

# Ensemble Plans in Redistricting

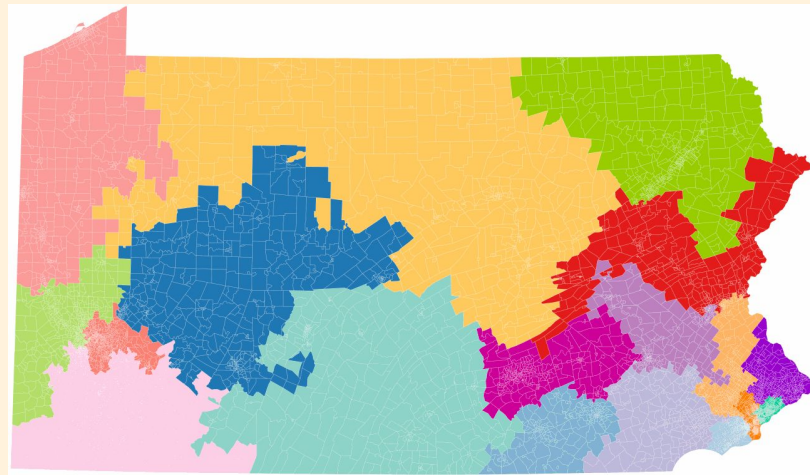
# Overview

Ensembles Intro (~15 minutes)

Review coding notebook (~45 minutes)

Wrap-up

- Answer any lingering questions
- If time/interest allows: begin discussing data preprocessing steps



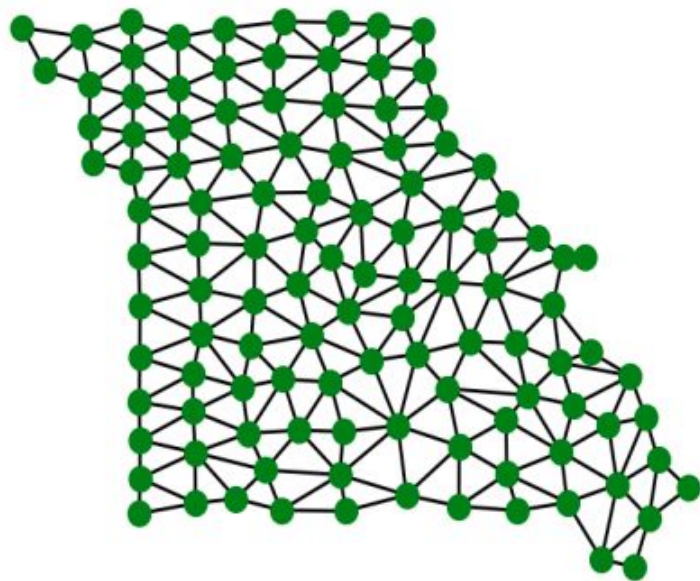
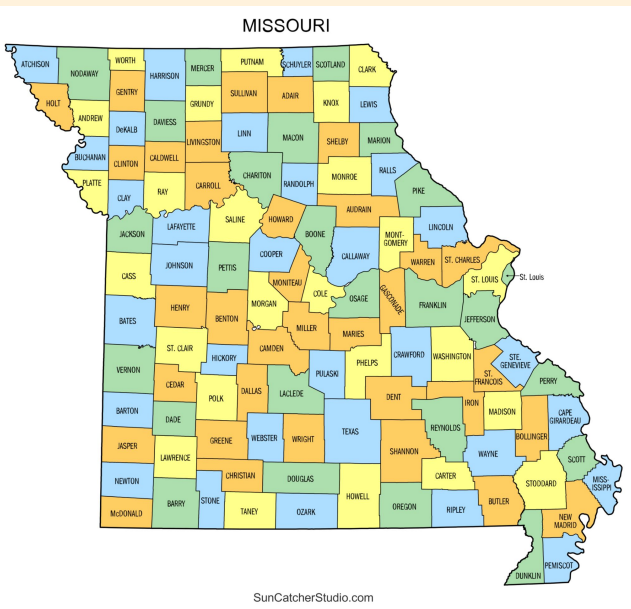
Source: DeFord et al.,  
"Recombination: A Family of Markov Chains for Redistricting," 2021,  
<https://hdr.mitpress.mit.edu/pub/1ds8ptxu#nvm1kj8eiiq>

