Cost Aware Best Arm Identification

Reinforcement Learning Conference 2024

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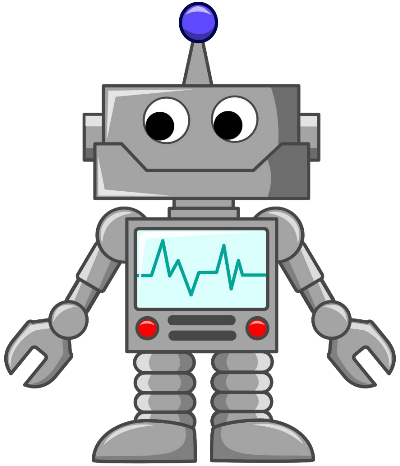
Outline

1. Recap MAB Motivations
2. Introduce New Formulation
3. Tight Asymptotic Lower Bound
4. Simple Algorithm
5. Results

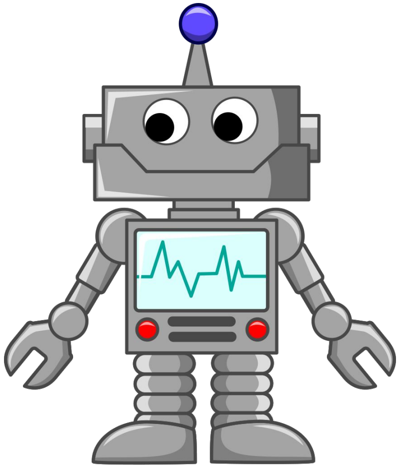
Scenario 1: Gambling Robot

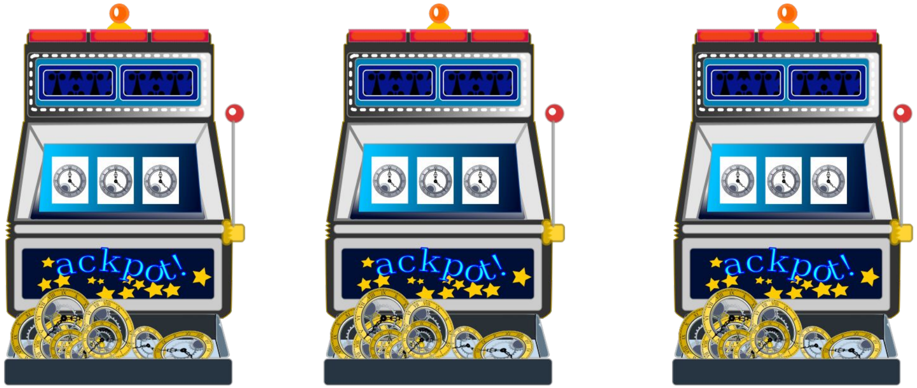


Scenario 1: Gambling Robot

$X_{i,t}$ = reward arm i at time t

$\mu_i = \mathbb{E}\left[X_{i,t}\right]$





μ_1 μ_2 μ_3

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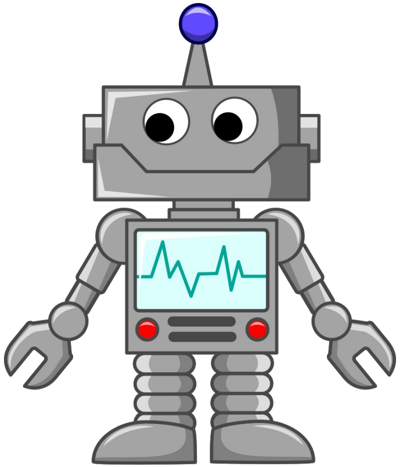
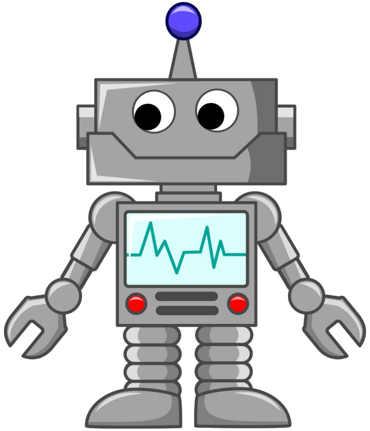


μ_1 μ_2 μ_3

Goal: Make as much money as possible

Scenario 1: Gambling Robot



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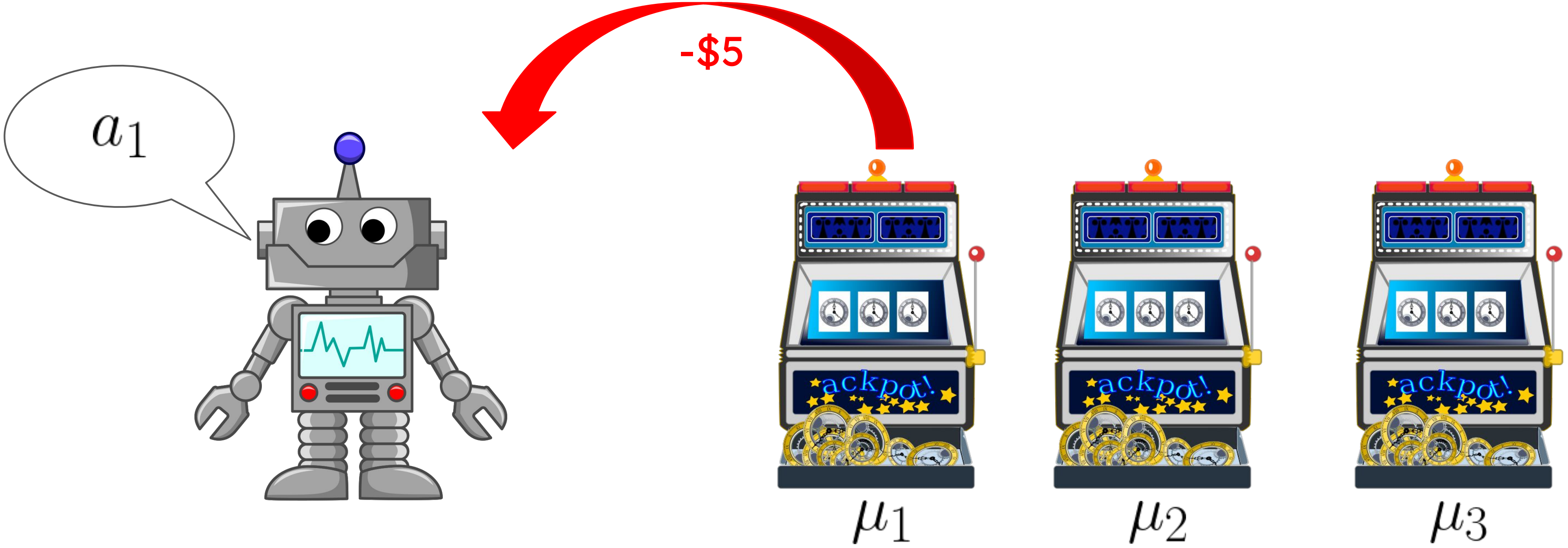

