ESO204A, Fluid Mechanics and rate Processes

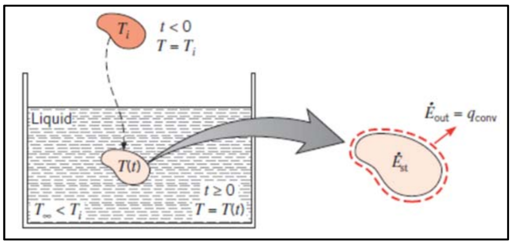
Transient Conduction

Chapter 4 of Cengel

Quenching of a hot metal forging

Assumptions: $T = T(t), T_\infty = \text{constant}$

$$\dot{E}_{st} = \frac{dE_{st}}{dt} = \cancel{\dot{E}_{in}} - \dot{E}_{out} + \cancel{\dot{E}_{gen}}$$



$$\frac{d(mcT)}{dt} = -hA(T - T_\infty)$$

$$\frac{dT}{dt} = -\frac{hA}{\rho Vc}(T - T_\infty)$$

Initial condition: $T(t=0) = T_i$

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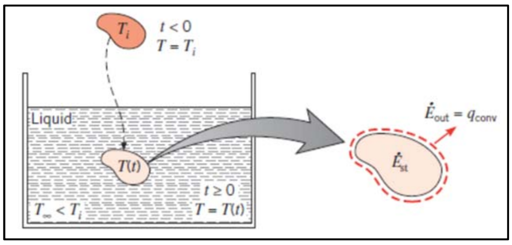
$$\frac{dT}{dt} = -\frac{hA}{\rho Vc}(T - T_\infty)$$

$$T(t=0) = T_i$$



Nondimensionalization:

$$\theta = \frac{T - T_\infty}{T_i - T_\infty} \qquad dT = (T_i - T_\infty)\, d\theta \qquad \frac{d\theta}{dt} = -\frac{hA}{\rho Vc}\theta = -\frac{\theta}{t_c} \qquad t_c = \frac{\rho Vc}{hA}$$

$$\tau = t/t_c \qquad \frac{d\theta}{d\tau} = -\theta \qquad \theta(\tau = 0) = 1 \qquad \theta = \exp(-\tau)$$

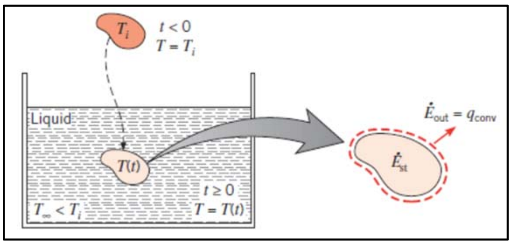
Time scale

Quenching of a hot metal forging

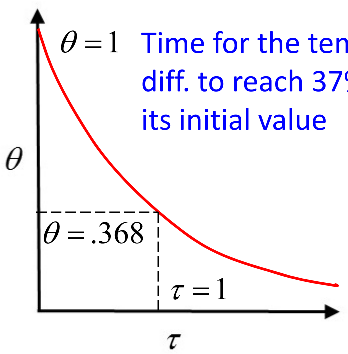
$$\theta = \frac{T - T_\infty}{T_i - T_\infty} \qquad t_c = \frac{\rho Vc}{hA}$$

