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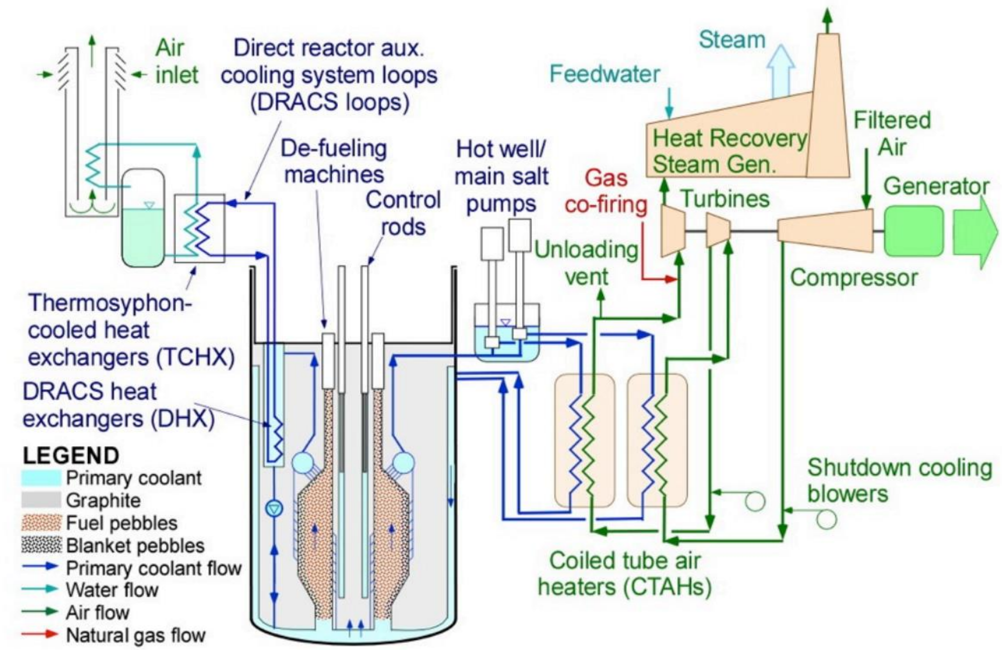
DEPARTMENT OF Engineering Physics
UNIVERSITY OF WISCONSIN–MADISON

INVESTIGATING THE STABILITY BOUNDARY OF A NATURAL CIRCULATION LOOP USING THE SYSTEM CODE SAM



1. Background and Motivation
2. Stability Analysis (Summary)
3. Loop Model in a System Code
4. Remove Sources of Numerical Instability
5. Impose Transients, Observe Behavior
6. Path Forward

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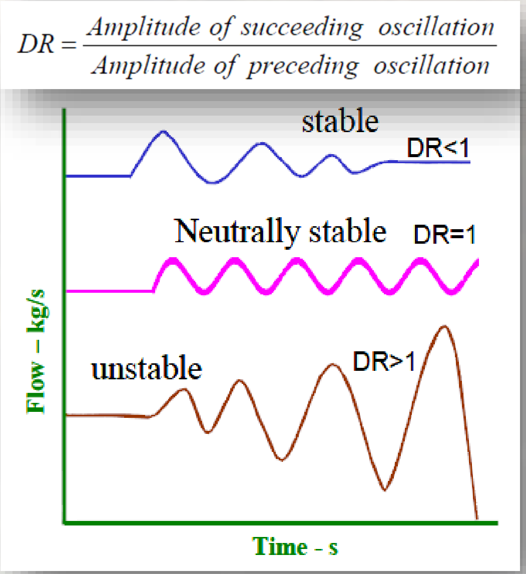
Graduate Student Seminar
21 September 2018
University of Wisconsin-Madison

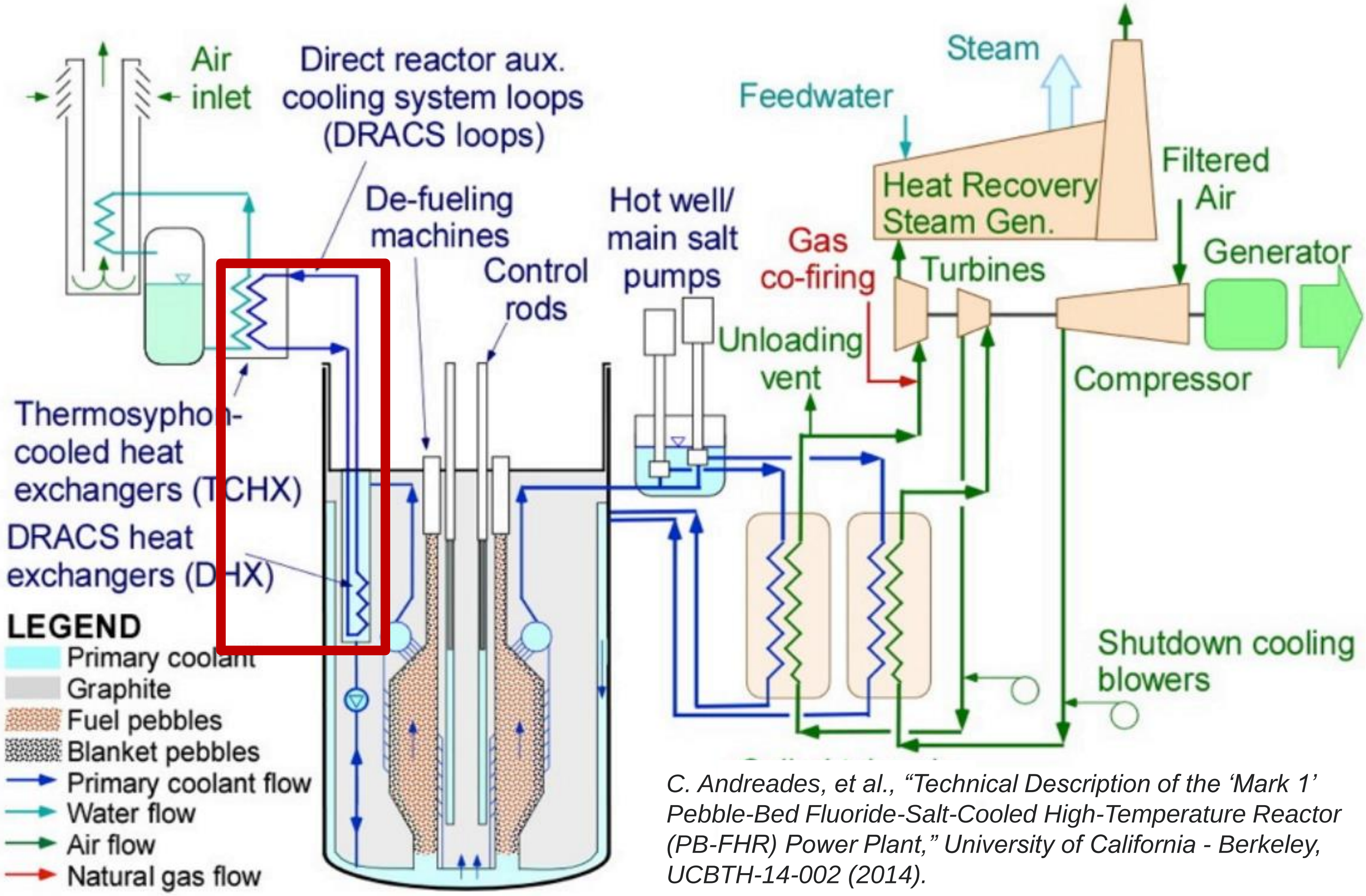
1. Background and Motivation



Instability definition: Flow versus time for damped, neutral and unstable systems (P.K. Vijayan)

Case Study: Mk1 PB-FHR

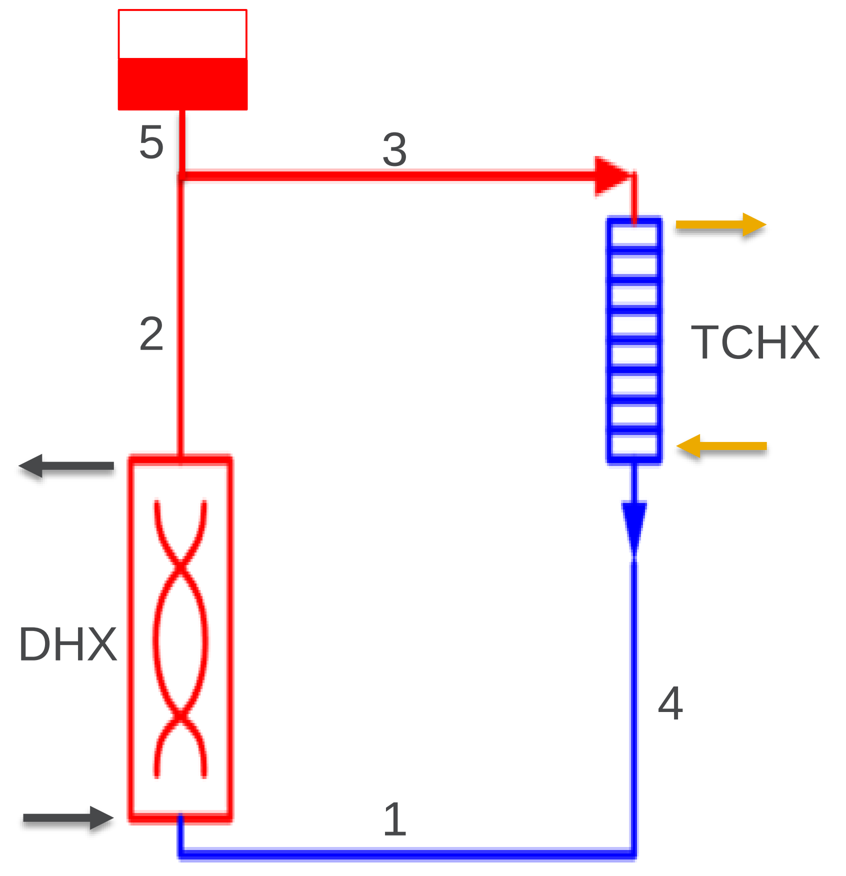
“Mark 1” Pebble-Bed Fluoride-Salt-Cooled High-Temperature Reactor



C. Andreades, et al., “Technical Description of the ‘Mark 1’ Pebble-Bed Fluoride-Salt-Cooled High-Temperature Reactor (PB-FHR) Power Plant,” University of California - Berkeley, UCBTH-14-002 (2014).

Passive Safety Systems: Physics Driven

- Primary advantage is execution of cooling functionality without operator intervention and without power
- Concern: instabilities because of strongly coupled flow, boundary conditions, and thermophysical properties (→ buoyant driving force)
- Can all components endure thermal stresses from continued oscillation? Are safety margins maintained?



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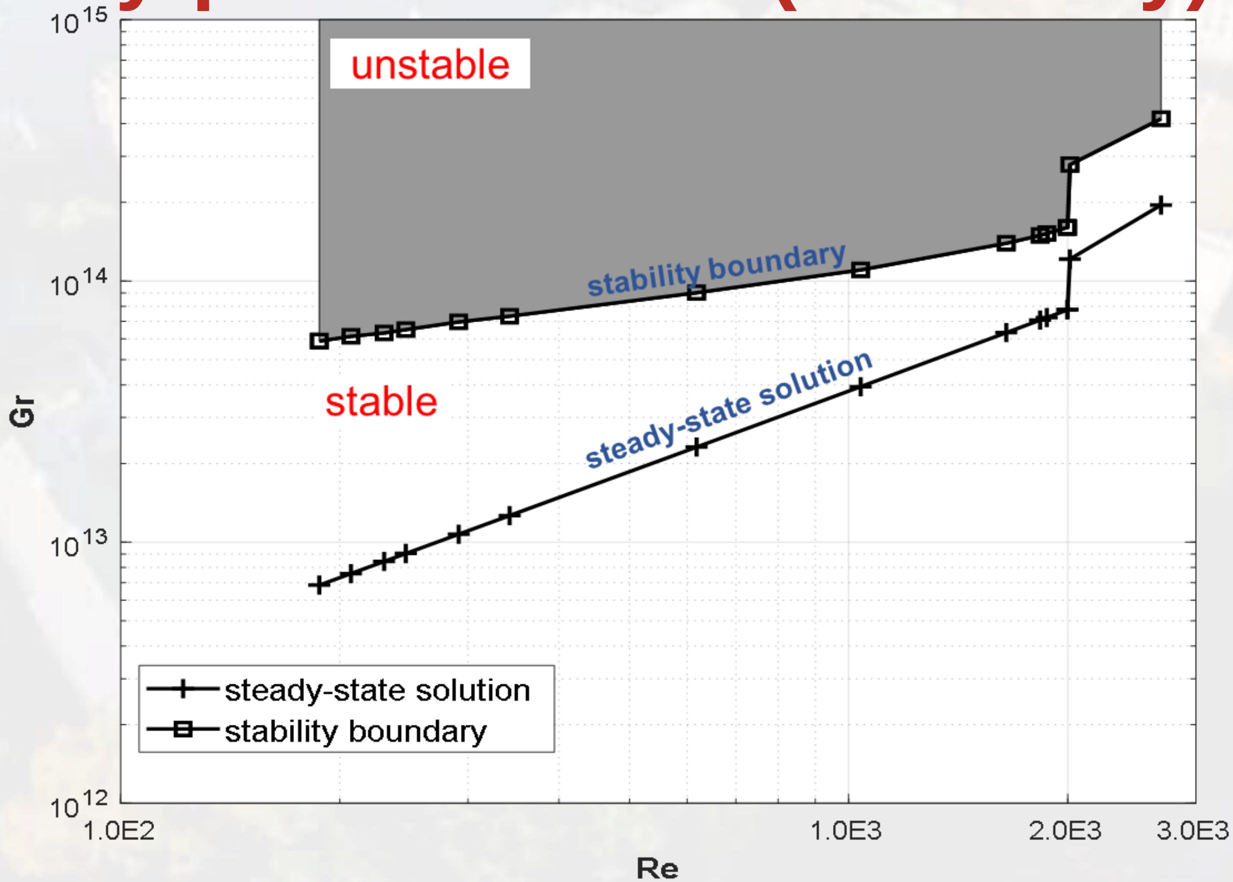
2. Stability Analysis using the Nyquist Criterion (Summary)



1D Linear Stability Analysis

- Integrate fluid momentum over the loop:

$$0 = \oint \beta g_s (T - T_o) ds - \sum_i^{loop} \frac{1}{2}\left(f_i \frac{L_i}{D_i} + K\right) u_i^2$$

- Superimpose disturbances:

$$\underline{u} = u + \varepsilon \mathrm{e}^{\omega t}, \qquad \underline{T} = T + \lambda e^{\omega t}$$

- Perturbed momentum equation:

$$\omega \sum_i^{loop} \varepsilon_i L_i = \oint \beta g_s \lambda \, ds - \sum_i^{loop} \left(f_i \frac{L_i}{D_i} + K\right) \varepsilon_i u_i$$

M. Abou Dbai, "Stability Analysis of a Molten FLiBe Natural Circulation Loop Using the Nyquist Criterion", UW-Madison, in Transactions of the American Nuclear Society, Vol. 118, Philadelphia, Pennsylvania, June 17-21, 2018

1D Linear Stability Analysis

- Energy equation (iterate with momentum to find constant properties):

$$\dot{m}c\frac{dT}{ds} = \begin{cases} \dot{q}''_w(\pi D) & \text{heated section (DHX)} \\ -U(T - T_\infty)(\pi D) & \text{cooled section (TCHX)} \\ -\pi k Nu\left(T - T_f\right) & \text{freezing section (TCHX)} \\ 0 & \text{adiabatic sections (hot and cold legs)} \end{cases}$$

- Characteristic equation:

$$\Phi(\omega) = \omega \sum_i^{loop} \varepsilon_i L_i - \beta g \left[\int^{H_{DHX}} \lambda dx + \int^{H_{HotLeg}} \lambda ds - \frac{H_{TCHX}}{L_{TCHX}}\left(\int^{S_{Rf}} \lambda ds + \int^{L_{TCHX}-S_{Rf}} \lambda ds\right) - \int^{H_{ColdLeg}} \lambda ds\right] + \sum_i^{loop}\left(f_i \frac{L_i}{D_i} + K\right)\varepsilon_i u_i$$

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1D Linear Stability Analysis

- Perturbed energy equation:

$$\lambda\omega + u\frac{\partial\lambda}{\partial s} + \varepsilon\begin{cases}\dfrac{\pi D\dot{q}_w''}{\dot{m}_h c} \\ (T_o + \Delta T - T_\infty)\left(\dfrac{-\pi DU}{\dot{m}_c c}\right)e^{\frac{-\pi DU}{\dot{m}_c c}s} \\ (T_i - T_f)\left(\dfrac{-\pi Nu\, k}{\dot{m}_c c}\right)e^{\frac{-\pi Nu\, k}{\dot{m}_c c}(s-s_{Rf})} \\ 0\end{cases} = \begin{cases}0 & \text{DHX} \\ -\dfrac{U}{\rho c}\dfrac{P}{A}\lambda & \text{TCHX} \\ -\dfrac{Nu\, k}{\rho c R_f^2}\lambda & \text{freezing} \\ 0 & \text{adiabatic}\end{cases}$$