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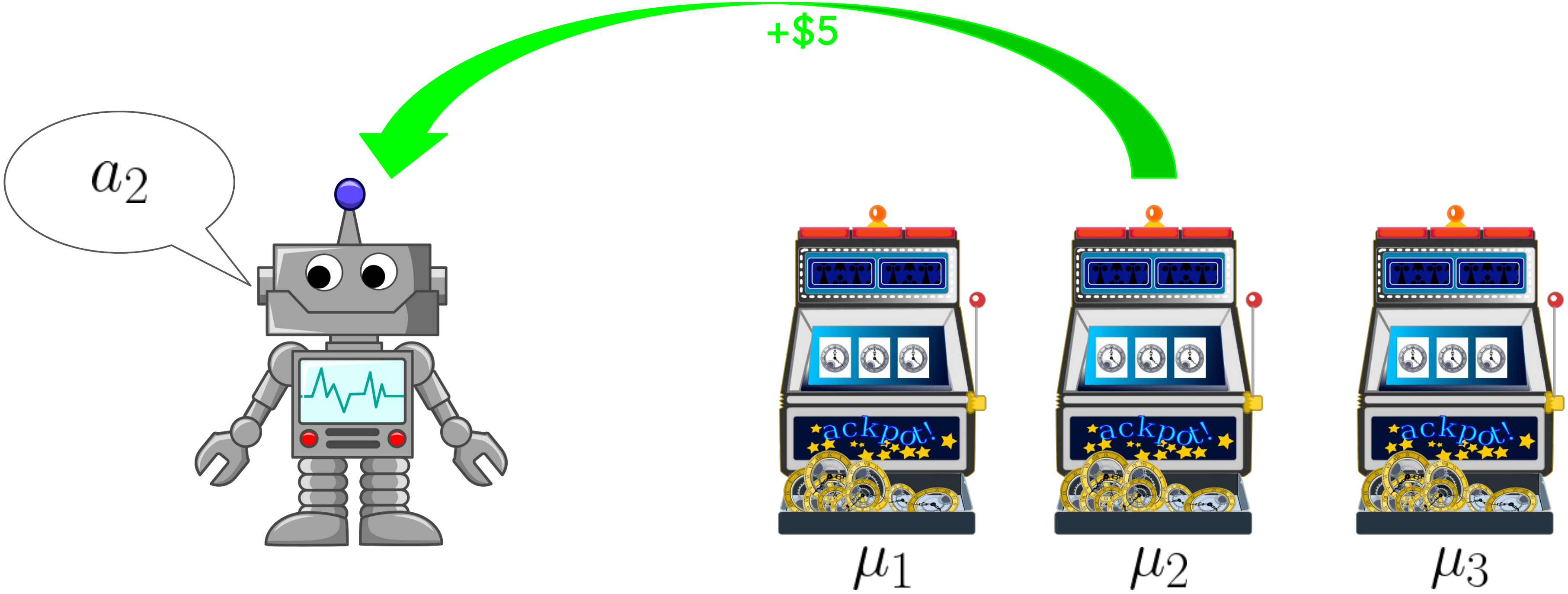

