DSE 210

Homework 3

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1 Generative models 2

1.1 Worksheet 6

1. (a) Uncorrelated
   (b) Positively Correlated
   (c) Negatively Correlated

3. (a) Unique Bivariate Gaussian, parameterized bymean: $\mu = \binom{2}{2}$ and Covariance Matrix:

$$Cov(x, y) = \begin{bmatrix} 1 & -0.25 \\ -0.25 & 0.25 \end{bmatrix}$$

   Because each standard deviation squared provides the variance, we are able to get the diagonals of the matrix. Calculating the covariance comes from the correlation formula: $corr(x, y) = \frac{cov(x,y)}{std(x)*std(y)}$.

   (b) Unique Bivariate Gaussian, parameterized bymean: $\mu = \binom{1}{1}$ and Covariance Matrix:

$$Cov(x, y) = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

   Because each standard deviation squared provides the variance (1:1), we are able to get the diagonals of the matrix. Since $y = x$, intuitively we know that their covariance must equal 1.

Please see attached Jupyter notebook for questions 4 and 5.

2 Linear Algebra Primer

2.1 Worksheet 7

1. $||x|| = \sqrt{1^2 + 2^2 + 3^2} = \sqrt{14}$. Hence the unit vector in the same direction as x is: $\vec{x_u} = (\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}})$.

2. Dot product must equal 0 for orthogonality, hence need to see a relation like: $\binom{1}{1} \cdot \binom{-1}{1} = -1 + 1 = 0$. Taking the unit vectors, we get: $(\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$ and $(\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}})$.

3. $x \cdot x = 25$, hence $||x||^2 = x \cdot x$ and thus 5 is the magnitude. In 2 dimensions, we know that this is a circle with radius 5 and in 3 dimensions this would be a sphere with radius 5. Because we have d dimensions, this would be classified as a hypersphere with radius 5.

4.

$$w = \begin{bmatrix} 2 \\ -1 \\ 6 \end{bmatrix} \tag{2.0.1}$$

5. Following matrix multiplication properties, we know $A = 10 \times 30$ and $B = 30 \times 20$.

6. (a) $n \times d$.
   (b) $n \times n$.
   (c) $x^i \cdot x^j$.

8. $x^T x = ||x||^2$. Hence, we have $\sqrt{1^2 + 3^2 + 5^2}^2 = 35$. xx^T is a matrix multiplied by its transpose:

$$xx^T = \begin{bmatrix} 1 & 3 & 5 \\ 3 & 9 & 15 \\ 5 & 15 & 25 \end{bmatrix}$$

10. Writing a symmetric matrix, we have:

$$M = \begin{bmatrix} 3 & 1 & -2 \\ 1 & 0 & 0 \\ -2 & 0 & 6 \end{bmatrix}$$

11. a) and c). a) because of multiplicative properties and c) because of additive properties. Not d) because of negative values within the matrix.

13. (a) UU^T yields the $d \times d$ identity matrix, I^d.
    (b) Following part a), we know from matrix properties that any matrix M, multiplied by its inverse M^{-1}, yields the identity matrix. Hence, we can conclude that $U^{-1} = U^T$.

14. $z = 6$, since the discriminant must $= 0$. Leaves us with $z - 6 = 0$ and 6.

3 Classification with Generative Models 3

3.1 Worksheet 8

Please see attached Jupyter notebook for digit classification.