- Characteristic equation:

$$\Phi(\omega) = \omega\sum_i^{loop}\varepsilon_i L_i - \beta g\left[\int^{H_{DHX}}\lambda dx + \int^{H_{HotLeg}}\lambda ds - \frac{H_{TCHX}}{L_{TCHX}}\left(\int^{s_{Rf}}\lambda ds + \int^{L_{TCHX}-s_{Rf}}\lambda ds\right) - \int^{H_{ColdLeg}}\lambda ds\right] + \sum_i^{loop}\left(f_i\frac{L_i}{D_i} + K\right)\varepsilon_i u_i$$

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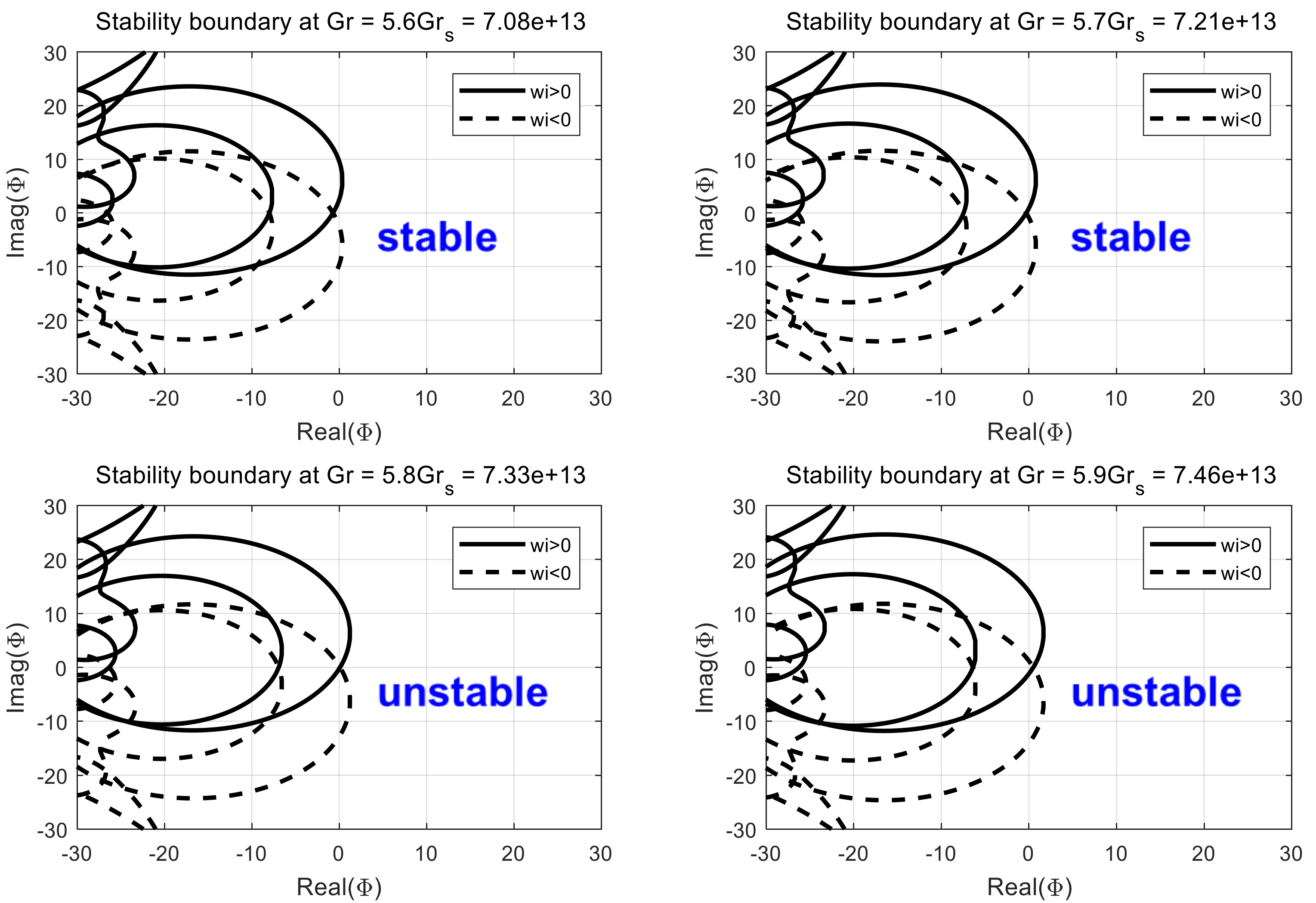
Stability Boundary Using the Nyquist Criterion



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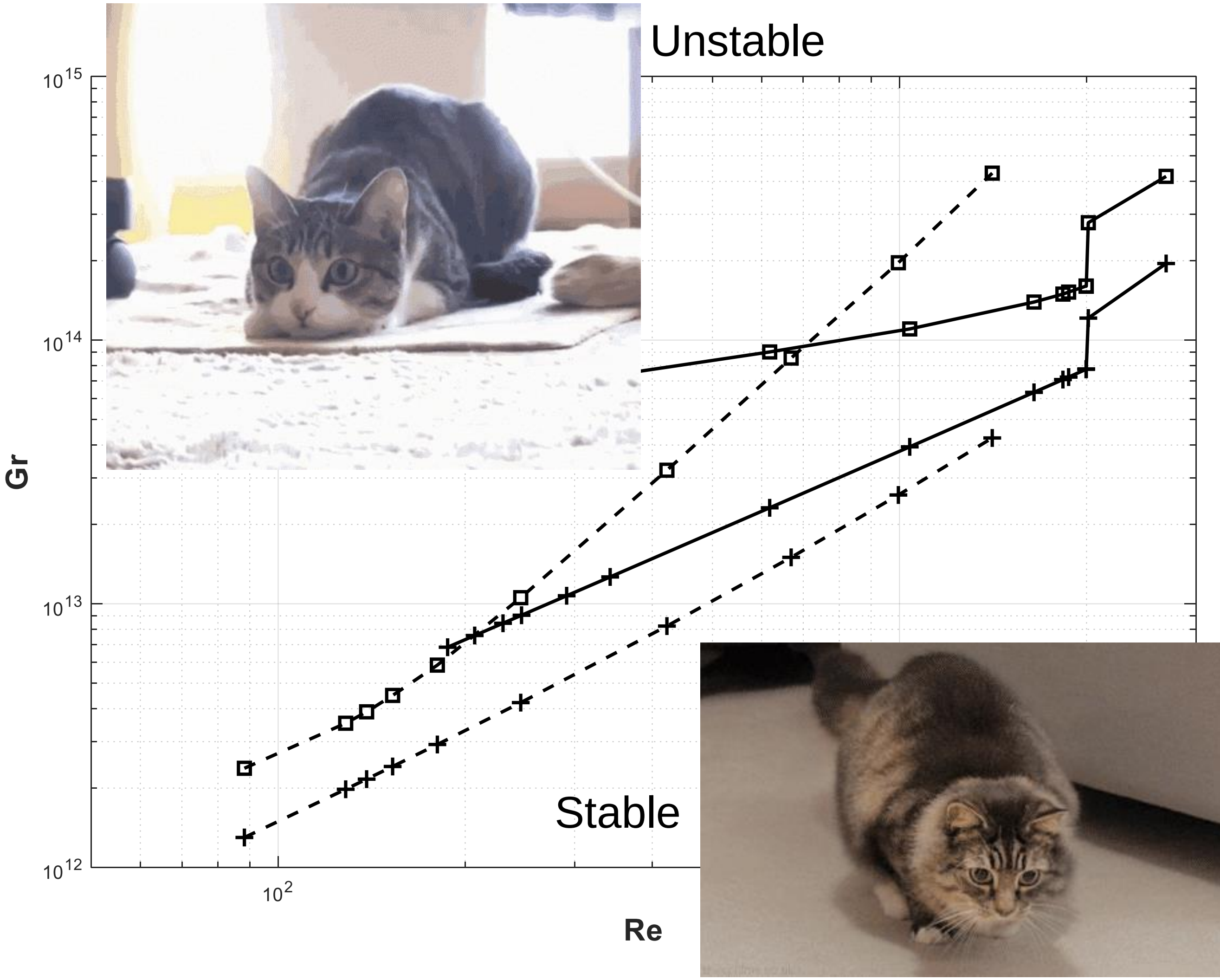
Stability Boundary Predicted for two Designs

Gr	$\rho^2 g\beta\Delta H^3\Delta T/\mu^2$
ΔH	**elevation difference between the centers of DHX and TCHX**
ΔT	**temperature rise across DHX**
Re	$\rho vD/\mu$ **at TCHX inlet**

$$\boxed{\begin{array}{c} Nt^*_{TCHX} = 4Nt_{TCHX} \\ L^*_{TCHX} = L_{TCHX}/4 \end{array}}$$



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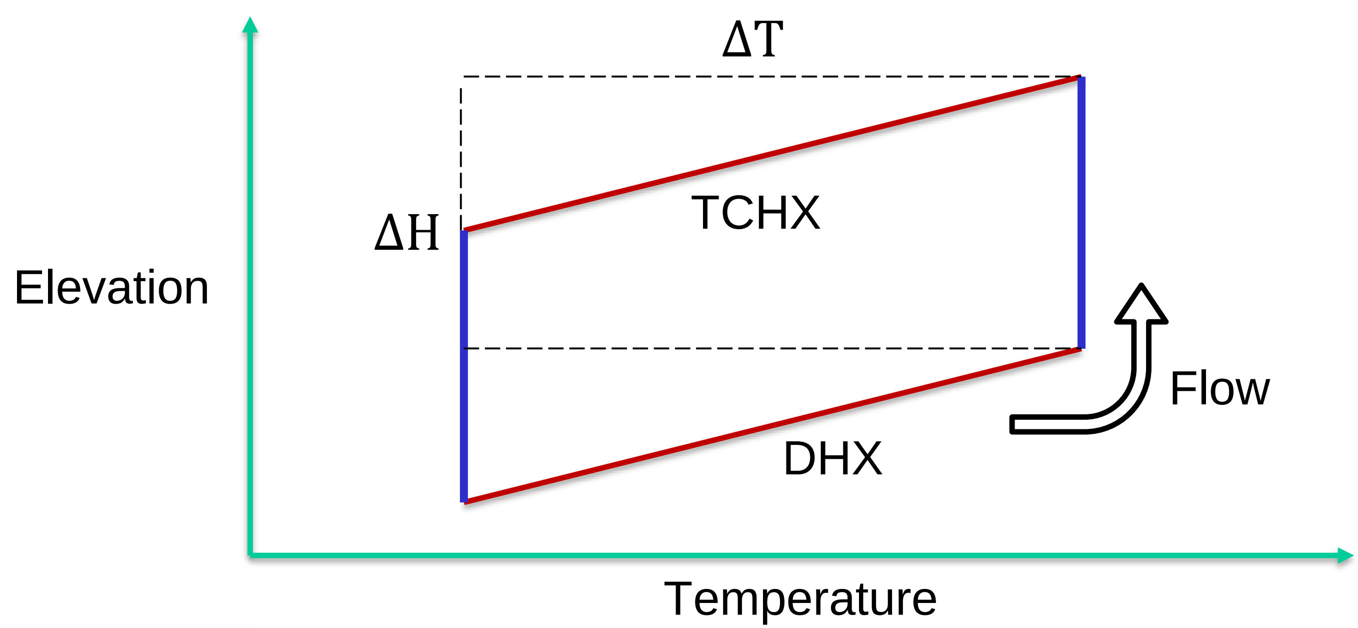
Aside: Grashof Number

- Note buoyancy term and loss term in the momentum equation:

$$0 = \oint \beta g_s (T - T_o) ds - \sum_i^{loop} \frac{1}{2}\left(f_i \frac{L_i}{D_i} + K\right) u_i^2$$

- $Gr = \rho^2 g \beta \Delta H^3 \Delta T / \mu^2$
- Ratio of buoyant forces to viscous forces



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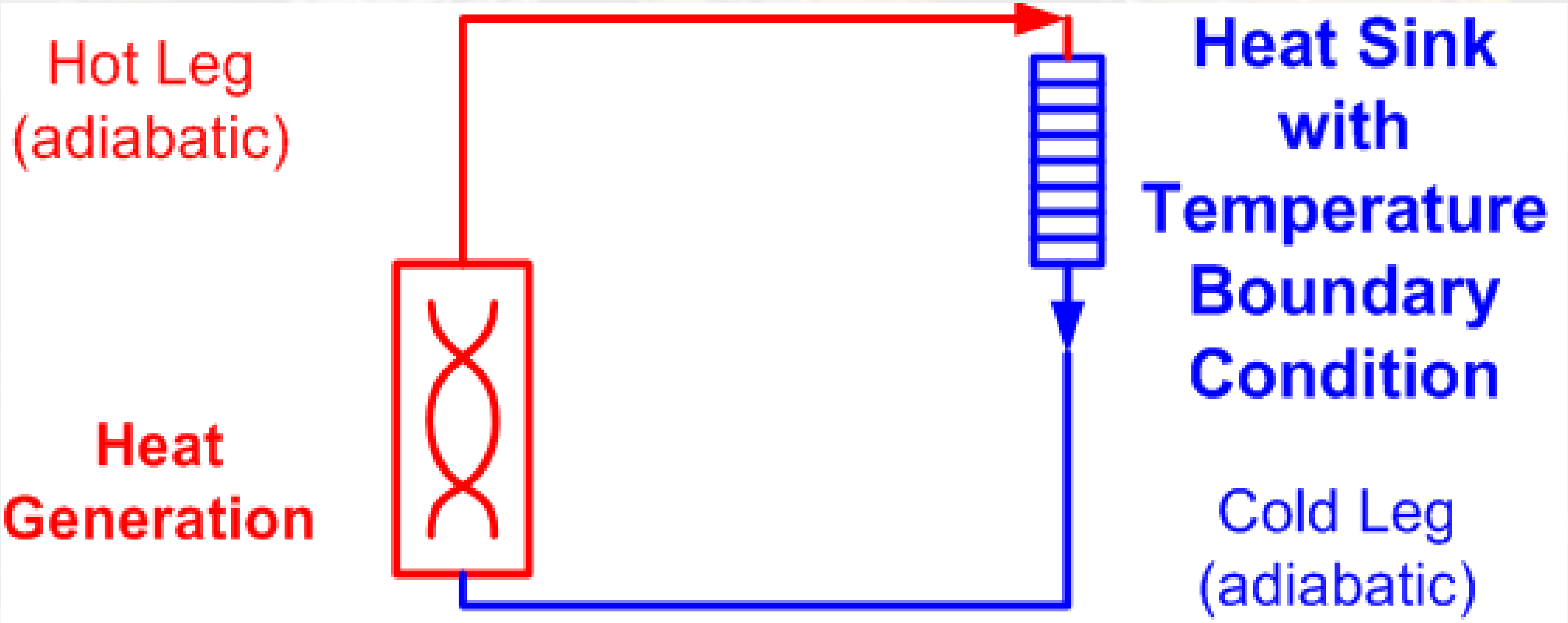
3. Loop Model in a System Code



System Analysis Module (SAM)

- Plant-level system tool for transient scenario safety analysis

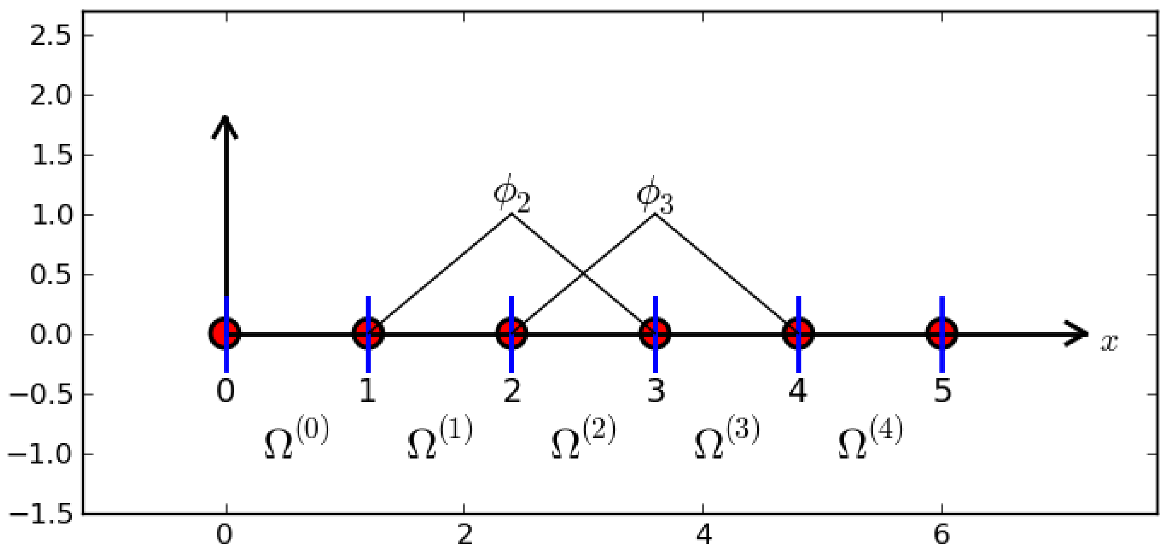
- Built on MOOSE (finite element framewor
  - High-order finite element method