# What are ensembles?

*Ensembles* = collection of valid alternative redistricting plans

- compact,
  - contiguous,
  - population-balanced plans
- 
- representative sample
  - Help create a baseline for comparing plans

*Ensemble method* = creating random sample of plans



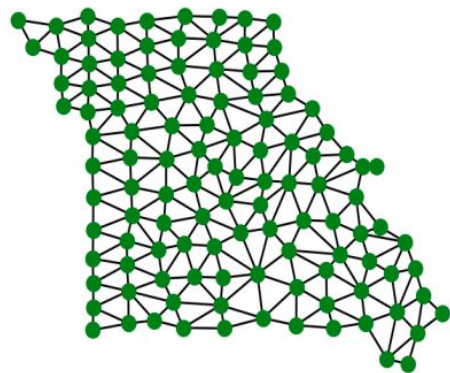
(a) County

Node = county

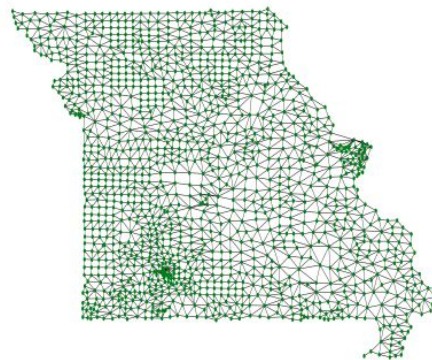
We can attach Census information to each node, such as:

- Total Population
- Population by demographic group
- VAP
- CVAP
- ... etc

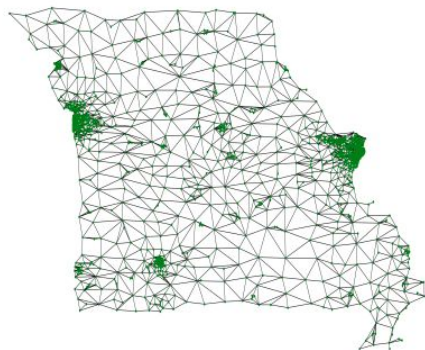
Source: DeFord et al., "Recombination: A Family of Markov Chains for Redistricting," 2020,  
<https://mggg.org/uploads/ReCom.pdf>



