PLUME MODEL EQUATIONS

- $\frac{dr}{dz} = \frac{4\alpha\omega^2 - br}{2\omega^2}$, $\frac{d\omega}{dz} = \frac{-2\alpha\omega^2 + br}{r\omega}$, $\frac{dT}{dz} = \frac{-2\alpha(T - T_{\text{amb}})}{r}$, $\frac{dS}{dz} = \frac{-2\alpha(S - S_{\text{amb}})}{r}$

  (4 equations: conservation of mass, heat, and salt & momentum change by forcing)
- $b = g(\alpha_T\,(T - T_{\text{amb}}) - \beta_S\,(S - S_{\text{amb}}))$ (linear buoyancy)
- $b = \frac{g(\rho_{\text{amb}} - \rho_{\text{plume}})}{\rho_{\text{amb}}}$ where $\rho_{\text{amb}} = f(S_{\text{amb}}, T_{\text{amb}}, P)$, $\rho_{\text{plume}} = f(S, T, P)$ (non-linear buoyancy using gsw_rho **in-situ density** function)
- $P = P_0 - (\rho g)z$ (pressure at depth in dbar)
- $\alpha_T = f(S, T, P)$ (using gsw_alpha function)

Constants:

$\alpha = 0.072$ (entrainment), $g = 0.113\ {}^{m}/_{s^2}$, $\rho g = 0.113/9.78$ (pressure factor),

$\alpha_T = 5 * 10^{-5}$ (thermal expansivity of plume), $\beta_S = 7 * 10^{-4}$ (haline contractivity)

T_{amb} =1e-6 (low), 1e-1 (high), $S_{\text{amb}} = 4$ (low), 40 (high)

Init. conditions:

$r_0 = 1\ or\ 10\ m,\ q_0 = 10\ {}^{m^3}/_{s},\ w_0 = \frac{q_0}{\pi {r_0}^2} = 0.03183\,\frac{\text{m}}{\text{s}}$ or $3.183\,\frac{\text{m}}{\text{s}}$

$T_0 = 100$℃ or 10℃ or 0.01℃, $S_0 = S_{\text{amb}} + 10$

P_0=678 dbar (pressure at source vent)

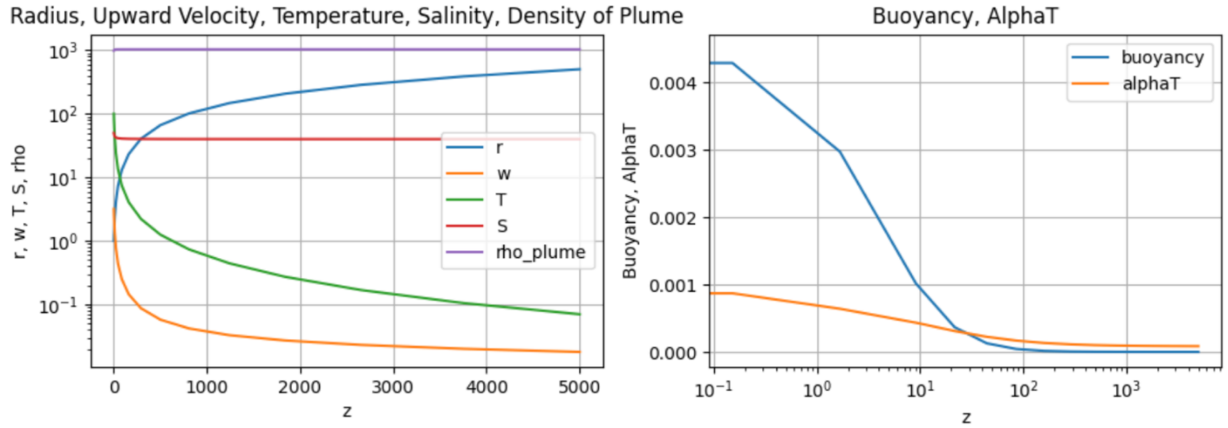
Stop condition: $\omega < 0.01$

Model params:

