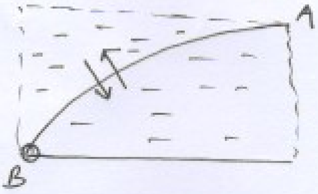
Quiz 1 – Solution

Method 1

Quiz 1 → first → Vertical force on the gate (F_V)

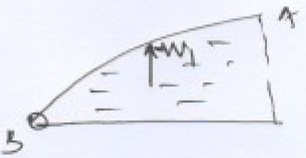
"if water were also present above the gate ⇒ No net force on a massless gate"



Pressure forces will be both from above and below the gate ⇒ They will cancel out

w_1 = weight of fluid on top of the gate

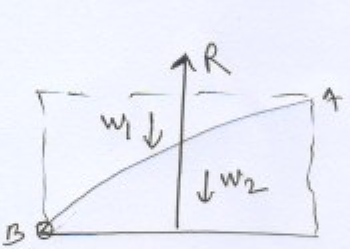
Actual case: No water on top of the gate



F_V = Vertical force on the gate

$F_V = -w_1$

Let's again take the case that water is also on top of gate



w_2 = weight of water below the gate

$\vec{R} = \vec{w_1} + \vec{w_2}$

$\vec{F_V} = -\vec{w_1}$

$\vec{w_1} = \vec{R} - \vec{w_2}$

$-\vec{F_V} = \vec{R} - \vec{w_2}$

$-\vec{F_V} = (\vec{w_1} + \vec{w_2}) - \vec{w_2}$ —— (i)

Thus we can find both magnitude and line of action and point of action of vertical force F_V on the gate by eqⁿ (i)

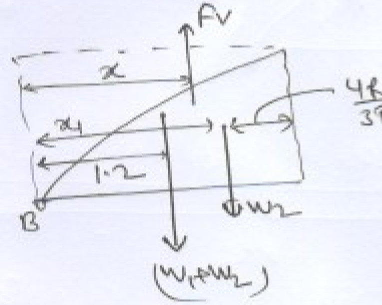
magnitude of F_V

$$|-F_V| = (W_1+W_2) - W_2$$
$$= (2.4)(2.4)\times 3\times 1000\times 9.81 - \frac{1}{4}\pi(2.4)^2\times 3\times 1000\times 9.81$$
$$= 36378.61662 \text{ N}$$

Line of action of F_V

from (i)

moment of $(-F_V)$ about B = moment of $\overrightarrow{(W_1+W_2)}$ about B − moment of $\overrightarrow{W_2}$ about B



$$x_1 = 2.4 - \frac{4R}{3\pi} = 1.381408 \text{ m}$$

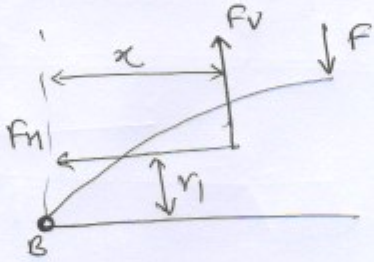
$$|-F_V|\cdot x = (W_1+W_2)\times 1.2 - W_2\times x_1$$
$$36378.61662\, x = (2.4)(2.4)(3)\times 1000\times 9.81\times 1.2 - \frac{1}{4}\pi(2.4)^2\times 3\times 1000\times 9.81\times 1.381408$$
$$= 19502.00837$$
$$x = 0.536084 \text{ m}$$

Horizontal force F_H

$$F_H = \rho g h_{cg} A = 1000\times 9.81\times 1.2\times 2.4\times 3 = 84758.4 \text{ N}$$

$$y_{cp} = -\frac{I_{xx}\sin\theta}{h_{cg}A} = -\frac{\left(\frac{bL^3}{12}\right)\sin 90}{1.2\times 2.4\times 3} = -\frac{3\times\frac{(2.4)^3}{12}}{1.2\times 2.4\times 3} = -0.4$$