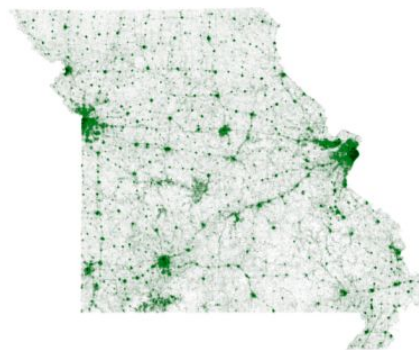
(a) County



(b) County Subunit



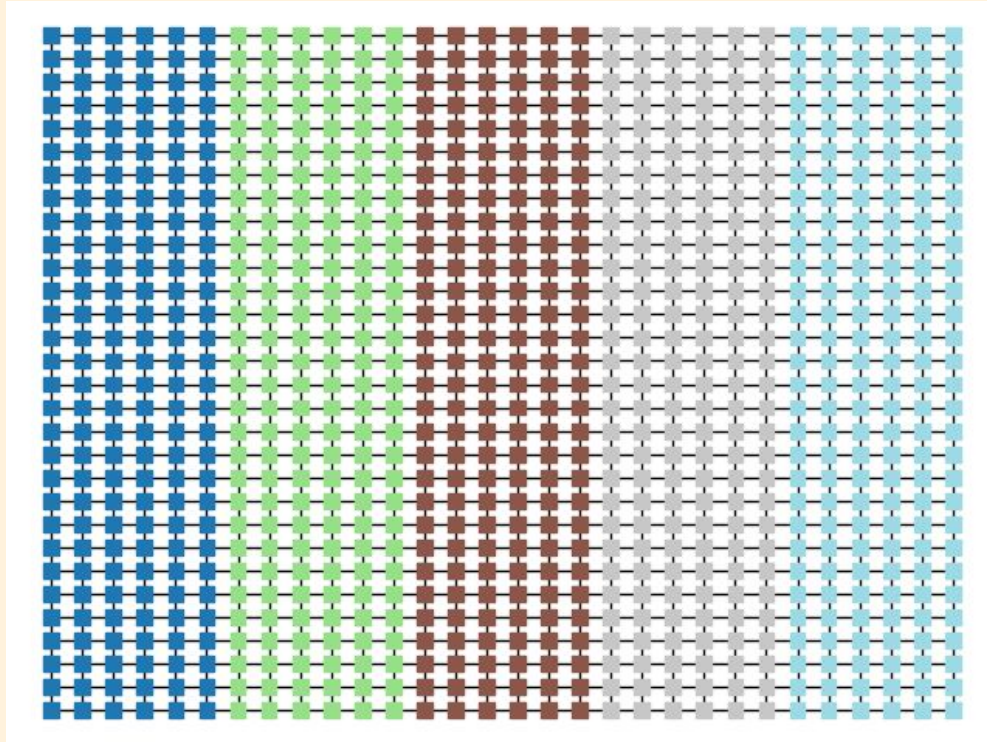
(c) Census Tract



(d) Census Block

Four dual graphs for Missouri at different levels of geography in the Census hierarchy. The graphs have 115, 1,395, 1,393, and 343,565 nodes respectively.

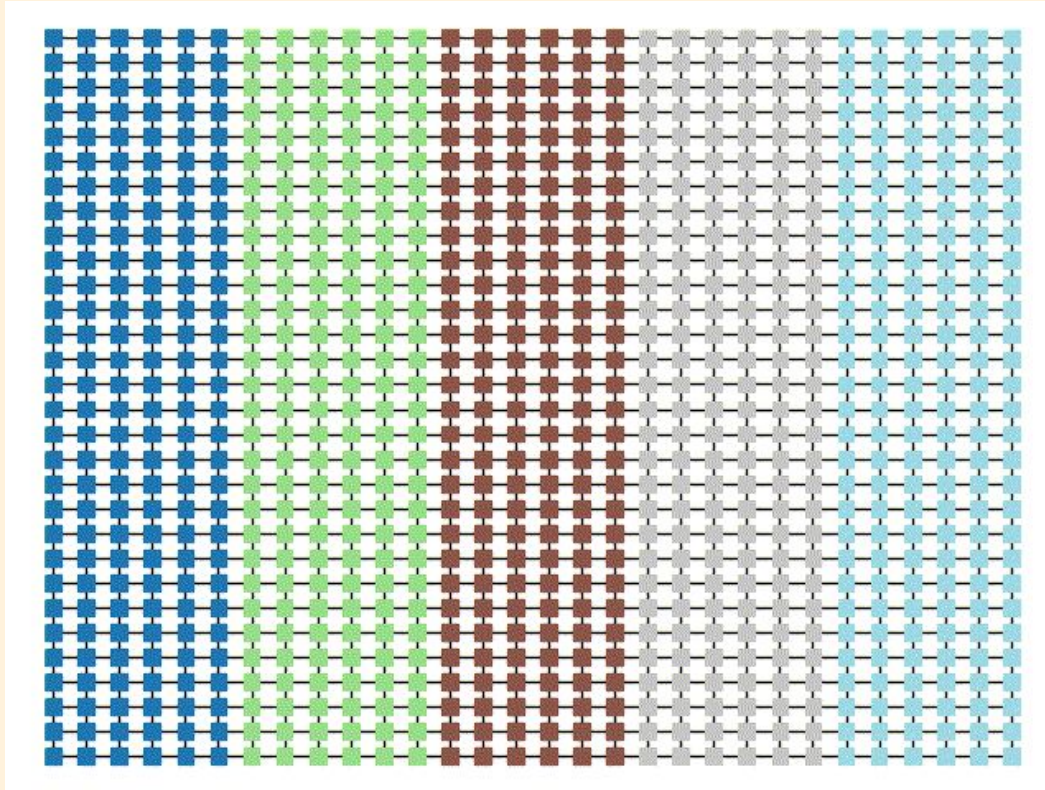
# Initial Partition (assigning each node to district)



Source: Daryl DeFord, "GerryChain-Templates," <https://github.com/drdeford/GerryChain-Templates?tab=readme-ov-file>