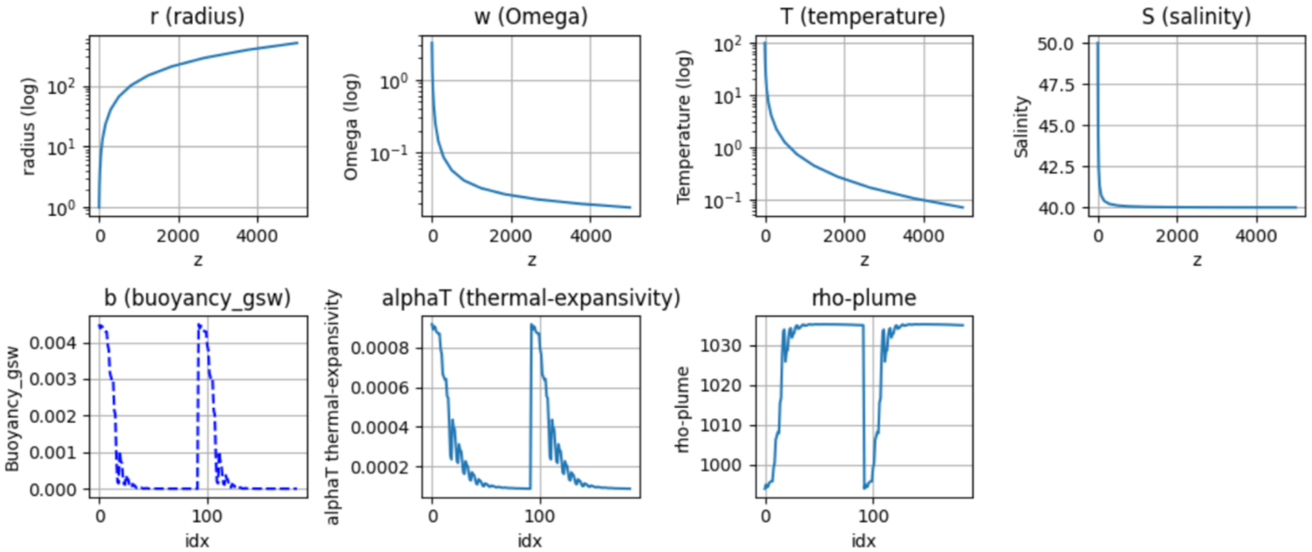
$\Delta z = 0.05\ m$, max iter $= 100{,}000$

NEW ODE SOLVER RESULTS (16 ITERATIONS)

.w,T,S,buoyancy,alphaT,plume density: [r0 = 1.000000 m, w0 = 3.183099 m^3/s, T0 = 100.000000 C, S0 = 50.00000 g/kg, S_amb = 40.000000 g/kg, T_amb = 0.000001 C]

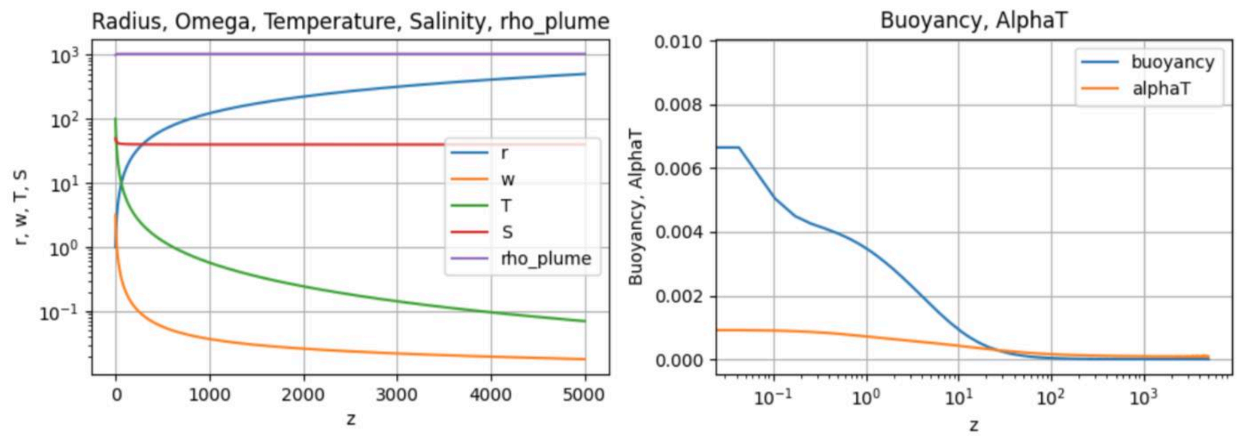


. w,T,S,buoyancy,alphaT,rho_plume_sol Solutions: [r0 = 1.000000 m, w0 = 3.183099 m^3/s, T0 = 100.000000 C, S0 = 50.00000 g/kg, S_amb = 40.000000 g/kg, T_amb = 0.000001 C]