Free body diagram of the gate



$$r_1 = (2.4) - (1.2 + 0.4) = 0.8$$

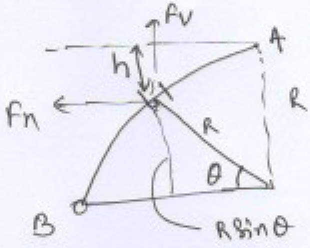
net moment about B $= 0$ for gate not to open

$$|F| \times 2.4 - |F_v| \cdot x - |F_h| \cdot r_1 = 0$$

$$|F| = \frac{36378.61662 \times 0.536084 + 84758.4 \times 0.8}{2.4}$$

$$= 36378.63\ N$$

Que 1 - Method 2



$F = \int p\,dA$

$F_H = \int p\,dA \cos\theta$

$F_V = \int p\,dA \sin\theta$

$F_H = \int (\rho g h)(R\,d\theta \times 3)\cos\theta \quad , \qquad h = R - R\sin\theta = R(1-\sin\theta)$

$= 3\rho g R^2 \int_0^{\pi/2} (1-\sin\theta)\cos\theta\, d\theta$

$= 3\rho g R^2 \int \left(\cos\theta - \frac{\sin 2\theta}{2}\right) d\theta$

$= 3\rho g R^2 \left\{ [\sin\theta]_0^{\pi/2} + \frac{1}{4}[\cos 2\theta]_0^{\pi/2} \right\}$

$= 3\rho g R^2 \left\{ 1 + \frac{1}{4}(-1) - \frac{1}{4} \right\} = 3\rho g R^2 \{1 - 1/2\} = \frac{3\rho g R^2}{2}$

$= \frac{3 \times 1000 \times 9.81 \times (2.4)^2}{2} = 84758.4\ N$

$F_V = \int (\rho g h)(R\,d\theta \times 3)\sin\theta$

$= 3\rho g R^2 \int (1-\sin\theta)\sin\theta\, d\theta = 3\rho g R^2 \left\{ \int_0^{\pi/2} \sin\theta\, d\theta - \int_0^{\pi/2} \left(\frac{1-\cos 2\theta}{2}\right) d\theta \right.$

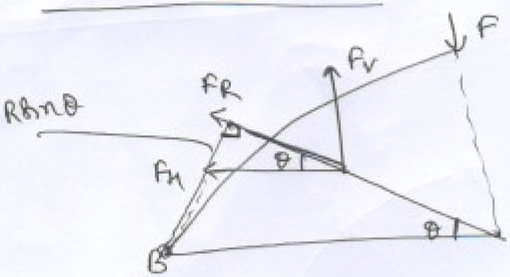
$= 3\rho g R^2 \left\{ [-\cos\theta]_0^{\pi/2} - \frac{1}{2}[\theta]_0^{\pi/2} + \frac{1}{4}[\sin 2\theta]_0^{\pi/2} \right\}$

$= 3\rho g R^2 \{ 0 + 1 - \frac{1}{2}\pi/2 + 0 \}$

$= 3\rho g R^2 \{ 1 - \frac{\pi}{4} \} = 3 \times 1000 \times 9.81 \times (2.4)^2 \left(1 - \frac{\pi}{4}\right)$

$= 36378.62\ N$

Resultant force F_R



$$F_R = \sqrt{F_H^2 + F_V^2} = \sqrt{(84758.4)^2 + (36378.62)^2}$$

$$= 92235.51 \text{ N}$$

$$\tan\theta = \frac{F_V}{F_H} = \frac{36378.62}{84758.4} \Rightarrow$$

$$\theta = 23.229^\circ$$

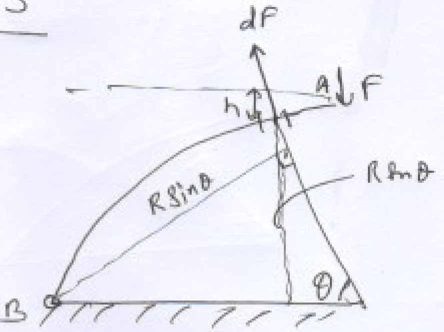
moment balance about B

$$F_R \cdot R\sin\theta = F \times 2.4$$

$$F = \frac{92235.51 \times 2.4 \sin 23.229^\circ}{2.4}$$

$$F = 36378.3 \text{ N}$$

Que 1 – Method 3



$h = R - R\sin\theta$

Σ moment of dF about B = moment of F about B

$$\int p\,dA \cdot R\sin\theta = F \times 2.4$$

$$F \times 2.4 = \int (\rho g h)(R\,d\theta \times 3)\, R\sin\theta \quad ; \quad h = R(1-\sin\theta)$$

