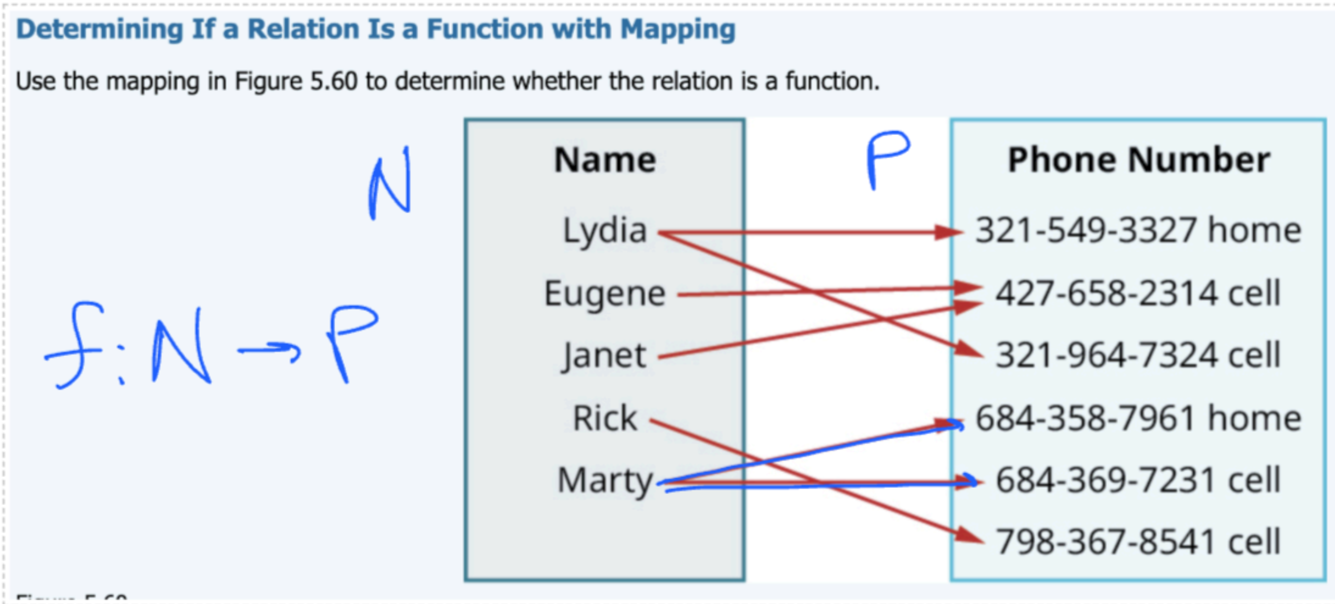
Home Work

First problem



The Key Definition

A relation is a **function** if and only if **each input (element in the domain) maps to exactly one output (element in the codomain)**.
==If a function for a single input produces two outputs, then it is not a function. It's ILLEGAL.

- Names on the left represent Domain (N)
- Phone numbers on the right represent Codomain (P)
- Mapping f:N -> P: The arrows show which name connects to which phone number(s)

Solution:

- NOT a function: Lydia, Marty
- Function: Eugene, Janet, Rick

Second Problem

Determine if each of the following equations are functions:

a. $y = x^2 + 1$

b. $y^2 = x + 1$

a. y = x^2 + 1 - function
y = 2^2 + 1 = 5 - one output
y = (-3)^2 + 1 = 10 - one output
y = (0)^2 + 1 = 1 - one output
b. y^2 = x + 1 - NOT a function
y = ±√x+1 - ± means this always will return two outputs

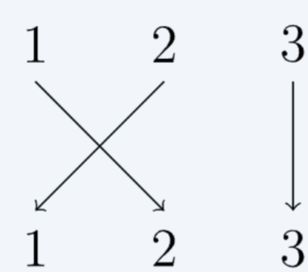
Solution:

Example a. - > Is a function, since one input has only one output
Example b. -> Is NOT a function, since one input has two possible outputs

Problem3

Which functions are surjective (i.e., onto)?

1. $f : \mathbb{Z} \to \mathbb{Z}$ defined by $f(n) = 3n$.
2. $g : \{1,2,3\} \to \{a,b,c\}$ defined by $g = \begin{pmatrix} 1 & 2 & 3 \\ c & a & a \end{pmatrix}$.
3. $h : \{1,2,3\} \to \{1,2,3\}$ defined as follows:



==*Surjective**: AT LEAST one input for each output (Also Output 1 has two possible Inputs a,c;)

1. f(n) = 3n -> NOT SURJECTIVE
   f(n) = 1 -> 1 = 3n -> n = 1/3 -> this is not an integer, so f(n)=1 would not have output
2. 1 - C, 2 - A, 3 - A -> NOT SURJECTIVE
   output B does not have any input

3. 1 - 1, 2 -2, 3 -3 -> IS SURJECTIVE -> each output has at least one input

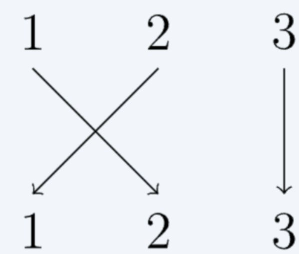
Solution:

Only function number 3 is a Surjective because each output has at least one input

Problem4

Which functions are injective (i.e., one-to-one)?