- Fluid dynamics implementation: Incompressible, thermally expandable
  - Inherent buoyancy-driven circulation

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial z} = 0\ ,$$

$$\frac{\partial(\rho u)}{\partial t} + \frac{\partial(\rho u u + p)}{\partial z} = -\rho g - \frac{f}{D_e}\frac{\rho u|u|}{2}\ ,$$

$$\frac{\partial(\rho H)}{\partial t} + \frac{\partial(\rho u H)}{\partial z} = q'''\ ,$$



System Code Simulation (most basic sense)

- Network of pipes and heat structures
- Illustrates a basic component simulated in a system code

Connect inlet/outlet of pipes to build entire flow system

Flow Area, Length
Hydraulic Diameter
Heated Perimeter

Specify form losses for area changes, junctions, branches

Friction loss, heat transfer coefficient:
Specified, or determined in code logic

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial z} = 0 \,,$$

$$\frac{\partial(\rho u)}{\partial t} + \frac{\partial(\rho u u + p)}{\partial z} = -\rho g - \frac{f}{D_e}\frac{\rho u|u|}{2} \,,$$

$$\frac{\partial(\rho \mathrm{H})}{\partial t} + \frac{\partial(\rho u \mathrm{H})}{\partial z} = q''' \,,$$

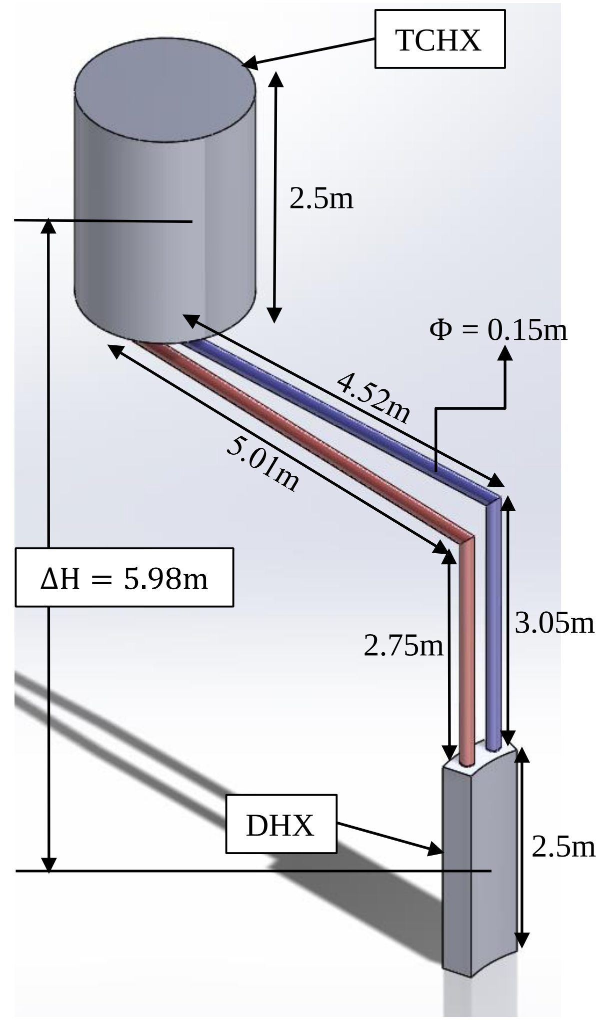
Model should be as Realistic as Possible

Mk1 design parameters for DHX (tube side) and TCHX (coiled tube side)

Parameter	DHX	TCHX	Unit
Water Temperature (T_∞)	-	100	°C
Outside diameter	0.0127	0.0127	m
Inside diameter	0.0109	0.0109	m
Number of tubes	984	936	-
Tube length	2.5	6	m
Overall U	291	22.6	W/m²-C
Heat Transfer Area	98	224	m²

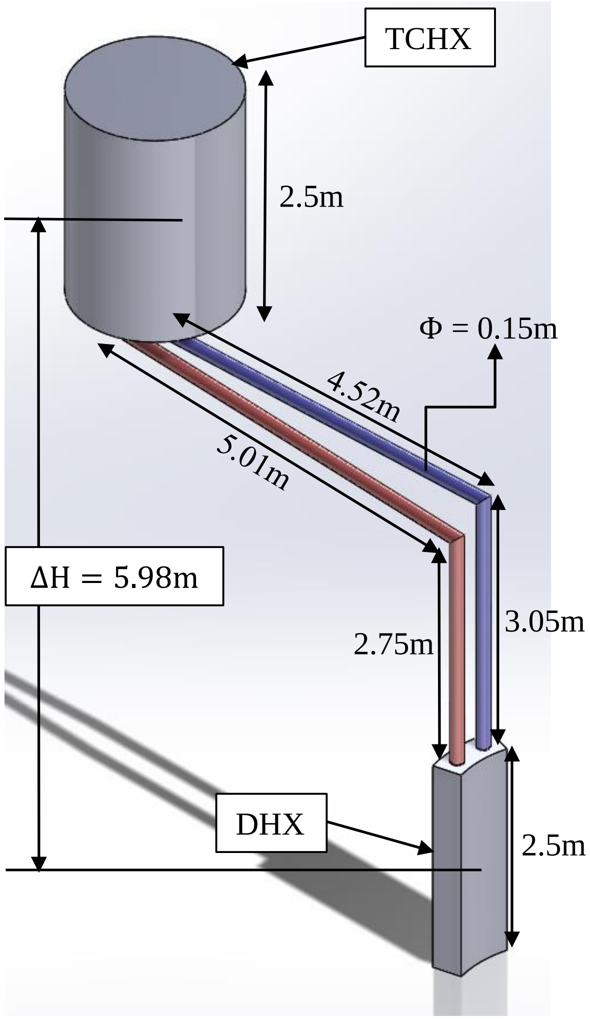
Liquid thermophysical properties for FLiBe (600-800°C)

Property	Correlation (T in °C)	Unit
Viscosity	$4.638 \cdot 10^5 / T^{2.79}$	kg/m-s
Specific Heat	2415.78	J/kg-C
Thermal Conductivity	$0.7662 + 0.0005 \cdot T$	W/m-C
Density	$2279.92 - 0.488 \cdot T$	kg/m³

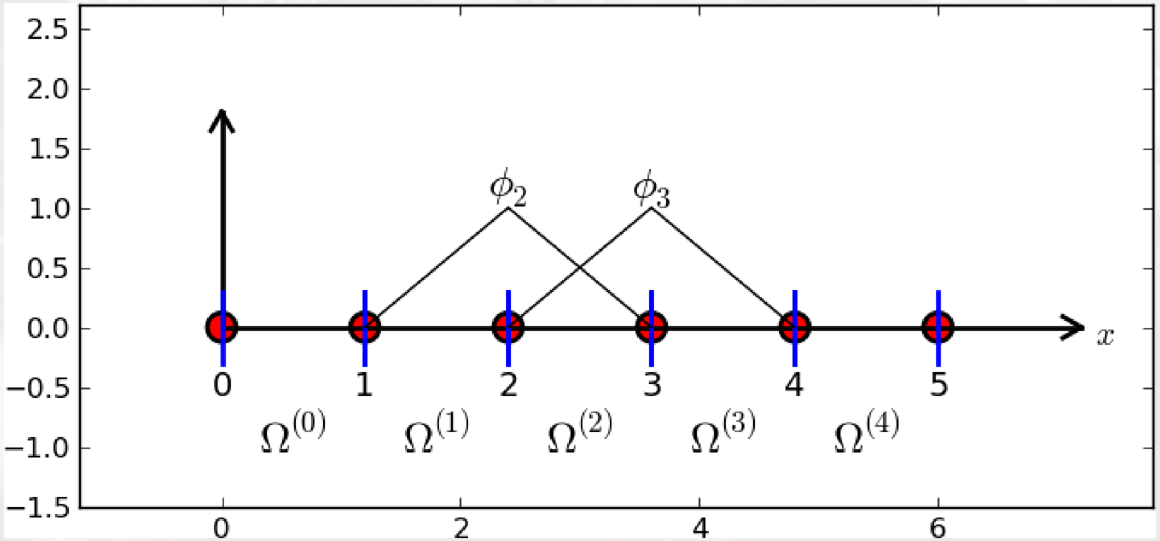


Simplifications for Comparison to Analysis

- Match flow lengths/heights/areas
- Match heat transfer areas, total heat addition at DHX, TCHX h-coefficient, etc.
- Make properties constant (except density)
- Emulate TCHX heat rejection fluid
- Make negligible the thermal inertia of solids and the junction losses
- Goal: corroborate linear stability analysis, demonstrate use of SAM for this purpose

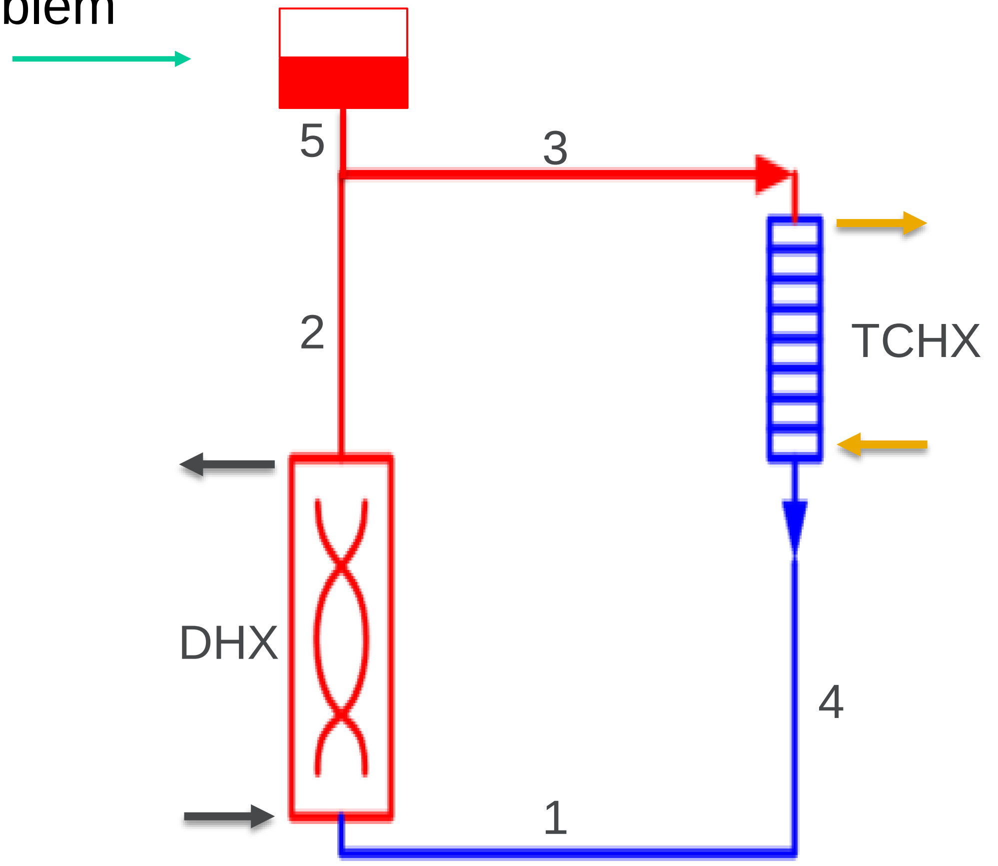


4. Remove Sources of Numerical Instability

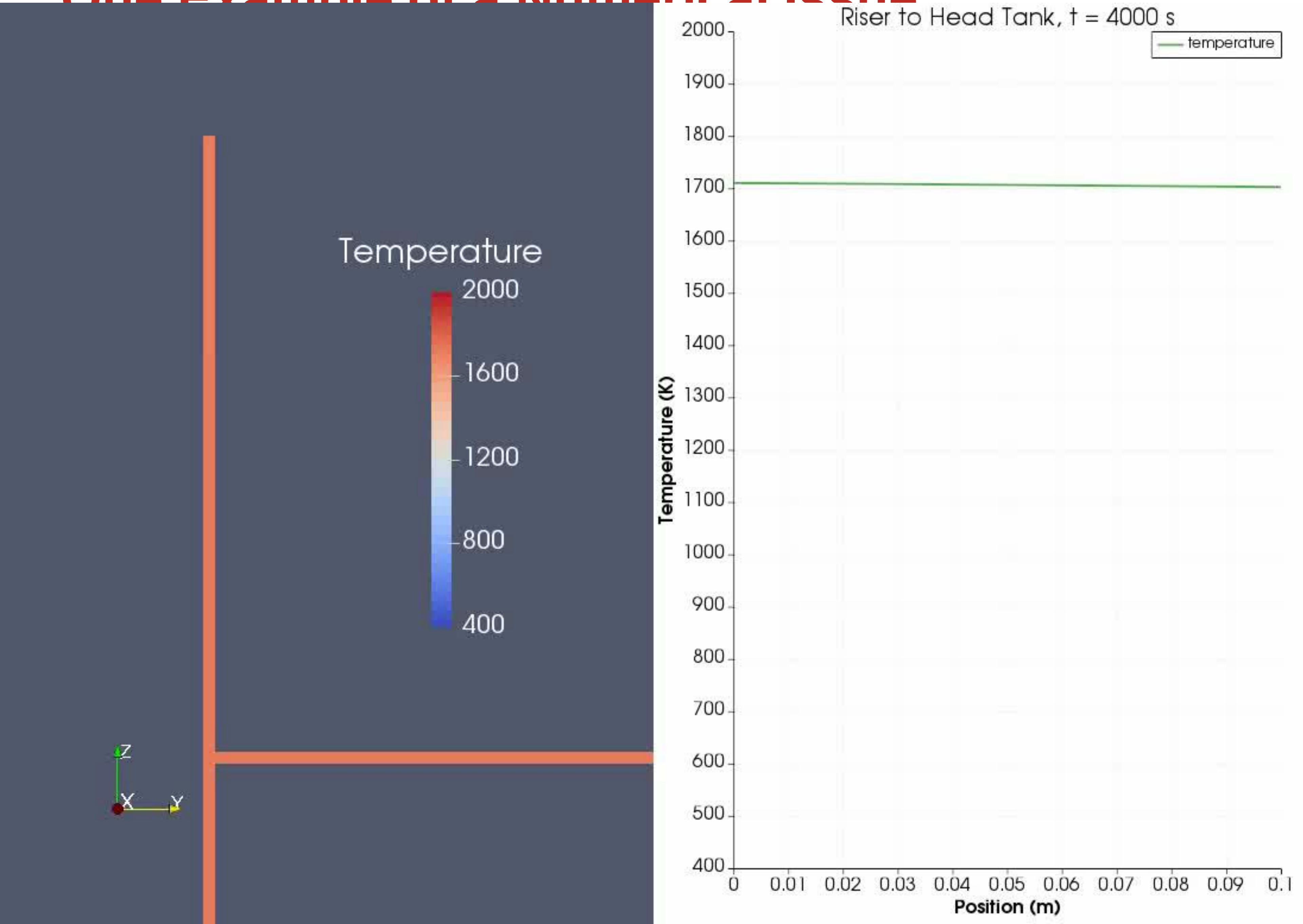


One Example of a Numerical Issue

- Tank outflow temperature problem

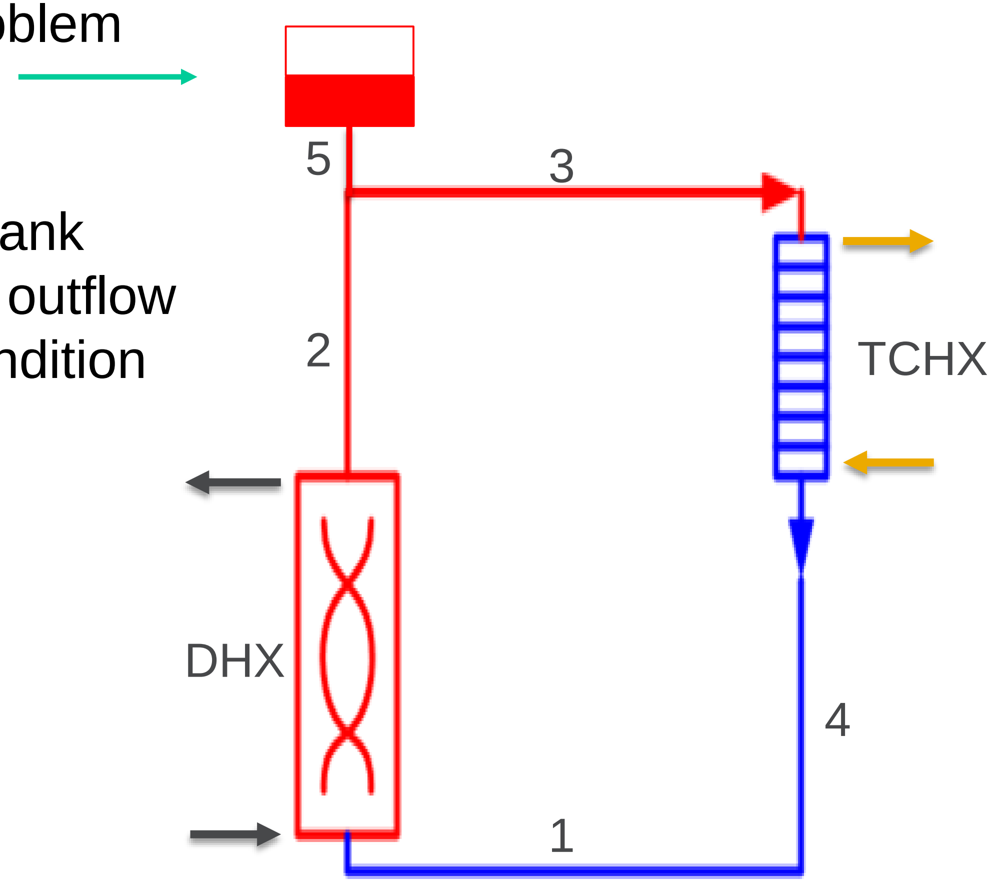


One Example of a Numerical Issue



One Example of a Numerical Issue

- Tank outflow temperature problem
- One possible approach:
  - Use a function to set the tank inflow temperature as the outflow temperature boundary condition



