Day 5 Homework

March 11, 2019

1 Worksheet 10: PCA & SVD

Problem 5:
Begin by leveraging previous work:

```
In [6]: # Imported Code from Professor:
        from struct import unpack
        import numpy as np
        import pandas as pd
        import matplotlib.pylab as plt

        def loadmnist(imagefile, labelfile):

            # Open the images with gzip in read binary mode
            images = open(imagefile, 'rb')
            labels = open(labelfile, 'rb')

            # Get metadata for images
            images.read(4)  # skip the magic_number
            number_of_images = images.read(4)
            number_of_images = unpack('>I', number_of_images)[0]
            rows = images.read(4)
            rows = unpack('>I', rows)[0]
            cols = images.read(4)
            cols = unpack('>I', cols)[0]

            # Get metadata for labels
            labels.read(4)
            N = labels.read(4)
            N = unpack('>I', N)[0]

            # Get data
            x = np.zeros((N, rows*cols), dtype=np.uint8)  # Initialize numpy array
            y = np.zeros(N, dtype=np.uint8)  # Initialize numpy array
            for i in range(N):
                for j in range(rows*cols):
                    tmp_pixel = images.read(1)  # Just a single byte
```

```
                tmp_pixel = unpack('>B', tmp_pixel)[0]
                x[i][j] = tmp_pixel
            tmp_label = labels.read(1)
            y[i] = unpack('>B', tmp_label)[0]

        images.close()
        labels.close()
        return (x, y)

    def displaychar(image):
        plt.imshow(np.reshape(image, (28,28)), cmap=plt.cm.gray)
        plt.axis('off')
        plt.show()
```

In [101]:
```
# Train
train_images = "/Users/kkannapp/Documents/DSE/DSE210-homework/day_3/train-images-idx3-
train_labels = "/Users/kkannapp/Documents/DSE/DSE210-homework/day_3/train-labels-idx1-
# Test
test_images = "/Users/kkannapp/Documents/DSE/DSE210-homework/day_3/t10k-images-idx3-ub
test_labels = "/Users/kkannapp/Documents/DSE/DSE210-homework/day_3/t10k-labels-idx1-ub

train_x,train_y = loadmnist(train_images,train_labels)
test_x,test_y = loadmnist(test_images,test_labels)
```

In [102]:
```
# Preview the data
train_x[0],train_y[0] # First entry is a 28 x 28 matrix of pixels, and is classified a
```

Out[102]:
```
(array([  0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,
          0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,
          0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,
          0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,
          0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,
          0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,
          0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,
          0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,
          0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,
          0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,
          0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,
          0,   0,   0,   0,   0,   0,   0,   0,   0,   3,  18,  18,  18,
        126, 136, 175,  26, 166, 255, 247, 127,   0,   0,   0,   0,   0,
          0,   0,   0,   0,   0,   0,   0,  30,  36,  94, 154, 170, 253,
        253, 253, 253, 253, 225, 172, 253, 242, 195,  64,   0,   0,   0,
          0,   0,   0,   0,   0,   0,   0,   0,  49, 238, 253, 253, 253,
        253, 253, 253, 253, 253, 251,  93,  82,  82,  56,  39,   0,   0,
          0,   0,   0,   0,   0,   0,   0,   0,   0,   0,  18, 219, 253,
        253, 253, 253, 253, 198, 182, 247, 241,   0,   0,   0,   0,   0,
          0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,
         80, 156, 107, 253, 253, 205,  11,   0,  43, 154,   0,   0,   0,
```

```
  0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,
  0,   0,   0,  14,   1, 154, 253,  90,   0,   0,   0,   0,   0,
  0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,
  0,   0,   0,   0,   0,   0,   0, 139, 253, 190,   2,   0,   0,
  0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,
  0,   0,   0,   0,   0,   0,   0,   0,   0,  11, 190, 253,  70,
  0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,
  0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,  35,
241, 225, 160, 108,   1,   0,   0,   0,   0,   0,   0,   0,   0,
  0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,
  0,   0,  81, 240, 253, 253, 119,  25,   0,   0,   0,   0,   0,
  0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,
  0,   0,   0,   0,   0,  45, 186, 253, 253, 150,  27,   0,   0,
  0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,
  0,   0,   0,   0,   0,   0,   0,   0,  16,  93, 252, 253, 187,
  0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,
  0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0, 249,
253, 249,  64,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,
  0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,  46, 130,
183, 253, 253, 207,   2,   0,   0,   0,   0,   0,   0,   0,   0,
  0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,  39, 148,
229, 253, 253, 253, 250, 182,   0,   0,   0,   0,   0,   0,   0,
  0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,  24, 114,
221, 253, 253, 253, 253, 201,  78,   0,   0,   0,   0,   0,   0,
  0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,  23,  66,
213, 253, 253, 253, 253, 198,  81,   2,   0,   0,   0,   0,   0,
  0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,  18, 171,
219, 253, 253, 253, 253, 195,  80,   9,   0,   0,   0,   0,   0,
  0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,  55, 172,
226, 253, 253, 253, 253, 244, 133,  11,   0,   0,   0,   0,   0,
  0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,
136, 253, 253, 253, 212, 135, 132,  16,   0,   0,   0,   0,   0,
  0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,
  0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,
  0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,
  0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,
  0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,
  0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,
  0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,   0,
  0,   0,   0,   0], dtype=uint8), 5)
```