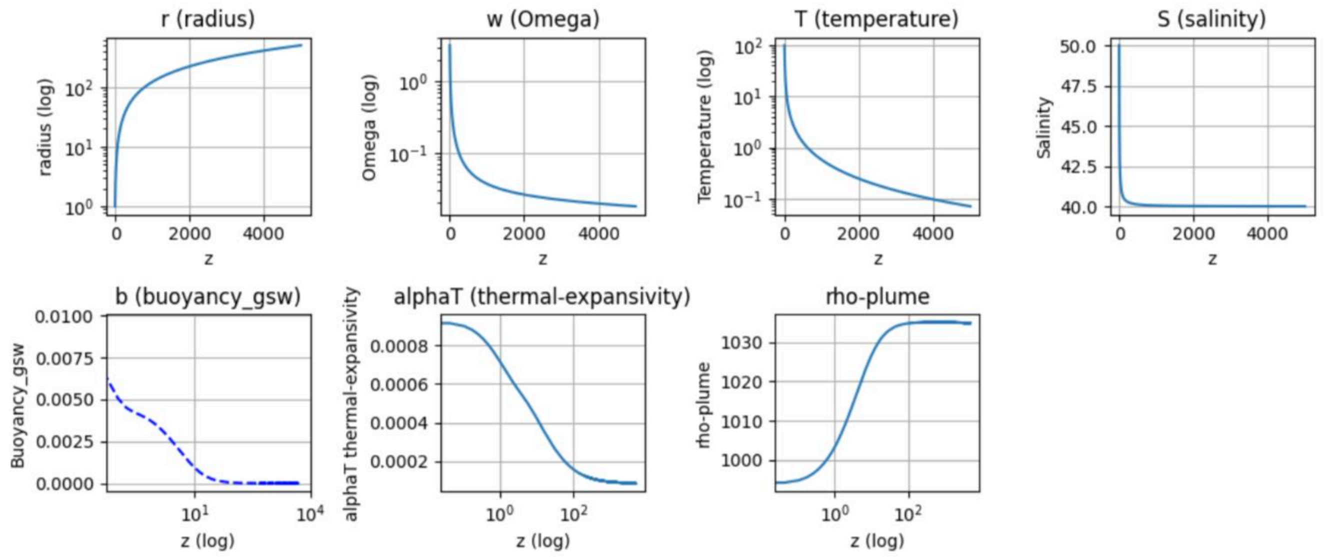


OLD ODE SOLVER RESULTS (30K ITERATIONS)

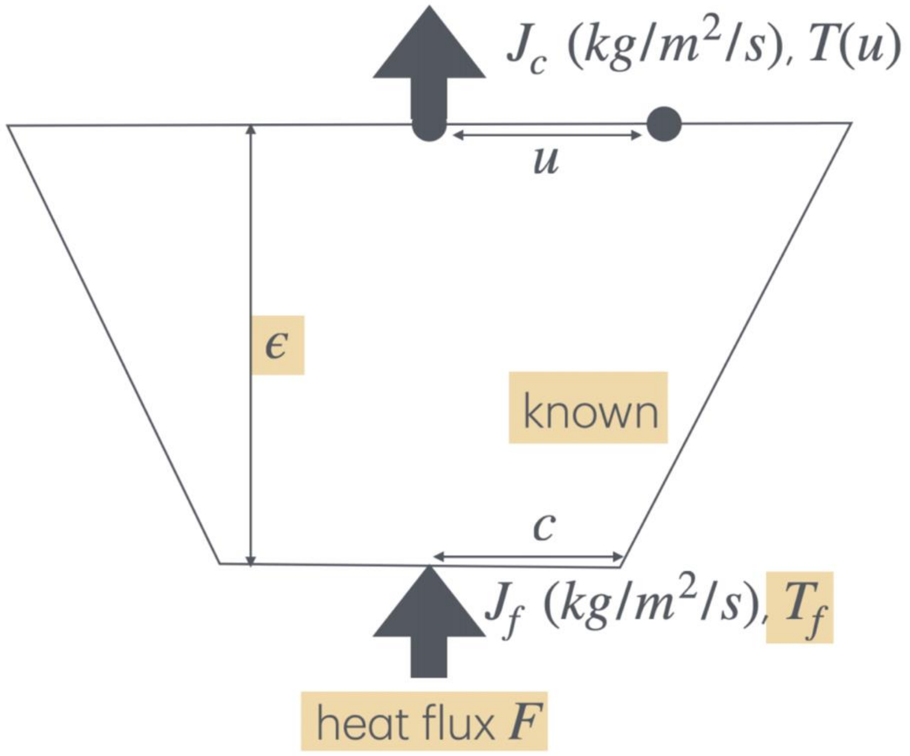
rho_plume Solutions: [r0=1.000000, w0=3.183099, T0=100.000000, S0=50.00000, b0=0.00959, alphaT0=0.000919, rho_plume0=993.9823855789817, S_amb=40.000000, T_amb=0.000001]