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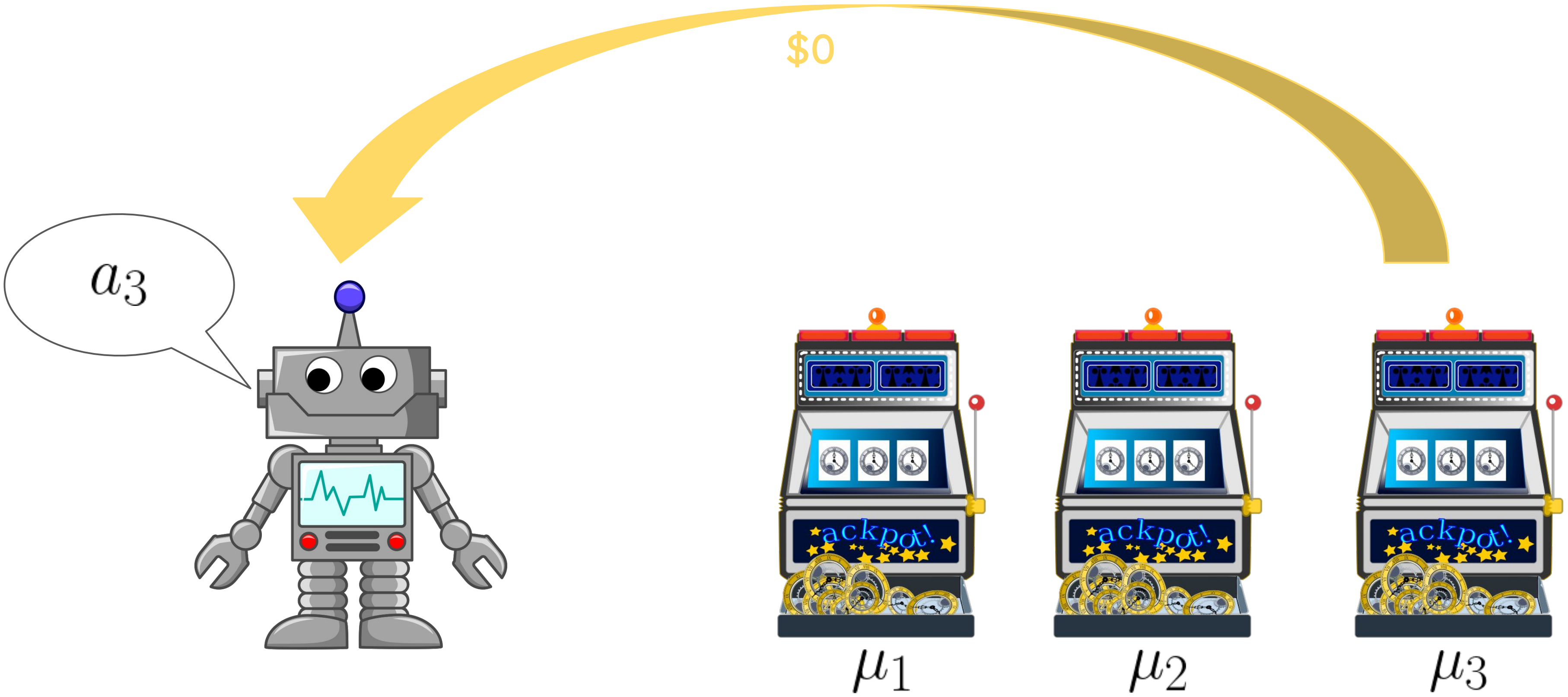
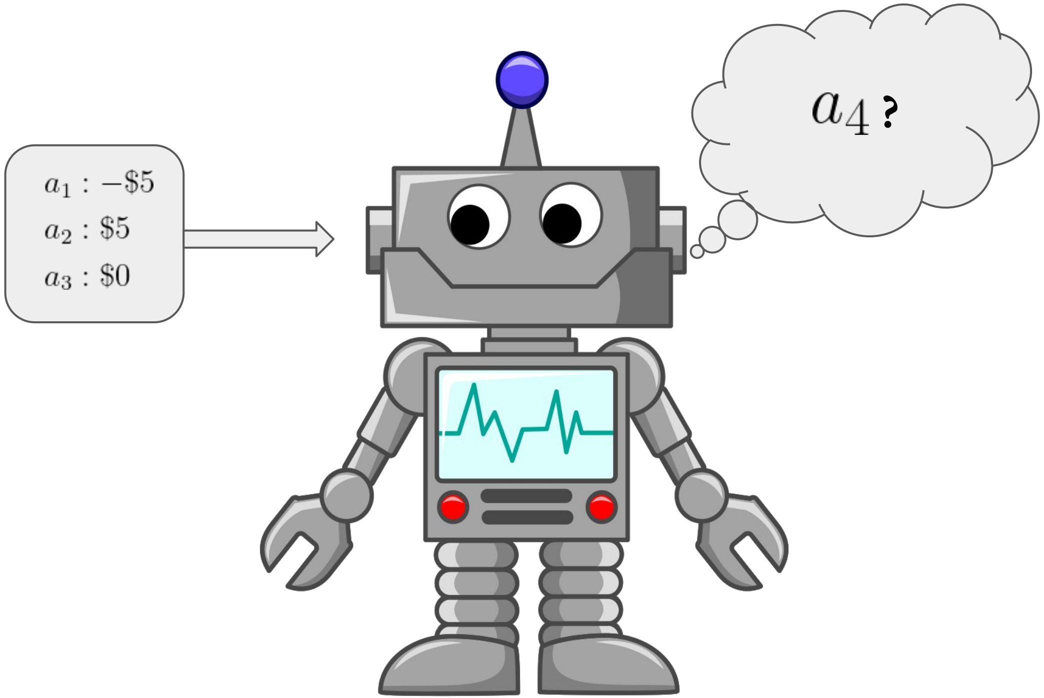


Scenario 1: Gambling Robot



Regret Minimization:

Inputs

Time Horizon

$$T \in \mathcal{N}$$

Components

Arm sampling rule

$$\pi : \mathcal{H} \to \mathcal{A}$$

Objective

Regret

$$\mathrm{Reg}_\pi(T) = \max_j \sum_{t=1}^{T} \mathbb{E}\left[X_{j,t} - X_{\pi(t),t}\right]$$

Scenario 2: Clinical Trials

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$S_i = i$th drug trial is a success.

$\mu_i = \mathbb{E}\left[\mathbf{1}_{\{S_i\}}\right]$

μ_1 μ_2 μ_3

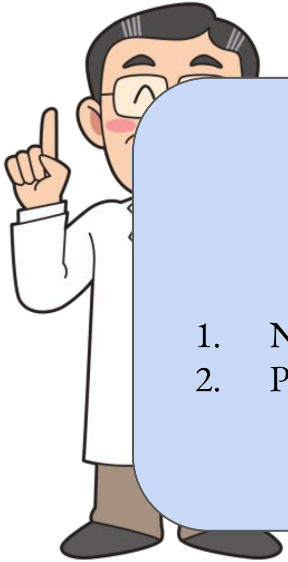
Scenario 2: Clinical Trials

$S_i = i$th drug trial is a success.
$\mu_i = \mathbb{E}\left[\mathbf{1}_{\{S_i\}}\right]$

μ_1 μ_2 μ_3

Goal: Identify best drug as fast as possible.

Scenario 2: Clinical Trials



Problem!

1. No fixed time horizon T.
2. Patients do not want to be the exploration arm pull.

μ_3

Best Arm Identification

Inputs

Time Horizon

$$T \in \mathcal{N}$$

Components

Arm sampling rule

$$\pi : \mathcal{H} \to \mathcal{A}$$

Objective

Regret

$$\mathrm{Reg}_\pi(T) = \max_j \sum_{t=1}^{T} \mathbb{E}\left[X_{j,t} - X_{\pi(t),t}\right]$$

Best Arm Identification

Inputs

Confidence Level

$$\delta \in [0, 1)$$

Components

Arm sampling rule

$$\pi : \mathcal{H} \to \mathcal{A}$$

Objective

Regret

$$\mathrm{Reg}_\pi(T) = \max_j \sum_{t=1}^{T} \mathbb{E}\left[X_{j,t} - X_{\pi(t),t}\right]$$

