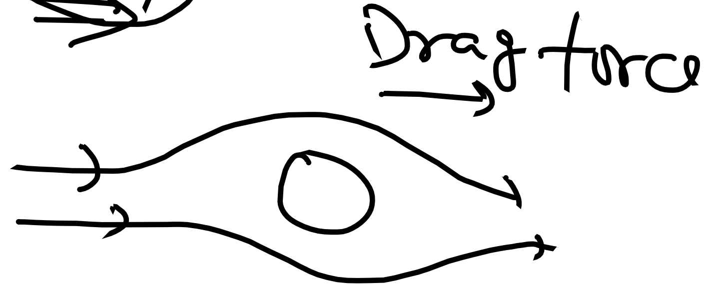
Happy Teacher's day → we exist because of you

- Now we known fundamental eq^n of fluid Mechanics
- But, these equations have limitations for application in practical problems of engineering
  - → navier-stokes' not valid for turbulent flow
  - → Even for laminar flow, solving N-S for a complicated geometry may be difficult
  - ⇒ Practical Approach → Dimensional Analysis ⇒ valid for any kind of flow
    - → couple fundamentals with experiments ⇒ called impirical Approacha

⇒ So Now we have tools to solve practical problems ⇐ we will focus on these problems

⇒ These problems are → (i) Calculating Pressure drop in internal flow

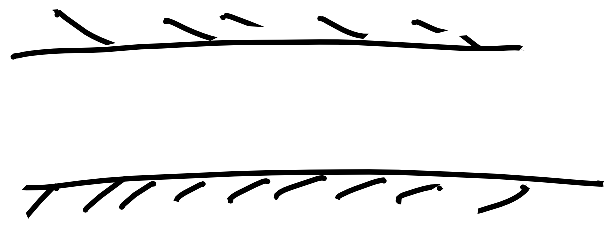
(ii) Calculating drag force on a solid in external flow