ho_plume_sol Solutions: [r0=1.000000, w0=3.183099, T0=100.000000, S0=50.00000, b0=0.00959, alphaT0=0.000919, rho_plume0=993.9823855789817, S_amb=40.000000, T_amb=0.000001]



AFFHOLDER PAPER, SECTION 2.1 (DEFINING PARAMETERS OF PHYSICAL MODEL)



$$C_i^{\mathrm{OP}} = \frac{\int_0^{\infty} J_c(u) C_i(u) 2\pi u du}{\int_0^{\infty} J_c(u) 2\pi u du}. \qquad (7)$$

Table 1
Parameters of the Physical Model of the Hydrothermal Environment

Parameter	Value	Unit	Description	Reference
T_o	275	K	Ocean temperature	
g	0.12	m s^{-2}	Enceladus gravitational acceleration	Choblet et al. (2017)
F	$5 \cdot 10^9$	W	Hydrothermal vent heat dissipation power	Choblet et al. (2017)
ϵ	1	m	ML thickness	
C_p	4200	J K^{-1} kg^{-1}	Specific heat capacity of liquid water	
α	3×10^{-4}	K^{-1}	Thermal expansion coefficient for liquid water	
ρ_o	1000	kg m^{-3}	Seawater mass density	

AFFHOLDER PAPER, SECTION 2.2

This section defines a step-by-step process to model chemical reaction rate & biomass growth rate coupled with the physical model:

- (1) Express value of **steady-state reaction quotient Q*** using Equations (13), (14), and (19)
- (2) Then, Equation (22) in [Affholder Nature paper] allows us to rewrite Equation (14) and solve numerically a system of three equations, one for each reactant and product in Equation (1), to obtain the **steady-state concentrations of CO_2, CH_4, and H_2.**
- (3) Ultimately, Equation (18) equal to zero is solved for **the steady-state values of bulk biomass concentration B*.**
- (4) Using Equation (8), combined with Equations (11) and (16), allows the derivation of the **steady-state cell density N* and Bc** where Bc is obtained by solving Equation (8) at the steady state of the population (Equation (12) equal to zero).
- (5) B* (steady state bulk biomass) **is integrated over each mixing layer** (represented by thin cylinder) to find Btot → **total bulk biomass over the entire plume**

STEP 1: EXPRESSIONS FOR REACTION QUOTIENT Q*

$$\Delta G_{\text{cat}} = \Delta G^0_{\text{cat}} + RT \ln Q, \qquad (13)$$

$$Q = \frac{[\mathrm{CH_4}]^{0.25}}{[\mathrm{H_2}][\mathrm{CO_2}]^{0.25}}. \qquad (14)$$

$$\Delta G_{\text{cat}}{}^*(T) = -\frac{\Delta G_{\text{diss}}}{q_{\text{cat}}(T)}(d + q_m), \qquad (19)$$

Constant as a function of T

Physical constant

$$q_{\text{cat}} = \tau q_e$$

$$q_e = \frac{k_{\text{cat}}}{1 + K_{\text{eq}}}$$

$$k_{\text{cat}} = \frac{k_B}{h} T e^{-\frac{\Delta G_{a,\text{cat}}}{RT}}$$

$$K_{\text{eq}} = e^{-\frac{\Delta H_{\text{eq}}}{R}\left(\frac{1}{T_{\text{eq}}} - \frac{1}{T}\right)}, \qquad (17)$$

Express value of **steady-state reaction quotient Q*** using Equations (13), (14), and (19).

