Boomerang Race in the State of Texas

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Introduction

Problem statement

‘Boomerang’ race involves starting from a certain city in a state and covering 14 other cities with different starting alphabets and returning to the initial city to close the circuit. We have chosen Texas as a required state and have chosen fifteen different cities in order of starting alphabets and Population. The aim is to minimize the time taken to complete the circuit and visit the cities only once while racing and return to the origin. Here, we applied two different solution algorithms, i.e. TSP by MTZ and Branch & Cut to the problem. Compared these two results and summarized them below.

Assumptions

1. While selecting cities, if there were multiple cities with the same letter the city with the highest population was chosen.
2. We used google maps to create the distance matrix, and the shortest distance was taken between any two cities, even it the time shown was more than the other distances. Hence shortest distance was preferred than the fastest time while retrieving distances from google maps.
3. It is assumed that the Shortest distance is correlated with the shortest time. i.e. you will reach faster with the shortest distance. This also means the model does not take into consideration the speed limits of different routes. i.e. Longer distances from city to city can have higher speed limits and can hence reach faster.
4. To avoid self-tours the distance value for the same city has been assigned as per the formula M = 5 * Max(All distance). Which came up to 4030 in our case.

Data Collection and Processing

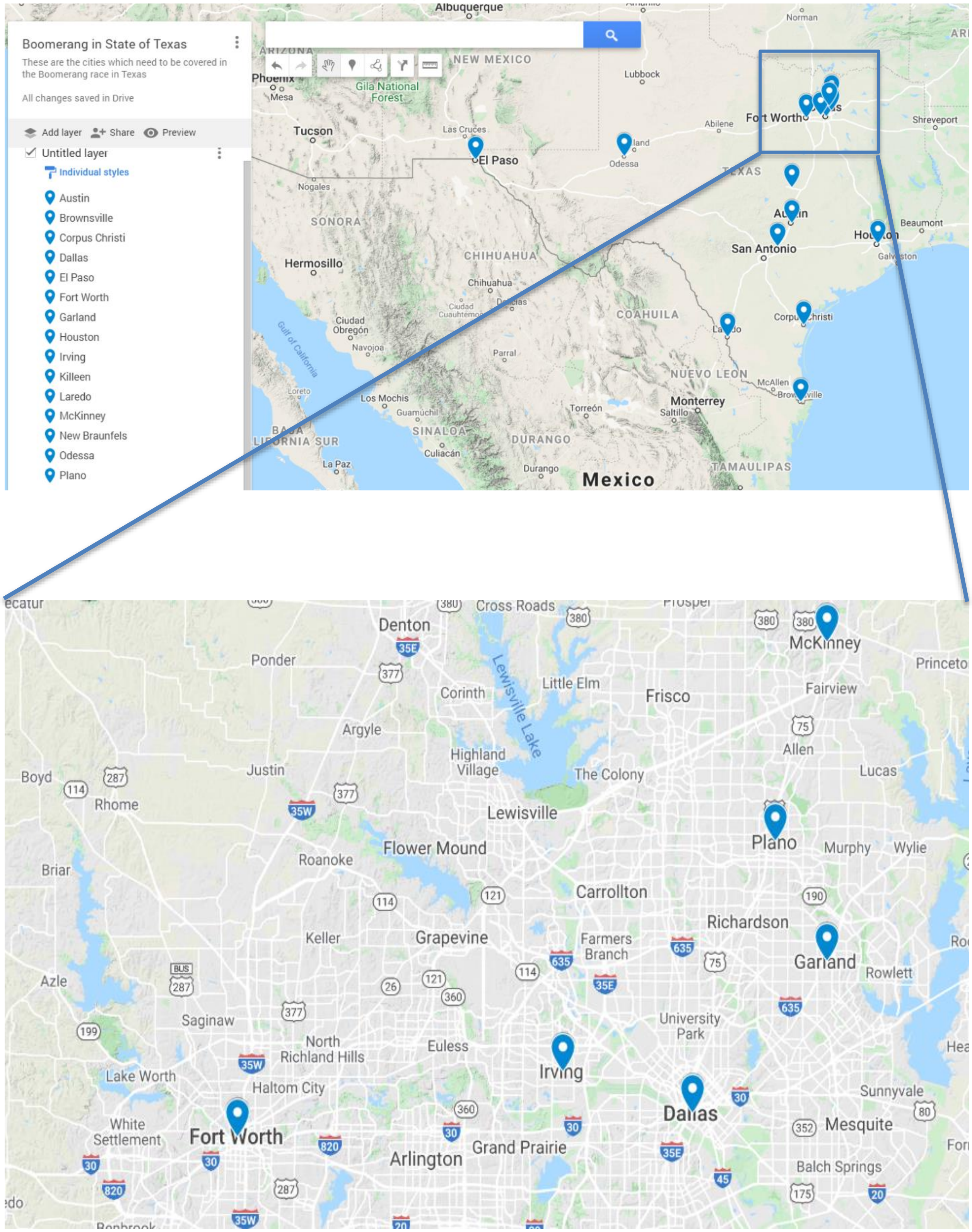
Cities were chosen in alphabetical order, with each city starting from a different letter. And the distance matrix was created by retrieving distances from google maps.