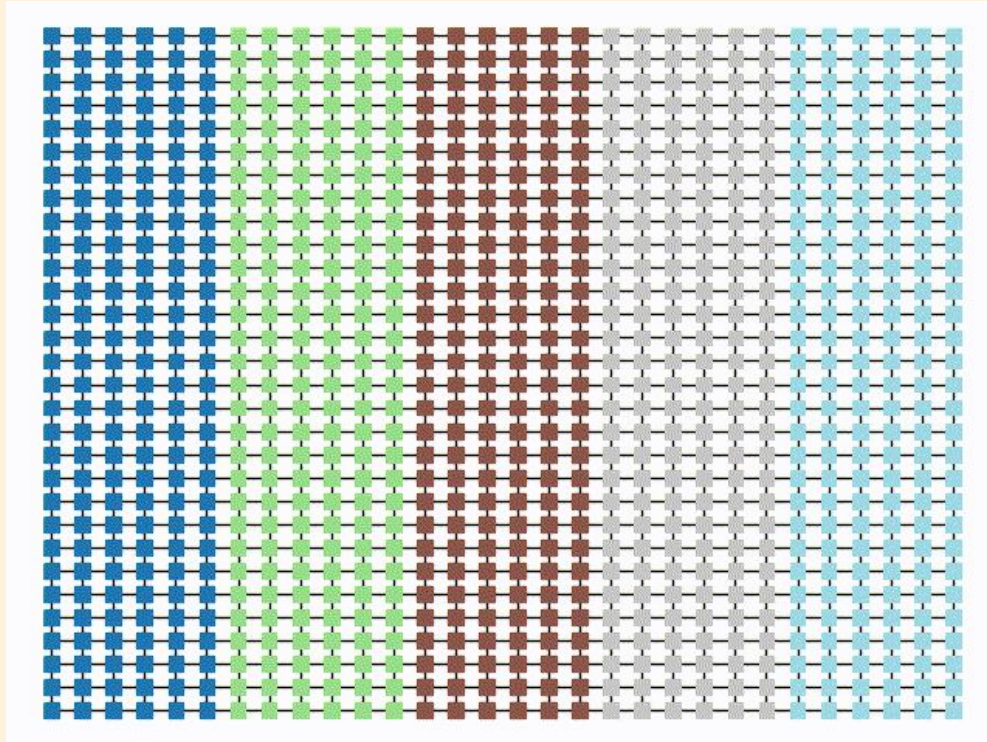


# Flip



Source: Daryl DeFord, "GerryChain-Templates," <https://github.com/drdeford/GerryChain-Templates?tab=readme-ov-file>

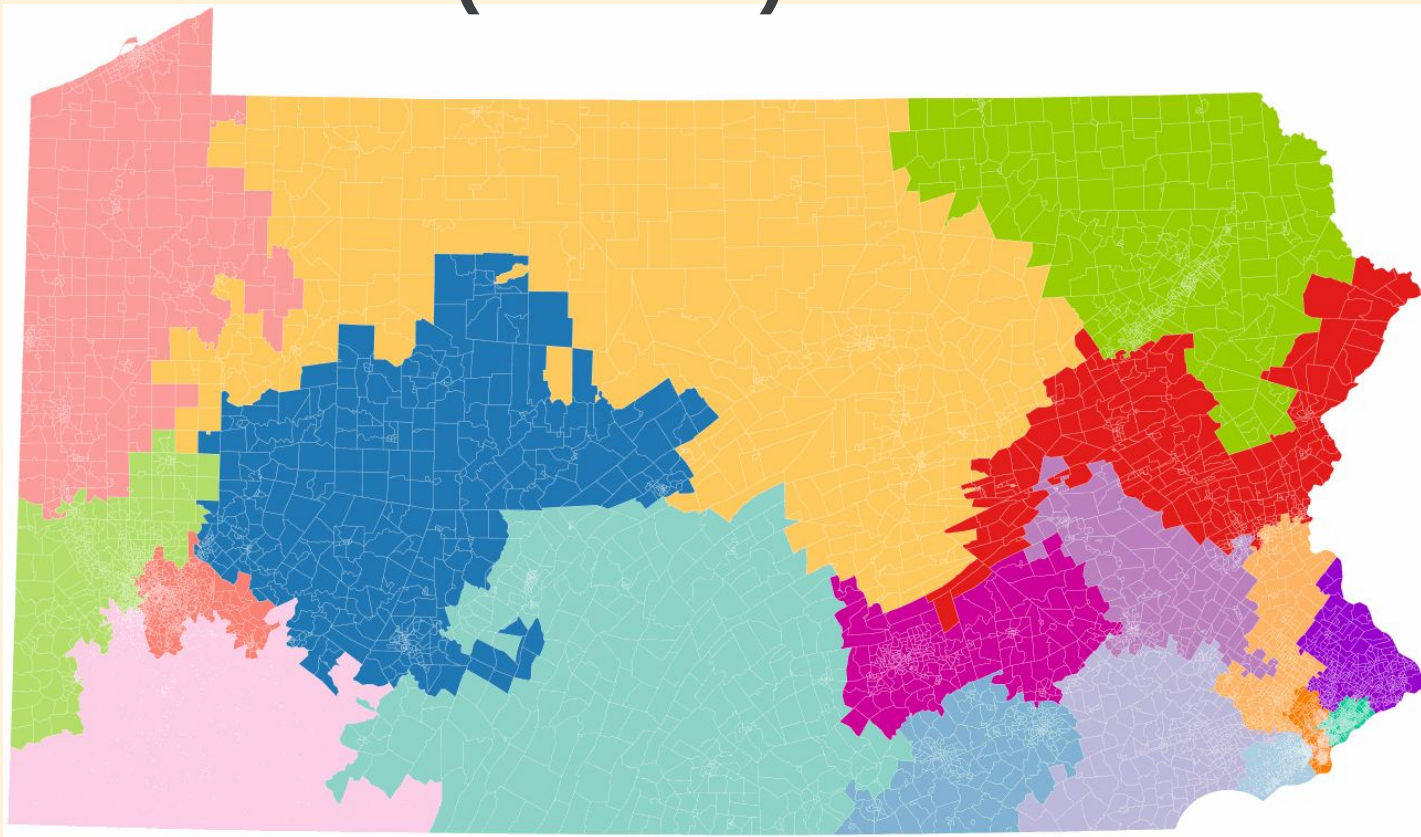
# ReCombination (ReCom)