$\Delta_r G^0_{S,\text{cat}}$	−32.6	kJ mol^{-1}	Standard Gibbs energy of the catabolic reaction
ΔG_{diss}	1088	kJ mol$_C^{-1}$	Required energy dissipation for biomass synthesis
d	0.03	day^{-1}	Baseline cell death rate

$q_m = e_m / \Delta G_{\text{diss}}$ (s^{-1})

T_{eq}	90	°C	Temperature at which activated and inactivated enzymes are in equal quantity
ΔH_{eq}	305	kJ mol^{-1}	Equilibrium enthalpy of enzyme deactivation
$e_m(T)$	$84\, e^{\frac{69{,}400}{R}\left(\frac{1}{298} - \frac{1}{T}\right)}$	kJ day^{-1} mol$_C^{-1}$	Specific cell maintenance energy

STEP 2: SOLVE FOR STEADY-STATE CONCENTRATIONS OF CO2, CH4, AND H2

Then, Equation (22) in [Affholder Nature paper] allows us to rewrite Equation (14) and solve numerically a system of three equations, one for each reactant and product in Equation (1), to obtain the **steady-state concentrations of CO2, CH4, and H2**

$$Q = \frac{[\mathrm{CH_4}]^{0.25}}{[\mathrm{H_2}][\mathrm{CO_2}]^{0.25}}. \tag{14}$$

In the absence of any other factors, the steady-state concentration of *i*, *Ci **, is given by

$$C_i^* = C_i^0 + Y_i^{\mathrm{cat}}(C_{\mathrm{eD}}^0 - C_{\mathrm{eD}}^*). \tag{22}$$

$$\begin{aligned} Q_{\mathrm{cat}}^* &= \exp\left[-\frac{1}{RT}\left(\Delta G_0 + \left(d + \frac{e_{\mathrm{m}}}{\Delta G_{\mathrm{diss}}}\right)\frac{\Delta G_{\mathrm{diss}}}{q_{\mathrm{cat}}}\right)\right] \\ \log_{\mathrm{e}} Q_{\mathrm{cat}}^* &= \log_{\mathrm{e}} K - \frac{1}{RT}\left(d + \frac{e_{\mathrm{m}}}{\Delta G_{\mathrm{diss}}}\right)\frac{\Delta G_{\mathrm{diss}}}{q_{\mathrm{cat}}} \end{aligned} \tag{20}$$

$$Q_{\mathrm{cat}}^* = \frac{1}{C_{\mathrm{eD}}^*}\prod_{i \neq \mathrm{eD}}\left(C_i^0 + Y_i^{\mathrm{cat}}(C_{\mathrm{eD}}^0 - C_{\mathrm{eD}}^*)\right)^{Y_i^{\mathrm{cat}}} \tag{23}$$

STEP 3: USE REACTION PROGRESS TO DERIVE BULK BIOMASS B*

Ultimately, Equation (18) equal to zero is solved for **the steady-state values of bulk biomass concentration B*.**

This step couples the biological reaction (biomass dynamics & catabolic rate) with the physical model (behavior of plume) and the chemical reaction progress (concentration of each reactant/product) using this central formula:

$$\frac{dC_i}{dt} = \frac{1}{\epsilon\rho}(J_f(C_f^i - C_o^i) + J_c(C_o^i - C_i)) + Y_i q_{\text{cat}} B, \qquad (18)$$

From biological reaction
From chemical reaction
From physical model of water
Constant

Y_i is the stoichiometric coefficient of molecule i in the catabolic reaction(e.g., Y_{H2}=-1)
C_i,o and C_i,f are the concentrations of i in the ocean and the HF, respectively
Epsilon is 1m, the mixing layer thickness
Rho, Jf, and Jc are derived from physical model and are constant within each ML
B is the bulk biomass concentration

STEP 4: FROM INDIVIDUAL CELL GROWTH RATE DERIVE BULK BIOMASS GROWTH RATE

Using Equation (8), combined with Equations (11) and (16), allows the derivation of the **steady-state cell density N* and Bc** where Bc is obtained by solving Equation (8) at the steady state of the population (Equation (12) equal to zero).

For a **single cell**, the division rate r (per second) as a function of internal biomass Bc (in moles of carbon, molC) derived as:

$$\begin{cases} r(B_c) = \dfrac{1}{60 \times 60 \times 24} \dfrac{r_{\max}}{1 + ((B_c - 2B_{\text{struct}})/B_{\text{struct}})^{-\theta}} & \text{if } B_c > 2B_{\text{struct}} \\ r(B_c) = 0 & \text{otherwise.} \end{cases} \quad (8)$$