$$\tau = t/t_c$$

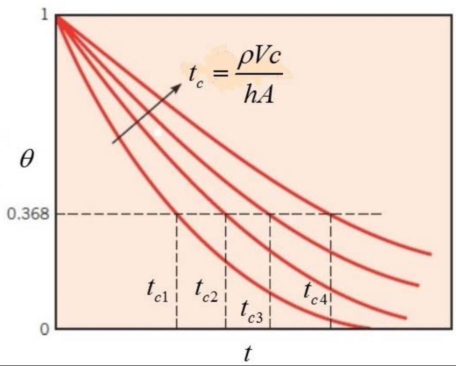
$$\theta = \exp(-\tau)$$



Time constant



Time for the temp. diff. to reach 37% of its initial value



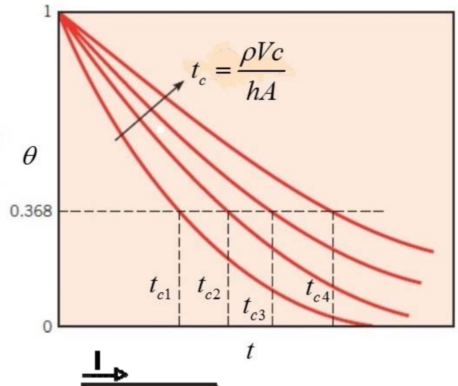
$$\theta = \frac{T - T_\infty}{T_i - T_\infty} = \exp(-t/t_c)$$

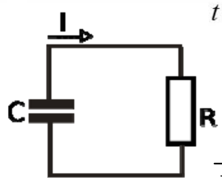
Time constant $t_c = \frac{\rho V c}{hA}$

Time constant indicates transient thermal response of an object

$$t_c = \frac{\rho V c}{hA} = \left(\frac{1}{hA}\right)(\rho V c)$$

Resistance **Capacitance**





$$C\frac{dE}{dt} + \frac{E}{R} = 0$$

$$\frac{E}{E_0} = \exp(-t/RC)$$

High t_c slows down transport

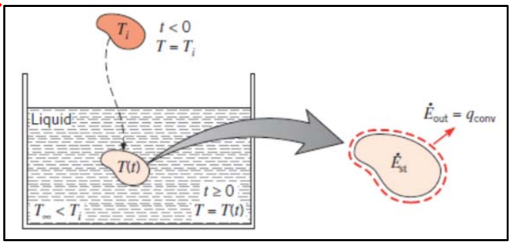
Heating cold object
~~Quenching~~ of a ~~hot~~ metal ~~forging~~

Assumption: $T = T(t), T_\infty = \text{constant}$

$$\dot{E}_{st} = \frac{dE_{st}}{dt} = \dot{E}_{in} - \cancel{\dot{E}_{out}} + \cancel{\dot{E}_{gen}}$$

$$\frac{d(mcT)}{dt} = -hA(T - T_\infty)$$

$$T(t=0) = T_{in}$$



The formulation remains same for heating/cooling

$T = T(t)$ assumption is known as **lumped approximation** **(or lumped capacitance approximation)**

Applicability of lumped approximation

$T = T(t)$ implies

$\boxed{\mathrm{Bi} = \dfrac{hL}{k}}$ $L = \dfrac{V}{A}$, usually

***k*: high/~~low~~**

***h*: ~~high~~/low**

***L*: ~~high~~/low**

Bi (Biot number) needs to be small (usually taken <=0.1) for lumped approximation to be applicable

$$\mathrm{Bi} = \frac{hL}{k} = \frac{hA\Delta T}{kA\Delta T/L} \approx \frac{\text{convection heat transfer rate}}{\text{conduction heat transfer rate}}$$

$$\mathrm{Bi} = \frac{hL}{k} = \frac{L/(kA)}{1/(hA)} \approx \frac{\text{conduction resistance}}{\text{convection resistance}}$$

Revisiting the lumped solution

$$\theta = \frac{T - T_\infty}{T_i - T_\infty} \quad \tau = t/t_c \quad \boxed{\theta = \exp(-\tau)} \quad \boxed{\theta = \exp(-\mathrm{Fo.Bi})}$$

$$t_c = \frac{\rho V c}{hA} = \frac{\rho L c}{h} = \frac{\rho c}{k}\frac{Lk}{h} = \frac{1}{\alpha}\frac{L^2 k}{hL} = \frac{L^2}{\alpha}\frac{1}{\mathrm{Bi}}$$

$$\tau = t/t_c = \frac{t}{\frac{L^2}{\alpha}\frac{1}{\mathrm{Bi}}} = \mathrm{Bi}\frac{\alpha t}{L^2} = \mathrm{Bi.Fo} \longrightarrow$$ **Fourier number**

J. Biot
1774-1862

Fourier
1768-1830