Source: Daryl DeFord, "GerryChain-Templates," <https://github.com/drdeford/GerryChain-Templates?tab=readme-ov-file>



# ReCombination (ReCom)



Steps in a ReCombination (ReCom) chain for Pennsylvania

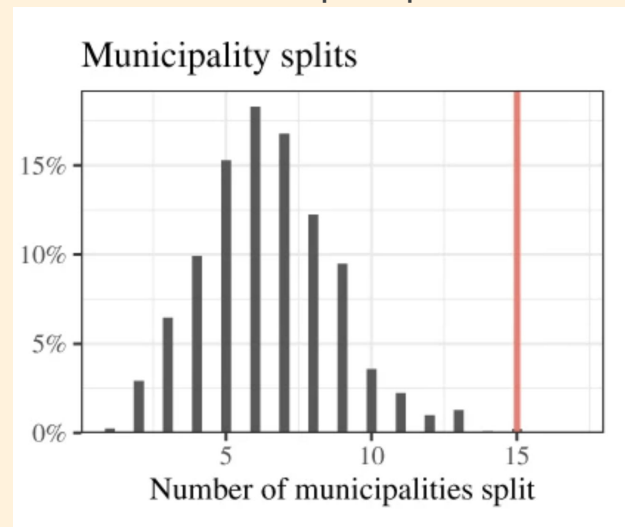
Source: DeFord et al., "Recombination: A Family of Markov Chains for Redistricting," 2021,

