DSE 210

Homework 3

Kevin Kannappan

February 10, 2019

1 Generative models 2

1.1 Worksheet 6

1. (a) Uncorrelated

   (b) Positively Correlated

   (c) Negatively Correlated

3. (a) Unique Bivariate Gaussian, parameterized bymean: $\mu = \binom{2}{2}$ and Covariance Matrix:

   $$Cov(x, y) = \begin{bmatrix} 1 & -0.25 \\ -0.25 & 0.25 \end{bmatrix}$$

   Because each standard deviation squared provides the variance, we are able to get the diagonals of the matrix. Calculating the covariance comes from the correlation formula: $corr(x, y) = \frac{cov(x,y)}{std(x)*std(y)}$.

   (b) Unique Bivariate Gaussian, parameterized bymean: $\mu = \binom{1}{1}$ and Covariance Matrix:

   $$Cov(x, y) = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

   Because each standard deviation squared provides the variance (1:1), we are able to get the diagonals of the matrix. Since $y = x$, intuitively we know that their covariance must equal 1.

Please see attached Jupyter notebook for questions 4 and 5.

2 Linear Algebra Primer

2.1 Worksheet 7

1. $||x|| = \sqrt{1^2 + 2^2 + 3^2} = \sqrt{14}$. Hence the unit vector in the same direction as x is: $\vec{x_u} = (\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}})$.

2. Dot product must equal 0 for orthogonality, hence need to see a relation like: $\binom{1}{1} \cdot \binom{-1}{1} = -1 + 1 = 0$. Taking the unit vectors, we get: $(\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$ and $(\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}})$.

3. $x \cdot x = 25$, hence $||x||^2 = x \cdot x$ and thus 5 is the magnitude. In 2 dimensions, we know that this is a circle with radius 5 and in 3 dimensions this would be a sphere with radius 5. Because we have d dimensions, this would be classified as a hypersphere with radius 5.

4.

$$w = \begin{bmatrix} 2 \\ -1 \\ 6 \end{bmatrix} \tag{2.0.1}$$

5. Following matrix multiplication properties, we know $A = 10 \times 30$ and $B = 30 \times 20$.

6. (a) $n \times d$.
   (b) $n \times n$.
   (c) $x^i \cdot x^j$.

8. $x^T x = ||x||^2$. Hence, we have $\sqrt{1^2 + 3^2 + 5^2}^2 = 35$. xx^T is a matrix multiplied by its transpose:

$$xx^T = \begin{bmatrix} 1 & 3 & 5 \\ 3 & 9 & 15 \\ 5 & 15 & 25 \end{bmatrix}$$

10. Writing a symmetric matrix, we have:

$$M = \begin{bmatrix} 3 & 1 & -2 \\ 1 & 0 & 0 \\ -2 & 0 & 6 \end{bmatrix}$$

11. a) and c). a) because of multiplicative properties and c) because of additive properties. Not d) because of negative values within the matrix.

13. (a) UU^T yields the $d \times d$ identity matrix, I^d.
    (b) Following part a), we know from matrix properties that any matrix M, multiplied by its inverse M^{-1}, yields the identity matrix. Hence, we can conclude that $U^{-1} = U^T$.

14. $z = 6$, since the discriminant must $= 0$. Leaves us with $z - 6 = 0$ and 6.

3 Classification with Generative Models 3

3.1 Worksheet 8

Please see attached Jupyter notebook for digit classification.

