

Q3.

$i=2$        $2^{2^k}$   
 a) while ( $i < n$ )  
    {  
        $i = i * i$       2, 4, 16  
    }

$$\begin{aligned}
 2^k \cdot \log 2 &= \log n \\
 \log(2^k \cdot \log 2) &= \log(\log n) \\
 k \cdot \log(2 \log 2) &= \log(\log n) \\
 \text{OK } k &\leftarrow \log(\log n)
 \end{aligned}$$

b)  $n=9$  the "if" triggers for  $i = 3, 6, 9$        $i = k\sqrt{n}$   
     $n=16$       "      "       $i = 4, 8, 12, 16$

$$T(n) = \sum_{i=1}^n \theta(i) + \sum_{k=1}^{\sqrt{n}} \sum_{j=0}^{i^3-1} \theta(i)$$

$$= \theta(n) + \sum_{k=1}^{\sqrt{n}} \theta(i^3)$$

$$= \theta(n) + \sum_{k=1}^{\sqrt{n}} \theta((k\sqrt{n})^3)$$

geometric?

$$\begin{aligned}
 \sum_{x=1}^y x^3 &= \frac{1}{4} y^2 (y+1)^2 \\
 \theta(n^{7/2}) &= \frac{1}{4} y^4 + \theta(y^3) \\
 &= \frac{1}{4} n^{4/2} \cdot n^{3/2} = n^{7/2} \\
 &\theta(n^2)
 \end{aligned}$$

$$c) \sum_{i=1}^n (\theta(n) + \theta(\log(n)))$$

$$\sum_{i=1}^n (\theta(n) + \sum_{j=1}^n (\theta(\log(n)))$$

$$\sum_{i=1}^n (\theta(n) + \theta(n \log(n)))$$

$$2 \cdot \theta(n^2) + \theta(n^2 \log(n))$$

$$\theta(n^2 \log(n)) > \theta(n^2)$$

$$= \theta(n^2 \log(n))$$

d.  $\sum_{i=0}^{n-1} (\Theta(1) + \sum_{j=1}^{2 \cdot \text{size}} \Theta(1))$  if statement  
best worst case  
runs twice

$$= \Theta(n) + \Theta(2n \cdot \text{size})$$
$$= \Theta(n + \text{size})$$