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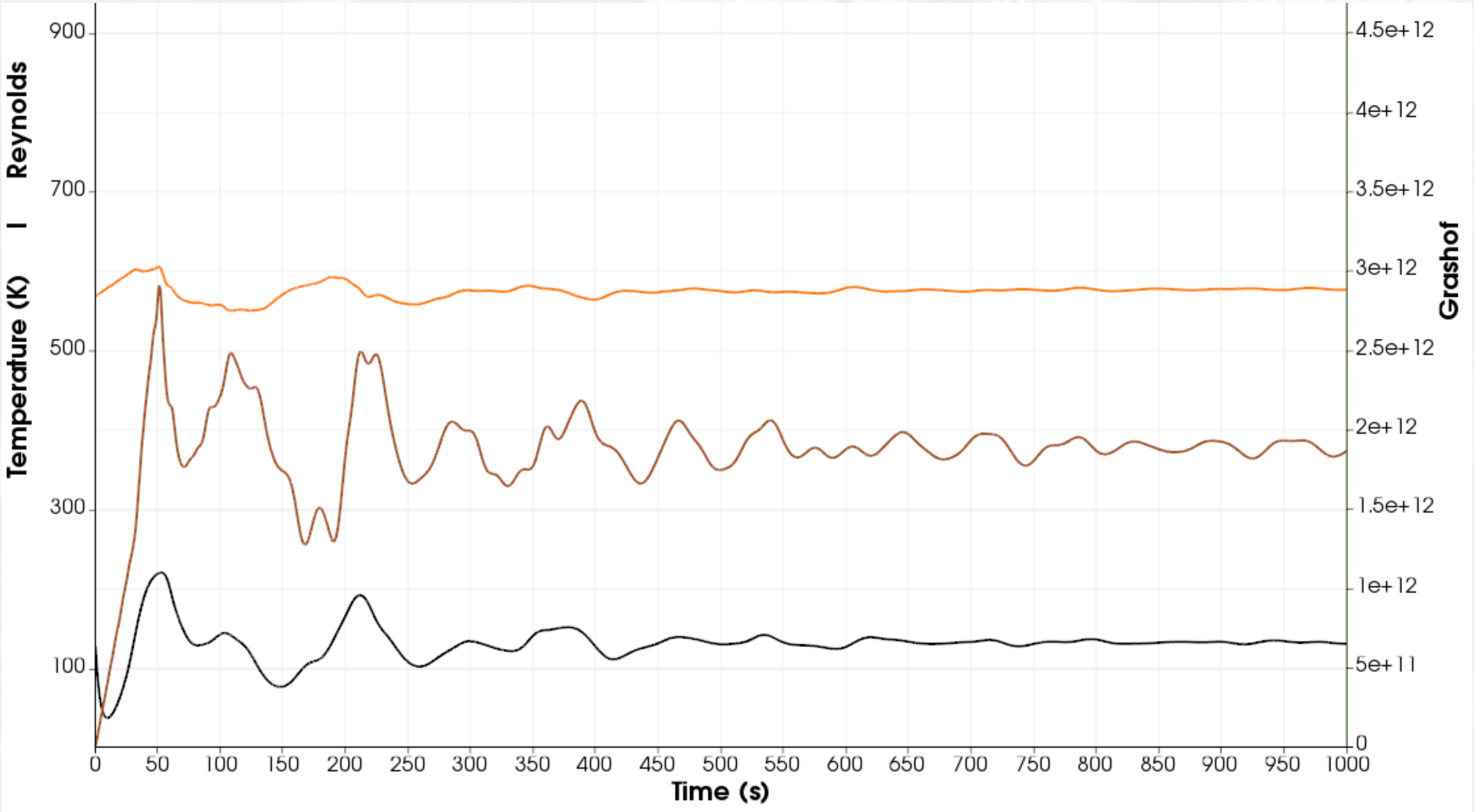
5. Impose Transients, Observe Behavior

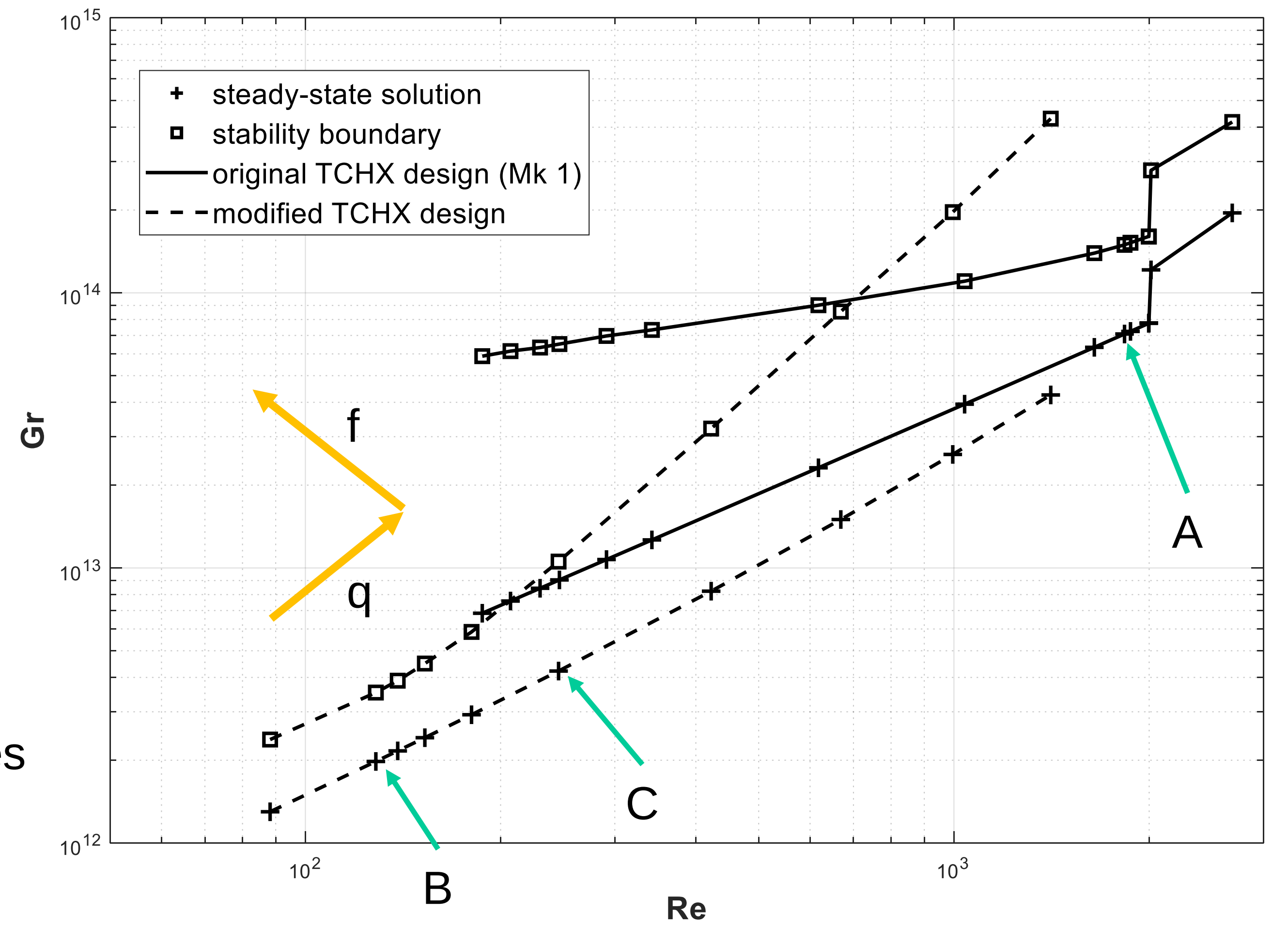


Stability Boundary: Three Test Samples

Gr	$\rho^2 g \beta \Delta H^3 \Delta T / \mu^2$
ΔH	**elevation difference between the centers of DHX and TCHX**
ΔT	**temperature rise across DHX**
Re	$\rho v D / \mu$ **at TCHX inlet**

$$Nt^*_{TCHX} = 4Nt_{TCHX}$$
$$L^*_{TCHX} = L_{TCHX}/4$$

f = increase in losses
q = increase in heat



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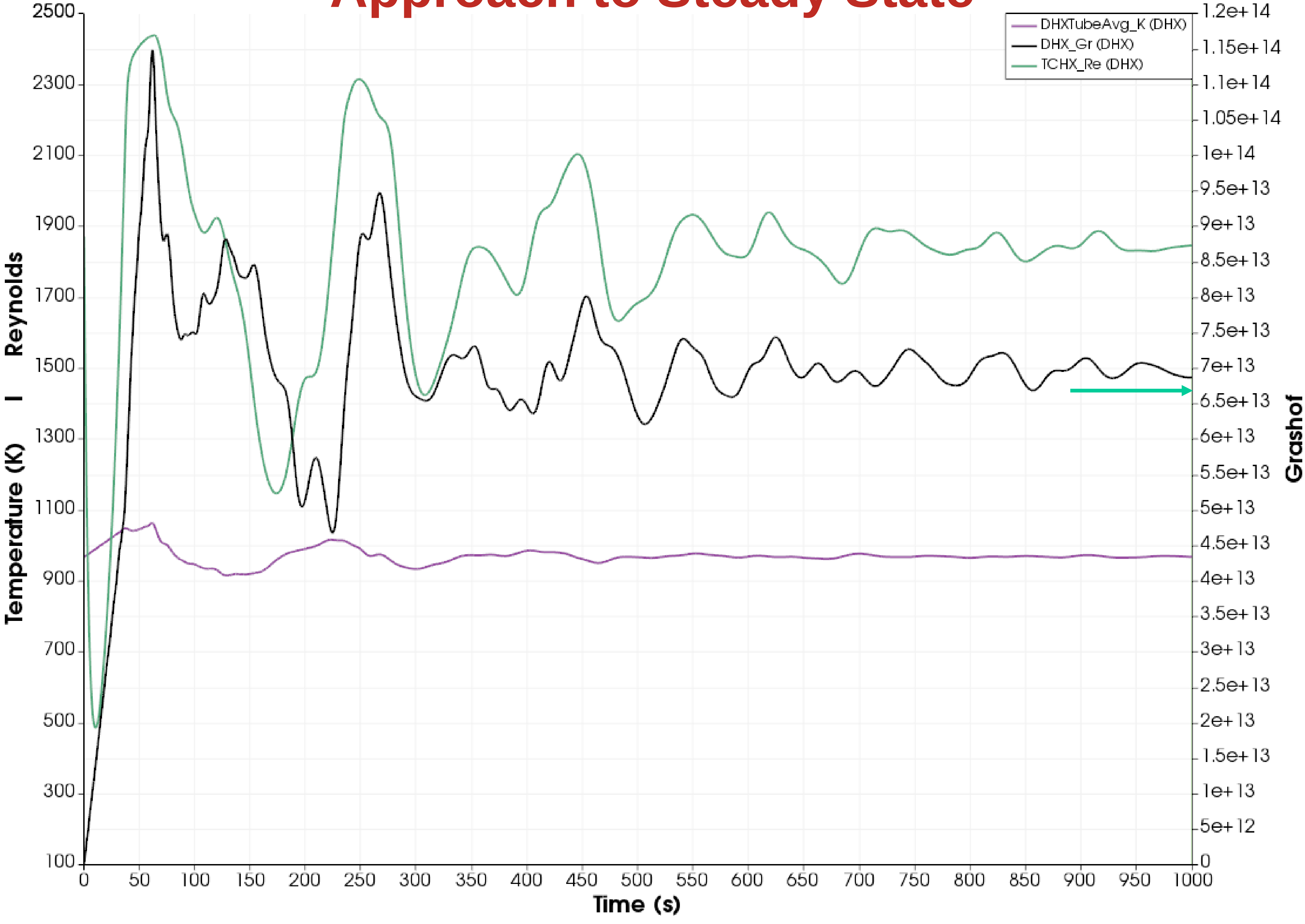
Approach to Steady State



Stability Boundary: Three Test Samples

Gr | $\rho^2 g\beta\Delta H^3\Delta T/\mu^2$

ΔH | **elevation difference between the centers of DHX and TCHX**

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Re | $\rho v D/\mu$ **at TCHX inlet**

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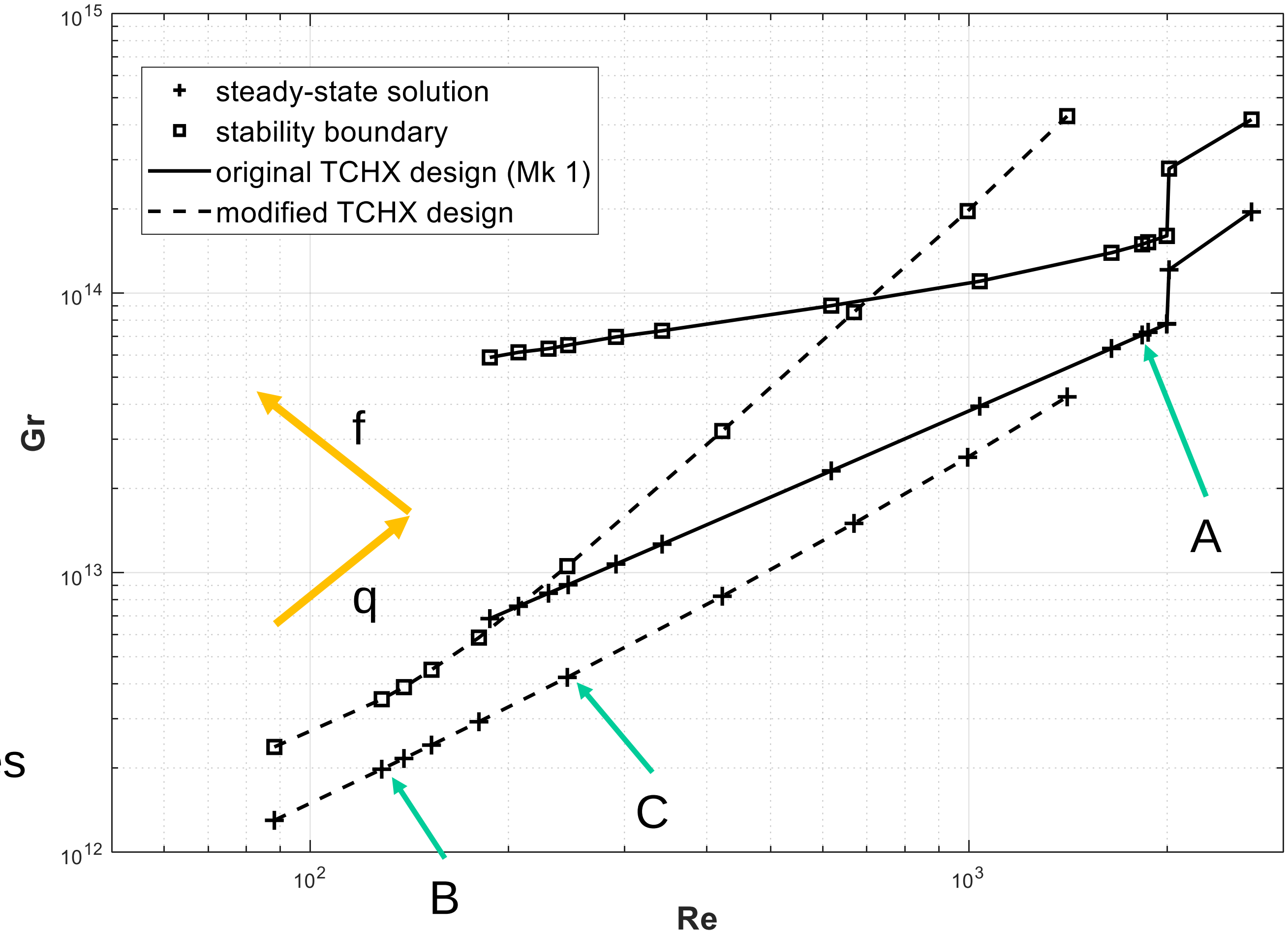
$$L^*_{TCHX} = L_{TCHX}/4$$