Day 3 Homework

February 11, 2019

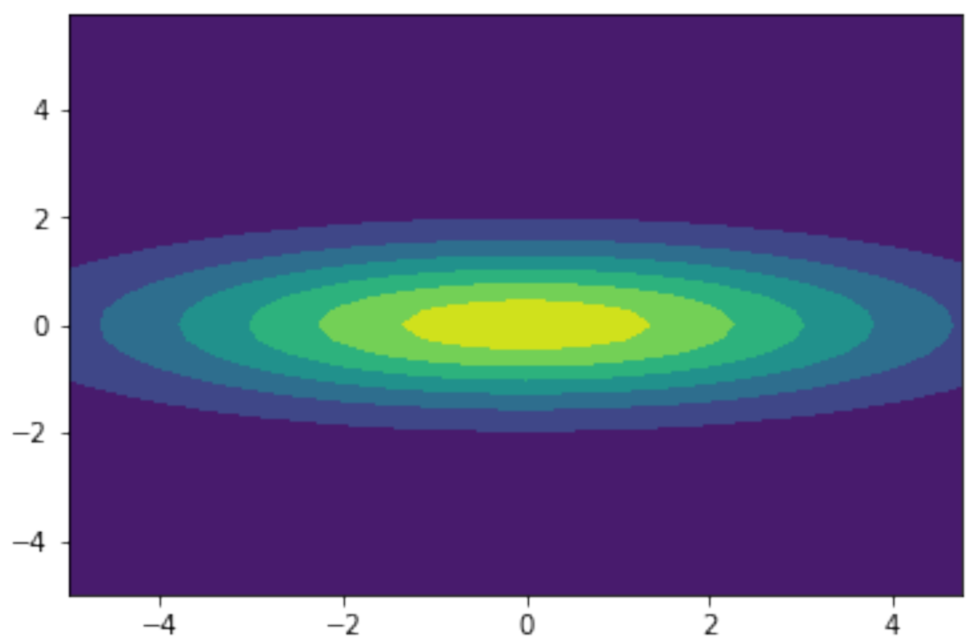
0.1 Worksheet 6

Problem 4:
Problem 5:

```
In [2]: import numpy as np
        import matplotlib.pyplot as plt
        from scipy.stats import multivariate_normal
```

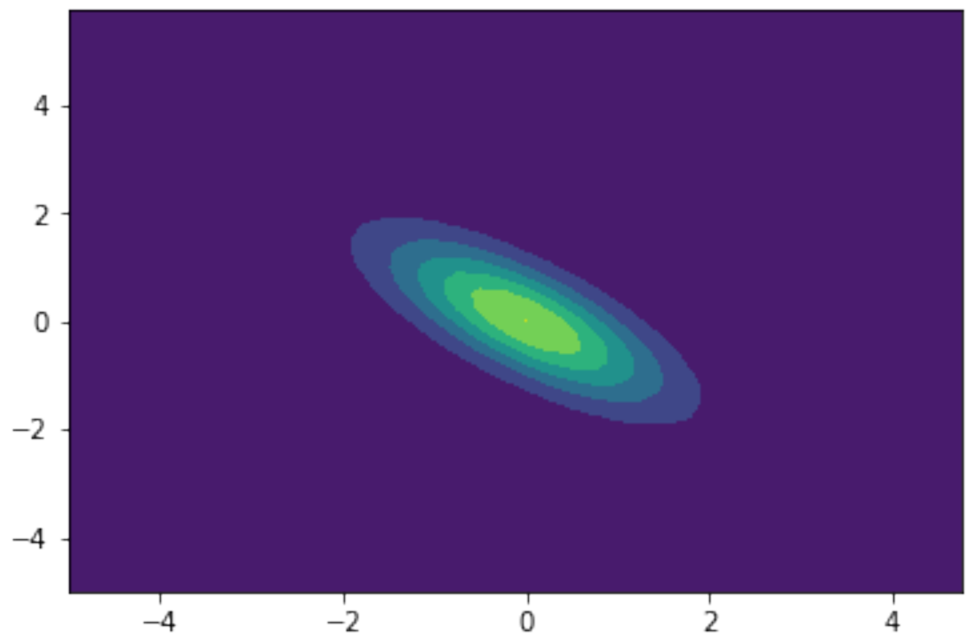
```
In [3]: # Problem a)
        %matplotlib inline

        x, y = np.mgrid[-5:5:.25, -5:6:.25]
        pos = np.empty(x.shape + (2,))
        pos[:, :, 0] = x; pos[:, :, 1] = y
        rv = multivariate_normal([0, 0], [[9.0, 0.0], [0.0, 1.0]])
        plt.contourf(x, y, rv.pdf(pos))
```

```
Out[3]: <matplotlib.contour.QuadContourSet at 0x119f911d0>
```