Best Arm Identification

Inputs

Confidence Level

$$\delta \in [0, 1)$$

Components

Arm sampling rule

$$\pi : \mathcal{H} \to \mathcal{A}$$

Stopping Rule

$$\tau : \mathcal{H} \to \{0, 1\}$$

Objective

Regret

$$\mathrm{Reg}_\pi(T) = \max_j \sum_{t=1}^{T} \mathbb{E}\left[X_{j,t} - X_{\pi(t),t}\right]$$

Best Arm Identification

Inputs

Confidence Level

$$\delta \in [0, 1)$$

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$$\pi : \mathcal{H} \to \mathcal{A}$$

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$$\tau : \mathcal{H} \to \{0, 1\}$$

Objective

Sample Complexity

$$\min \tau$$
$$\text{s.t } \Pr(\hat{a} = a^*) \geq 1 - \delta$$

Scenario 3: Corporate Health

Scenario 3: Corporate Health

$$c_i = \mathbb{E}\left[\text{cost of trial for drug } i\right]$$

c_1

μ_1

c_2

μ_2

c_3

μ_3

Scenario 3: Corporate Health

$$c_i = \mathbb{E}\left[\text{cost of trial for drug } i\right]$$

c_1 c_2 c_3

μ_1 μ_2 μ_3

Goal: Identify best drug as cheap as possible.

Cost Aware Best Arm Identification

Inputs

Confidence Level

$$\delta \in [0, 1)$$

Components

Arm sampling rule

$$\pi : \mathcal{H} \to \mathcal{A}$$

Stopping Rule

$$\tau : \mathcal{H} \to \{0, 1\}$$

Objective

Sample Complexity

$$\min \tau$$
$$\text{s.t } \Pr(\hat{a} = a^*) \geq 1 - \delta$$

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Cost Aware Best Arm Identification

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Confidence Level

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Arm sampling rule

$$\pi : \mathcal{H} \to \mathcal{A}$$

Stopping Rule

$$\tau : \mathcal{H} \to \{0, 1\}$$

Objective (Ours)

Cost Complexity

$$\min \sum_a \mathbb{E}\left[c_a N_a(\tau_\delta)\right]$$

$$\text{s.t } \Pr(\hat{a} = a^*) \geq 1 - \delta$$

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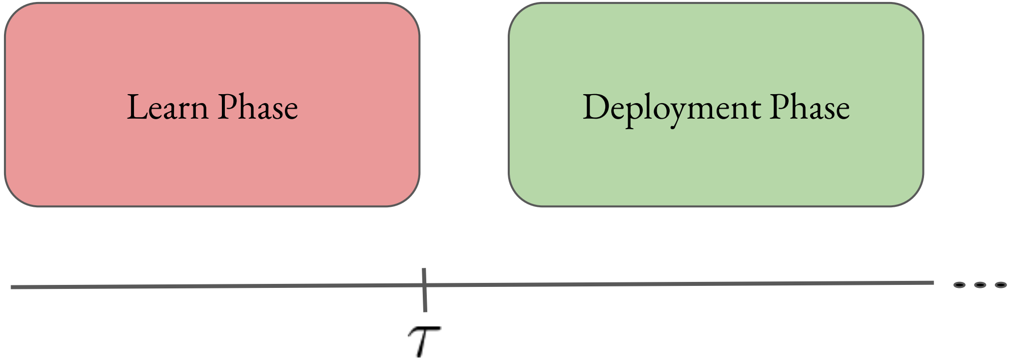
Cost Aware Best Arm Identification



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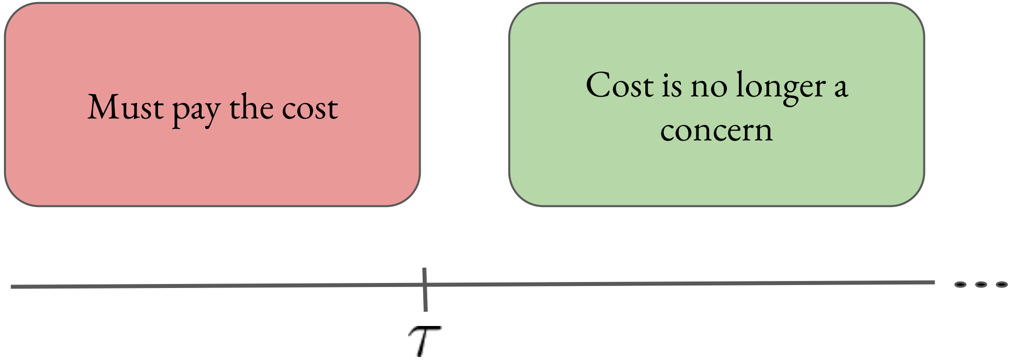
Cost Aware Best Arm Identification



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Cost Aware Best Arm Identification

Must pay the cost:
Clinical Trial

Cost is no longer a concern:
Cost is covered by insurance

τ ...

Objective (Ours)

Cost Complexity

$$\min \sum_a \mathbb{E}\left[c_a N_a(\tau_\delta)\right]$$

$$\text{s.t } \Pr(\hat{a} = a^*) \geq 1 - \delta$$

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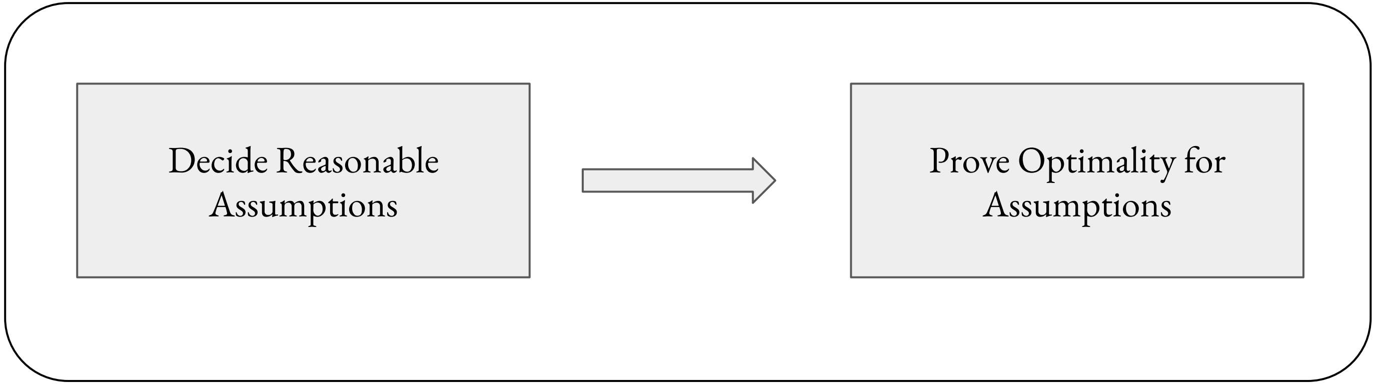
Optimal Cost Aware Best Arm Identification