In [103]:
```
# Part a)
# Leverage covariance matrix to gather eigenvalues & eigenvectors
cov=np.cov(train_x.T)
lamda, evectors = np.linalg.eig(cov)
lamda = np.float64(lamda)
evectors = np.float64(evectors)
total_var = sum(lamda)
```

```
/Users/kkannapp/anaconda/lib/python3.6/site-packages/ipykernel_launcher.py:5: ComplexWarning: Ca
  """
/Users/kkannapp/anaconda/lib/python3.6/site-packages/ipykernel_launcher.py:6: ComplexWarning: Ca
```

```
In [104]: # Provides the sum of lost variance
          sum(lamda[25::])
```

```
Out[104]: 1056647.6637139525
```

```
In [105]: # Let us compute a fraction of lost variance, for each value of k defined by F(k):
          def F(k):
              lost_variance = sum(lamda[k+1::])
              return lost_variance/total_var
```

```
In [106]: k_var = [(i, F(i)) for i in [200, 150, 100, 50, 25]]

          k_lost_var = pd.DataFrame(data = k_var, columns = ['k value', 'Variance Lost'])
          k_lost_var
```

Out[106]:

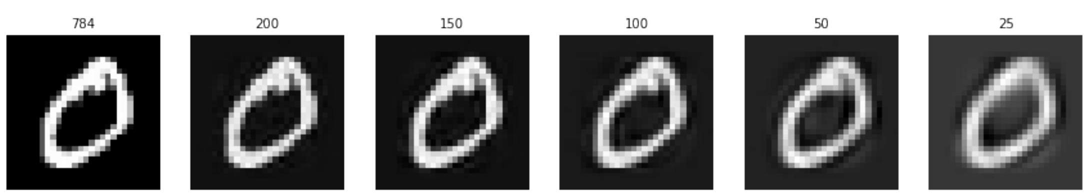
	k value	Variance Lost
0	200	0.033271
1	150	0.051177
2	100	0.084387
3	50	0.172163
4	25	0.299802

```
In [144]: # Part b)
          # Apply PCA formula to re-generate images, calculate class probabilities

          class PCA_fx(object):

              def __init__(self,train_x,train_y):
                  self.train_x = train_x
                  self.train_y = train_y
                  self.dict={}
                  self.dict = {num:{} for num in range(10)}

              def F(self, k):
                  lost_variance = sum(self.lamda[k+1::])
                  return lost_variance/self.total_var

              def reshape_proj(self, k, image, image_id):
                  U_matrix = self.evectors[:,:k+1]
                  transform_U = np.dot(U_matrix,U_matrix.T)
                  image = image.T
                  X = np.dot(transform_U,image).T[image_id]
                  X = X.reshape(28,28)
```

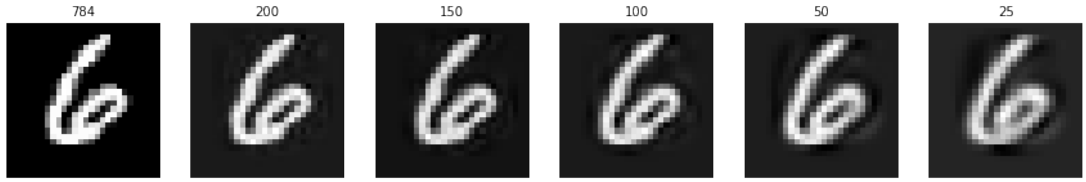
```
        return X

    def show_digit(self,X,k):
        plt.imshow(X, cmap=plt.cm.gray)
        plt.title('%i' % k, fontsize = 10)
        plt.axis('off')

    def computation(self):
        for num in range(10):
            x_train = []
            y_train = []
            for i, image in enumerate(self.train_x):
                if train_y[i] == num:
                    x_train.append(image)
                    y_train.append(num)
            x_train = np.array(x_train)
            cov = np.cov(x_train.T)
            lamda, evectors = np.linalg.eig(cov)
            self.lamda = np.float64(lamda)
            self.evectors = np.float64(evectors)
            self.total_var = np.sum(self.lamda)
            index = 0
            plt.figure(figsize=(15,5))
            for k in [784, 200, 150, 100, 50, 25]:
                self.dict[num][k]=self.F(k)
                image_reduced = self.reshape_proj(k, x_train[:10], 0)
                index += 1
                plt.subplot(1, 6, index)
                self.show_digit(image_reduced, k)
            plt.show();
```