In [4]:
```
# Problem b)
x, y = np.mgrid[-5:5:.25, -5:6:.25]
pos = np.empty(x.shape + (2,))
pos[:, :, 0] = x; pos[:, :, 1] = y
rv = multivariate_normal([0, 0], [[1.0, -0.75], [-0.75, 1.0]])
plt.contourf(x, y, rv.pdf(pos))
```

Out[4]:
```
<matplotlib.contour.QuadContourSet at 0x11a0010f0>
```



0.2 Worksheet 8

Problem 4:

In [5]:
```
# Imported Code from Professor:
from struct import unpack
import numpy as np
import matplotlib.pylab as plt

def loadmnist(imagefile, labelfile):

    # Open the images with gzip in read binary mode
    images = open(imagefile, 'rb')
    labels = open(labelfile, 'rb')
```

```
    # Get metadata for images
    images.read(4)  # skip the magic_number
    number_of_images = images.read(4)
    number_of_images = unpack('>I', number_of_images)[0]
    rows = images.read(4)
    rows = unpack('>I', rows)[0]
    cols = images.read(4)
    cols = unpack('>I', cols)[0]

    # Get metadata for labels
    labels.read(4)
    N = labels.read(4)
    N = unpack('>I', N)[0]

    # Get data
    x = np.zeros((N, rows*cols), dtype=np.uint8)  # Initialize numpy array
    y = np.zeros(N, dtype=np.uint8)  # Initialize numpy array
    for i in range(N):
        for j in range(rows*cols):
            tmp_pixel = images.read(1)  # Just a single byte
            tmp_pixel = unpack('>B', tmp_pixel)[0]
            x[i][j] = tmp_pixel
        tmp_label = labels.read(1)
        y[i] = unpack('>B', tmp_label)[0]

    images.close()
    labels.close()
    return (x, y)

def displaychar(image):
    plt.imshow(np.reshape(image, (28,28)), cmap=plt.cm.gray)
    plt.axis('off')
    plt.show()
```

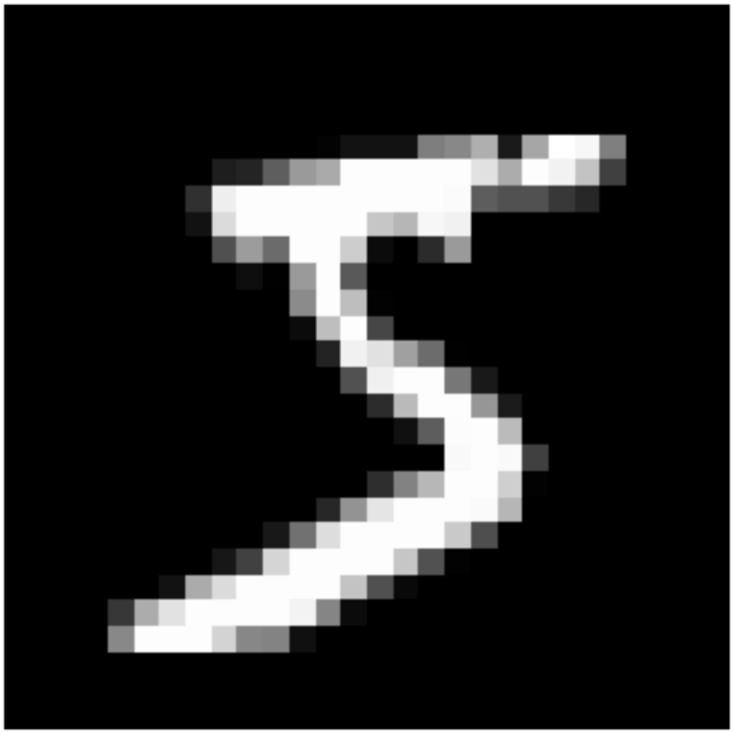
In [12]:
```
# Problem a)
# Train
train_images = "/Users/kkannapp/Documents/DSE/DSE210-homework/day_3/train-images-idx3-u
train_labels = "/Users/kkannapp/Documents/DSE/DSE210-homework/day_3/train-labels-idx1-u
# Test
test_images = "/Users/kkannapp/Documents/DSE/DSE210-homework/day_3/t10k-images-idx3-uby
test_labels = "/Users/kkannapp/Documents/DSE/DSE210-homework/day_3/t10k-labels-idx1-uby

x,y = loadmnist(train_images,train_labels)
test_x,test_y = loadmnist(test_images,test_labels)
```