<https://hdrs.mitpress.mit.edu/pub/1ds8ptxu#nvm1kj8eiiq>

# Use cases in redistricting

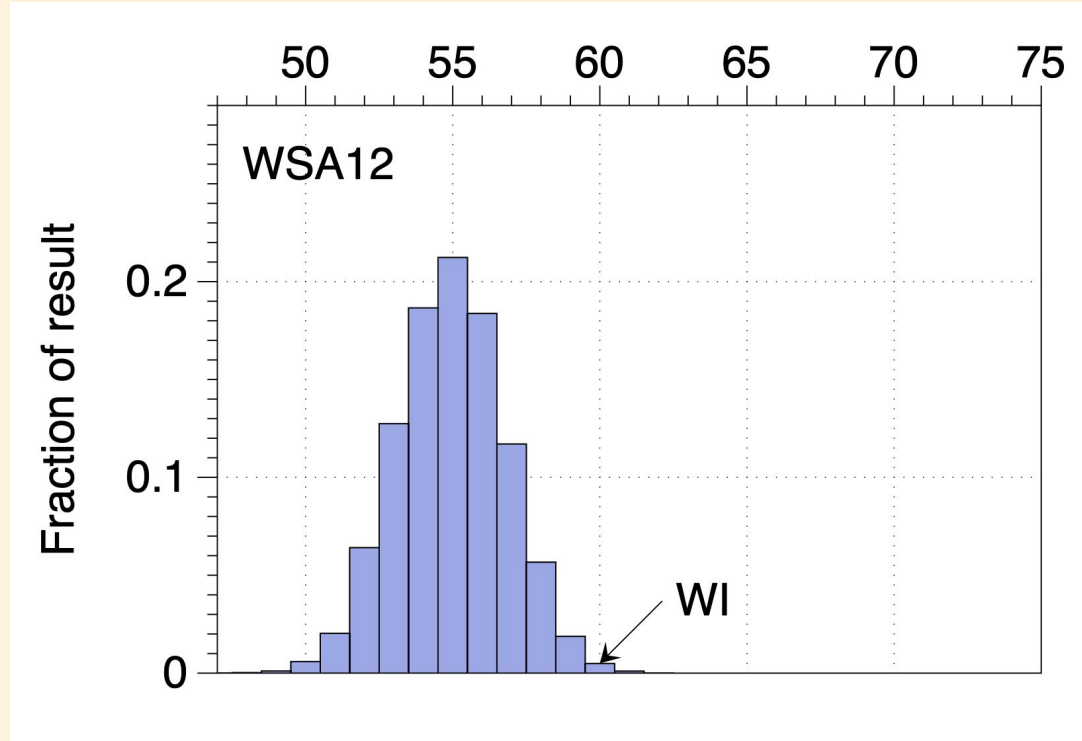
## 1. Outlier Analysis

- Ensembles create a baseline for us to understand and compare plans
- “Neutral” ensemble analysis



# Use cases in redistricting

## Outlier Analysis



# Use cases in redistricting

## Outlier Analysis

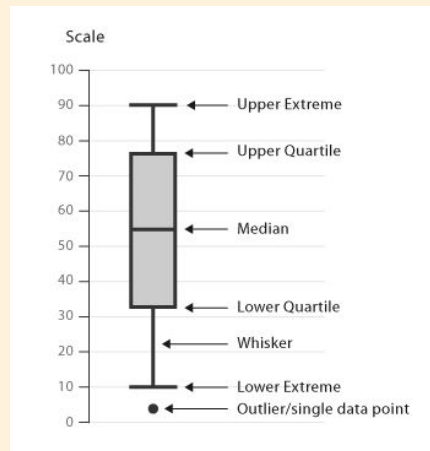
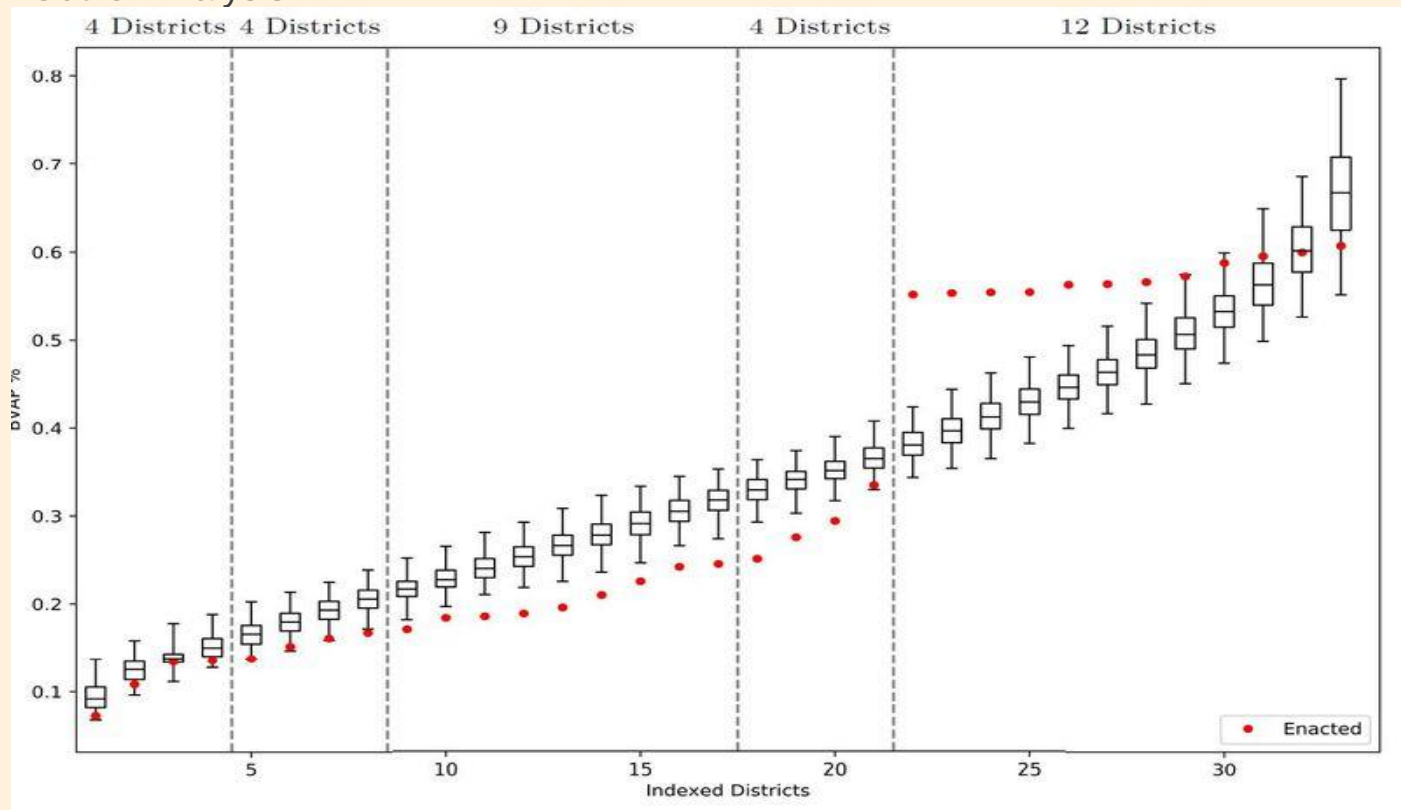