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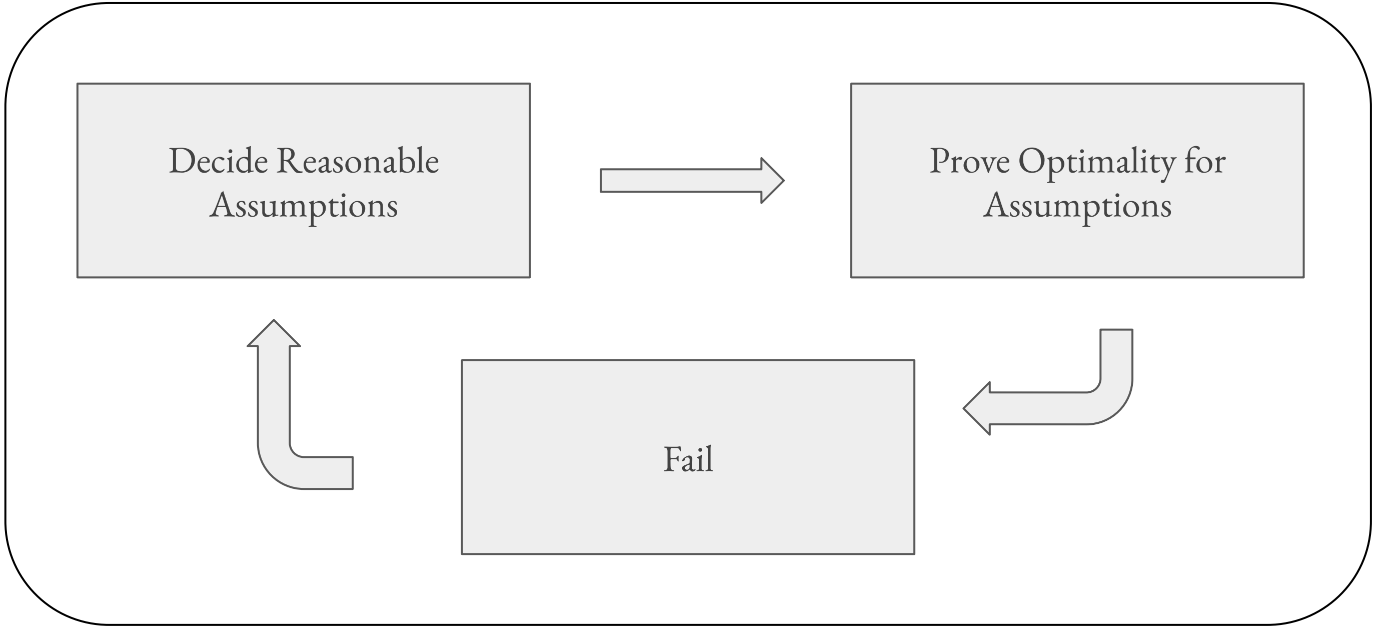

Kanarios, Kellen; Zhang, Qining; Ying, Lei. (2024). Cost Aware Best Arm Identification.

Optimal Cost Aware Best Arm Identification

Assumptions on Rewards

1. Single parameter exponential family.
2. Unique best arm.

Inherited from previous BAI work.

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Optimal Cost Aware Best Arm Identification

Assumptions on Rewards

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Assumptions on Costs

1. Costs are bounded.
2. Costs are strictly greater than 0.

Inherited from previous BAI work.

Ours

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Optimal Cost Aware Best Arm Identification

"Everything has a price"

"Nothing in life is free"

Assumptions on Costs

1. Costs are bounded.
2. Costs are strictly greater than 0.

Ours

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Optimal Cost Aware Best Arm Identification

Lower Bound. Let $\delta \in (0,1)$. For any δ - PAC algorithm and any bandit model $\boldsymbol{\mu} \in \mathcal{M}$, we have:

$$\sum_a \mathbb{E}_{\boldsymbol{\mu} \times \boldsymbol{c}}\left[c_a N_a\left(\tau_\delta\right)\right] \geq T^*(\boldsymbol{\mu}) \log \frac{1}{\delta} + o\left(\log \frac{1}{\delta}\right)$$

where $T^*(\boldsymbol{\mu})$ is the instance dependent constant satisfying:

$$T^*(\boldsymbol{\mu})^{-1} = \sup_{\boldsymbol{w} \in \Sigma_K} \inf_{\boldsymbol{\lambda} \in \{a^*(\boldsymbol{\lambda}) \neq a^*(\boldsymbol{\mu})\}} \sum_a \frac{w_a}{c_a} d\left(\mu_a, \lambda_a\right).$$

Kanarios, Kellen; Zhang, Qining; Ying, Lei. (2024). Cost Aware Best Arm Identification.

Stopping Rule

In CABAI, we still want to stop as soon as possible

- Can reuse **exact** stopping rule from BAI!

Garivier, A; Kaufmann, E. (2016). Optimal Best Arm Identification with Fixed Confidence.

Stopping Rule

In CABAI, we still want to stop as soon as possible

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$$\hat{\mu}_{a,b}(t) := \frac{N_a(t)}{N_a(t) + N_b(t)}\hat{\mu}_a(t) + \frac{N_b(t)}{N_a(t) + N_b(t)}\hat{\mu}_b(t)$$

$$Z_{a,b}(t) = N_a(t)d\left(\hat{\mu}_a(t), \hat{\mu}_{a,b}(t)\right) + N_b(t)d\left(\hat{\mu}_b(t), \hat{\mu}_{a,b}(t)\right)$$

$$\tau_\delta = \inf\left\{t \in \mathbb{N} : Z(t) := \max_{a\in\mathcal{A}} \min_{b\in\mathcal{A}\setminus\{a\}} Z_{a,b}(t) > \beta(t, \delta)\right\}$$

Garivier, A; Kaufmann, E. (2016). Optimal Best Arm Identification with Fixed Confidence.