$$= 3\rho g R^3 \int_0^{\pi/2} (1-\sin\theta)\sin\theta \, d\theta$$

$$= 3\rho g R^3 \left\{ \int \sin\theta \, d\theta - \int \left(\frac{1-\cos 2\theta}{2}\right) d\theta \right\}$$

$$= 3\rho g R^3 \left\{ [-\cos\theta]_0^{\pi/2} - \frac{1}{2}[\theta]_0^{\pi/2} + \frac{1}{4}[\sin 2\theta]_0^{\pi/2} \right\}$$

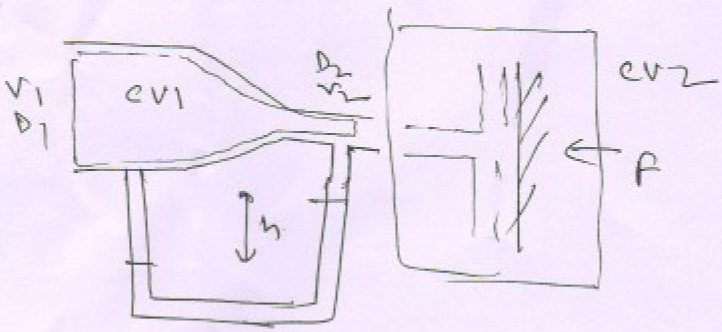
$$= 3\rho g R^3 \left\{ 1 - \frac{\pi}{4} + \frac{1}{4}[0-0] \right\}$$

$$= 3\rho g R^3 \left(1 - \frac{\pi}{4}\right)$$

$$F = \frac{3\rho g R^3 (1 - \frac{\pi}{4})}{2.4} = \frac{3 \times 1000 \times 9.81 \times (2.4)^3 (1 - \pi/4)}{2.4}$$

$$F = 36378.62\ N$$

Q2



(a) x-momentum balance on CV2

$$-F = \cancel{\frac{d}{dt}}\int(\ )^{0} + \int u\rho(V.n)\,dA$$

$$-70 = -\rho V_2 V_2 A_2 + 0$$

$$70 = 1000\, V_2^2 \frac{\pi}{4}(0.03)^2$$

$$V_2 = 9.95 \text{ m/s.}$$

mass balance on CV1

$$0 = \frac{d}{dt}\int \quad + \int \rho(V.n)\,dA$$

$$0 = 0 + \rho V_2 A_2 - \rho V_1 A_1$$

$$V_1 = (9.95)\frac{(0.03)^2}{(0.1)^2} = 0.8955 \text{ m/s.}$$

(b) Energy balance on CV1

$$\frac{P_1}{\gamma} + \frac{1}{2g}\alpha_1 V_1^2 + \cancel{z_1} = \frac{P_2}{\gamma} + \frac{1}{2g}\alpha_2 V_2^2 + \cancel{z_2} + \cancel{h_f} - \cancel{h_p} + \cancel{h_t}$$

assume frictionless $\Rightarrow h_f \to 0$

no pump or turbine $h_p = 0,\ h_t = 0$

assume turbulent flow $\alpha_1 = 1.0,\ \alpha_2 = 1.0$

$$\frac{P_1}{\rho g} = \frac{P_2}{\rho g} + \frac{1}{2g}V_2^2 - \frac{1}{2g}(V_1^2)$$

$$P_1 - P_2 = \rho\left(\tfrac{1}{2}\right)(V_2^2 - V_1^2) = \tfrac{1}{2}\times 1000 \times \left((9.95)^2 - (0.8955)^2\right) = 49100.3\text{ Pa}$$

$$(\rho_{Hg} - \rho_w)\, g * h = P_1 - P_2 \Rightarrow (13.6\times10^3 - 1000)\times 9.81\times h = 49100.3$$