Source: https://www.texas-demographics.com/cities_by_population

Table 1: Cities were chosen for the race

Sr No.	City	Population
1	Austin	935,755
2	Brownsville	182,679
3	Corpus Christi	324,692
4	Dallas	1,318,806
5	El Paso	680,354
6	Fort Worth	855,786
7	Garland	237,982
8	Houston	2,295,982
9	Irving	238,637
10	Killeen	143,070
11	Laredo	257,575
12	McKinney	173,460
13	New Braunfels	74,587
14	Odessa	118,582
15	Plano	284,579

Figure 1: Boomerang Race in the state of Texas: Cities



Model

TSP by MTZ

Decision Variables:

x_{ij} = {1, when city j is visited right after city i
= {0, Otherwise

Parameter:
c_{ij} = distance from city i to city j

Auxiliary variables:

u_i = Auxiliary variable corresponding to origin i
u_j= Auxiliary variable corresponding to destination j

Objective Function:

Minimize $z = \sum_{i=1}^{15} \sum_{j=1}^{15} c_{ij} x_{ij}$
Minimizing the total distance traveled.

Constraints:
$\text{s.t.}\ \ \sum_{i=1}^{15} x_{ij} = 1\ \forall\ j = 1, \dots, 15$ (Arrive at city j once)
$\sum_{j=1}^{15} x_{ij} = 1\ \forall\ i = 1, \dots, 15$ (Leave city i once)
$u_i - u_j + 15x_{ij} \leq 15 - 1 \text{ for } i \neq j \text{ where } i = 2, \dots 15 \text{ and } j = 2, \dots 15$
(Ensures that no sub-tours take place)
Where all $x_{ij} \in \{0,1\}$ and all $u_j \geq 0$ and $c_{ij} \geq 0$

Note:
Here, all c_{ii} are set to a big M. Where M is a large number as defined in our assumptions to avoid self-travel to cities.

Branch and Cut

Decision Variables:

x_{ij} = {1, when city j is visited right after city i
= {0, Otherwise

Parameter:
d_{ij} = distance from city i to city j

Objective Function:

Minimize $z = \sum_{i=1}^{15} \sum_{j=1}^{15} d_{ij} x_{ij}$
Minimizing the total distance traveled.

Constraints:
$\text{s.t.} \ \sum_{i=1}^{15} x_{ij} = 1 \ \forall \ j = 1, \dots, 15$ (Arrive at city j once)
$\sum_{j=1}^{15} x_{ij} = 1 \ \forall \ i = 1, \dots, 15$ (Leave city i once)
Where all $x_{ij} \in \{0,1\}$ and all $d_{ij} \geq 0$

Note:
Here, all d_{ii} are set to a big M. Where M is a large number as defined in our assumptions to avoid self-travel to cities.

Description of Solution Methods

TSP by MTZ

In MTZ formulation, the solution of an integer program is utilized to solve the TSP problem. This method aims at eliminating sub tours, but it becomes inefficient for large TSPs. Also, since it includes many decision variables; it can't be solved using the branch and bound method.

