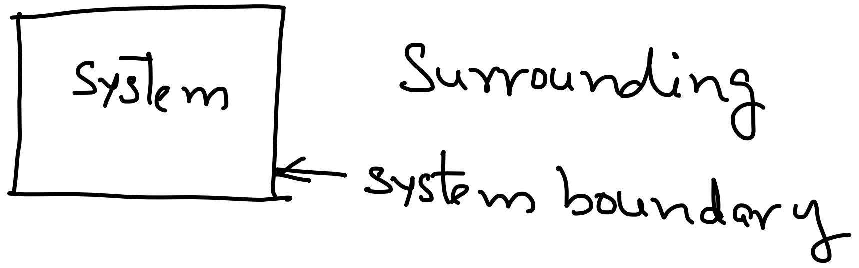
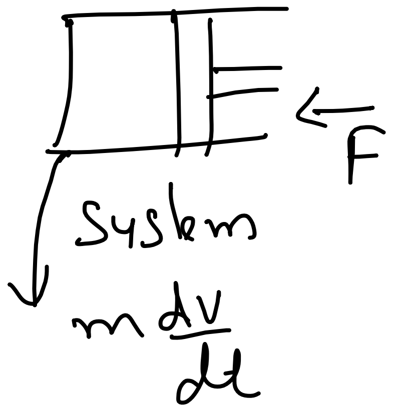
How to apply laws of mechonics to a control Volume

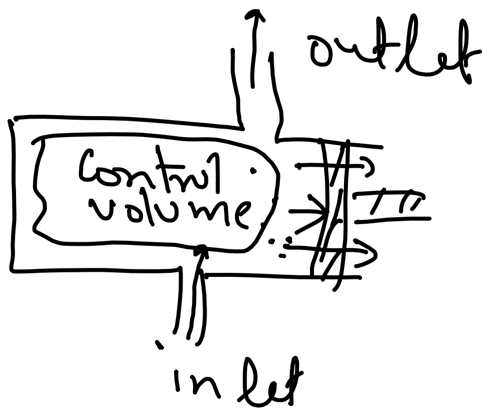
- All laws of mechonics are written for a System
- System: An arbitary quantity of mass of fixed identity



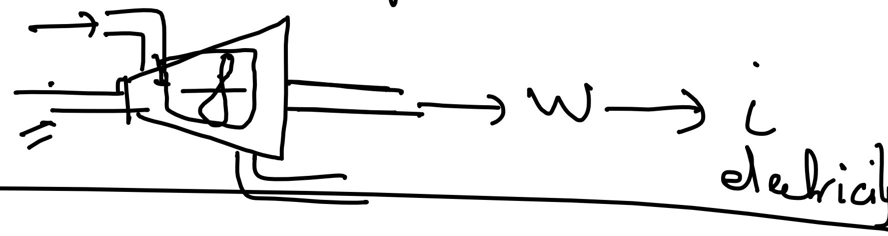
system $\xleftrightarrow[\text{Mechonic}]{\text{laws of}}$ Surrounding

Control Volume





control fixed region



Laws of mechonics

1. Mass conservation

$$\frac{dm_{sys}}{dt} = 0$$

2. Momentum conservation

$$F = m a = m\frac{dV}{dt} = \frac{d}{dt}(mV)$$

3. Angular momentum conservation

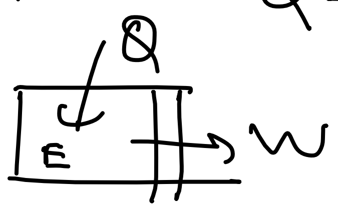
$$M = \frac{dH}{dt}$$

M = moment about the center of mass

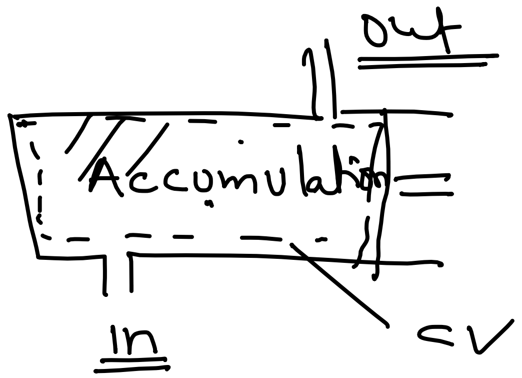
H = angular momentum

4. Energy conservation

$$\dot{Q} - \dot{W} = \frac{dE}{dt}$$



The Reynolds transport theorem



v = velocity

$\forall$ = volume

General balance

rate (in − out) + rate of generation = rate of Accumulation

Property = B , intensive property = $\frac{dB}{dm} = \beta$

rate of generation = $\frac{dB_{sys}}{dt}$

rate of Accumulation = $\frac{d}{dt}\int \beta\, dm = \frac{d}{dt}\int \beta \rho\, d\forall$

rate of out B $= \int \beta \rho (v\cos\theta)\, dA = \int \beta \rho (v.n)\, dA$