In [18]:
```
# Preview the data:
print(x[0])
```

```python
plt.figure(figsize=(10,5))
plt.title('Label is: %i' % (y[0]), fontsize = 10)
displaychar(x[0])
print(x.shape)
print(y.shape)
```

```
[   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
    0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
    0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
    0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
    0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
    0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
    0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
    0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
    0   0   0   0   0   0   0   0   3  18  18  18 126 136 175  26 166 255
  247 127   0   0   0   0   0   0   0   0   0   0   0   0  30  36  94 154
  170 253 253 253 253 253 225 172 253 242 195  64   0   0   0   0   0   0
    0   0   0   0   0  49 238 253 253 253 253 253 253 253 253 251  93  82
   82  56  39   0   0   0   0   0   0   0   0   0   0   0   0  18 219 253
  253 253 253 253 198 182 247 241   0   0   0   0   0   0   0   0   0   0
    0   0   0   0   0   0   0   0  80 156 107 253 253 205  11   0  43 154
    0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
    0  14   1 154 253  90   0   0   0   0   0   0   0   0   0   0   0   0
    0   0   0   0   0   0   0   0   0   0   0   0   0 139 253 190   2   0
    0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
    0   0   0   0   0  11 190 253  70   0   0   0   0   0   0   0   0   0
    0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0  35 241
  225 160 108   1   0   0   0   0   0   0   0   0   0   0   0   0   0   0
    0   0   0   0   0   0   0   0   0  81 240 253 253 119  25   0   0   0
    0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
    0   0  45 186 253 253 150  27   0   0   0   0   0   0   0   0   0   0
    0   0   0   0   0   0   0   0   0   0   0   0   0  16  93 252 253 187
    0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
    0   0   0   0   0   0   0 249 253 249  64   0   0   0   0   0   0   0
    0   0   0   0   0   0   0   0   0   0   0   0   0   0  46 130 183 253
  253 207   2   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
    0   0   0   0  39 148 229 253 253 253 250 182   0   0   0   0   0   0
    0   0   0   0   0   0   0   0   0   0   0   0  24 114 221 253 253 253
  253 201  78   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
    0   0  23  66 213 253 253 253 253 198  81   2   0   0   0   0   0   0
    0   0   0   0   0   0   0   0   0   0  18 171 219 253 253 253 253 195
   80   9   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
   55 172 226 253 253 253 253 244 133  11   0   0   0   0   0   0   0   0
    0   0   0   0   0   0   0   0   0   0 136 253 253 253 212 135 132  16
    0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
    0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
    0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
    0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
```

```
0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
0   0   0   0   0   0   0   0   0   0]
```




```
(60000, 784)
(60000,)
```

```
In [19]: # Problem b)
         from sklearn.model_selection import train_test_split

         # Split training into train and validation
         train_x, valid_x, train_y, valid_y = train_test_split(x, y, test_size=0.16666666666, ra
         print('Training set shape',train_x.shape)
         print('Training label shape',train_y.shape)
         print('Validation set shape',valid_x.shape)
         print('Validation label shape',valid_y.shape)
```

```
Training set shape (50000, 784)
Training label shape (50000,)
Validation set shape (10000, 784)
```

```
Validation label shape (10000,)
```

```
In [20]: # Problem c)
         # Leverage a library found at http://www.eggie5.com/68-mnist-gaussian-classifier to hel
         from sklearn.naive_bayes import GaussianNB
         gnb = GaussianNB()
         gnb.fit(train_x, train_y)

Out[20]: GaussianNB(priors=None)
```

```
In [21]: # Leverage library to gather prior probabilities
         prior_prob = gnb.class_prior_
         nums = len(prior_prob)
         for i in range(nums):
             perc = (prior_prob[i])
             print('The class probability of image %s is' %i, '{0:.4f}'.format(perc))
```

```
The class probability of image 0 is 0.0994
The class probability of image 1 is 0.1118
The class probability of image 2 is 0.0996
The class probability of image 3 is 0.1020
The class probability of image 4 is 0.0972
The class probability of image 5 is 0.0901
The class probability of image 6 is 0.0981
The class probability of image 7 is 0.1042
The class probability of image 8 is 0.0978
The class probability of image 9 is 0.0997
```

```
In [22]: # Gather classes data groups, for use in generating class-wise statistics:
         classes = gnb.classes_

         def groups(class_id):
             grouping = []
             for i, group in enumerate(train_x):
                 if train_y[i] == class_id:
                     grouping.append(group)
             grouping = np.matrix(grouping)
             return grouping

         # Create covariance matrices and means, by class - use to calculate Gaussians:
         post_prob = []
         for c in classes:
             grouping = groups(c)
             mean_x = np.array(grouping.mean(0))[0]
             cov_x = np.cov(grouping.T)
             Px = multivariate_normal(mean_x, cov_x, allow_singular=True)
             post_prob.append(Px)
```