Sampling Rule

Proof Snippet:

$$\begin{aligned}\mathrm{kl}(\delta, 1-\delta) &\leq \mathbb{E}_{\boldsymbol{\mu}\times\boldsymbol{c}}\left[C\left(\tau_\delta\right)\right] \inf_{\lambda\in\mathrm{Alt}(\boldsymbol{\mu})}\left(\sum_{a=1}^{K}\frac{\mathbb{E}_{\boldsymbol{\mu}\times\boldsymbol{c}}\left[c_a N_a\left(\tau_\delta\right)\right]}{\mathbb{E}_{\boldsymbol{\mu}\times\boldsymbol{c}}\left[c_a C\left(\tau_\delta\right)\right]} d\left(\mu_a, \lambda_a\right)\right)\\ &\leq \mathbb{E}_{\boldsymbol{\mu}}\left[C\left(\tau_\delta\right)\right] \sup_{w\in\Sigma_K}\inf_{\lambda\in\mathrm{Alt}(\boldsymbol{\mu})}\left(\sum_{a=1}^{K}\frac{w_a}{c_a} d\left(\mu_a, \lambda_a\right)\right),\end{aligned}$$

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$$\leq \mathbb{E}_{\boldsymbol{\mu}}\left[C\left(\tau_\delta\right)\right] \boxed{\sup_{w\in\Sigma_K}} \inf_{\lambda\in\mathrm{Alt}(\boldsymbol{\mu})}\left(\sum_{a=1}^{K}\frac{w_a}{c_a} d\left(\mu_a, \lambda_a\right)\right),$$

Key insight 1: Can compute $\boxed{w^*(\boldsymbol{\mu})}$ for given $\boldsymbol{\mu}$

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Key insight 1: Can compute $\boxed{w^*(\boldsymbol{\mu})}$ for given $\boldsymbol{\mu}$

Key insight 2: Proportion should match $w^*(\boldsymbol{\mu})$

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Sampling Rule

Proof Snippet:

$$\mathrm{kl}(\delta, 1-\delta) \leq \mathbb{E}_{\boldsymbol{\mu}\times\boldsymbol{c}}\left[C\left(\tau_\delta\right)\right] \inf_{\lambda \in \mathrm{Alt}(\boldsymbol{\mu})}\left(\sum_{a=1}^{K} \boxed{\frac{\mathbb{E}_{\boldsymbol{\mu}\times\boldsymbol{c}}\left[c_a N_a\left(\tau_\delta\right)\right]}{\mathbb{E}_{\boldsymbol{\mu}\times\boldsymbol{c}}\left[c_a C\left(\tau_\delta\right)\right]}} d\left(\mu_a, \lambda_a\right)\right)$$

$$\leq \mathbb{E}_{\boldsymbol{\mu}}\left[C\left(\tau_\delta\right)\right] \boxed{\sup_{w \in \Sigma_K}} \inf_{\lambda \in \mathrm{Alt}(\boldsymbol{\mu})}\left(\sum_{a=1}^{K} \frac{w_a}{c_a} d\left(\mu_a, \lambda_a\right)\right),$$

Key insight 1: Can compute $\boxed{w^*(\boldsymbol{\mu})}$ for given $\boldsymbol{\mu}$

Key insight 2: Proportion should match $w^*(\boldsymbol{\mu})$

Sampling Rule: $\pi(t) \in \arg\max_i \left|C(t) w^*(\widehat{\boldsymbol{\mu}}(t)) - \widehat{c}_i(t) N_i(t)\right|$

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Optimal Cost Aware Best Arm Identification

$$\pi(t) \in \arg\max_i \left| C(t) w^*(\widehat{\boldsymbol{\mu}}(t)) - \widehat{c}_i(t) N_i(t) \right|$$

+

$$\tau_\delta = \inf \left\{ t \in \mathbb{N} : Z(t) := \max_{a \in \mathcal{A}} \min_{b \in \mathcal{A} \setminus \{a\}} Z_{a,b}(t) > \beta(t, \delta) \right\}$$

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$+$

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Asymptotic Optimality in Expectation.

For suitably chosen $\beta(t,\delta)$ we have

$$\limsup_{\delta\to 0} \frac{\mathbb{E}\left[C(\tau_\delta)\right]}{\log(1/\delta)} \leq T^*(\boldsymbol{\mu})$$

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$$\tau_\delta = \inf \left\{ t \in \mathbb{N} : Z(t) := \max_{a \in \mathcal{A}} \min_{b \in \mathcal{A} \setminus \{a\}} Z_{a,b}(t) > \beta(t, \delta) \right\}$$

Algorithm	Optimal?	$w_1(t)$ $(1.5, 1)$	$w_2(t)$ $(1, 0.1)$	$w_3(t)$ $(0.5, 0.01)$
TAS	$\times$	0.46	0.46	0.08
CTAS	✓	0.23	0.72	0.05

Optimal proportions of Cost Aware vs. non Cost Aware algorithm.

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A heuristic: $\sqrt{c_i}$

- Solving lower bound optimization - HARD
- Computing confidence regions - EASY

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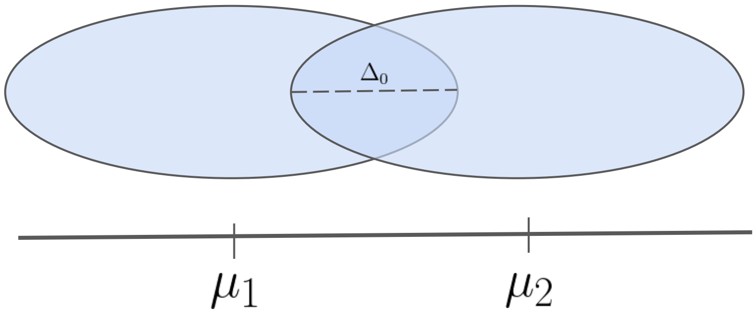