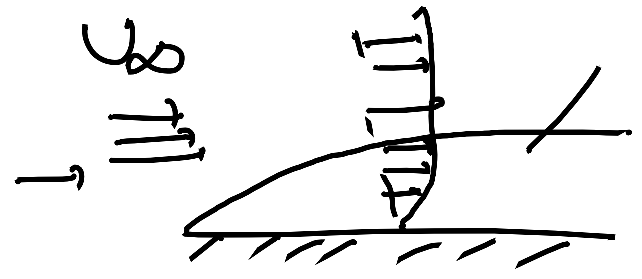


$u_r, u_\theta, \to \tau_{ij} \to F$

→ Engine

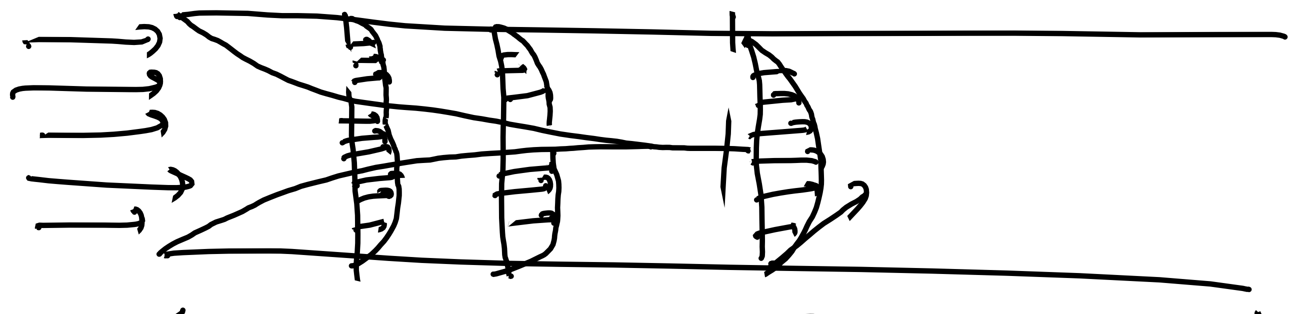
Internal flow





Boundary layer

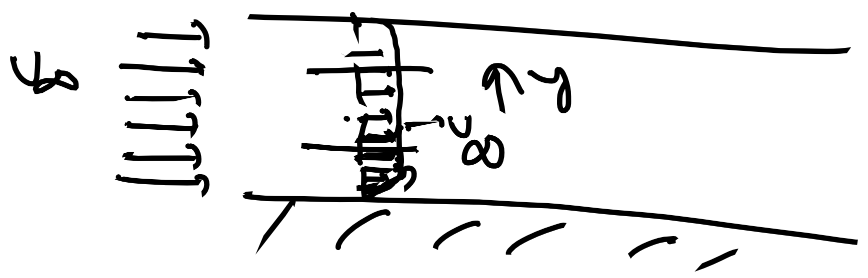
↳ Boundary layer theory ← Simplification of NS- for the Boundary layer



entry length Fully developed flow

Le

Turbulent



$$Le = f(\rho, \mu, V, d)$$

$$\frac{Le}{D} = g\left(\frac{\rho V d}{\mu}\right) = g(Re)$$

Correlation

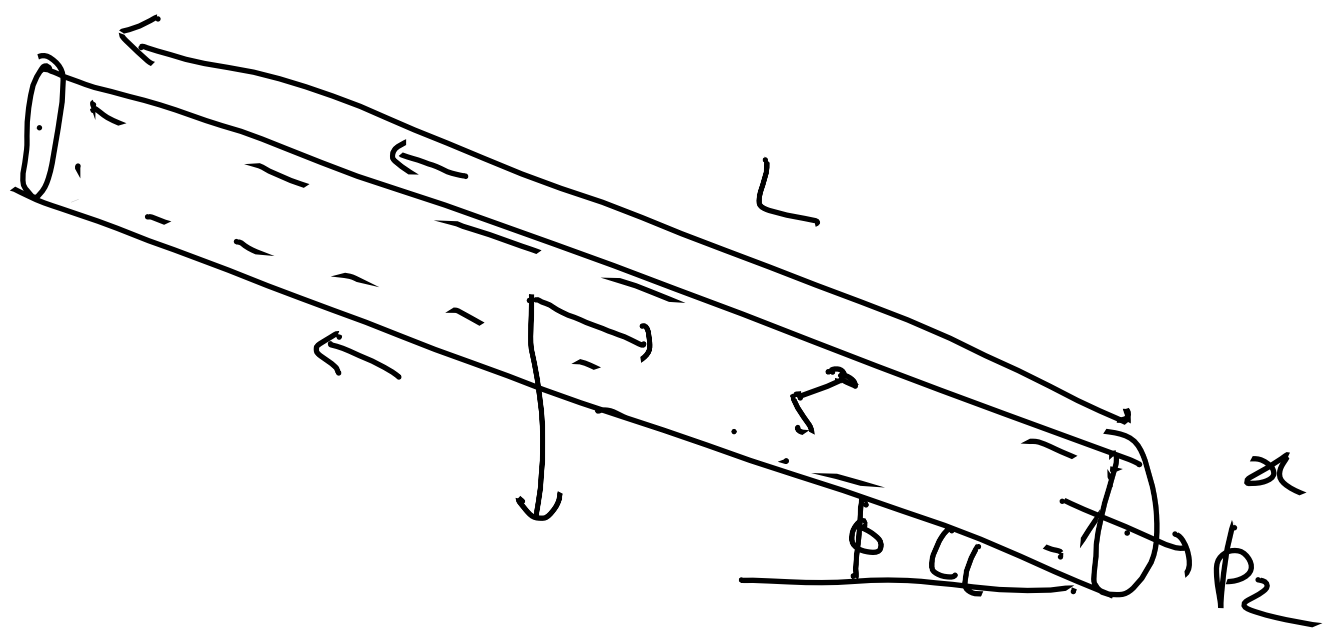
$$\frac{Le}{d} = 0.06\, Re_d \quad \leftarrow \text{laminar flow}$$

$$\frac{Le}{d} = 1.6\, Re_d^{1/4} \qquad 4000 \leq Re_d \leq 10^7$$

→ (entry Length)$_{\text{Turbulent}}$ < (entry length) laminar

→ Find the pressure drop in a internal flow (Avg V ↳ $\frac{Q}{A}$)