In [23]:
```
# Now fit a Gaussian with a Naive Bayes classifier:
Y = []
for i in test_x:
    bayes_prob = []
    for c in classes:
        fin_prob = [c, np.log(prior_prob[c]) + post_prob[c].logpdf(i)]
        bayes_prob.append(fin_prob)
    prediction = max(bayes_prob, key= lambda a: a[1])
    Y.append(prediction[0])
```

In [25]:
```
# Accuracy without smoothing yields 81%, pretty good fit considering there was no addit

miss = (test_y != Y).sum()
total = test_x.shape[0]
print("Accuracy: = %f" %(100*((total-miss)/total)),'%')
```

```
Accuracy: = 81.380000 %
```

In [26]:
```
# Now let us get to part d), where we add a smoothing component to the class variances
# Do not disrupt co-variances, as to just increase variance along the diagonal
# Observed output and selected relevant c's to demonstrate selection...
```

In [27]:
```
constant = [1,1000,1500,2100,2300,2400,2500,2600,2700,2800,3500,4000]
error_rates = []
for co in constant:
    post_prob = []
    for c in classes:
        grouping = groups(c)
        meanx = np.array(grouping.mean(0))[0]
        covx = np.cov(grouping, rowvar=0)
        cov_smoothed = covx + (co * np.eye(meanx.shape[0]))
        p_x = multivariate_normal(meanx, cov_smoothed, allow_singular=True)
        post_prob.append(p_x)

    Y = []
    for x in valid_x:
        bayes_prob = []
        for c in classes:
            fin_prob = [c, np.log(prior_prob[c]) + post_prob[c].logpdf(x)]
            bayes_prob.append(fin_prob)
        prediction = max(bayes_prob, key= lambda a: a[1])
        Y.append(prediction[0])

    miss = (valid_y != Y).sum()
    total = valid_x.shape[0]
    error_rate = 100*(miss/float(total))
    error_rates.append(error_rate)
    print("Error rate for c= %s: %d/%d = %f" %((co, miss, total, error_rate)),'%')
```

```
Error rate for c= 1: 1641/10000 = 16.410000 %
Error rate for c= 1000: 532/10000 = 5.320000 %
Error rate for c= 1500: 521/10000 = 5.210000 %
Error rate for c= 2100: 510/10000 = 5.100000 %
Error rate for c= 2300: 502/10000 = 5.020000 %
Error rate for c= 2400: 498/10000 = 4.980000 %
Error rate for c= 2500: 496/10000 = 4.960000 %
Error rate for c= 2600: 490/10000 = 4.900000 %
Error rate for c= 2700: 489/10000 = 4.890000 %
Error rate for c= 2800: 490/10000 = 4.900000 %
Error rate for c= 3500: 496/10000 = 4.960000 %
Error rate for c= 4000: 501/10000 = 5.010000 %
```

In [28]:
```
print("The best c value for me was c= 2700","with a error rate of:",min(error_rates), "
```

```
The best c value for me was c= 2700 with a error rate of: 4.89 %
```