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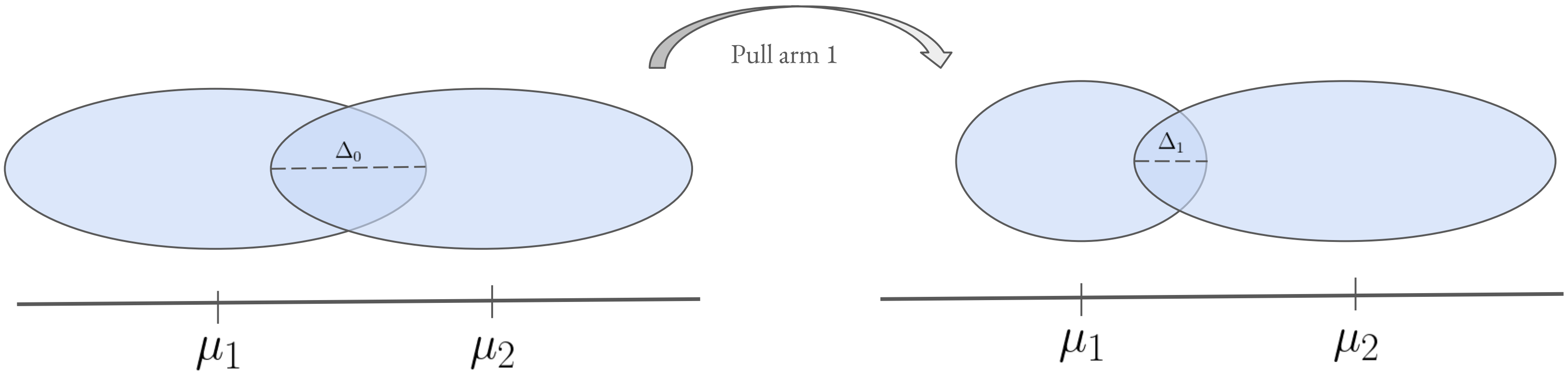

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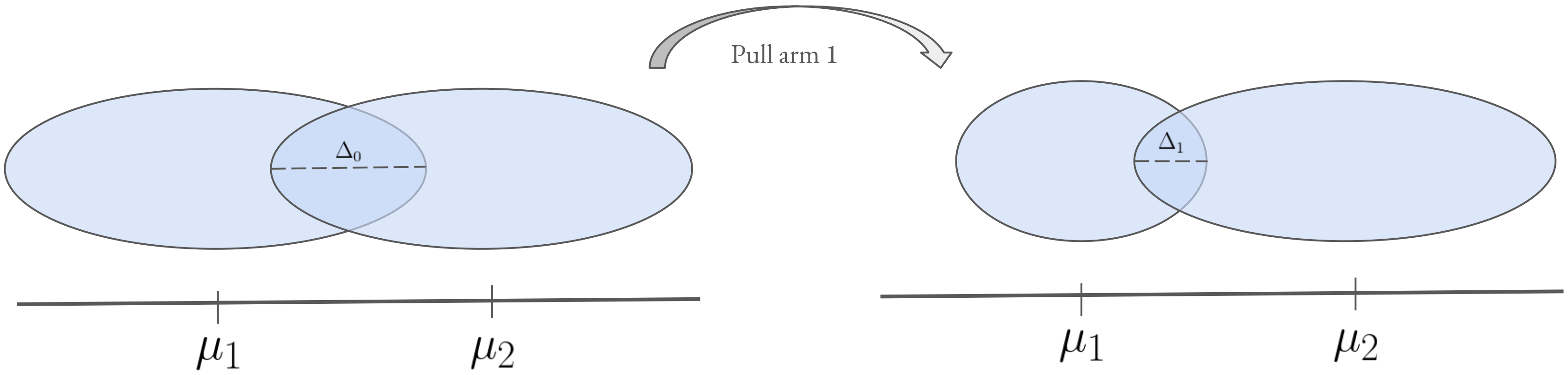
A heuristic: $\sqrt{c_i}$

- **Idea**: Most reduced overlap for the cost $\dfrac{\Delta_1 - \Delta_0}{c_1}$



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A heuristic: $\sqrt{c_i}$

- **Idea**: Most reduced overlap for the cost $\dfrac{\Delta_1 - \Delta_0}{c_1}$

Sampling Rule:

$$a_t \in \arg\max_i \frac{\Delta_{t+1}^{\hat{a},a_i} - \Delta_t^{\hat{a},a_i}}{c_i}$$

Elimination Rule:

Eliminate i at t if $\Delta_t^{\hat{a},a_i} = 0$

Stopping Rule:

Stop at t if $\max_i \Delta_t^{\hat{a},a_i} = 0$

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A heuristic: $\sqrt{c_i}$

- **Observation**: sampling rule can be approximated by $a_t \in \arg\max_i \sqrt{c_i} N_i(t)$

Theoretical Result. For any 2-armed Gaussian bandit model with rewards $\{\mu_1, \mu_2\}$ and costs $\{c_1, c_2\}$, we have with probability 1:

$$\limsup_{\delta \to 0} \frac{\sum_a \mathbb{E}\left[c_a N_a(\tau_\delta)\right]}{\log(1/\delta)} \leq \frac{2\alpha\left(\sqrt{c_1} + \sqrt{c_2}\right)^2}{(\mu_1 - \mu_2)^2}$$

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A heuristic: $\sqrt{c_i}$

- **Observation**: sampling rule can be approximated by $a_t \in \arg\max_i \sqrt{c_i} N_i(t)$

Much cheaper! ⟹

	CO	CTAS	TAS	d-LUCB
Gaussian	85	1712	2410	82
Bernoulli	58	1995	2780	60
Poisson	96	3260	4633	101

Process time (sec) over 1000 trajectories.