Branch and Cut Algorithm

As we are solving the TSP as an assignment problem, the initial solution would satisfy the constraints, but would not be feasible as it would create sub tours. Hence, the Branch and Cut method aim at eliminating sub-tours at each stage of the model by making sure that one of the sub-tours does not occur again. This is ensured by adding the cities in the sub-tour selected to a set "S" and adding the remaining cities to a complement set "S'" and creating a constraint between the two sets to have at least one arc, which is set by constraint to be greater than or equal to 1. These constraints are also known as cut-set constraints.

Results

TSP by MTZ

Results

The MTZ formulation, gives us the objective function value of 2034 miles, as the shortest distance to be traveled with 106 Simplex iterations.

Figure 2: AMPL TSP MTZ Results

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CPLEX 12.10.0.0: optimal integer solution; objective 2034
106 MIP simplex iterations
0 branch-and-bound nodes
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Visualizations

Figure 3: The optimal route with the MTZ formulation

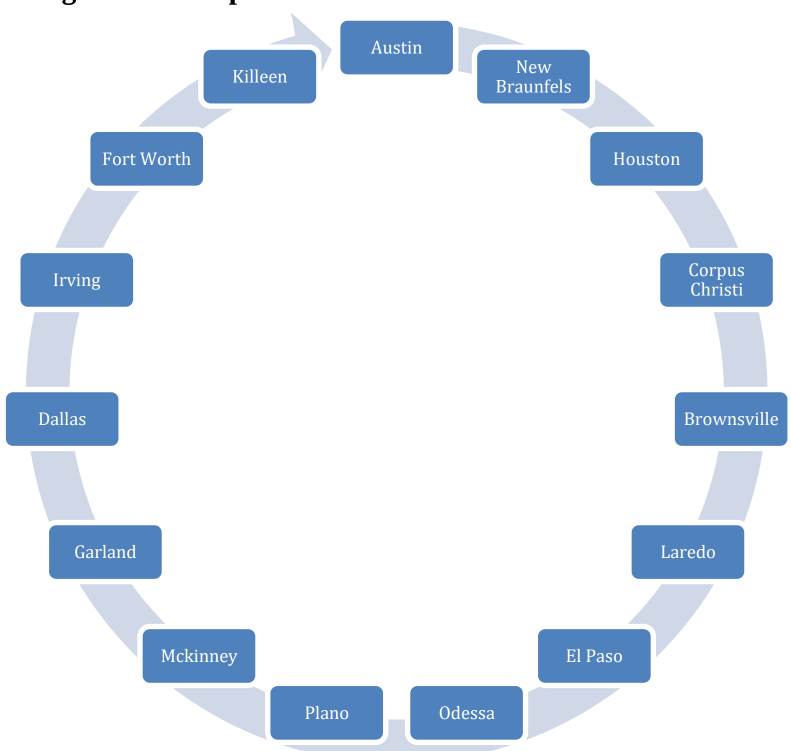
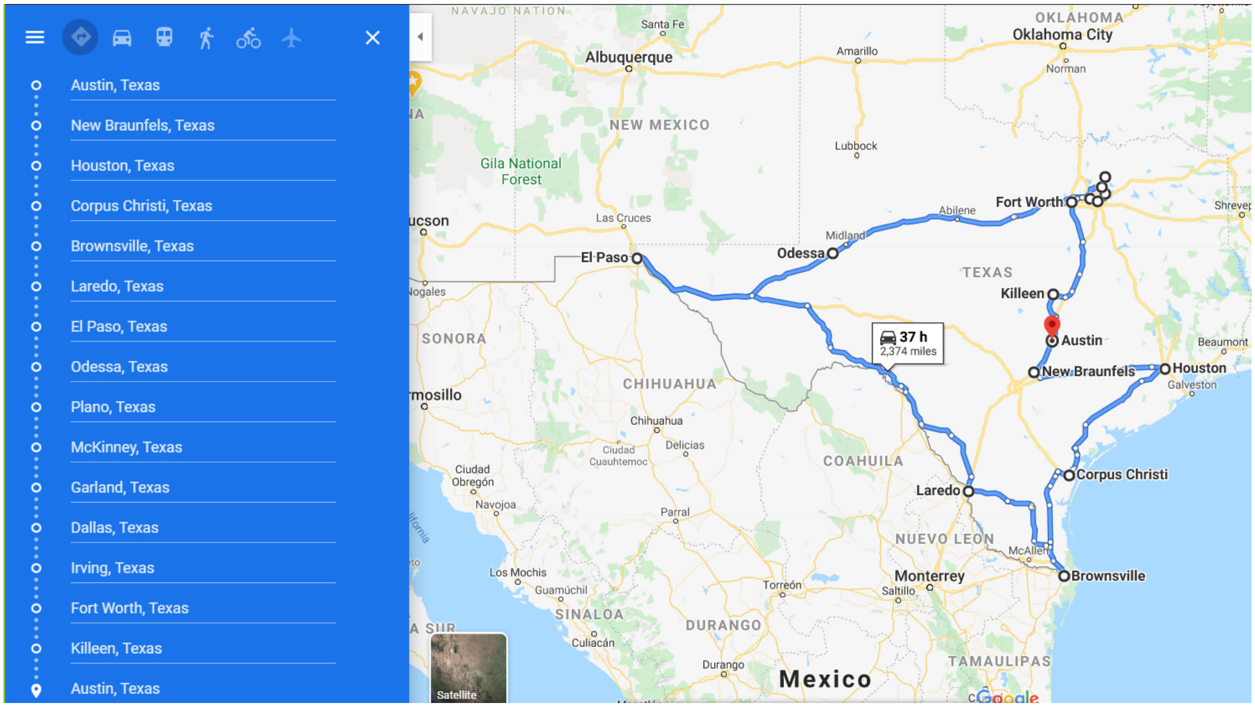


