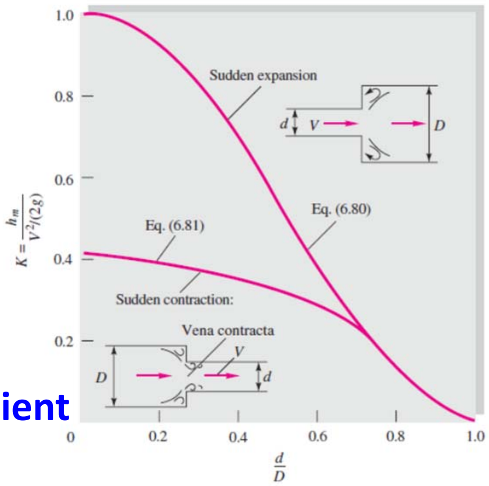
Such approach fails for sudden contraction

Minor losses: sudden expansion/contraction

$$K_{SE} = \left(1 - \frac{d^2}{D^2}\right)^2$$

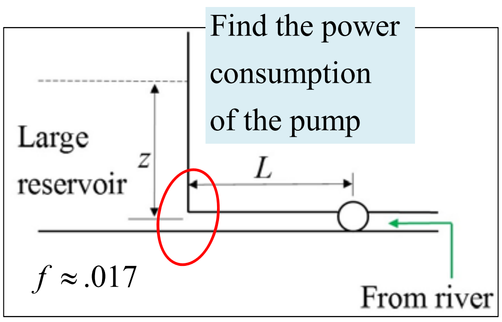
$$K_{SC} \approx 0.42\left(1 - \frac{d^2}{D^2}\right)$$

For sudden contraction, the minor loss coefficient is obtained from experiments



Example cont.

$$\dot{W} = \dot{m}g\left(z + f\frac{L}{d}\frac{u_1^2}{2g} - \frac{u_1^2}{2g}\right)$$

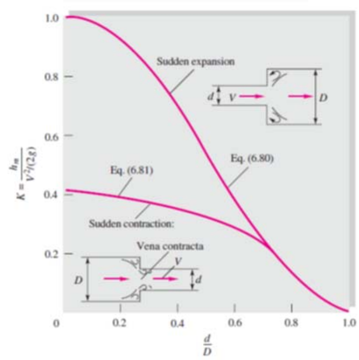


Including minor loss due to sudden expansion

$$\dot{W} = \dot{m}g\left(z + f\frac{L}{d}\frac{u_1^2}{2g} + K\frac{u_1^2}{2g} - \frac{u_1^2}{2g}\right)$$

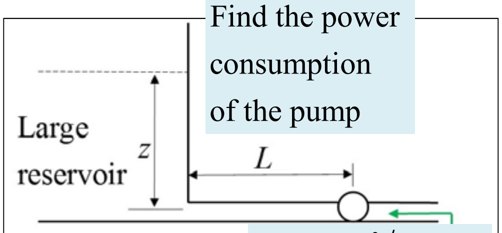
here $K \approx 1$

$$\dot{W} = \dot{m}g\left(z + f\frac{L}{d}\frac{u_1^2}{2g}\right) = 689\text{W}$$



Example problem summary: energy budget

Find the power consumption of the pump



$f \approx .017$

$Q = .01\,\text{m}^3/\text{s}$
$z = 10\text{m},\ L = 100\text{m}$
$\text{dia} = 75\text{mm}$

Elevation head = 10m

$$\dot{W} = \dot{m}g\left(z + f\frac{L}{d}\frac{u_1^2}{2g} + K\frac{u_1^2}{2g} - \frac{u_1^2}{2g}\right)$$

Major loss = 58.9m

Minor loss = 2.6m

For long pipe, **L/d** is the primary source of loss

f is always < 0.1

For short pipe, loss is often neglected

Kinetic energy correction factor

Kinetic energy at a cross-section

$$\frac{1}{2}\int_A \rho u^3 dA \neq \frac{1}{2}\dot{m}u_{\text{av}}^2 \qquad \frac{1}{2}\int_A \rho u^3 dA = \alpha\frac{1}{2}\dot{m}u_{\text{av}}^2$$

Value of the correction factor depends on the nature of velocity profile

Laminar flow: $u = u_{\max}\left(1 - \frac{r^2}{R^2}\right) \Rightarrow \alpha = 2$

Turbulent flow: $u = u_{\max}\left(1 - \frac{r}{R}\right)^{\frac{1}{n}}$

$n = 5 \Rightarrow \alpha = 1.106$
$n = 7 \Rightarrow \alpha = 1.058$
$n = 9 \Rightarrow \alpha = 1.037$

Coming back to the example problem

Pump power $\dot{W} = \dot{m}g\left(z + f\frac{L}{d}\frac{u_1^2}{2g} + K\frac{u_1^2}{2g} + \alpha\frac{u_1^2}{2g} \right)$

For turbulent flow contribution of the correction factor is usually very small

Unless otherwise specified, you may assume 1/7-th profile for the turbulent pipe flow

$$\alpha = 1.058$$

Momentum correction factor (not necessary in present case)

You may use similar correction factors (β) in integral momentum Equation as well

$$\int_A \rho u^2 dA = \beta \dot{m} u_{av}$$

Laminar flow: $\beta = 4/3$

Turbulent flow: $n = 5, 7, 9 \Rightarrow \alpha = 1.037, 1.020, 1.013$

Once again, for turbulent flow, contribution of the correction factor is usually very small