f = increase in losses
q = increase in heat



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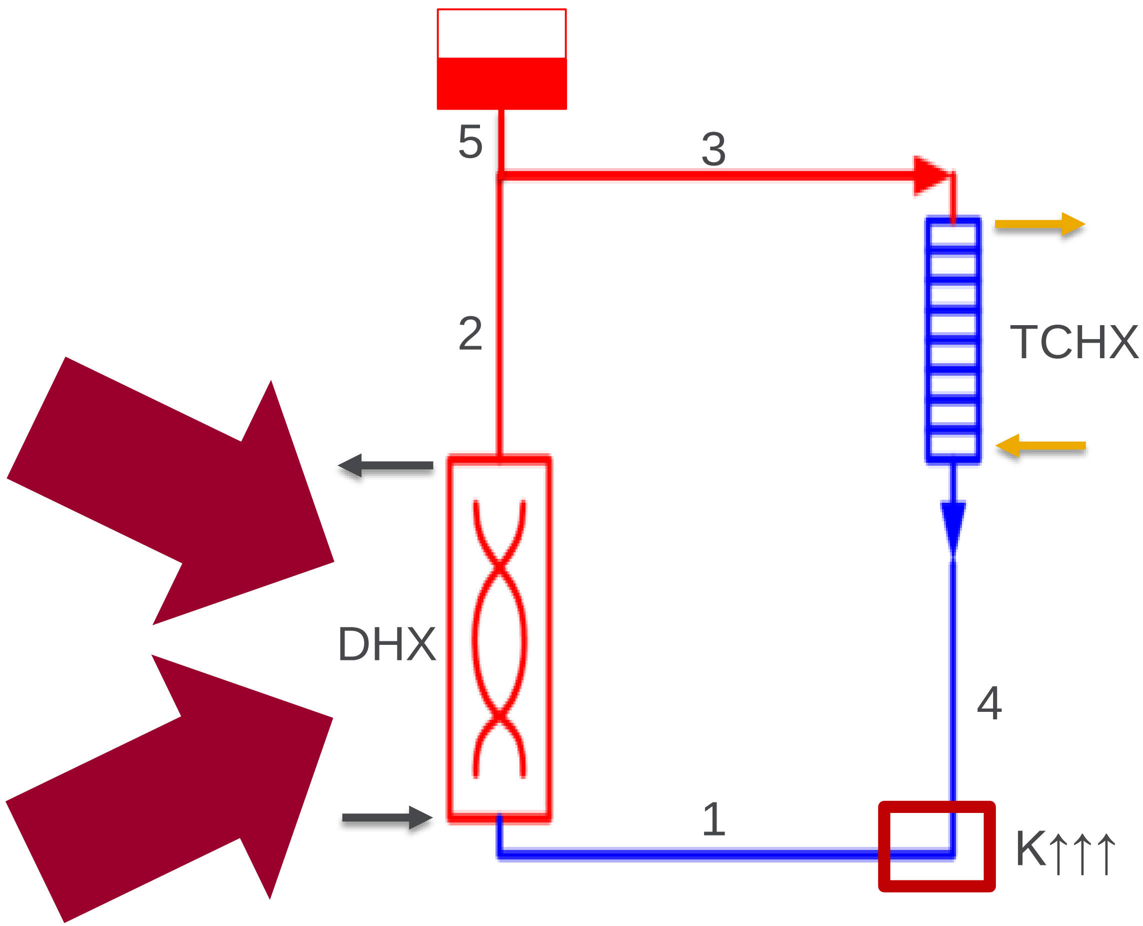
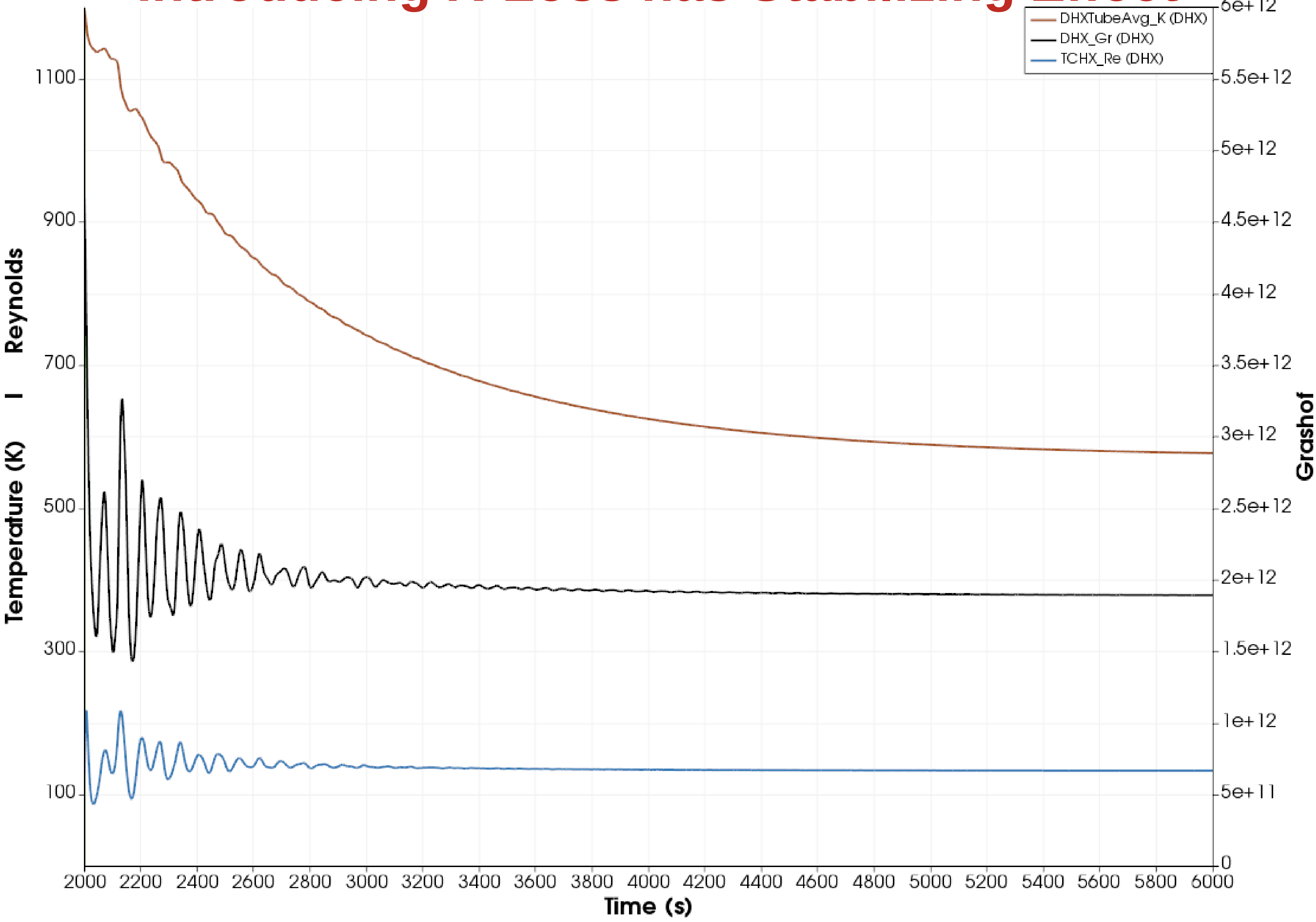
Impose Transient: Increase q and flow losses

- Increase heat addition at DHX, add K-loss factors at junctions

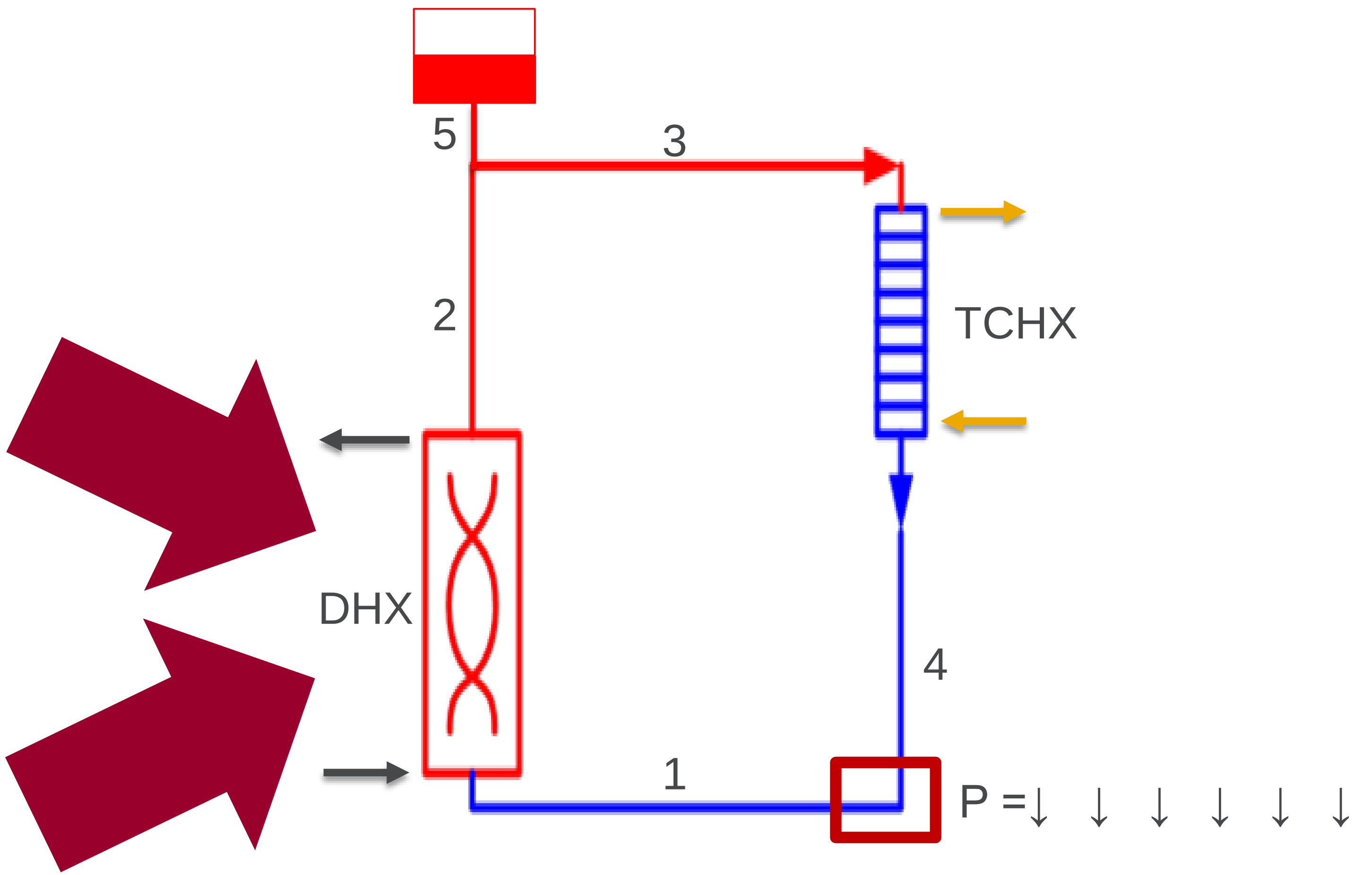


Introducing K-Loss has Stabilizing Effect



Better Idea: Artificial Head Loss

- Ramp up heat addition at DHX, ramp up anti-pump at junction



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$$Nt^*_{TCHX} = 4Nt_{TCHX}$$
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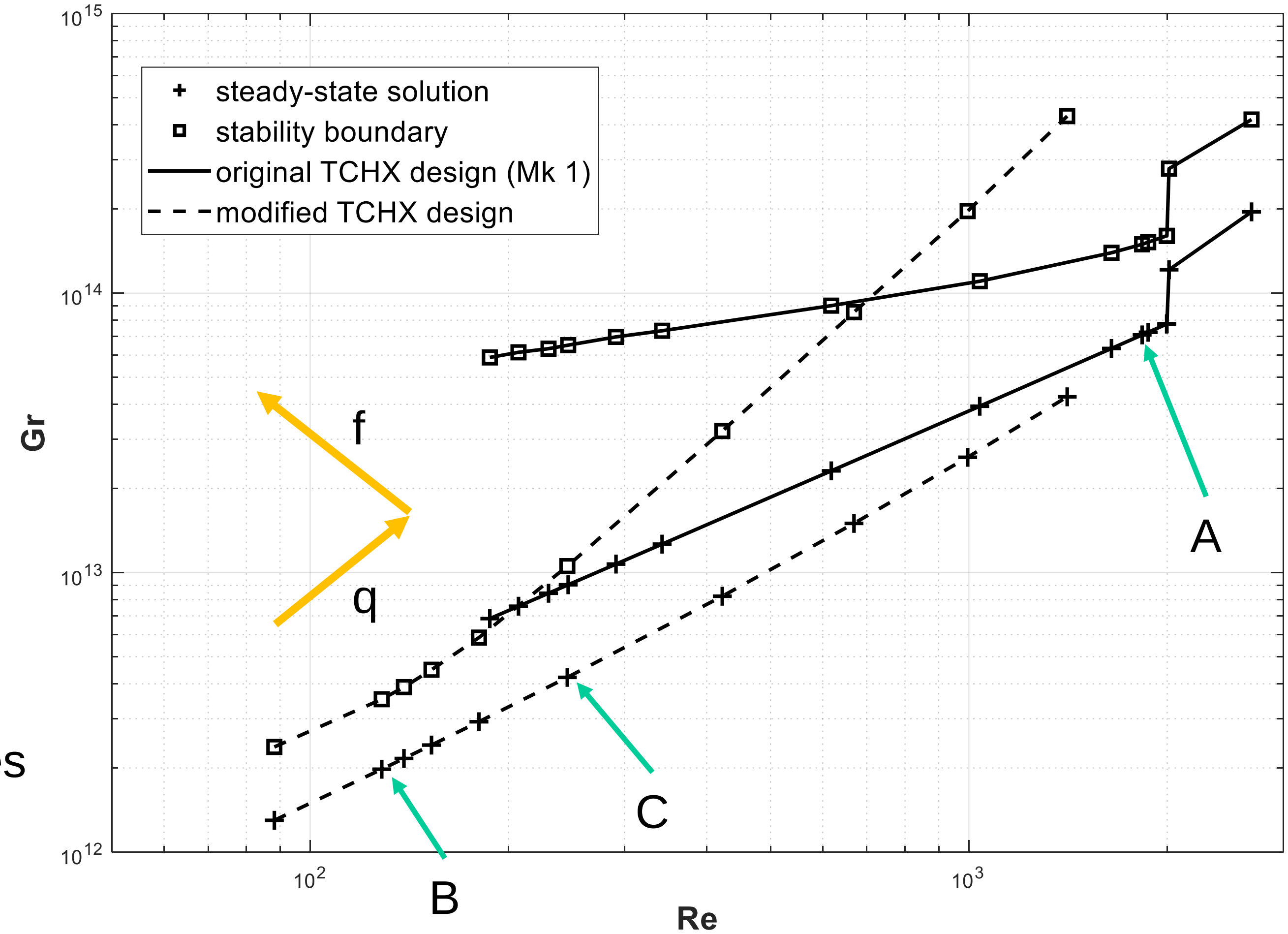
f = increase in losses
q = increase in heat