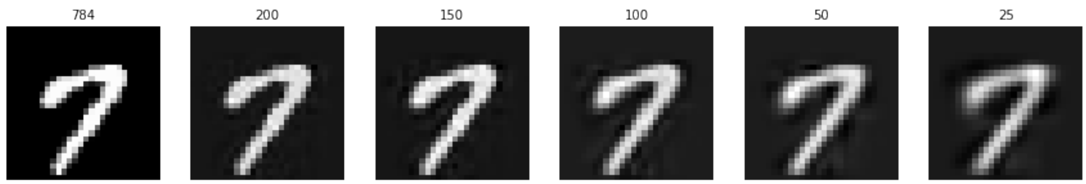
In [145]:
```
PCA_Class = PCA_fx(train_x, train_y)
PCA_Class.computation()
```

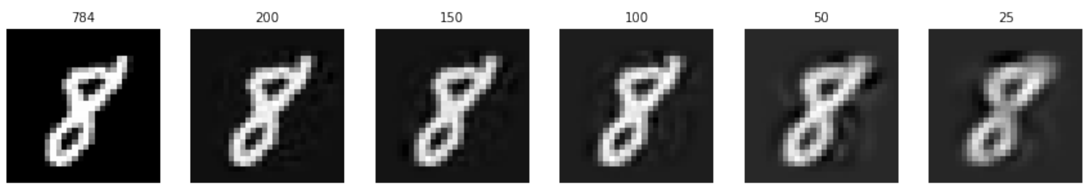
```
/Users/kkannapp/anaconda/lib/python3.6/site-packages/ipykernel_launcher.py:20: ComplexWarning: C
/Users/kkannapp/anaconda/lib/python3.6/site-packages/ipykernel_launcher.py:21: ComplexWarning: C
```

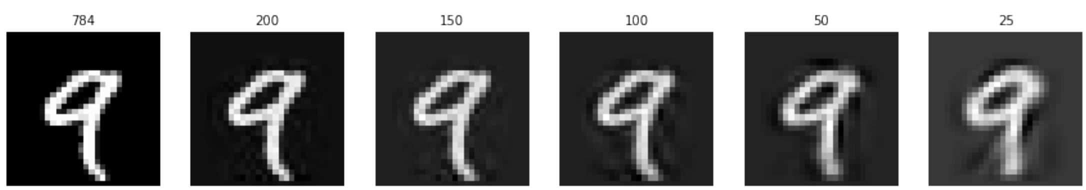



784
200
150
100
50
25
784
200
150
100
50
25
784
200
150
100
50
25
784
200
150
100
50
25
784
200
150
100
50
25









In [146]:
```
lost_var_table = pd.DataFrame(PCA_Class.dict)
lost_var_table
```

Out[146]:

	0	1	2	3	4	5	6	\
25	0.201817	0.127616	0.276346	0.268917	0.246501	0.256309	0.216404	
50	0.117534	0.071766	0.158965	0.156145	0.145469	0.147014	0.120032	
100	0.058764	0.030534	0.077597	0.075243	0.072825	0.070057	0.057224	
150	0.034165	0.014634	0.045578	0.043139	0.042213	0.039591	0.032374	
200	0.020834	0.006765	0.028082	0.026002	0.025398	0.023609	0.018940	
784	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	

	7	8	9
25	0.216041	0.291628	0.218670
50	0.126723	0.164755	0.121179
100	0.062837	0.076202	0.056898
150	0.035918	0.042682	0.030738
200	0.021194	0.024901	0.016958
784	0.000000	0.000000	0.000000

Based on this table, I would expect 1 to be the digit most amenable to dimensionality reduction. Intuitively, this makes the most sense.