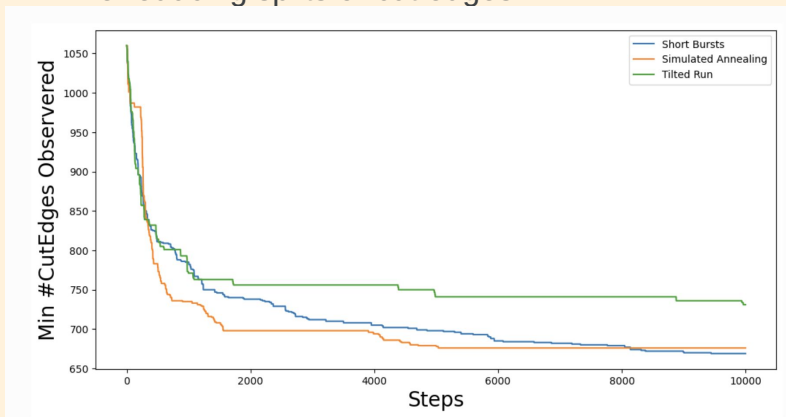
Image from  
[https://datavizcatalogue.com/methods/box\\_plot.html](https://datavizcatalogue.com/methods/box_plot.html)

# Use cases in redistricting

## Optimization ensembles

### 2. Optimization

- “Heuristic optimization methods can be used to find example plans with properties [that are rare to encounter in a neutral run] and to explore the trade-offs between them.” <https://gerrychain.readthedocs.io/en/latest/user/optimizers/#>
  - I.e reducing splits or cut edges



progression of the best score as the chain moves along

Source: MGGG, “Optimization Methods of GerryChain,” <https://gerrychain.readthedocs.io/en/latest/user/optimizers/#>



# Use cases in redistricting

## Optimization ensembles

- First condition of Gingles I
  - demonstrating that the minority population is sufficiently large and geographically compact to constitute a majority of the voting-age population in a district
- Optimizing for the creation of majority-minority district(s)
  - Gingles 1
  - “Heuristic optimization methods can be used to find example plans with these properties [that are rare to encounter in a neutral run] and to explore the trade-offs between them.” <https://gerrychain.readthedocs.io/en/latest/user/optimizers/#>

# Summary

What?

- using an **ensemble method**, we can use sampling algorithms to generate a large collection of valid redistricting plans called **ensembles**

Why?

