ESO204A, Fluid Mechanics and rate Processes

Conduction Heat Transfer

Chapter 2 of Cengel

General Eq. of heat conduction $\frac{\partial T}{\partial t} = \alpha \nabla^2 T + \frac{\dot{e}_{\text{gen}}}{\rho c}$

1-D Cartesian, no generation: $\frac{d^2 T}{dx^2} = 0$

Conduction heat flux: $q'' = -k \frac{dT}{dx}$

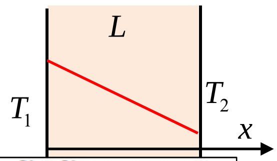
Convective heat flux: $q'' = h\left(T - T_\infty\right)$

Radiative heat flux: $q'' = h_r\left(T - T_\infty\right)$

$$h_r = \varepsilon\sigma\left(T + T_\infty\right)\left(T^2 + T_\infty^2\right)$$

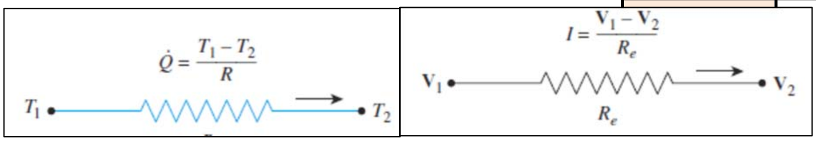
Steady, 1-D Heat Transfer in a Slab: Electrical Analogy

$$T_1 - T_2 = \Delta T \qquad \dot{Q} = \frac{kA(T_1 - T_2)}{L}$$



$$\Delta T = \dot{Q}\left(\frac{L}{kA}\right) \equiv \Delta V = IR \qquad R_{\text{cond}} = \frac{L}{kA}$$

voltage current resistance



Similarly $\dot{Q}_{\text{conv}} = hA\Delta T$ $\qquad R_{\text{conv}} = \frac{1}{hA}$ $\qquad R_{\text{rad}} = \frac{1}{h_{\text{rad}}A}$

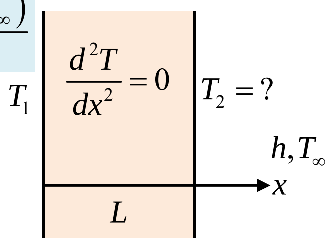
$\dot{Q}_{\text{rad}} = h_{\text{rad}} A\Delta T$

We can now have series and parallel combination as in electrical circuit

Steady, 1-D Heat Transfer in a Slab

$$T = T_1 + \frac{h(T_\infty - T_1)}{k + hL}x$$

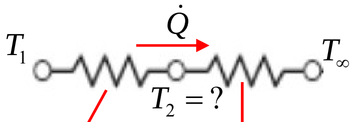
$$T_2 = T_1 + \frac{hL(T_\infty - T_1)}{k + hL} \qquad \dot{Q} = \frac{kAh(T_1 - T_\infty)}{k + hL}$$



Electrical analogy

$$\dot{Q} = \frac{T_1 - T_\infty}{L/(kA) + 1/(hA)}$$

To find T_2

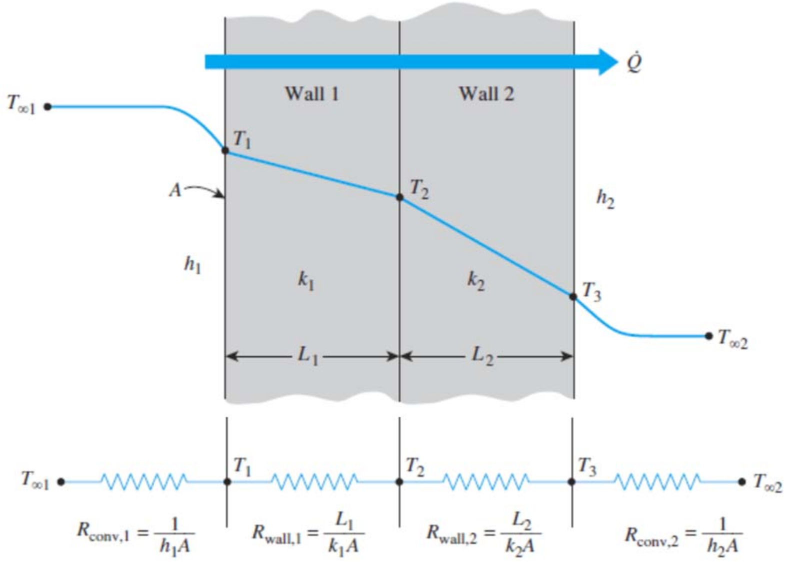


$$\dot{Q} = \frac{T_1 - T_\infty}{L/(kA) + 1/(hA)} = \frac{T_1 - T_2}{L/(kA)}$$

$$\frac{T_2 - T_1}{T_\infty - T_1} = \frac{L/(kA)}{L/(kA) + 1/(hA)} = \frac{hL}{k + hL}$$