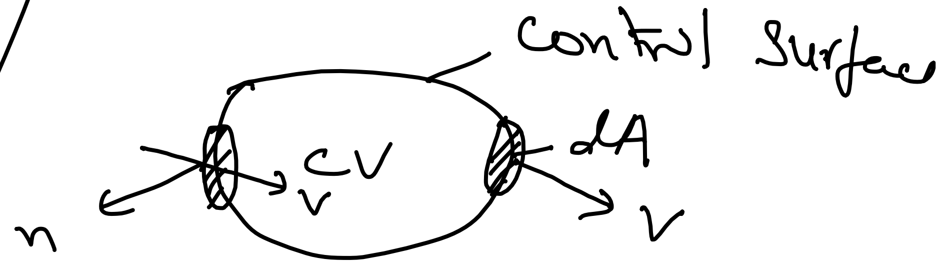
rate in B $= \int \beta \rho\, v\cos\theta\, dA = -\int \beta \rho\, v\cos(180-\theta)\, dA$

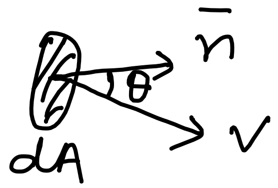
$= -\int \beta \rho (v.n)\, dA$

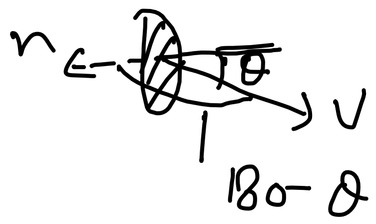
$$\frac{dB_{sys}}{dt} = \frac{d}{dt}\int \beta \rho\, d\forall + \int_{out} \beta \rho (v.n)\, dA + \int_{in} \beta \rho (v.n)\, dA$$

$$\dot{Q} - \dot{w} = \frac{dB_{sys}}{dt} = \frac{d}{dt}\int_{cv} \beta \rho\, d\forall + \int_{cs} \beta \rho (v.n)\, dA$$

RTT



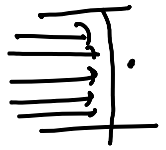




Mass conservation

$B = m \qquad \beta = \frac{dm}{dm} = 1$

$$\frac{dm_{sys}}{dt} = 0 = \frac{d}{dt}\int_{CV} \rho \, d\forall + \int_{CS} \rho \, (\underline{V}\cdot\underline{n}) \, dA$$

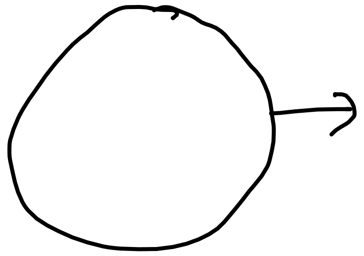
one-dimensional  $V \neq V(r)$

$V = \text{constant}$

$$= \frac{d}{dt}\int \rho \, d\forall + (\rho V A)_{out} - (\rho V A)_{in}$$

Steady state $\frac{d}{dt} = 0$

$(\rho VA)_{out} = (\rho VA)_{in}$ Steady and one-dimension in/out

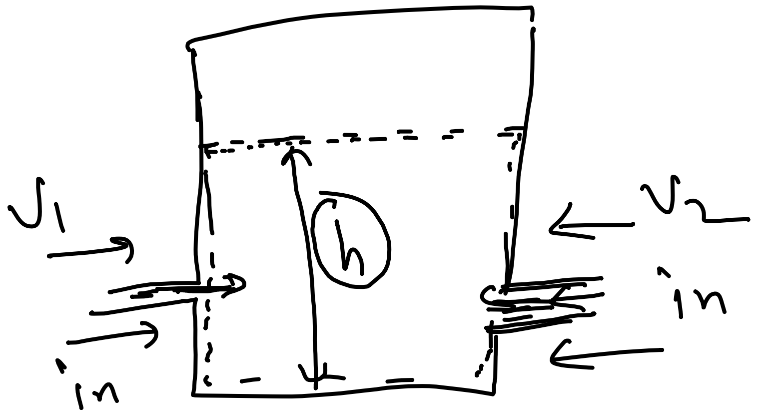


control surface velocity $= V_s$

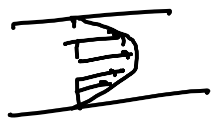
$$0 = \frac{d}{dt}\int \rho \, d\forall + \int \rho \, (V_r \cdot n) \, dA$$

$V_r = V - V_s$

Example 1 : The tank in fig is being filled with water by two one-dimensional inlets. The water height is h. Find an expression for change in water height.



A_t = Cross-section Area



mass conservation

$$0 = \frac{d}{dt}\int_{cv} \rho \, d\forall + \int_{cs} \rho \, (\underline{v} \cdot \underline{n}) \, dA$$

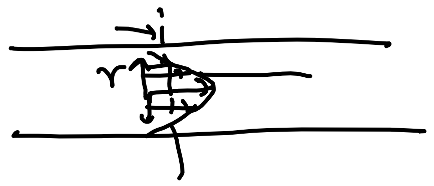
$$= \frac{d}{dt}(\rho A_t h) + (-)(\rho V_1 A_1) + (-)\, \rho V_2 A_2 = 0$$

$$\frac{dh}{dt} = \frac{A_1 V_1 + A_2 V_2}{A_t}$$

Example 2 = For steady viscous flow through a circular tube the axial velocity profile is given by

$$u = U_0\left(1 - \frac{r}{R}\right)^m$$

so u varies from zero at the wall ($r=R$) or no slip up to maximum $u = U_0$ at the centerline. for highly viscous (laminar) flow $m = 1/2$ while for less viscous (turbulent) flow $m = 1/7$. Compute average velocity if density is constant.



$$u = U_0\left(1 - \frac{r}{R}\right)^m$$

Solution $$V_{avg} = \frac{\int u \, 2\pi r \, dr}{A} = U_0 \frac{2}{(1+m)(2+m)}$$