P_1



→ find h_f ←

→ mass balance $\rho A_1 V_1 = \rho A_2 V_2$ $V_1 = V_2$

→ Energy balance → $\frac{P_1}{\rho g} + \frac{1}{2g}\cancel{\alpha_1 V_1^2} + z_1 = \frac{P_2}{\rho g} + \frac{1}{2g}\cancel{\alpha_2 V_2^2} + z_2 + h_f$

$h_f = \frac{P_1 - P_2}{\rho g} + (z_1 - z_2) = \frac{\Delta p}{\rho g} + \Delta z$

Δp = Pressure drop $= (P_1 - P_2)$
Δz = height drop

→ what is the dependence of $h_f = f(- - -)$

→ Momentum balance

$\Delta p \pi R^2 + \rho g (\Delta L) \pi R^2 \sin\phi - (\tau_w) 2\pi R \Delta L = \underbrace{\frac{d}{dt}\int \quad)}_{=0} + \cancel{(\beta_2 \rho V_2^2 A_2)} - \cancel{(\beta_1 \rho V_1^2 A_1)}$

$\frac{\Delta p}{\rho g} + \Delta L \sin\phi = \frac{\tau_w 2\pi R \Delta L}{\pi R^2 \rho g} = \frac{4\tau_w}{\rho g}\frac{L}{d}$

$h_f = \frac{\Delta p}{\rho g} + \Delta z = \frac{4\tau_w}{\rho g}\left(\frac{L}{d}\right)$ ← $\Rightarrow h_f \propto \frac{L}{d}$

Experimental $\quad \underline{h_f} = \frac{\Delta p}{\rho g} + \Delta z \propto \underline{V^2} \longleftarrow \frac{Q}{A}$ $\quad V =$ Avg Velocity

$$h_f \propto V^2$$

$$h_f \propto \left(\frac{L}{d}\right)\left(\frac{V^2}{2g}\right)$$

$$h_f = f\left(\frac{L}{d}\right)\frac{V^2}{2g}$$

$\underline{f} =$ friction factor

→ So if we know f ~~for~~ for any flow condition $\Rightarrow \underline{h_f} \Rightarrow \underline{\Delta p}$

$\Delta p \rightarrow \underline{f} = f(\underline{Re_d}, \underline{\frac{\varepsilon}{d}}, \underline{\text{duct shape}})$

Roughness ε material of pipe
factor

valid for any prototype

$$h_f = f\frac{L}{d}\frac{V^2}{2g} = \frac{4\tau_w}{\rho g}\frac{L}{d} \Rightarrow f_{Darcy} = \frac{8\tau_w}{\rho V^2}$$

$Re = \frac{\text{inertial stress}}{\text{Viscous}}$

$$f_{fanning} = \frac{1}{4} f_{Darcy} = \left(\frac{\tau_w}{\frac{1}{2}\rho V^2}\right) \leftarrow f \rightarrow \text{experimentally}$$

→ But for laminar flow

Laminar fully developed pipe flow

$$u = u_{max}\left(1 - \frac{r^2}{R^2}\right) \; ; \; u_{max} = \left(-\frac{dp}{dx}\right)\frac{R^2}{4\mu}$$

$$V = \frac{u_{max}}{2}$$

$$f_{darcy} = \frac{\tau_w}{\frac{1}{8}\rho V^2}$$

$$\tau_w = \left|\mu \frac{du}{dr}\right|_{r=R} = \frac{2u_{max}\mu}{R} = \frac{4V\mu}{R} = \frac{8V\mu}{d}$$

$$\boxed{f_{darcy} = \frac{(8)\left(\frac{8V\mu}{d}\right)}{\rho V^2} = \frac{64\mu}{\rho V d} = \frac{64}{Re_d}}$$

$$f = f\left(\overset{\checkmark}{Re_d}, \overset{\times}{\frac{\varepsilon}{d}}, \overset{\checkmark}{\underline{\text{shape}}}\right)$$

ε = Protrusions

$\frac{\varepsilon}{d}$ = Roughness factor

Turbulent flow

f