ESO204A, Fluid Mechanics and rate Processes

## Kinematics of Fluid Flow

### Flow Visualization

Vector plot, streamline, streakline, pathline, timeline

(Chapter 1 of F M White)

### Reynolds Transport Theorem

Connection between Eulerian and Lagrangian descriptions

(Chapter 3 of F M White)

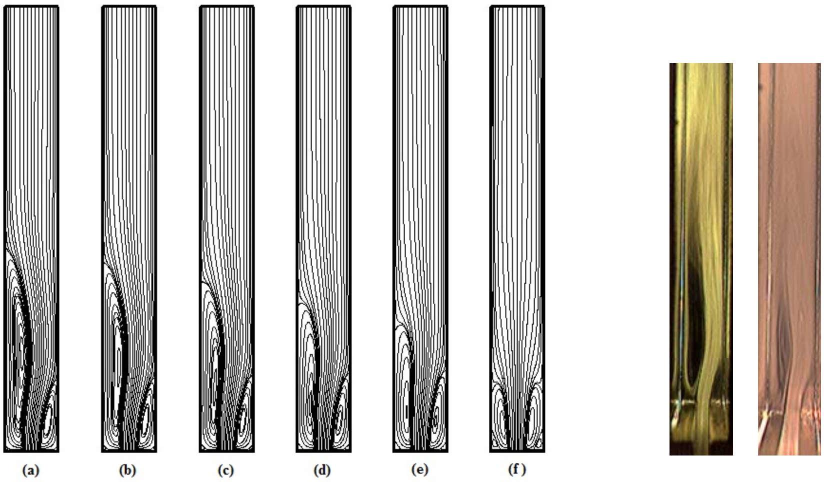
**Streamline:** imaginary lines in the flow field; tangent at any point indicates velocity direction

**Streakline:** locus of fluid particles that have passed sequentially through a prescribed point

**Pathline:** actual path travelled by an individual fluid particle over a prescribed period of time

**Timeline:** set of adjacent fluid particles that were marked at the same (earlier) instant of time

## An example: flow with sudden expansion




Computed streamlines (left) and smoke visualization experiments (right) show flow asymmetry with the increase in $\mathrm{Re}$

Find the Equations of streamline, streakline, and pathline for the following velocity field, at time $t = 0$

$$\vec{u} = (1+2t)x\vec{i} + y\vec{j}$$

All the lines passes point $(1,1)$ at time $t = 0$

**Equation of the streamline** $\dfrac{dx}{u} = \dfrac{dy}{v}$

$$\frac{dx}{x(1+2t)} = \frac{dy}{y} \qquad \Rightarrow \ln x = (1+2t)\ln y + \ln c \qquad \Rightarrow x = cy^{1+2t}$$

IC: streamline passes through $(1,1)$ at $t = 0$

Equation of the streamline at $t = 0$ $y = x$

Find the streamline, streakline, pathline for the following velocity field

$$\vec{u} = (1+2t)x\vec{i} + y\vec{j}$$

Streamline passes point $(1,1)$ at time $t=0$ $y = x$

**For streakline and pathline** $\frac{dx}{dt} = u; \frac{dy}{dt} = v$

$$\frac{dx}{dt} = (1+2t)x \quad \Rightarrow x = c_1 \exp(t+t^2)$$

$$\frac{dy}{dt} = y \quad \Rightarrow y = c_2 \exp(t)$$

Suppose a fluid particle passes point $(1,1)$ at time $t=\tau$; find $c_1$ and $c_2$ for such case

**Streakline and pathline** $x = c_1 \exp(t+t^2); y = c_2 \exp(t)$

at $t=\tau$, $(x,y) = (1,1)$ $c_1 = \exp(-\tau-\tau^2), c_2 = \exp(-\tau)$

$$y = \exp(-\tau)\exp(t) = \exp(t-\tau)$$

$$\text{similarly } x = \exp(-\tau-\tau^2)\exp(t+t^2) = \exp\left[(t-\tau)+(t^2-\tau^2)\right]$$

$$= \exp\left[(t-\tau)(1+t+\tau)\right]$$

$$\text{therefore, } \ln x = (t-\tau)(1+t+\tau) = (1+t+\tau)\ln y = \ln y^{1+t+\tau}$$

$$\Rightarrow x = y^{1+t+\tau}$$

**Streakline and pathline are governed by** $x = y^{1+t+\tau}$

**Recall** $y = \exp(t-\tau)$

we may write $t = \tau + \ln y$ or $\tau = t - \ln y$

**Now eliminating $t$** $x = y^{1+t+\tau} \Rightarrow x = y^{1+2\tau+\ln y}$

The above Equation shows the **path of a fluid particle** that passes through point $(1,1)$ at time $t=\tau$

Putting $\tau = 0$, we find the pathline traced by the fluid particle that was at location $(1,1)$ at time $t=0$

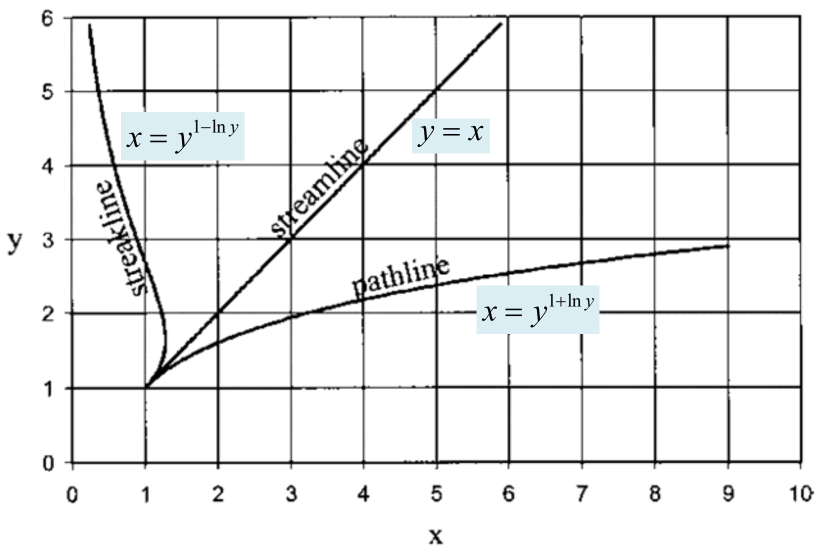
$$x = y^{1+\ln y}$$

**Suppose we eliminate $\tau$** $x = y^{1+t+\tau} \Rightarrow x = y^{1+2t-\ln y}$

Any point on the above line indicates a fluid particle that has passed through point $(1,1)$ at some time $t=\tau$

Putting $t= 0$, we find the steakline, at time $t = 0$, traced by the fluid particles passed through the point $(1,1)$

$$x = y^{1-\ln y}$$




Comparison of the streamline, streakline and pathline passing through $(1,1)$ at $t = 0$