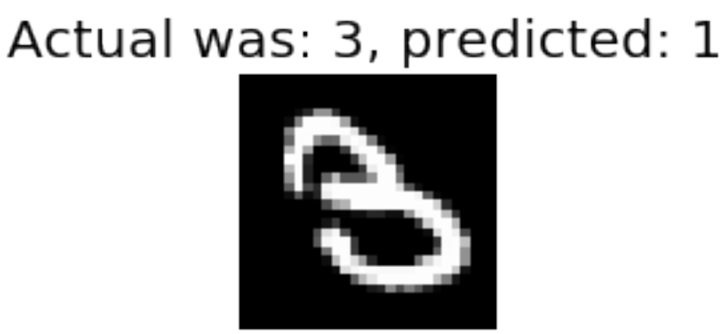
In [29]:
```
# Part e) Submit code, use value of c on test set, and pick 5 misclassified
```

In [30]:
```
# Replicate part d, replace validation set with test set
constant = 2700
post_prob = []
for c in classes:
    grouping = groups(c)
    meanx = np.array(grouping.mean(0))[0]
    covx = np.cov(grouping, rowvar=0)
    cov_smoothed = covx + (constant * np.eye(meanx.shape[0]))
    p_x = multivariate_normal(meanx, cov_smoothed, allow_singular=True)
    post_prob.append(p_x)

Y = []
for x in test_x:
    bayes_prob = []
    for c in classes:
        fin_prob = [c, np.log(prior_prob[c]) + post_prob[c].logpdf(x)]
        bayes_prob.append(fin_prob)
    prediction = max(bayes_prob, key= lambda a: a[1])
    Y.append(prediction[0])

miss = (test_y != Y).sum()
total = test_x.shape[0]
error_rate = 100*(miss/float(total))
error_rates.append(error_rate)
print("Error rate for c= %s: %d/%d = %f" %((constant, miss, total, error_rate)),'%')
```

```
Error rate for c= 2700: 437/10000 = 4.370000 %
```

In [31]:
```
index_error = np.array(np.where((test_y != Y)==True))[0]
```

In [35]:
```
incorrect = index_error[:5]
actual = test_y[incorrect]
miss = []
for x in incorrect:
    bayes_prob = []
    for c in classes:
        fin_prob = [c, np.log(prior_prob[c]) + post_prob[c].logpdf(x)]
        bayes_prob.append(fin_prob)
    prediction = max(bayes_prob, key= lambda a: a[1])
    miss.append(prediction[0])

count = 0
for i in range(len(incorrect)):
    bayes_prob = []
    for c in classes:
        fin_prob = [c, np.log(prior_prob[c]) + post_prob[c].logpdf(test_x[incorrect[i]]
        bayes_prob.append(fin_prob)
    count += 1
    plt.figure(figsize=(10,5))
    plt.subplot(1, 5, count)
    plt.title('Actual was: %i, predicted: %i' % (actual[i], miss[i]), fontsize = 20)
    displaychar(test_x[incorrect[i]])
    print("Probabality of each number: ",bayes_prob)
```

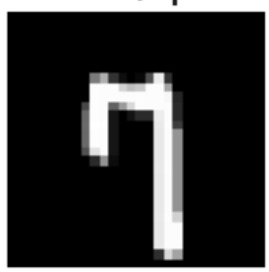



```
Probabality of each number:  [[0, -4215.2891664339231], [1, -4629.7571621129564], [2, -4105.2901
```

Actual was: 9, predicted: 5

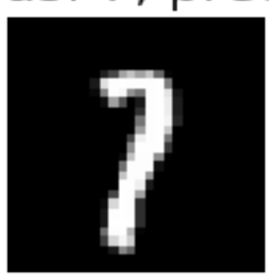
```
Probabality of each number:  [[0, -4126.0736693739054], [1, -4099.1776924240921], [2, -4072.1907
```

Actual was: 7, predicted: 5



```
Probabality of each number:  [[0, -4153.7620084166192], [1, -4214.5211688603258], [2, -4132.5787
```

Actual was: 7, predicted: 2



```
Probabality of each number:  [[0, -4095.6362065586577], [1, -3954.1446728082788], [2, -4021.7226
```


```
Probabality of each number:  [[0, -4179.2841727026398], [1, -4497.9209520738677], [2, -4132.1600
```