Counting

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1. UNUSUAL

3U
1N
1S
1A
1L

• 5-elt-subsets ⟶ 3Us {U,U,U...}

$\binom{4}{2}$

0Us | 1Us | 2Us

0 | {U,__ (1 set) | {U,U,__} $\binom{4}{3}$

1 + 4 + 6 = 11

strings (length 5)

1Us: 5!

2Us: $\binom{5}{2} \cdot 3!$ or $\frac{5!}{2!}$

3Us: $\binom{5}{3} \cdot 2!$ or $\frac{5!}{3!}$

$5! \cdot \frac{5!}{2!} \cdot \frac{5!}{3!} = 200$

120
60
+ 20
200

3. 7 tables, 16 songs

$x_1, \ldots, x_7$ = # songs for tables 1, ..., 7

* | $x_1 + \ldots + x_7 = 16$, $x_i \geq 0$, $x_1 \leq 1$

$x_1 = 0$ | $x_1 = 1$

$x_2 + \ldots + x_7 = 16$, $x_i \geq 0$

$n = 16$ $r = 6$

$\binom{n+r-1}{r-1}$

$x_2 + \ldots + x_7 + 1 = 16$, $x_i \geq 0$

$n = 15$ $r = 6$

$\binom{n+r-1}{r-1}$

$= \binom{21}{5} + \binom{20}{5}$

#2 is on the back

2. 5 cards AABBC 1-13

choose A, B $\binom{13}{2}$ (rank repeated)

chose C: 11 (rank not repeated)

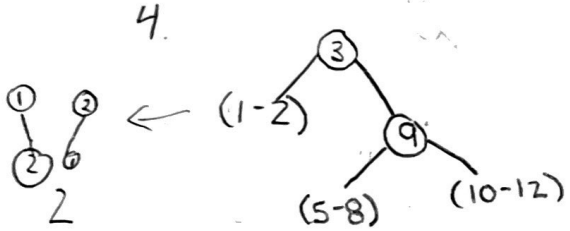
$\binom{4}{2} \cdot \binom{4}{2}$ $\binom{4}{1}$

$= \binom{13}{2} \cdot 11 \cdot 6 \cdot 6 \cdot 4$

$= 123,552$

or $\binom{13}{3} \cdot 3 \cdot \binom{4}{2} \cdot \binom{4}{2} \cdot 4$

4.

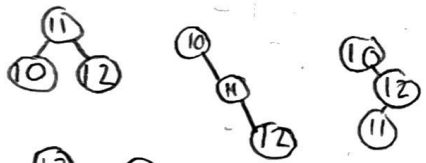


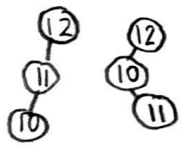
2

(5-8) = x

(10-12) = y

total combinations: 2xy





y potential subtrees

y = 5

5 nodes: $\frac{(2 \cdot 5)!}{5!\,6!} = 42$

total combinations = 2(42)(5) = 420

5. ~~3 nurses~~ → x_1, x_2, x_3 | $x_1 + x_2 + x_3 = 10$, $x_i \geq 1$ → $n = 10$ $r = 3$ → $\binom{n-1}{r-1} = \binom{9}{2}$

4 nurses → x_1, x_2, x_3, x_4 | $x_1 + x_2 + x_3 + x_4 = 10$, $x_i \geq 1$ $= \binom{9}{3}$

$\binom{9}{2} + \binom{9}{3} = 120$

5.

case : 4 nurses

(7,1,1,1), (6,2,1,1), (5,3,1,1), (5,2,2,1) (4,4,1,1)(4,3,2,1) : 9
(4,2,2,2), (3,2,2,3) (3,3,3,1)

case : 3 nurses

(8,1,1), (7,2,1), (6,3,1), (6,2,3), (5,4,1), (5,3,2), (4,4,2), (3,3,4) : 8

9+8 = 17