DSE 210

Homework 1

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1 Sets and Counting

1.1 Worksheet 1

1. (a) A $= \{a, b, c, d, e\}$; $|A| = 5$
   (b) $A^3 = A \times A \times A$
   (c) $5^3 = 125$
2. 2^{500}
3. (a) 7, $|A| + |B|$
   (b) 4, $A \subset B$
   (c) Largest $= 3$, $A \subset B$; Smallest $= \emptyset$, no intersection
4. $4! = 4 * 3 * 2 = 24$, order 4 separate animals.
5. $26^5 = 11,881,376$, alphabet sequence length 5.
6. Pick 3 items out of 10 possibilities: $\binom{10}{3} = \frac{10*9*8}{3*2} = 120$
7. Pick 5 items out of 10 possibilites, order 5 separate ways. $\binom{10}{5} * 5! = \frac{10!}{5!*5!} = 10 * 9 * 8 * 7 * 6 = 30,240$

2 Probability Spaces

2.1 Worksheet 2

2. (a) $\Omega = \{H, T\}^{200}$; $|\Omega| = 2^{200}$
3. (a) $A \cap B \cap C$
   (b) $A \cup B \cup C$
   (c) $A \cap B \cap C^c$, where C^c is the complement of C
4. (a) $Pr(c) = 1 - (Pr(a) + Pr(b)) = \frac{1}{6}$
   (b) 8; $\big\{\{\}, \{a\}, \{b\}, \{c\}, \{ab\}, \{ac\}, \{bc\}, \{abc\}\big\}$
   (c) $Pr(\{\}) = 0$, $Pr(\{a\}) = \frac{1}{2}$, $Pr(\{b\}) = \frac{1}{3}$, $Pr(\{c\}) = \frac{1}{6}$, $Pr(\{ab\}) = \frac{5}{6}$, $Pr(\{ac\}) = \frac{2}{3}$, $Pr(\{bc\}) = \frac{1}{2}$, $Pr(\{abc\}) = 1$
5. (a) $E_1 =$ "The first shot is heads." $Pr(E_1) = \frac{4}{8} = \frac{1}{2}$
   (b) $E_2 =$ "All three tosses are the same result." $Pr(E_2) = \frac{2}{8} = \frac{1}{4}$
   (c) $E_3 =$ "There is exactly one tails." $Pr(E_3) = \frac{3}{8}$

6. $Pr(A \cup B) = Pr(A) + Pr(B) - Pr(A \cap B)$
   $Pr(A^c) = 1 - Pr(A)$
   $\frac{1}{3} = 1 - Pr(A)$; $Pr(A) = \frac{2}{3}$. Substitute values:
   $Pr(A \cup B) = \frac{2}{3} + \frac{1}{2} - \frac{1}{4} = \frac{11}{12}$

7. $|\Omega| = 6^2$, $\Omega = \{1, 2, 3, 4, 5, 6\}$; $6x \times \frac{1}{6^2} = \frac{6}{36} = \frac{1}{6}$

9. $100 = x + 2x + 3x + 4x + 5x + 6x$
   $100 = 21x$; $Pr(1) = 1/21)$. Hence, $Pr(even) = Pr(2, 4, 6) = \frac{2}{21} + \frac{4}{21} + \frac{6}{21}$
   $= \frac{12}{21}$

11. Only one possibility. Because the five people have different heights, the formula for the individuals being arranged in increasing order of height is $1 \times \frac{1}{5!} = \frac{1}{120}$

14. $Pr(all\,apples\,good) = \frac{\frac{90!}{80!}}{\frac{100!}{90!}} \approx 0.33$

3 Multiple events, conditioning, and independence

3.1 Worksheet 3

1. (a) Looking for $Pr(\{HT\}, \{TH\})$. Hence, $Pr(2H|H) = \frac{1}{2}$
   (b) Looking for $Pr(\{HH\})$. Hence, $Pr(2H|T) = \frac{1}{4}$
   (c) Looking for $Pr(\{T\})$. Hence, $Pr(2H|2H) = \frac{1}{2}$

5. (a) $Pr(Heart|Red) = \frac{1}{2}$. Even possibility with a Diamond.
   (b) $Pr(> 10|Heart) = \frac{4}{13}$. 13 Heart cards, 4 higher than 10.
   (c) $Pr(Jack| > 10) = \frac{1}{4}$. 4 suits of cards, each with equal likelihood of being > 10.

7. (a) $Pr(> 7|4) = Pr(\{4, 5, 6\}) = \frac{1}{2}$
   (c) $Pr(> 7|3) = \frac{Pr(>7 \cap >3)}{Pr(>3)}$
   12 possibilities on the intersection (counted them up), hence: $\frac{\frac{12}{36}}{\frac{1}{2}} = \frac{2}{3}$

9. (a) $Pr(D) = (0.05) * (0.25) + (0.04) * (0.35) + (0.02) * (0.4) = 0.0345$
   (b) $Pr(F_1|D) = \frac{Pr(D|F_1) * Pr(F_1)}{Pr(D)}$
   $= \frac{(0.05)*(0.25)}{.0345} = 0.3623$

10. $Pr(M|C) = \frac{Pr(C|M) * Pr(M)}{Pr(C)}$,
   $Pr(C) = (0.5) * (0.05) + (0.5) * (0.1) = 0.03$, inserting into original equation:
   $\frac{(0.05)*(0.5)}{0.03} = \frac{5}{6}$

12. $Pr(Trick|6H) = \frac{Pr(6H|Trick)*Pr(Trick)}{Pr(6H)}$
$= \frac{Pr(6H|Trick)*Pr(Trick)}{Pr(6H|Trick)*Pr(Trick) + Pr(6H|Fair)*Pr(Fair)}$
$= \frac{1*\frac{1}{65}}{(1*\frac{1}{65})+(\frac{1}{64}*\frac{64}{65}}$
$= \frac{\frac{1}{65}}{\frac{2}{65}} = \frac{1}{2}$

14. $Pr(B|S) = \frac{Pr(S|B)*Pr(B)}{Pr(S)}$,
$Pr(S) = (0.75)*(0.1) + (0.25)*(0.6) = 0.225$, inserting into original equation:
$\frac{(0.6)*(0.25)}{0.225} = \frac{2}{3}$

15. (a) Event pairs (1) and (2). Work for (2):
$Pr(A \cup D) = Pr(A) * Pr(D)$ for independence:
$Pr(A) = \frac{1}{2}$, $Pr(D) = \frac{2}{8} = \frac{1}{4}$
$Pr(A \cup D) = \frac{1}{2} * \frac{1}{4} + 0$ as the first roll must be a H, and thus the next 2 rolls must also be H.
Hence, $\frac{1}{2} * \frac{1}{4} = \frac{1}{2} * \frac{1}{4} = \frac{1}{8}$. Events A and D are independent.

16. Event paris (2) and (4). Work for (4):
$Pr(A \cup B) = Pr(B) * Pr(B)$ for independence:
$Pr(A) = \frac{1}{4}$, $Pr(B) = \frac{1}{13}$
$Pr(A \cup B) = \frac{1}{4} * \frac{4}{51} + \frac{1}{52} * \frac{3}{51}$. In other words, the probability of the intersection is equal to the probability of the first card being a Heart and the second one a 10 *plus* the first card being both a Heart and a 10.
Hence, $\frac{1}{4} * \frac{4}{51} + \frac{1}{52} * \frac{3}{51} = \frac{1}{4} * \frac{1}{13} = \frac{1}{52}$. Events A and B are independent.