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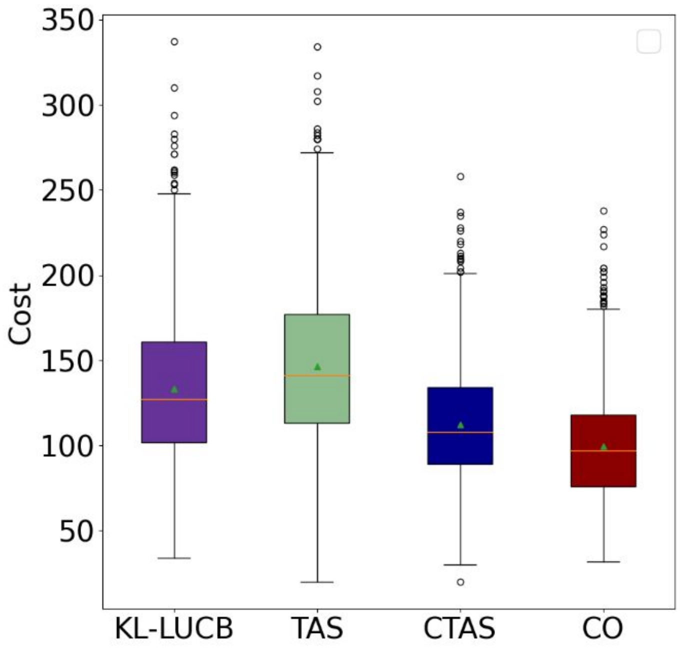
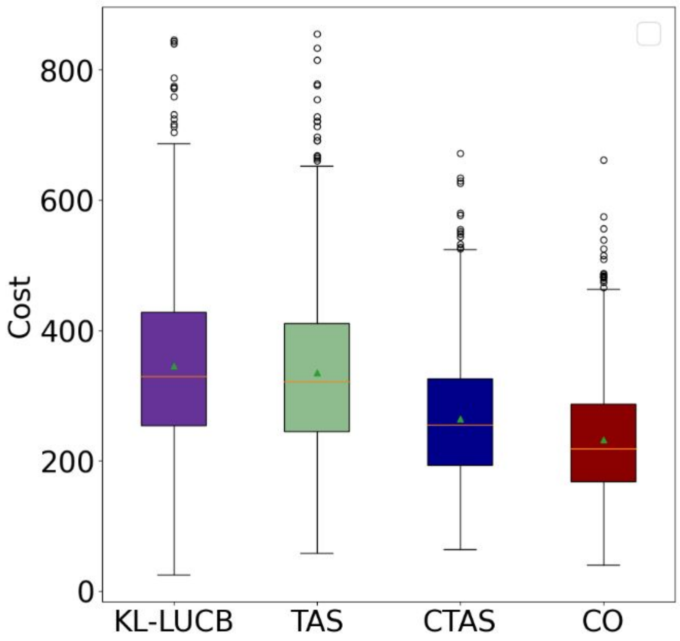
Results



350
300
250
200
150
100
50
Cost
KL-LUCB
TAS
CTAS
CO



800
600
400
200
0
Cost
KL-LUCB
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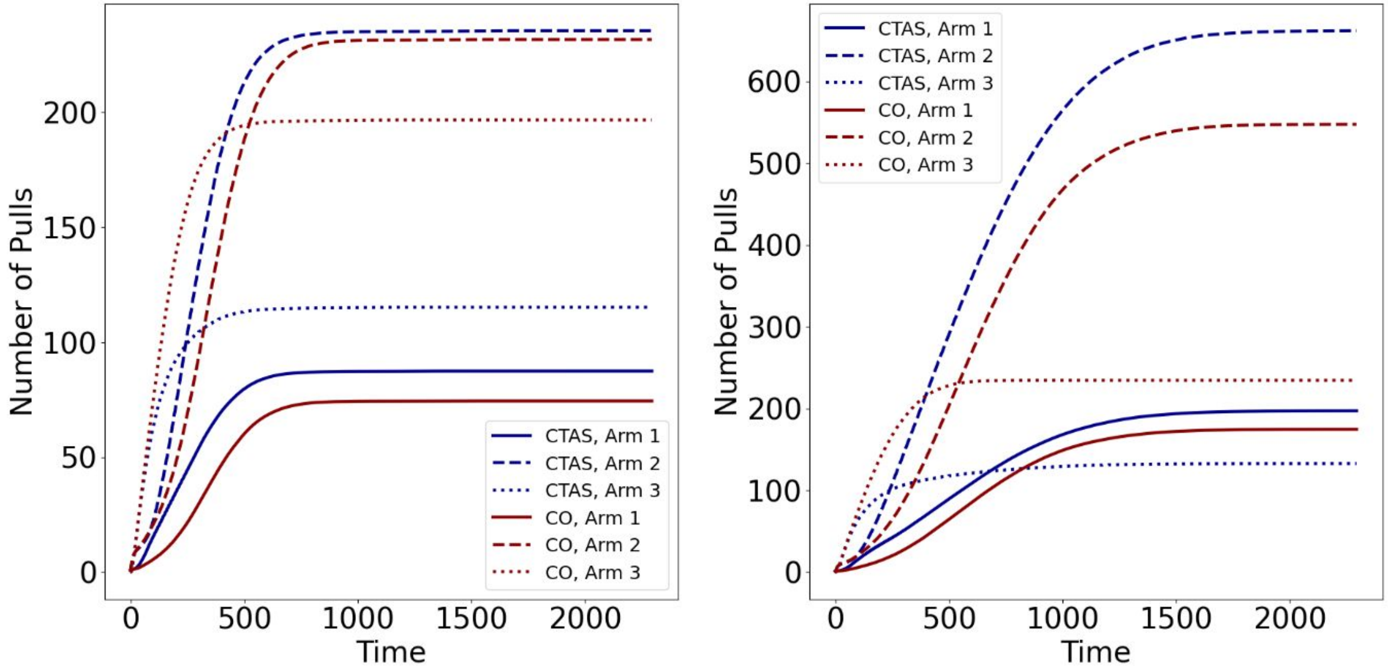
CTAS
CO

Results



Number of Pulls
Time
0
50
100
150
200
0
500
1000
1500
2000
CTAS, Arm 1
CTAS, Arm 2
CTAS, Arm 3
CO, Arm 1
CO, Arm 2
CO, Arm 3
Number of Pulls
Time
0
100
200
300
400
500
600
0
500
1000
1500
2000
CTAS, Arm 1
CTAS, Arm 2
CTAS, Arm 3
CO, Arm 1
CO, Arm 2
CO, Arm 3

Thanks!