$$R_{\text{cond}} = \frac{L}{kA} \qquad R_{\text{conv}} = \frac{1}{hA}$$

1-D steady conduction in a multilayer plane wall

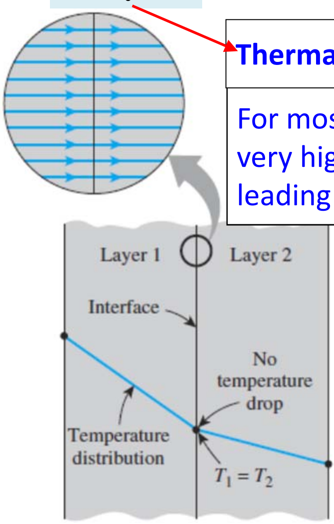


$\dot{Q} = h_c A \Delta T$

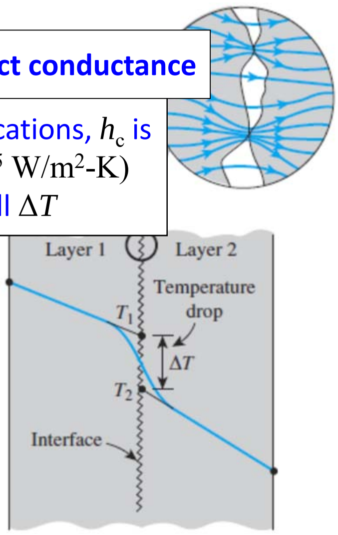
Thermal contact

Thermal contact conductance

For most applications, h_c is very high ($\sim 10^5$ W/m^2-K) leading to small ΔT



Surface to surface contact (ideal)

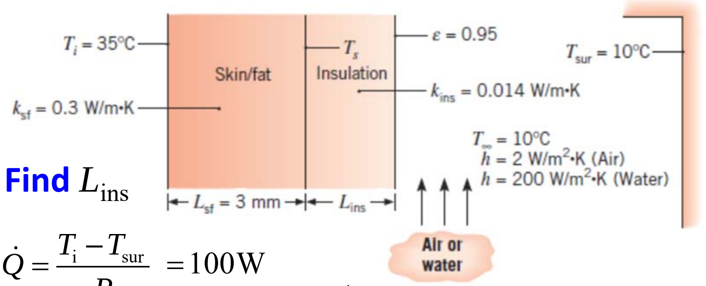


Surface to surface contact (actual)

Designing an winter garment

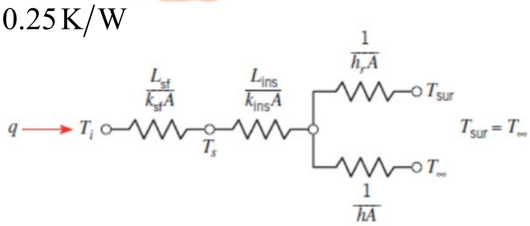
Heat generation in human body in rest condition is about 100 W

Find L_{ins}



$$\dot{Q} = \frac{T_i - T_{sur}}{R} = 100\,\text{W} \Rightarrow R = 0.25\,\text{K/W}$$

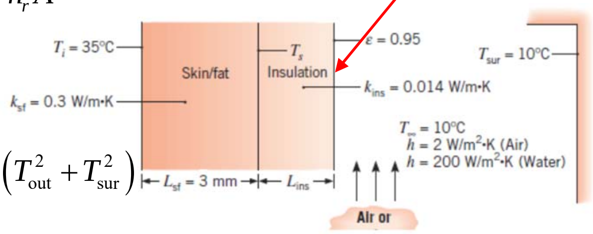
$$R = \frac{L_{sf}}{k_{sf}A} + \frac{L_{ins}}{k_{ins}A} + \frac{1}{\frac{1}{1/hA} + \frac{1}{1/h_r A}}$$



$$\frac{L_{sf}}{k_{sf}A} + \frac{L_{ins}}{k_{ins}A} + \frac{1}{hA + h_r A} = .25\,\text{K/W}$$

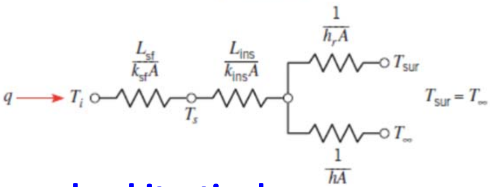
T_{out}

Find L_{ins}



$$h_r = \varepsilon\sigma\left(T_{out} + T_{sur}\right)\left(T_{out}^2 + T_{sur}^2\right)$$

$$\frac{T_i - T_{out}}{\frac{L_{sf}}{k_{sf}A} + \frac{L_{ins}}{k_{ins}A}} = \dot{Q} = 100\,\text{W}$$

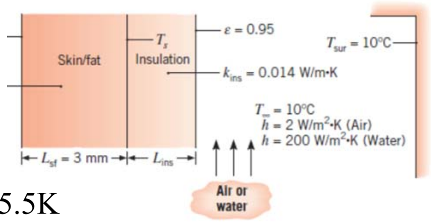


Above three Eqns to be solved itertively to find h_r, T_{out}, L_{ins}

Designing an winter garment

Find L_{ins}

$$\frac{L_{\text{sf}}}{k_{\text{sf}}A}+\frac{L_{\text{ins}}}{k_{\text{ins}}A}+\frac{1}{hA+h_rA}=.25\,\text{K/W}$$



assume $T_{\text{out}} \approx 273+\frac{35+10}{2}=295.5\text{K}$

$$h_r=\varepsilon\sigma\left(T_{\text{out}}+T_{\text{sur}}\right)\left(T_{\text{out}}^2+T_{\text{sur}}^2\right)=5.2\,\text{W/m}^2\text{-K}$$

$h=2\,\text{W/m}^2\text{-K}$

$L_{\text{ins}}=4.1\text{mm}$

$h=200\,\text{W/m}^2\text{-K}$

$L_{\text{ins}}=6.1\text{mm}$

We may now recalculate T_{out} and improve the result itertively

$$\frac{T_{\text{i}}-T_{\text{out}}}{\frac{L_{\text{sf}}}{k_{\text{sf}}A}+\frac{L_{\text{ins}}}{k_{\text{ins}}A}}=100\text{W}$$