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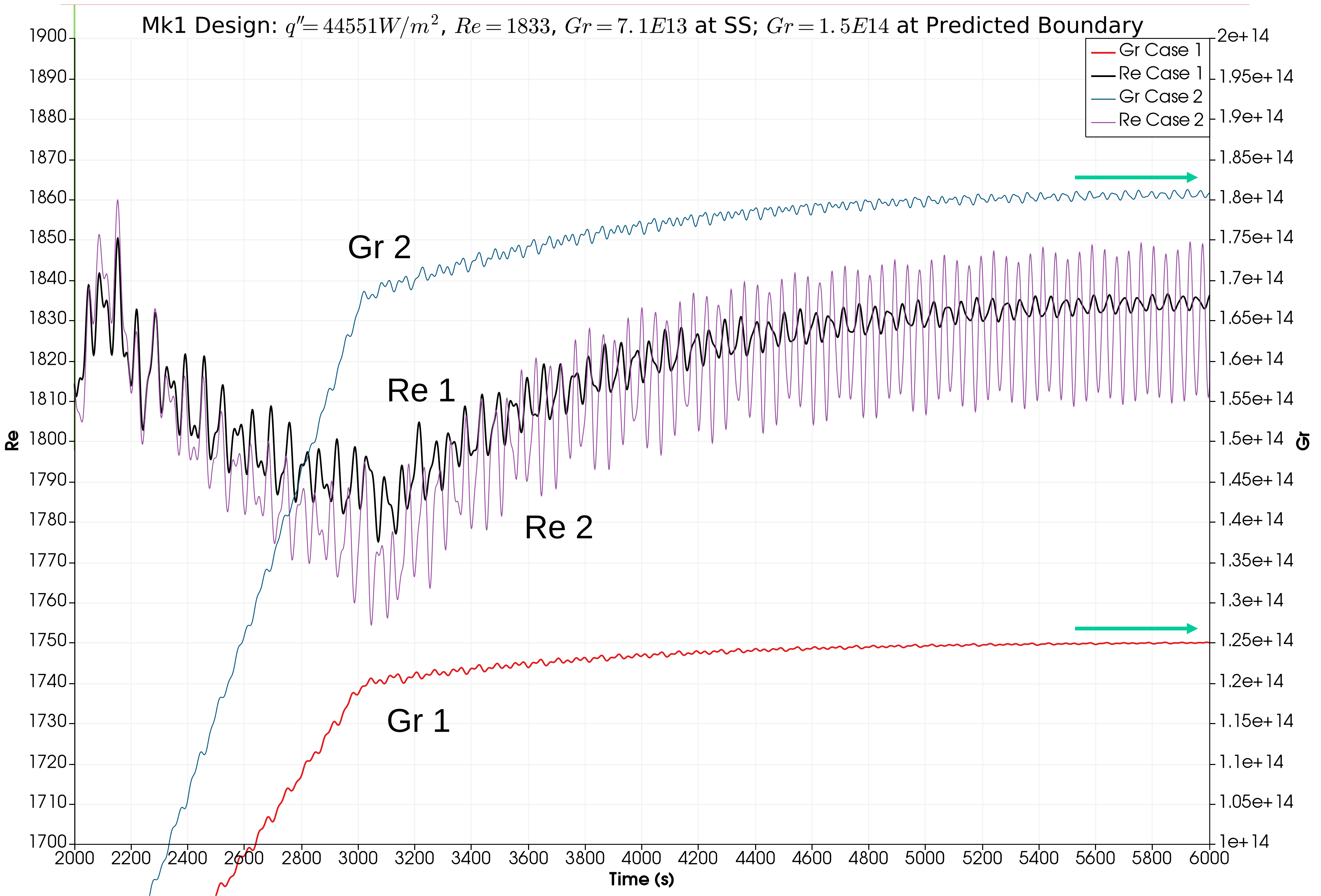
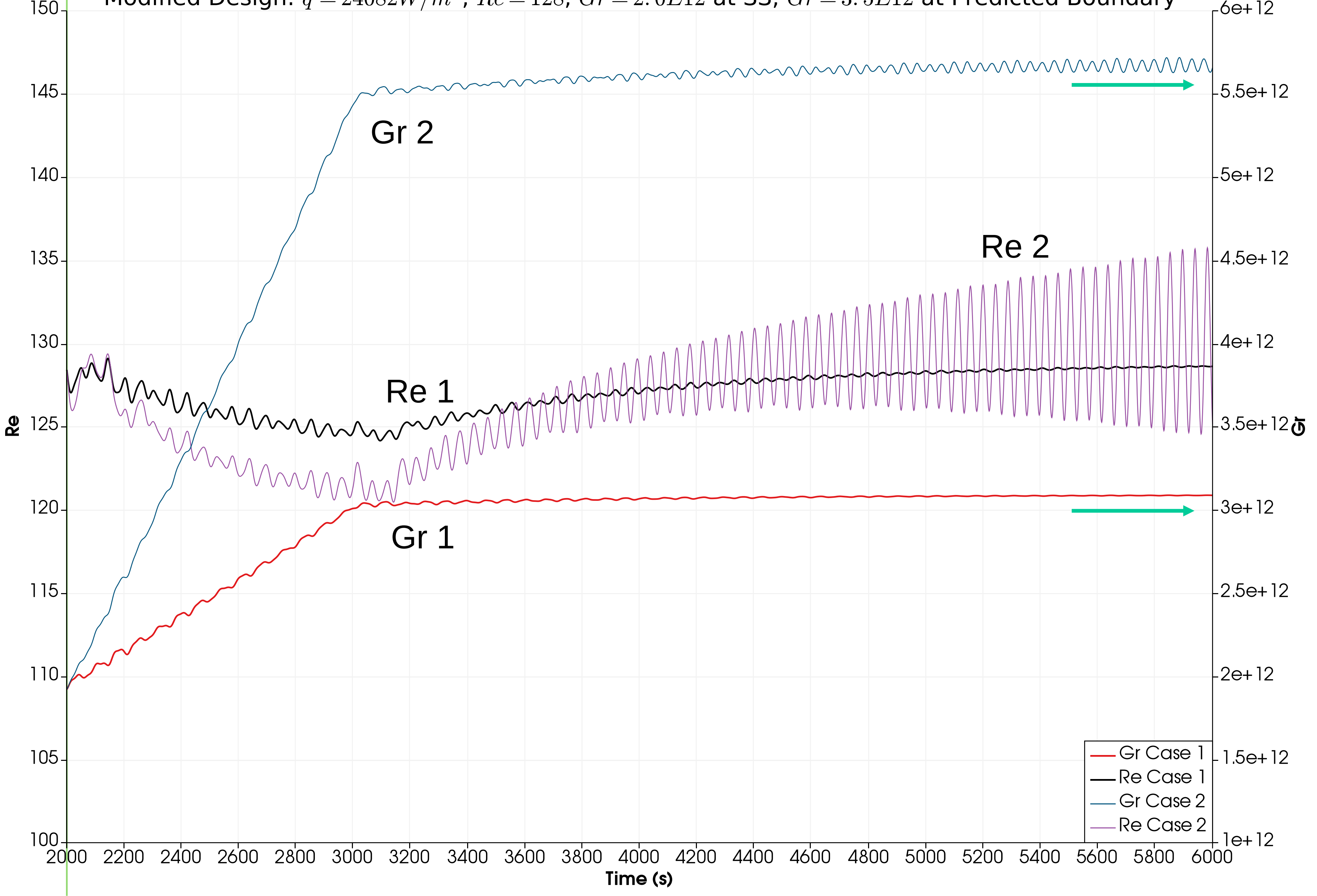
Test A

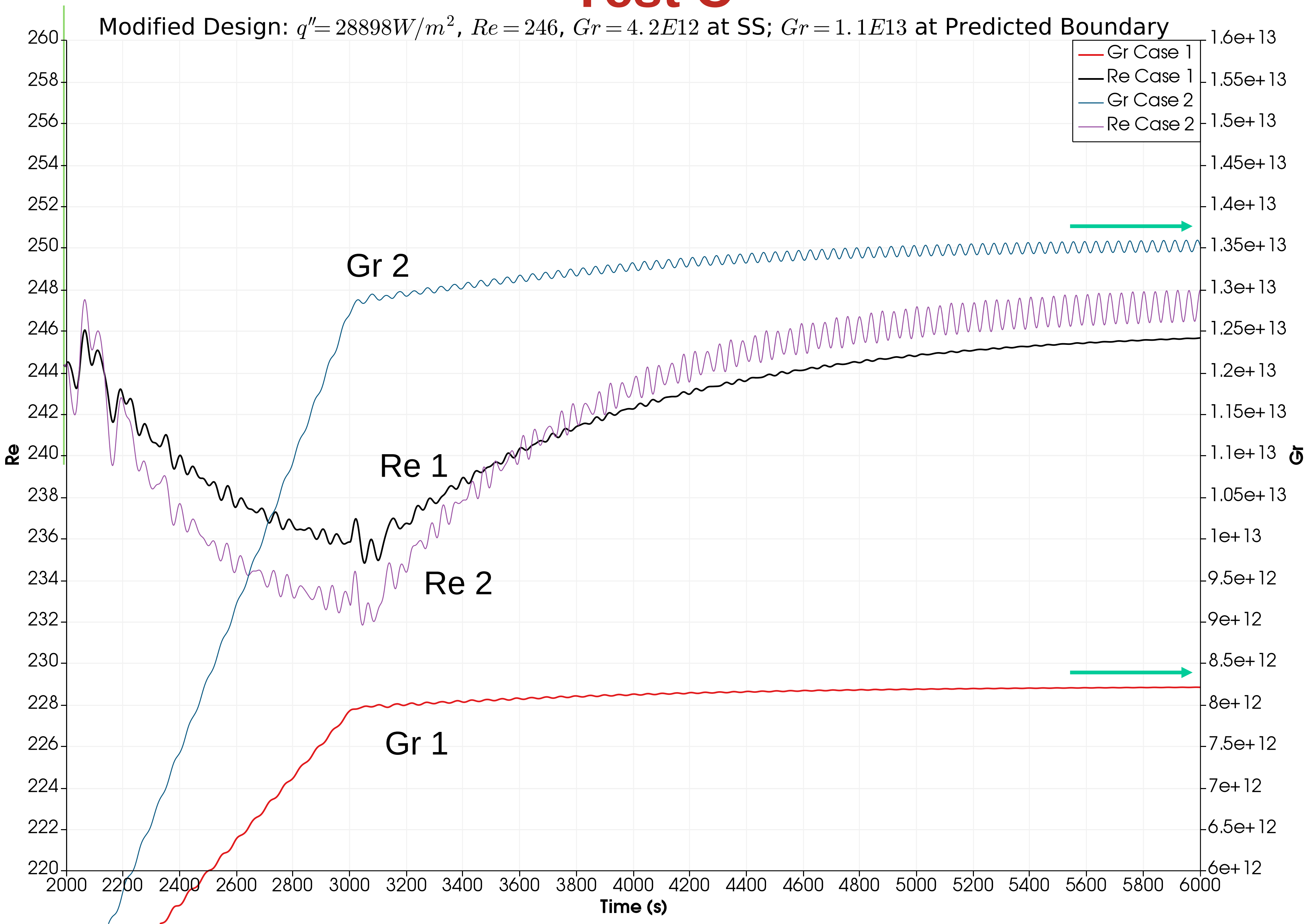

Mk1 Design: $q''=44551W/m^2$, $Re=1833$, $Gr=7.1E13$ at SS; $Gr=1.5E14$ at Predicted Boundary
Gr Case 1
Re Case 1
Gr Case 2
Re Case 2
Gr 2
Re 1
Re 2
Gr 1
Re
Gr
Time (s)
1900
1890
1880
1870
1860
1850
1840
1830
1820
1810
1800
1790
1780
1770
1760
1750
1740
1730
1720
1710
1700
2e+14
1.95e+14
1.9e+14
1.85e+14
1.8e+14
1.75e+14
1.7e+14
1.65e+14
1.6e+14
1.55e+14
1.5e+14
1.45e+14
1.4e+14
1.35e+14
1.3e+14
1.25e+14
1.2e+14
1.15e+14
1.1e+14
1.05e+14
1e+14
2000
2200
2400
2600
2800
3000
3200
3400
3600
3800
4000
4200
4400
4600
4800
5000
5200
5400
5600
5800
6000

Test B

Modified Design: $q'' = 24082 W/m^2$, $Re = 128$, $Gr = 2.0E12$ at SS; $Gr = 3.5E12$ at Predicted Boundary

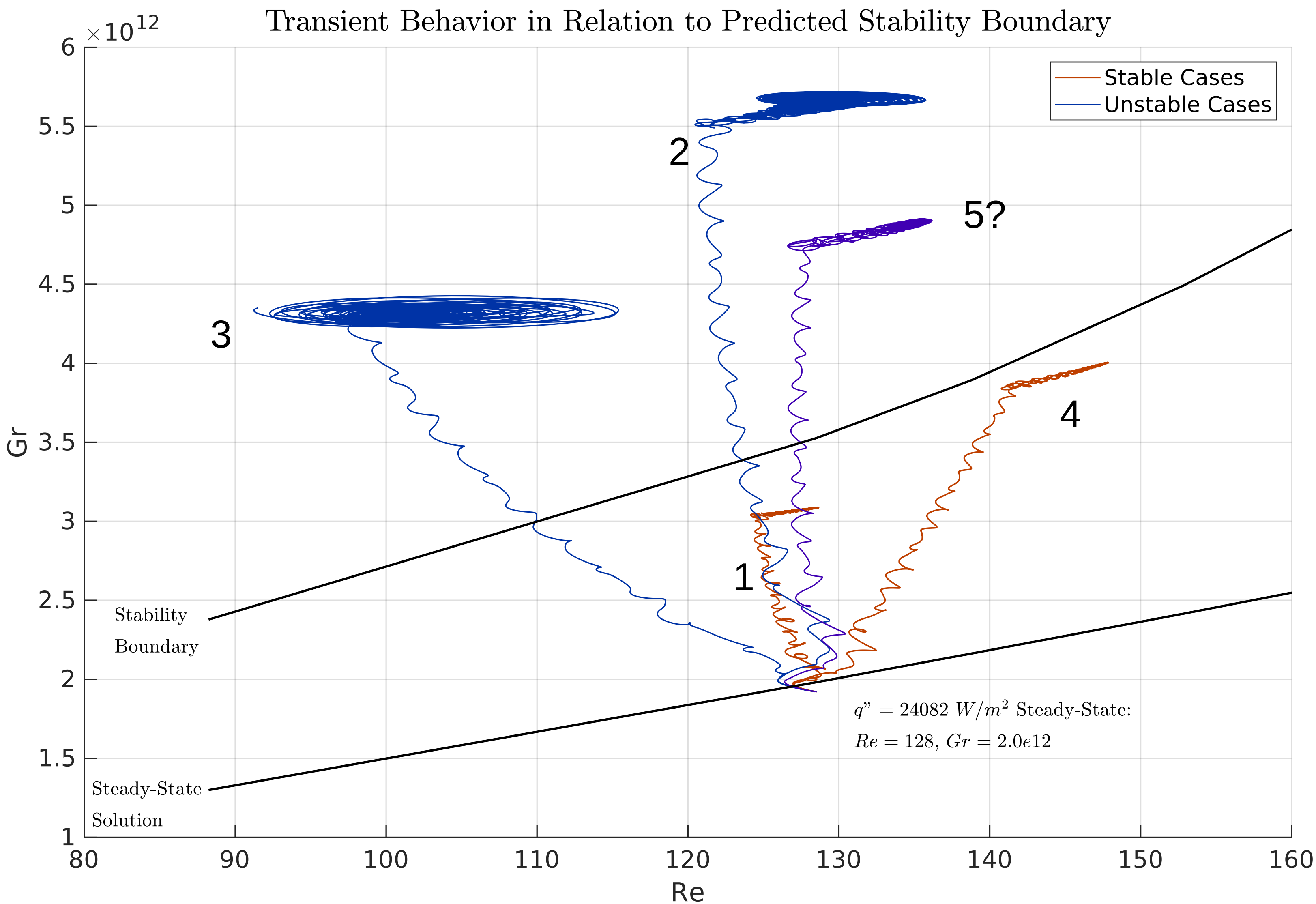


Test C



Modified Design: $q''=28898W/m^2$, $Re=246$, $Gr=4.2E12$ at SS; $Gr=1.1E13$ at Predicted Boundary
Gr Case 1
Re Case 1
Gr Case 2
Re Case 2
Gr 2
Re 1
Re 2
Gr 1
Re
Gr
Time (s)
260
258
256
254
252
250
248
246
244
242
240
238
236
234
232
230
228
226
224
222
220
1.6e+13
1.55e+13
1.5e+13
1.45e+13
1.4e+13
1.35e+13
1.3e+13
1.25e+13
1.2e+13
1.15e+13
1.1e+13
1.05e+13
1e+13
9.5e+12
9e+12
8.5e+12
8e+12
7.5e+12
7e+12
6.5e+12
6e+12
2000
2200
2400
2600
2800
3000
3200
3400
3600
3800
4000
4200
4400
4600
4800
5000
5200
5400
5600
5800
6000

