Name: Kamalpreet Kaur

Rollno: 102103259

Group: 3COE9

Assignment : parameter Estimation

1. let $(x_1, x_2, \ldots)$ be a random sample of size n taken from a normal population with parameter mean $= \theta_1$, variance $= \theta_2$. find Maximum likelihood estimates of these two parameters.

Normal distribution function, for mean $= \theta_1$ and variance $= \theta_2$

$$f(x; \theta_1, \theta_2) = \frac{1}{\sqrt{2\pi\theta_2}} \exp\left(\frac{-(x-\theta_1)^2}{2\theta_2}\right)$$

likelihood function :

$$L(x_1, x_2, \ldots; \theta_1, \theta_2) = \prod_{i=1}^{n} f(x_i, \theta_1, \theta_2)$$

$$L = \frac{1}{(2\pi\theta_2)^{n/2}} \exp\left[-\frac{1}{2}\sum_{i=1}^{n}\left(\frac{x_i-\theta_1}{\theta_2}\right)^2\right]$$

taking natural log on both sides

$$\ln L = -\frac{n}{2}\ln(2\pi) - \frac{n}{2}\ln\theta_2 - \frac{1}{2}\sum_{i=1}^{n}\frac{(x_i-\theta_1)^2}{\theta_2}$$

taking derivative of above equation with respect to θ_1, and the for θ_2

$$\frac{1}{L}\frac{dL}{d\theta_1} = -\frac{1}{2}\sum_{i=1}^{n}\frac{(x_i-\theta_1)^2 \times 2(-1)}{\theta_2}$$

$$\frac{1}{L}\cdot\frac{dL}{d\theta_1} = \sum_{i=1}^{n}\frac{(x_i-\theta_1)}{\theta_2}$$

$$\frac{1}{L}\cdot\frac{dL}{d\theta_2} = -\frac{n}{2\theta_2} + \left(\frac{-1}{2}\right)\sum_{i=1}^{n}\frac{(x_i-\theta_1)^2 \times (-1)}{(\theta_2)^2}$$

$$\frac{1}{L}\cdot\frac{dL}{d\theta_2} = \frac{-n}{2\theta_2} + \frac{1}{2(\theta_2)^2}\sum_{i=1}^{n}(x_i-\theta_1)^2$$

Now, $\frac{dL}{d\theta_1} = \frac{dL}{d\theta_2} = 0$

$$\frac{dL}{d\theta_1} = L\left(\sum_{i=1}^{n}\frac{(x_i-\theta_1)}{\theta_2}\right) = 0$$

$$\sum_{i=1}^{n}(x_i-\theta_1) = 0$$

$$\sum_{i=1}^{n}x_i - n\theta_1 = 0$$

$$\theta_1 = \frac{\sum_{i=1}^{n}x_i}{n} \quad \text{] MLE of mean}$$

$$\frac{dL}{d\theta_2} = \frac{-n}{2\theta_2} + \frac{1}{2(\theta_2)^2}\sum_{i=1}^{n}(x_i-\theta_1)^2 = 0$$

$$= -n + \frac{1}{\theta_2}\sum_{i=1}^{n}(x_i-\theta_1)^2 = 0$$

$$\frac{1}{\theta_2}\left(\sum_{i=1}^{n}(x_i-\theta_1)^2\right)=n$$

$$\left.\theta_2=\sum_{i=1}^{n}\frac{(x_i-\theta_1)^2}{n}\quad\right]$$ MLE of variance, θ_1 = mean of population

2. let $x_1, x_2, \dots, x_n$ be a random sample from $B(m,\theta)$ distribution, where $\theta \in (0,1)$ is unknown and m is a positive integer. compute value of θ using the MLE.

Binomial Distribution formula:

$$B(m,\theta) = {}^{n}C_{m}\, \theta^{m}(1-\theta)^{n-m}$$

Likelihood function:

$$L(m,\theta) = B(m,\theta)$$

$$L = {}^{n}C_{m}\, \theta^{m}(1-\theta)^{n-m}$$

taking natural log on both side

$$\ln L = \ln({}^{n}C_{m}) + m\ln\theta + (n-m)\ln(1-\theta)$$

taking derivate with respect to θ

$$\frac{1}{L}\cdot\frac{dL}{d\theta} = 0 + \frac{m}{\theta} + \frac{(n-m)(-1)}{(1-\theta)}$$

$$\frac{1}{L}\cdot\frac{dL}{d\theta} = \frac{m}{\theta} - \frac{(n-m)}{(1-\theta)}$$

$$\frac{1}{L} \cdot \frac{dL}{d\theta} = \frac{m(1-\theta) - (n-m)\theta}{\theta(1-\theta)}$$

$$\frac{dL}{d\theta} = 0$$

$$\frac{m(1-\theta) - (n-m)\theta}{\theta(1-\theta)} = 0$$

$$m(1-\theta) - (n-m)\theta = 0$$

$$m(1-\theta) = (n-m)\theta$$

$$m - \cancel{m\theta} = n\theta - \cancel{m\theta}$$

$$\frac{m}{n} = \theta \quad \Big] \text{ MLE of } \theta$$