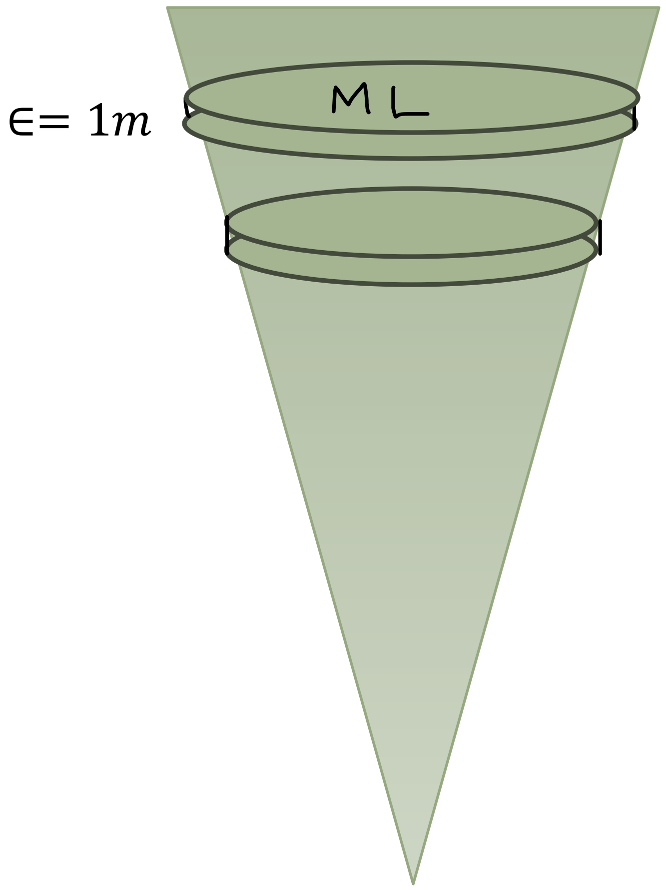
Steady-state is defined as **dBc/dt = 0**. **Anabolic** (individual cell internal growth) **reaction rate** derived as:

$$q_{\text{ana}} = r(B_c{}^*), \quad (11)$$

Anabolic reaction rate (Qana) relates to catabolic rate (Qcat)and Qm, the enzymatically accelerated rate of the catabolic reaction through this formula.
This relates single-cell-level growth to the larger reaction rate & catabolic rate.

$$q_{\text{ana}} = \lambda q_{\text{cat}} - q_m, \quad (16)$$

STEP 5: INTEGRATE OVER THE PLUME



- The last step is to divide the plume into 1m thick cylinders and integrate to find total biomass and total cell density
- <u>**Ex:**</u> **B*** (**steady state** bulk biomass) integrated over each mixing layer (represented by thin cylinder) to find **Btot** → **total** bulk biomass over the entire plume

$$\begin{cases} N_{\text{tot}} = \rho\epsilon \int N^*(u) 2\pi u du \\ B_{\text{tot}} = \rho\epsilon \int B^*(u) 2\pi u du. \end{cases} \tag{21}$$

AFFHOLDER PAPER, SECTION 2.3

- **This section defines "productivity" P of a population as quantity of biomass produced per unit time**
- By definition, @ steady state, productivity = mortality rate = quantity of biomass that leaves the stock of living cells per unit time
- P found through this integral

$$P_B = \left(\frac{e_m}{\Delta G_{\text{diss}}} + d \right). \qquad (22)$$

$$\boldsymbol{P_{B,\text{tot}}} = \rho\epsilon \int \boldsymbol{P_B(u)B^*(u)} 2\pi \boldsymbol{u} d\boldsymbol{u}. \qquad (23)$$

AFFHOLDER PAPER, SECTION 2.4

- **This section defines some formulas to track # of dead cells and productivity at steady state (where the population perfectly replenishes itself)**

$$N_d{}^* = \frac{\rho\epsilon}{J_c} N^* d,$$

$$B_d{}^* = \frac{\rho\epsilon}{J_c} P_B B^*.$$

- Concentrations in the initial ocean plume are obtained by combining Equations (7), (25), and (27). Discusses **assumption that initial concentrations across the entire plume = init. concentrations @ the ocean floor***

AFFHOLDER PAPER, SECTION 2.5

- **This section discusses simulations conducted using the previous formulas to find both initial conditions that can support life (priors) and long-term steady-state conditions of such life (posteriors)**
- 1) Prior densities (init. conditions) are estimated by running 20,000 simulations with parameters (composition and temperature of the HF and ocean) randomly drawn from the distributions defined in Table 2 of [Affholder Nature Paper]
- 2) Only 8763 simulations that produced habitable conditions were retained. They use these habitable conditions to simulate biological activity and the probability of presence of biological material, sustained growth, etc.
- (… how they conducted later simulations can be studied further)