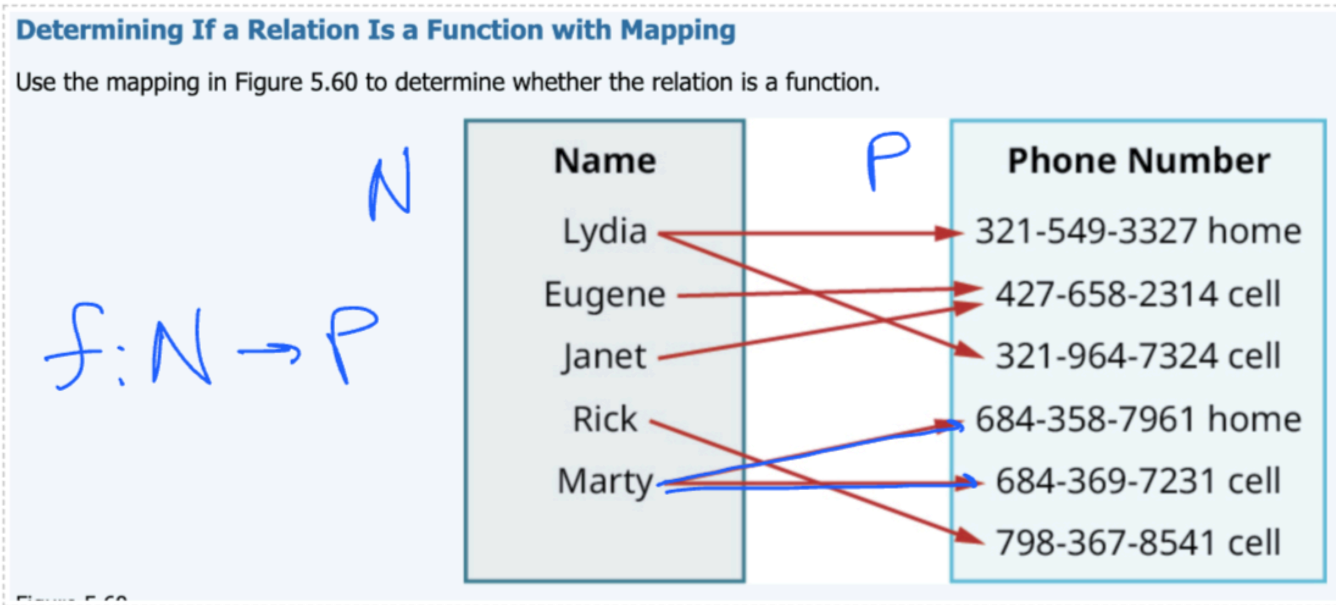
Home Work

First problem



The Key Definition

A relation is a **function** if and only if **each input (element in the domain) maps to exactly one output (element in the codomain)**.
==If a function for a single input produces two outputs, then it is not a function. It's ILLEGAL.

- Names on the left represent Domain (N)
- Phone numbers on the right represent Codomain (P)
- Mapping f:N -> P: The arrows show which name connects to which phone number(s)

Solution:

- NOT a function: Lydia, Marty
- Function: Eugene, Janet, Rick

Second Problem

Determine if each of the following equations are functions:

a. $y = x^2 + 1$
b. $y^2 = x + 1$

a. y = x^2 + 1 - function
y = 2^2 + 1 = 5 - one output
y = (-3)^2 + 1 = 10 - one output
y = (0)^2 + 1 = 1 - one output
b. y^2 = x + 1 - NOT a function
y = ±√x+1 - ± means this always will return two outputs

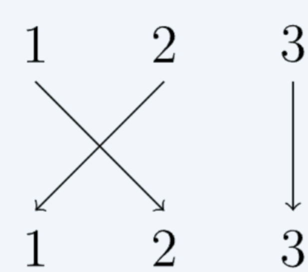
Solution:

Example a. - > Is a function, since one input has only one output
Example b. -> Is NOT a function, since one input has two possible outputs

Problem3

Which functions are surjective (i.e., onto)?

1. $f : \mathbb{Z} \to \mathbb{Z}$ defined by $f(n) = 3n$.
2. $g : \{1,2,3\} \to \{a,b,c\}$ defined by $g = \begin{pmatrix} 1 & 2 & 3 \\ c & a & a \end{pmatrix}$.
3. $h : \{1,2,3\} \to \{1,2,3\}$ defined as follows:



==*Surjective**: AT LEAST one input for each output (Also Output 1 has two possible Inputs a,c;)

1. f(n) = 3n -> NOT SURJECTIVE
   f(n) = 1 -> 1 = 3n -> n = 1/3 -> this is not an integer, so f(n)=1 would not have output
2. 1 - C, 2 - A, 3 - A -> NOT SURJECTIVE
   output B does not have any input

3. 1 - 1, 2 -2, 3 -3 -> IS SURJECTIVE -> each output has at least one input

Solution:

Only function number 3 is a Surjective because each output has at least one input

Problem4

Which functions are injective (i.e., one-to-one)?

1. $f : \mathbb{Z} \to \mathbb{Z}$ defined by $f(n) = 3n$.
2. $g : \{1, 2, 3\} \to \{a, b, c\}$ defined by $g = \begin{pmatrix} 1 & 2 & 3 \\ c & a & a \end{pmatrix}$.
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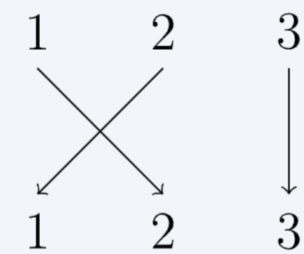

==*Injective** : For each output it has at MOST one input. (OR output can have ONLY one input or NOT AT ALL)