- 1. This allows us to create a baseline of possible plans and compare/understand plans in relation to a baseline
  - **Outlier Analysis**
- 2. Optimize for example plans with certain properties
  - **Heuristics Optimization methods**

# Questions?

## Let's code! Notebook [here](#)

- You will want to click “Copy to Drive” to make a copy of the notebook, so any changes you make are saved in your personal GDrive

Data preprocessing notebook [here](#)

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Data preprocessing notebook [here](#)

Want to run the code locally?

1. I recommend installing an IDE, like [VSCode](#), to more easily view and run your code
2. Similar to in the notebook above, you will need to install all necessary packages
  - See “Installing GerryChain” steps in this [Guide](#), as an outline for installing GerryChain and other packages. If you run into problems, let me know and I can help debug! Package dependency can sometimes be finicky
3. Open the .ipynb file (coding notebook) or create a new one and code away!

# References

## *Ensembles in redistricting*

- Daryl DeFord and Moon Duchin, “Redistricting Reform in Virginia: Districting Criteria in Context,” 2019, <https://mggg.org/VA-criteria.pdf>
- Duchin et al., “Amicus Brief of Mathematicians, Law Professors, and Students in Support of Appellees and Affirmance,” Rucho v. Common Cause (2019), <https://mggg.org/SCOTUS-MathBrief.pdf>
- Jowei Chen and Jonathan Rodden, “Report on Computer Simulations of Florida Congressional Districting Plans,” 2013, <https://sites.tufts.edu/vrdi/files/2018/06/Chen-FL.pdf>
- Duchin et al., “Locating the Representational Baseline: Republicans in Massachusetts,” 2019, <https://doi.org/10.1089/elj.2018.0537>
- “The Algorithm-Assisted Redistricting Methodology (ALARM) Project,” <https://alarm-redist.org/about.html>
- MGGG, “Voting Rights Act Research,” 2022, <https://mggg.org/vra>
  - Includes references to several other papers and resources
- Moon Duchin and Douglas M. Spencer, “Models, Race, and the Law,” 2021, <https://mggg.org/publications/ModelsRaceLaw.pdf>



# References

## *Intro to Markov Chain Monte Carlo (MCMC) methods*

- Deford et al., “Recombination: A family of Markov chains for redistricting,” 2020, <https://mggg.org/uploads/ReCom.pdf>
- Daryl DeFord, “Introduction to Discrete MCMC for Redistricting,” 2019, [https://people.csail.mit.edu/ddeford/MCMC\\_Intro\\_plus.pdf](https://people.csail.mit.edu/ddeford/MCMC_Intro_plus.pdf)
  - Accompanying interactive guide: [https://github.com/drdeford/MCMC\\_Intro](https://github.com/drdeford/MCMC_Intro)
- MGGG, “Welcome to Gridlandia! An interactive introduction to the math of redistricting,” 2018, <https://mggg.org/metagraph/>
- MGGG, “Recombination and other samplers,” 2024, <https://mggg.org/samplers>
- Joshua S. Speagle, “A Conceptual Introduction to Markov Chain Monte Carlo Methods,” 2020, <https://arxiv.org/pdf/1909.12313>
- Marco Taboga, “Markov Chain Monte Carlo (MCMC) methods,” <https://www.statlect.com/fundamentals-of-statistics/Markov-Chain-Monte-Carlo>

# References

## *GerryChain Documentation and Guides*

- Metric Geometry and Gerrymandering Group, “GerryChain,” <https://gerrychain.readthedocs.io/en/latest/>
  - Optimization methods: <https://gerrychain.readthedocs.io/en/latest/user/optimizers/#using-gingleator>
- Maup – geospatial toolkit for redistricting
  - <https://maup.readthedocs.io/en/latest/>
- GerryTools – companion to GerryChain for analyze redistricting plans
  - <https://gerrytools.readthedocs.io/en/latest/>
- MGGG, “Recombination and other samplers,” 2024, <https://mggg.org/samplers>
  - Links to various papers on sampling graph partitions

## *GerryChain Guides*

- Daryl DeFord, “How to build districting ensembles: A guide to GerryChain,” 2019, [https://people.csail.mit.edu/ddeford//GerryChain\\_Guide.pdf](https://people.csail.mit.edu/ddeford//GerryChain_Guide.pdf)
- Daryl DeFord, “Computational Approaches for Political Redistricting,” 2019, <https://people.csail.mit.edu/ddeford/CAPR.php>
  - Created and compiled many resources for understanding and implementing gerrychain
- “GerryChain-Templates,” <https://github.com/drdeford/GerryChain-Templates/tree/master>