Probability

1.

$$\binom{15}{8} = \frac{15!}{8!(15-8)!} = 259459200 \rightarrow \frac{259459200}{15^8} = 0.101237$$

2.

* on another paper *

3. $P(A \cap B) = P(A) \cdot P(B)$, then A & B are indep.

$P(\text{atleast 2 dice show 4 or above}) = P(A)$

$P(A) = P(X=2) + P(X=3) = 1/2$

$P(\text{all 3 dice show the same value}) = \frac{6}{6^3} = \frac{1}{36}$

$P(A) \cdot P(B) = \frac{1}{2} \cdot \frac{1}{36} = \frac{1}{72}$

$P(A \cap B) = \frac{1}{6^3} + \frac{1}{6^2} + \frac{1}{6^3} = \frac{1}{72}$

$P(A) \cdot P(B) = P(A \cap B)$

$\frac{1}{72} = \frac{1}{72}$ ✓ independent

Probability

2.

$$\left.\begin{array}{l} 100-1000 : 100 \\ 1000-10000 = 700 \\ 10000-99999 = 4200 \end{array}\right\} 5000$$

$$P = \frac{5000}{10^5} = 0.05$$

$$P(S) = 8(0.05)^5(1-0.05)^3$$
$$= 1.5004 \times 10^{-5}$$

4. $P(\text{flush on 5 card hand}) = \dfrac{\binom{13}{5}\cdot 4}{\binom{52}{5}} = 0.001980792$

Find expected num of hands to play to get a flush. Let's call this X

$X \sim \text{Geometric}(p)$

$p = 0.001980792$

$\dfrac{1}{p} = \dfrac{1}{0.001980792} = E[X] \sim 504.85$

5. win = W

super star plays = S = .75

no super star plays = S^c = 0.25

$P(W|S) = .7$

$P(W|S^c) = .5$

$P(S) = 0.75$ → for next 5 games

$P(W\,4/5\,|\,S) = \binom{5}{4} \cdot (0.7)^4 (.3) = 0.36015$

$P(W\,4/5\,|\,S^c) = \binom{5}{4} \cdot (0.5)^4 = 0.15625$

$P(S\,|\,W\,4/5) = ?$

$$\begin{aligned} P(W\,4/5) &= P(W\,4/5\,|\,S) \cdot P(S) + P(W\,4/5\,|\,S^c) \cdot P(S^c) \\ &= 0.36015(.7) + 0.15625(0.25) \\ &= 0.309175 \end{aligned}$$

$$P(S\,|\,W\,4/5) = \frac{P(W\,4/5\,|\,S) \cdot P(S)}{P(W\,4/5)} = \frac{0.36015(.75)}{0.309175}$$

$= 0.8737$