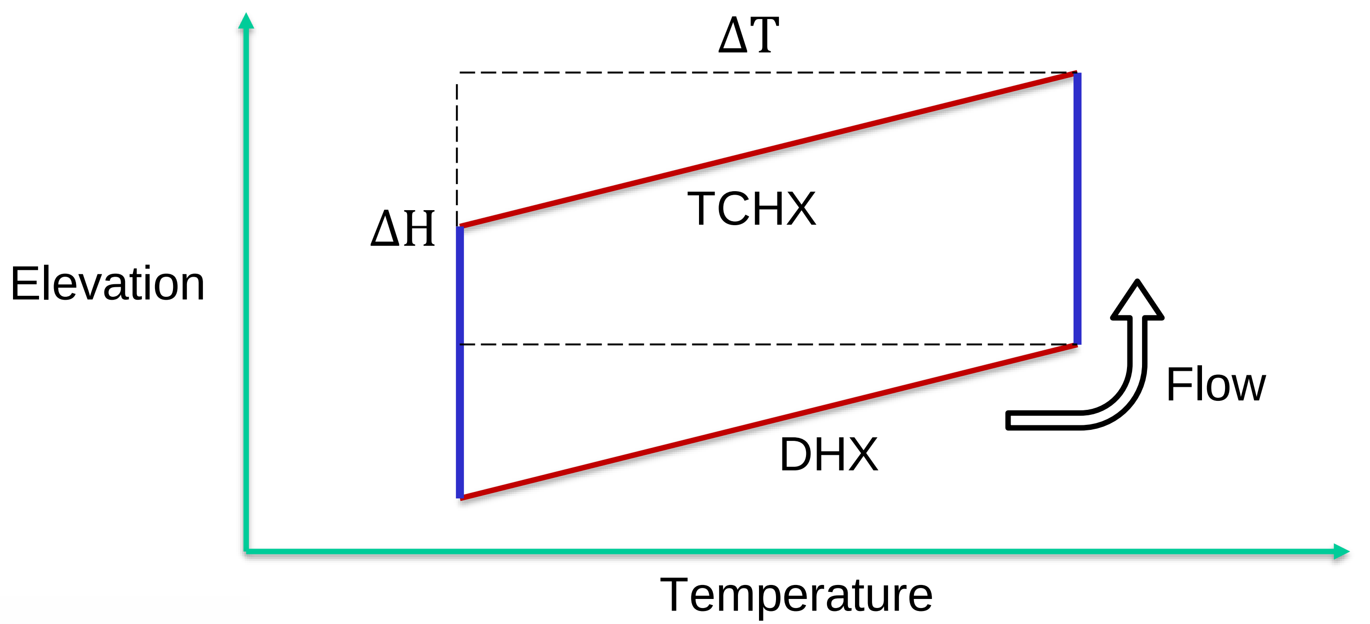
Test B: Parametric Gr vs Re Plot



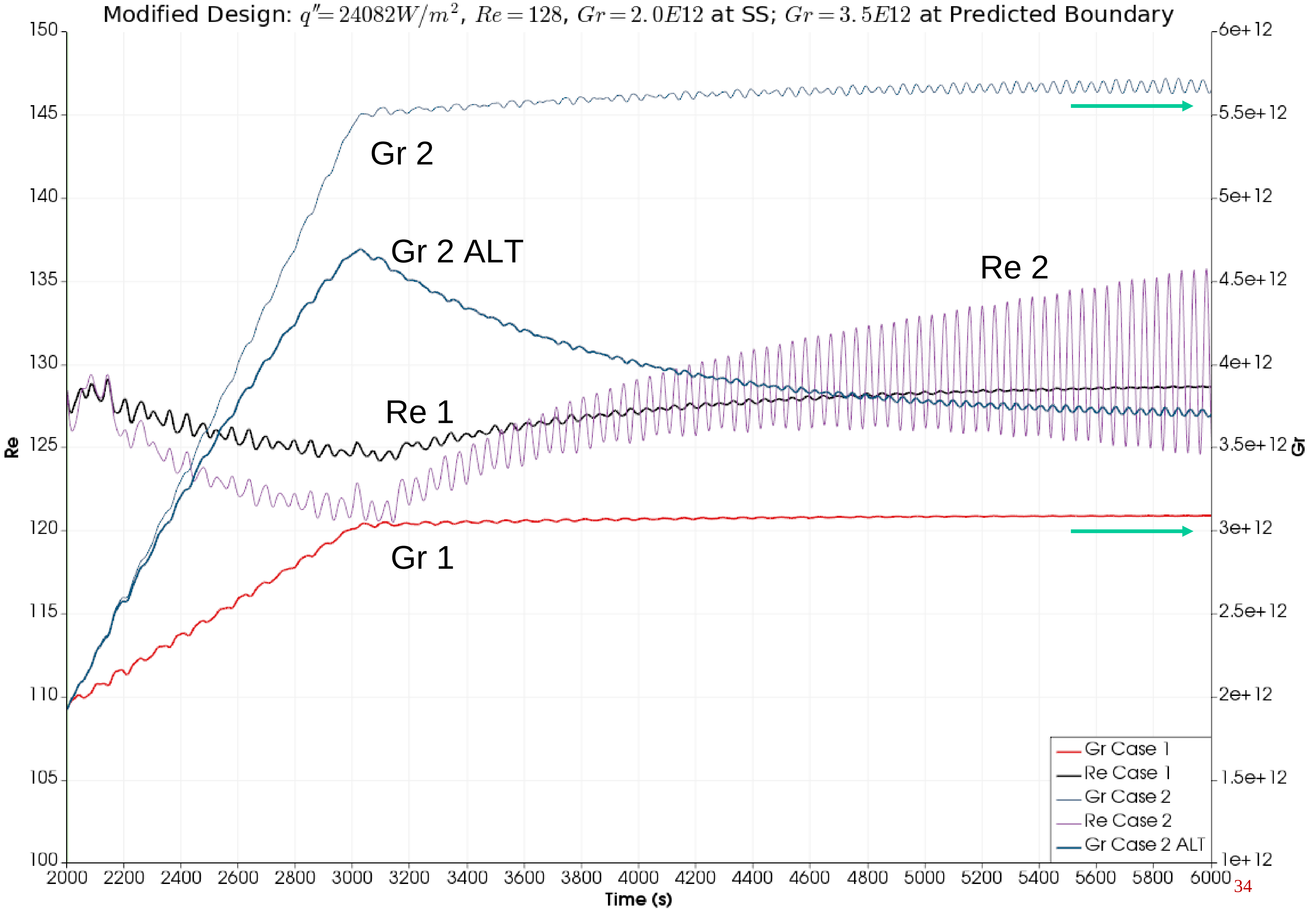
Consider the Definition of Re and Gr

- $Re = \rho v D/\mu = \dot{m}D/\mu A$
- Define with mass flow, this is actually fine

- $Gr = \rho^2 g\beta \Delta H^3 \Delta T/\mu^2$
- Should be calculated from an integral, not constant properties
- First approximation: treat DHX as linear, use inlet and outlet ρ



Test B: More Realistic Calculation of Gr



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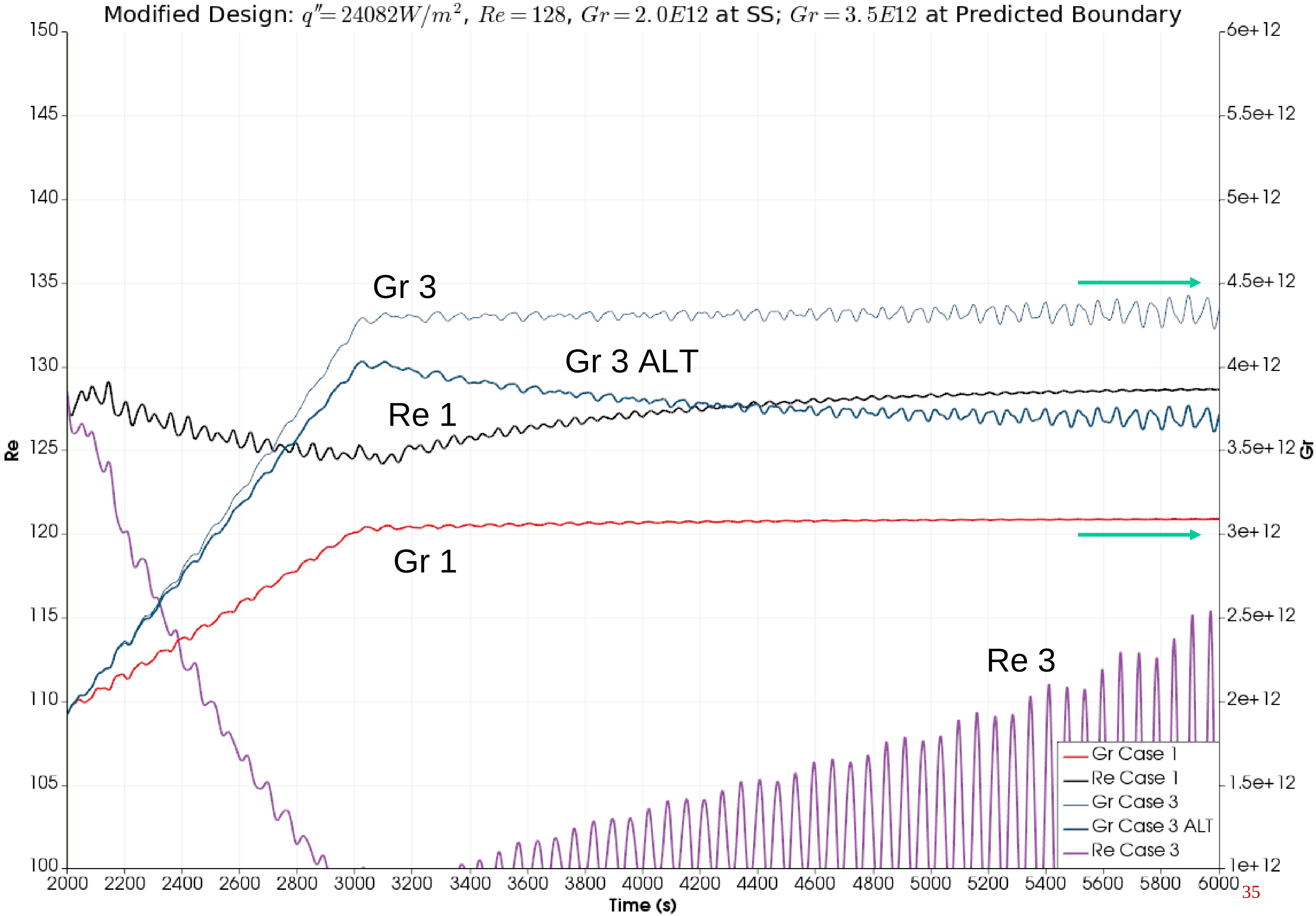

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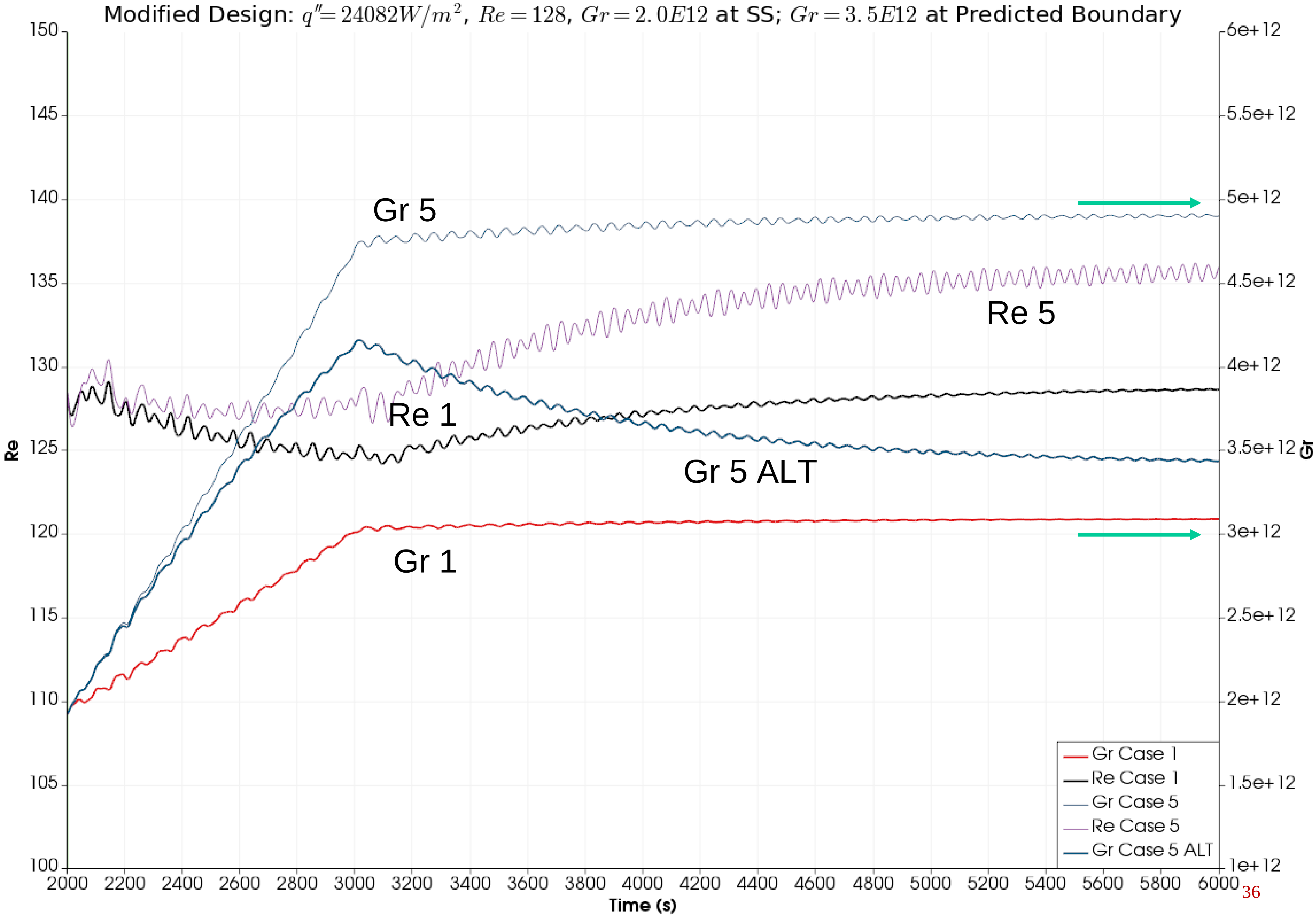
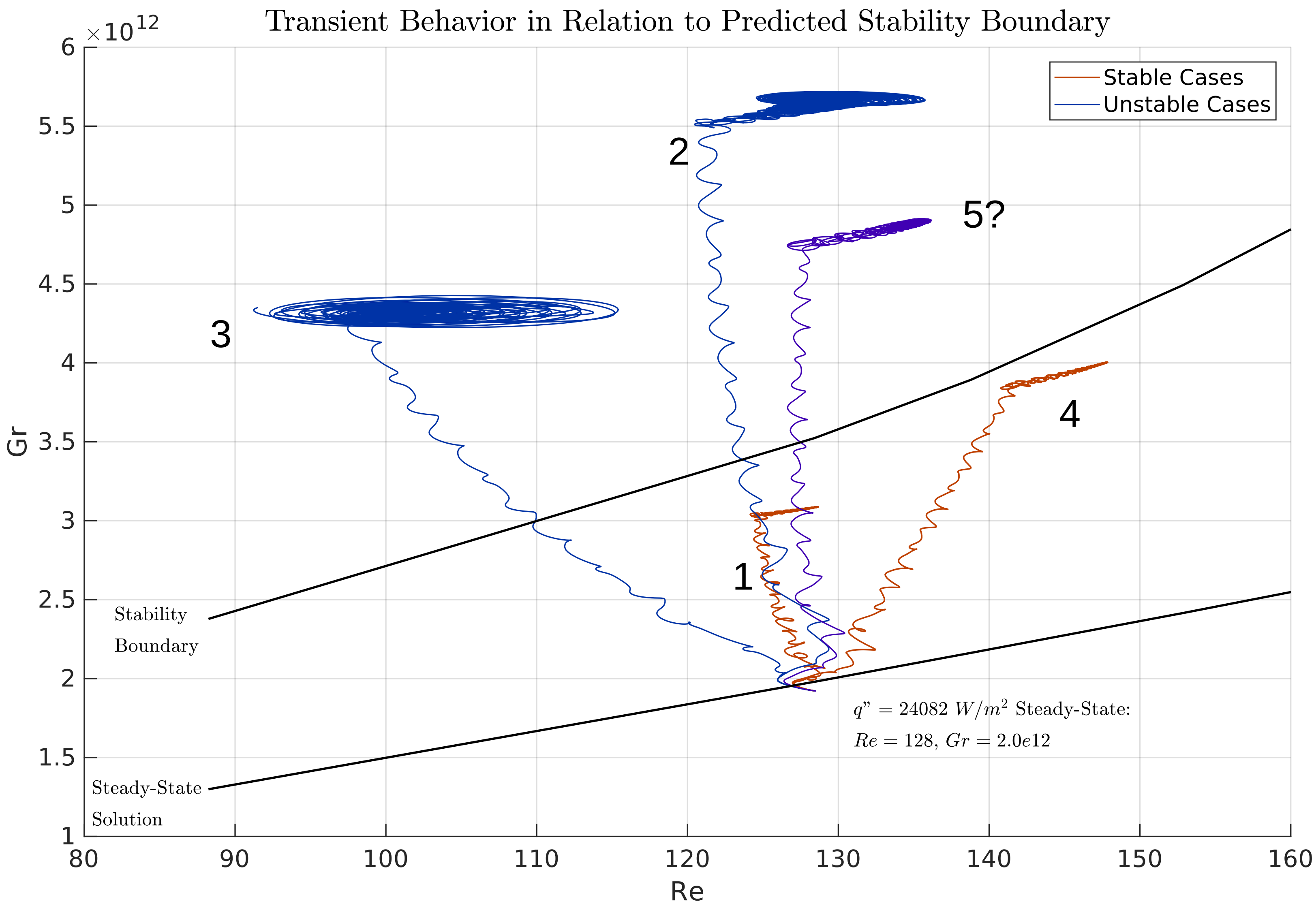
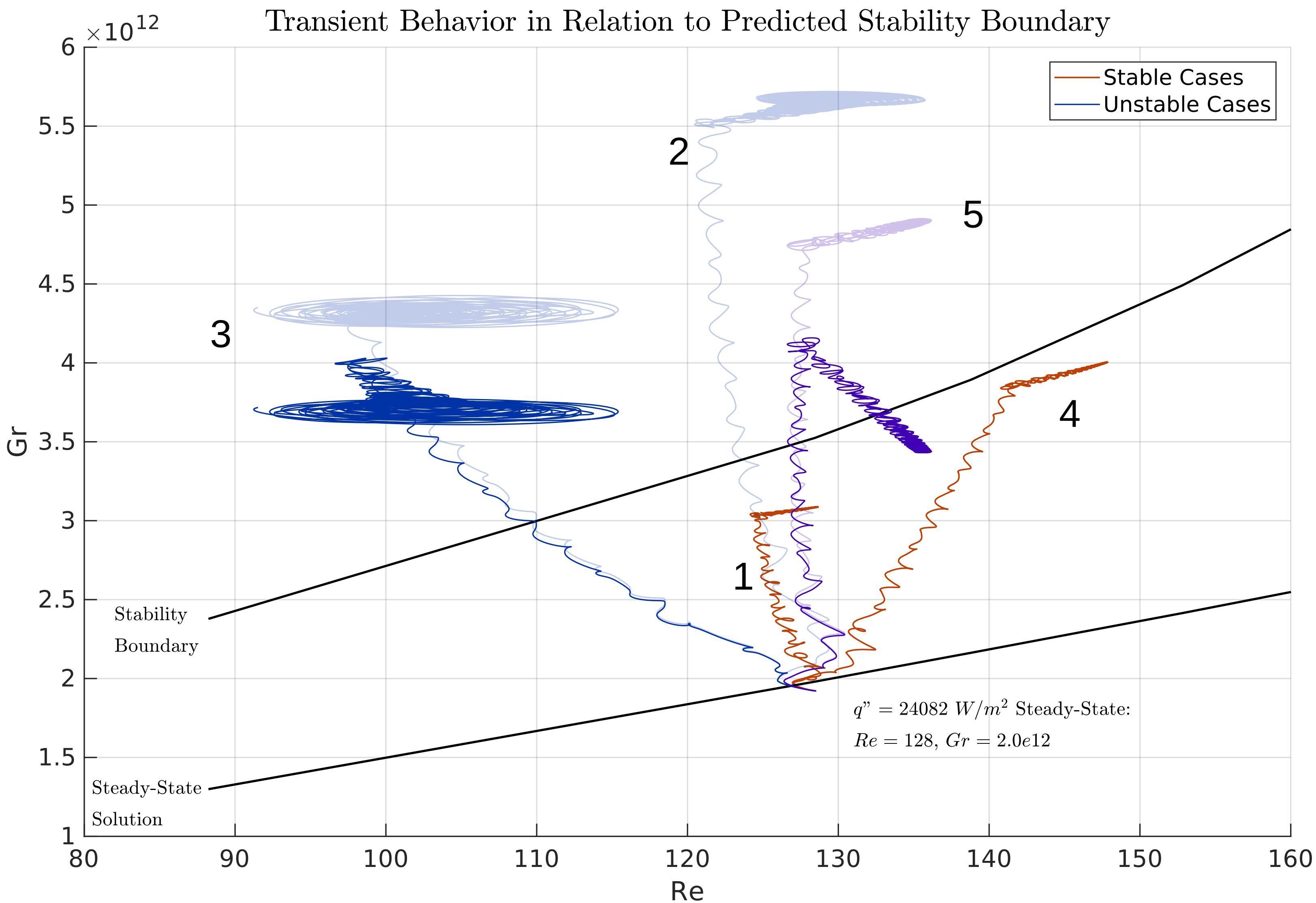


