ESO204A, Fluid Mechanics and rate Processes

Transient Conduction

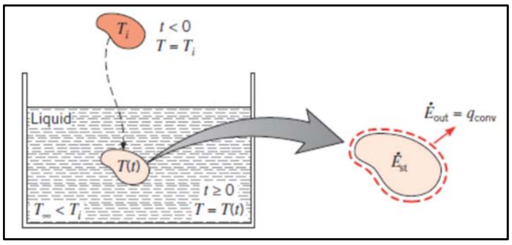
Chapter 4 of Cengel

Lumped approximation: heating/cooling of a solid in a large pool of fluid

$$T = T(t), T_\infty = \text{constant}$$

$$\frac{dT}{dt} = -\frac{hA}{\rho Vc}(T - T_\infty)$$

$$T(t=0) = T_i$$



$$\theta = \frac{T - T_\infty}{T_i - T_\infty} \qquad \tau = t/t_c$$

Time constant $t_c = \dfrac{\rho Vc}{hA}$

$$\frac{d\theta}{d\tau} = -\theta \qquad \theta(\tau = 0) = 1 \qquad \theta = \exp(-\tau)$$

Applicability of lumped approximation

Bi needs to be small (usually <=0.1) for lumped approximation to be applicable

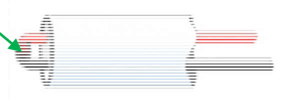
$$\mathrm{Bi} = \frac{hL}{k} \qquad L = \frac{V}{A}$$

$$\mathrm{Bi} = \frac{hL}{k} = \frac{\text{convection heat transfer rate}}{\text{conduction heat transfer rate}} = \frac{\text{conduction resistance}}{\text{convection resistance}}$$

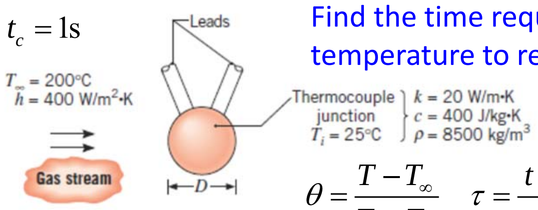
$$t_c = \frac{\rho L c}{h} \qquad \tau = \frac{t}{t_c} = \frac{hL}{k}\frac{\alpha t}{L^2} = \mathrm{Bi.Fo} \qquad \theta = \exp(-\mathrm{Fo.Bi})$$

Example: lumped problem, thermocouple measurement

Bead



$t_c = 1\mathrm{s}$



Find the time required for the junction temperature to reach 199°C

$$\theta = \exp(-\tau)$$

$$\theta = \frac{T - T_\infty}{T_i - T_\infty} \qquad \tau = \frac{t}{t_c} \qquad t_c = \frac{\rho V c}{hA} \Rightarrow \frac{V}{A} = \frac{h t_c}{\rho c}$$

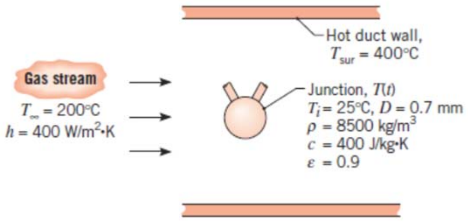
$$\mathrm{Bi} = \frac{h(V/A)}{k} = \frac{h^2 t_c}{\rho c k} = 2.35\times 10^{-3}$$

Lumped approximation is applicable

$$\theta = \frac{T - T_\infty}{T_i - T_\infty} = \frac{199-200}{25-200} = \frac{1}{175} \qquad \theta = \exp(-\tau) \Rightarrow \tau = 5.16 \quad t = 5.16\mathrm{s}$$

Not a very effective thermocouple for highly unsteady flow

$$\rho c V \frac{dT}{dt} = -hA(T - T_\infty) - \varepsilon\sigma\left(T^4 - T_{sur}^4\right)$$



Solution of the above Eq. requires numerical techniques

Is this a good experimental setup?

Steady-state solution

$$0 = -hA(T - T_\infty) - \varepsilon\sigma\left(T^4 - T_{sur}^4\right)$$

This too requires numerical solution

In general, we can expect $T_\infty < T < T_{sur}$

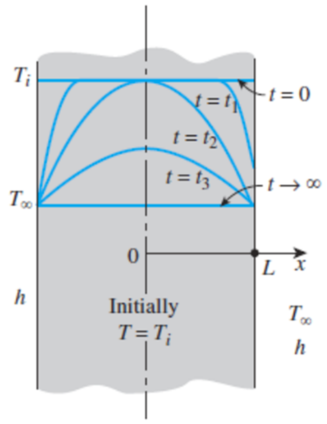
In the present case, steady solution: $T = 218^\circ\mathrm{C}$

1-D transient conduction, Cartesian

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2}$$

$$T(t=0) = T_i \qquad x = 0: \frac{\partial T}{\partial x} = 0$$

$$x = L: -k\frac{\partial T}{\partial x} = h(T - T_\infty)$$



Nondimensionalization

$$\theta = \frac{T - T_\infty}{T_i - T_\infty} \qquad X = \frac{x}{L}$$

$$\text{length scale: } \frac{V}{A} = \frac{2L.H.1}{2H.1} = L$$

1-D transient conduction, Cartesian

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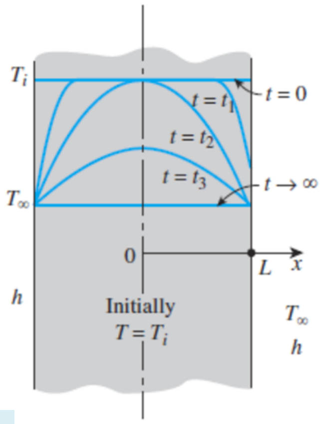

Nondimensionalization

$$\theta = \frac{T - T_\infty}{T_i - T_\infty} \qquad X = \frac{x}{L} \qquad \tau = \frac{\alpha t}{L^2} = \text{Fo}$$

$$\partial T = (T_i - T_\infty)\partial\theta \qquad \partial x = L\partial X \qquad \frac{\partial \theta}{\partial t} = \frac{\alpha}{L^2}\frac{\partial^2 \theta}{\partial X^2} \qquad \frac{\partial \theta}{\partial \tau} = \frac{\partial^2 \theta}{\partial X^2}$$