DSE 210

Homework 2

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1 Random Variable, Expectation, and Variance

1.1 Worksheet 4

1. Let $X = min(X_1, X_2)$ where X_1 and X_2 are the outcomes of respective dice rolls. $\Omega = \{1, 2, 3, 4, 5, 6\}$. Beginning with $6 = min(X_1, X_2)$, we know that there is one possibility, $\{6, 6\}$.
Hence, we can define the following distribution of X, decreasing from 6:

Distribution of X						
$min(X_1, X_2)$	1	2	3	4	5	6
$Pr(X = x)$	$\frac{11}{36}$	$\frac{1}{4}$	$\frac{7}{36}$	$\frac{5}{36}$	$\frac{1}{12}$	$\frac{1}{36}$

2. 6. Since, $6 \times \frac{1}{6} = 1$.

4. (a) $\binom{n}{1} * \frac{1}{10} * \frac{9}{10}^{n-1}$ as there are $\binom{n}{1}$ ways of selecting an individual to get off at a particular floor with equal probability $\frac{1}{10}$, with the remaining $n - 1$ individuals selecting from $\frac{9}{10}$ floors.

   (b) $10 * \binom{n}{1} * \frac{1}{10} * \frac{9}{10}^{n-1}$. Since each floor has an equal probability, it is like a coin flip, hence it is $n * p$, for which $n = 10$.

6. Let X represent the number of people who end up in their own bed. Hence, $X = x_1 + x_2 + \cdots + x_n$ where

$$x_i = \begin{cases} 1, & \text{if person ends in the 1st bed} \\ 0, & \text{otherwise.} \end{cases} \tag{1.0.1}$$

Now, we have $Pr(x_i = 1) = \frac{1}{n}$, which, by the linearity of expectation yields $E(x_i) = \frac{1}{n} * n = 1$. The expected number of students who end up in their own bed is 1.

7. (a) $Pr(X) = \frac{1}{n}$ and $Pr(Y) = \frac{1}{n}$. If $X = Y$, then $Pr(X \cap Y) = 0$ because $X \neq Y$ by permutation principles. Hence, X and Y are dependent.

   (c) $Pr(X \cap Y) = \frac{1}{52}$, since there is only one card that is both a 9 and a heart. If the result is independent, then:
   $Pr(X \cap Y) = Pr(X) * Pr(Y)$ Computing the probabilities we have:
   $Pr(X) = \frac{1}{13}$, $Pr(Y) = \frac{1}{4}$. Lastly,
   $Pr(X \cap Y) = \frac{1}{13} * \frac{1}{4} = \frac{1}{52}$. X and Y are independent.

8. (a) $E(Z) = 1 \times \frac{1}{8} + 2 \times \frac{1}{8} + 3 \times \frac{1}{8} + 4 \times \frac{1}{8} + 5 \times \frac{1}{4} + 6 \times \frac{1}{4}$, or
   $E(Z) = \frac{10}{8} + \frac{10}{8} + \frac{12}{8} = 4$. $E(Z) = 4$.
   $Var(Z) = E(Z^2) - \mu^2$. Computing it out we have:
   $Var(Z) = 1 \times \frac{1}{8} + 4 \times \frac{1}{8} + 9 \times \frac{1}{8} + 16 \times \frac{1}{8} + 25 \times \frac{1}{4} + 36 \times \frac{1}{4} - 4^2$
   $Var(Z) = 19 - 16 = 3$. $Var(Z) = 3$.

(b) By expected value and variance rules, I expect $E(X) = 40$ and $Var(X) = 30$. Written out we see:
$E(X) = E(X_1) + E(X_2) + \cdots + E(X_{10})$. Or, $E(X) = 10 * 4$. For variance of X, we have:
$Var(X) = Var(X_1) + Var(X_2) + \cdots + Var(X_{10})$. Or, $Var(X) = 10 * 3$.

(c) Because we are taking the average of all the rolls, A, we know that it is the sum of the rolls, divided by the number of times the dice is rolled, n. $E(A) = 4$ because it is the definition of expectation. For variance, we know by the rules of variance that $var(aX+b) = a^2 var(X)$. Leveraging this property, we can conclude $Var(A) = \frac{3}{n}$.

12. There is a way to solve this problem using an infinite sum, considering that there is always a possibility of starting over (next coin mismatches the previous). However, by indicating that there is an initial dead toss, toss +1, then there is an expected number of tosses 2 to generate that particular outcome. Hence, the expected number of tosses to yield a consecutive outcome is $1 + 2 = 3$.

2 Classification with generative models 1

2.1 Worksheet 5

1. (a) As the probability of talking a little, $Pr(\text{talks a little}) = \frac{1}{6}$, is the same for both happy and sad classes, we can reason that his most likely mood is happy, since it has a higher fraction of the data with that label.

   (b) $\frac{1}{4}$, since it is the probability of not being happy (sad).

2.

$$h^*(X, Y) = \begin{cases} Y = 1, & \text{for } X \in [-1, 0) \\ Y = 3, & \text{for} X \in [0, 1]. \end{cases} \tag{2.0.1}$$