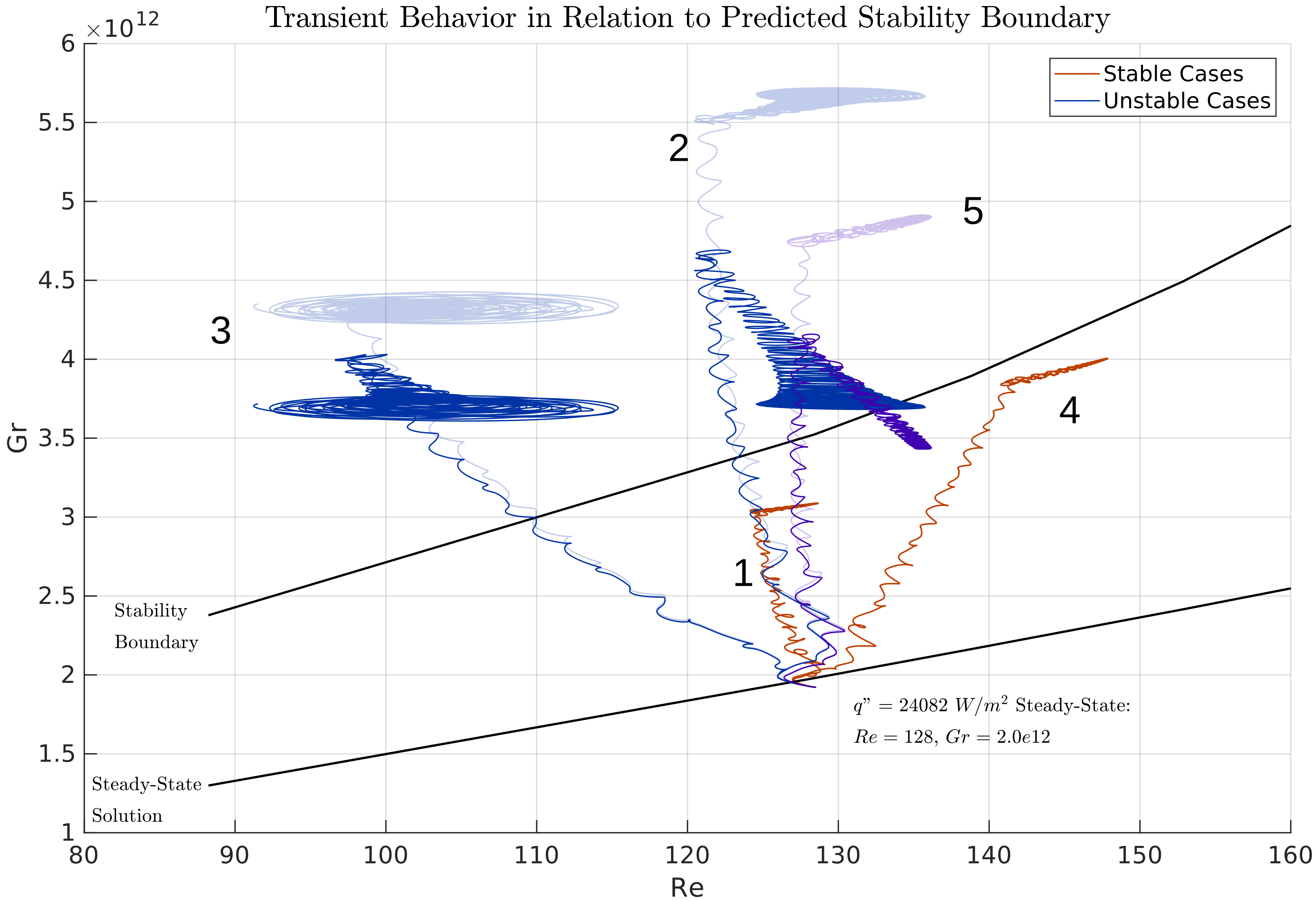
Test B: Parametric Gr vs Re Plot



Test B: Parametric Improved Gr vs Re Plot



Test B: Parametric Improved Gr vs Re Plot



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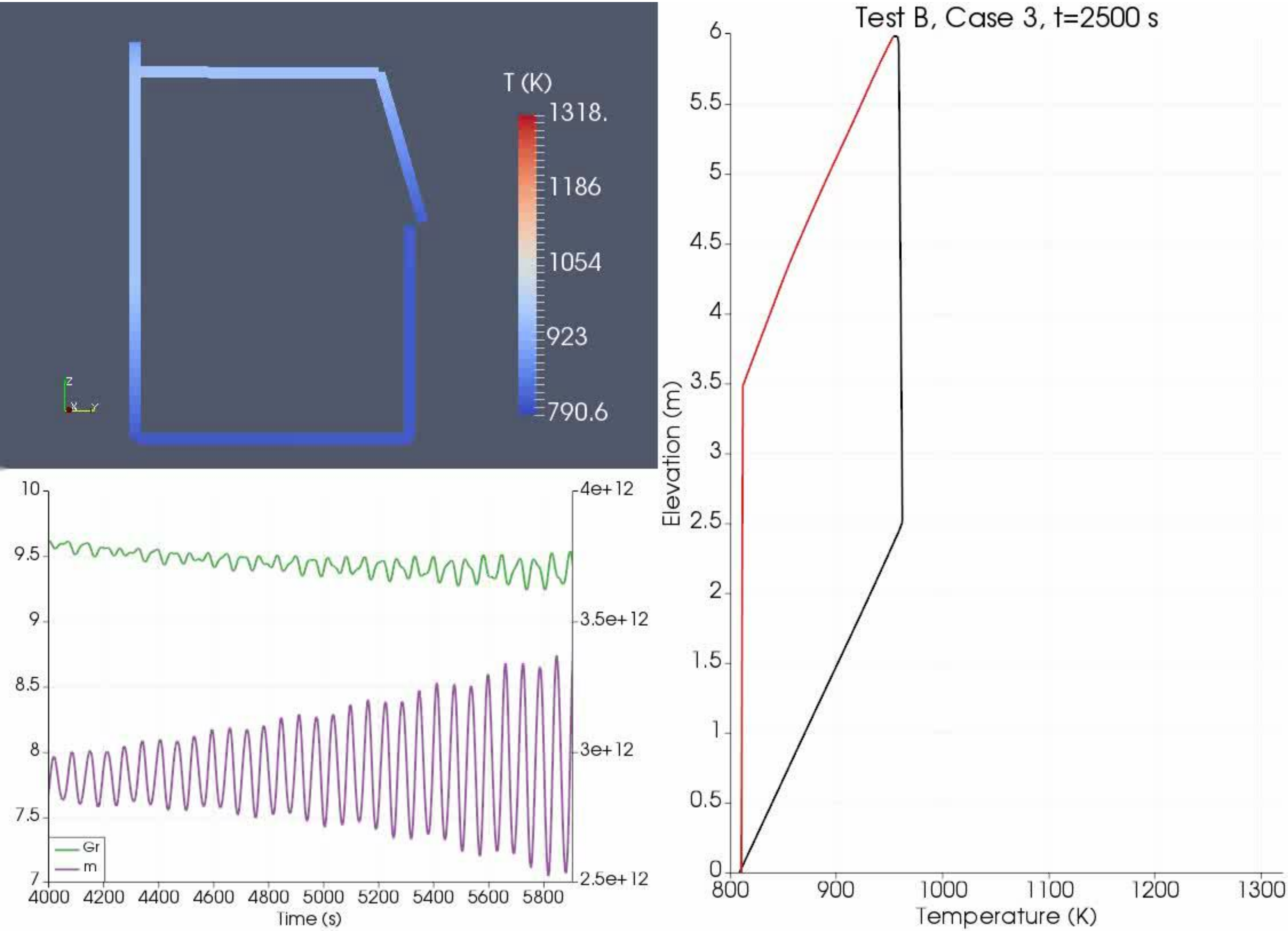
T (K)
1318.
1186
1054
923
790.6
Z
X
Y
10
9.5
9
8.5
8
7.5
7
4e+12
3.5e+12
3e+12
2.5e+12
Gr
m
4000
4200
4400
4600
4800
5000
5200
5400
5600
5800
Time (s)
Test B, Case 3, t=2500 s
Elevation (m)
6
5.5
5
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
800
900
1000
1100
1200
1300
Temperature (K)

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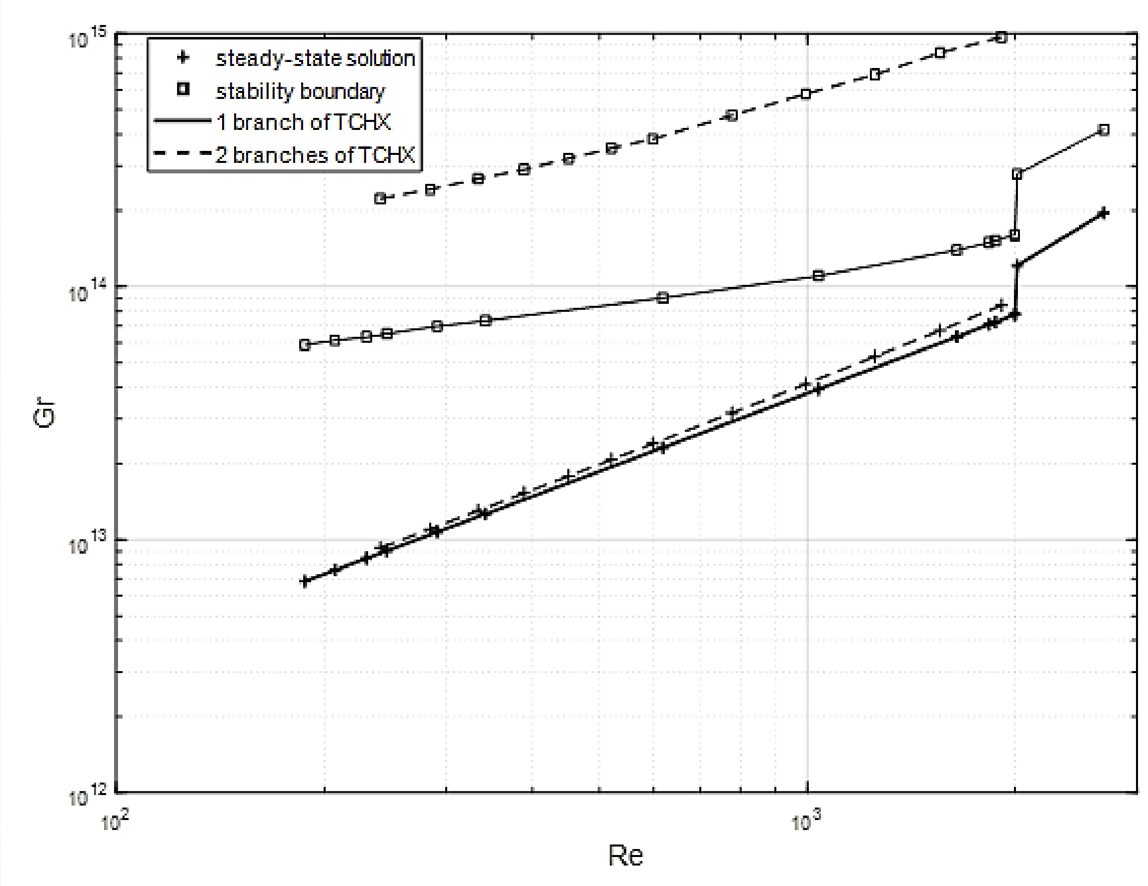
6. Path Forward



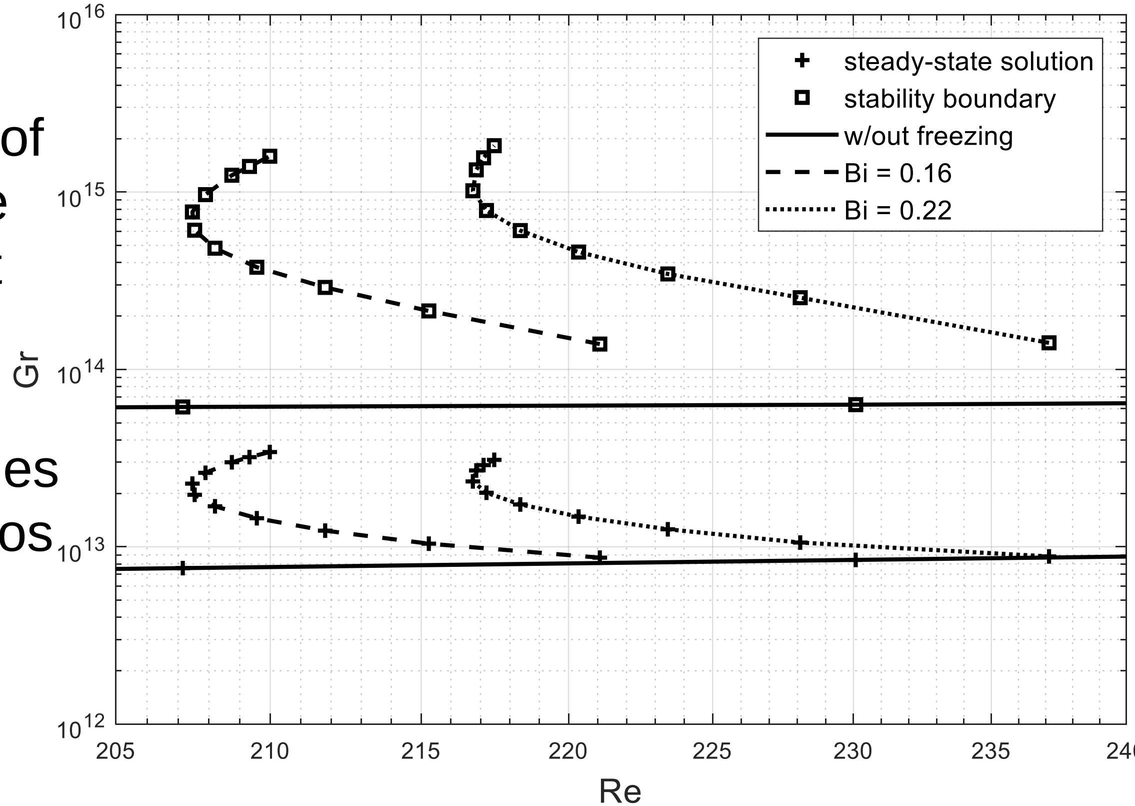
Conclusions and Next Steps

- This code-to-theory comparison corroborates the use of linear stability analysis using the Nyquist Criterion for salt loops

- Exploration of the stability boundary with SAM is demonstrated, this is an integral effects verification test case

- Motivates continued use of stability analysis for more complex cases (add heat structure masses, etc.)

- Guide SAM stability studies of many possible scenarios using analytical results



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