1. $f : \mathbb{Z} \to \mathbb{Z}$ defined by $f(n) = 3n$.
2. $g : \{1,2,3\} \to \{a,b,c\}$ defined by $g = \begin{pmatrix} 1 & 2 & 3 \\ c & a & a \end{pmatrix}$.
3. $h : \{1,2,3\} \to \{1,2,3\}$ defined as follows:



==***Injective** : For each output it has at MOST one input. (OR output can have ONLY one input or NOT AT ALL)

1. f(n) = 3n is injective ->
   a) for it to have more than one input it x1 should be = x2
   b) but it is defined x1 cannot be = x2
2. g: is not injective -> because output "a" has two inputs which contradict the defenetition
3. h: is injective -> because each output has at most one input

Solution: Functions #1 and #3 are injective functions, because each output has at most one input which aligns with the definition

Problem5

If $f(x) = \frac{1}{x+2}$ and $g(x) = \frac{1}{x} - 2$, is $g = f^{-1}$?

1. Validate if f(g(x)) = x ->
   a. let: g(3) = - 5/3
   g(3) = 1/3 -2 = -5/3

c. *$f(g(3) = 3$ which aligns with $f(g(x)) = x$*
f(-5/3) = 1/ (-5/3 +2) = 1 / (1/3) = 3

2. Validate if g(f(x)) =x
    a. let: f(3) =
    f(3) = 1/ (3+2) = 1/5
    b. g(f(3)) = 3 which aligns with g(f(x)) = x
    g(1/5) = 1(1/5) -2 = 3
3. Validate if g = f^-1
    a. f^-1 = 1/x - 2
    y = 1 / (x+2)
    y (x + 2) = 1 -> x + 2 = 1/y -> x = 1/y - 2
    * f^-1(1/y - 2) = 1/x - 2
    b. g(x) = 1/x - 2 and f^-1 = 1/x - 2

Solution: g = f^-1 is true because g(x) = 1/x - 2 and f^-1 = 1/x - 2

What this problem is asking to validate?

==This problem is asking to determine if above two functions are inverses (g = f^-1)

Definition: Two functions are inverses if and only if:

1. **f(g(x)) = x** for all x in the domain of g, AND
2. **g(f(x)) = x** for all x in the domain of f

f(g(x)) = x
This is called a **composition of functions**. You're plugging g(x) into f(x).

- Start with some input value x

- Apply function g to get g(x)
- Take that result and apply function f to get f(g(x))
- If you end up back at x, then f "undoes" what g did

**Example
If g(x) = x + 5 and f(x) = x - 5

- Start with x = 3
- Apply g: g(3) = 3 + 5 = 8
- Apply f: f(8) = 8 - 5 = 3
- We're back to 3! So f(g(x)) = x ✓

The Verification Process

1: **Compute f(g(x)) and simplify completely**

- If you get x, condition 1 passes ✓
  **2: Compute g(f(x)) and simplify completely
- If you get x, condition 2 passes ✓
  Both must equal x
- If both work → they ARE inverses
- If even one fails → **they are NOT inverses**
  3: Compute f^-1(x) and compare with g(x)

Problem6

Find the inverse of the function $f(x) = 2 + \sqrt{x - 4}$.

$y = 2 + \sqrt{(x-4)}$
$y - 2 = \sqrt{(x-4)}$ -> multiply ^{2}
$(y - 2)^2 = x - 4$
$x = (y - 2)^2 + 4$
$(f^{-1}(x)) = (x - 2)^2 + 4$

Domains: IN PROGRESS -> OUTSTANDING TASK
$x - 4 \geq 0$ -> $x \geq 4$
$2 + \sqrt{x - 4} \geq 2$ ->

Verification

To verify, check that $f(f^{-1}(x)) = x$
$f(f^{-1}(x)) = f((x - 2)^2 + 4) = x$

- $2 + \sqrt{((x - 2)^2 + 4 - 4)} = 2 + \sqrt{(x-2)^2} = 2 + |x - 2| = x$ (if $x \geq 2$) $= 2 + (x - 2) = x$

Solution: $(f^{-1}(x)) = (x - 2)^2 + 4$