Figure 4: TSP route with the MTZ Algorithm



TSP by Branch and Cut

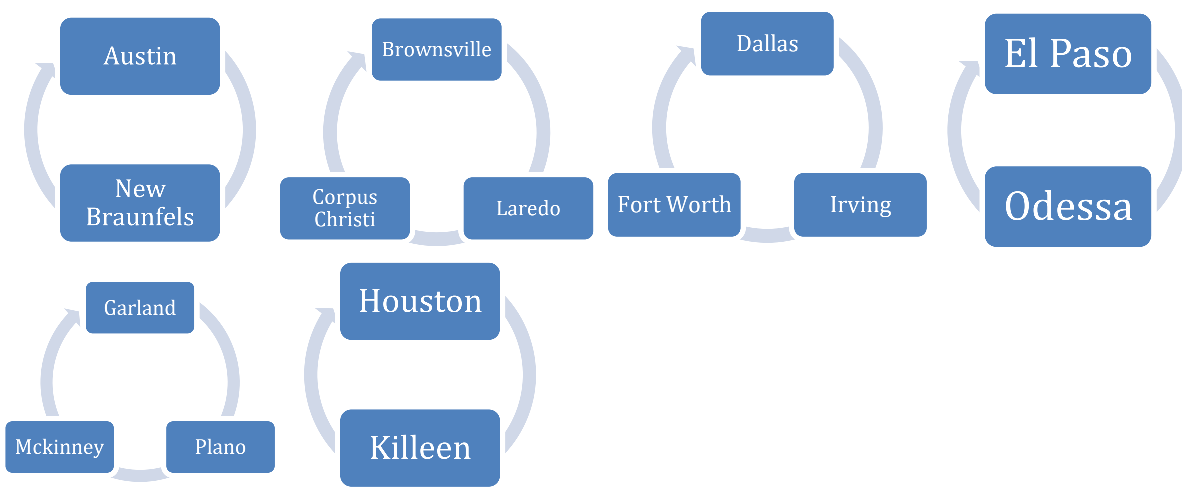
Model progression and Results

After performing the initial run on the AMPL solver, the following sub tours were found:

Table 1: Subtours after Initial Run

Iteration 0	Z	1674			
Sub tour 1	A	N	A		
Sub tour 2	B	L	C	B	
Sub tour 3	D	I	F	D	
Sub tour 4	E	O	E		
Sub tour 5	G	P	M	G	
Sub tour 6	H	K	H		

Figure 5: Subtours after initial Run



After this, cities in Subtour 1, was set to a new Set called “set cut11” and all the remaining cities to its complement set called “set cut12”. After introducing a new constraint to break sub tour 1 the model was solved in. This process was repeated until no sub tours existed.

The table below shows all the iterations and the identified sub tours. The initial of each city are present in the table. And color indicates which sub tour is chosen to break. And all the remaining cities go to the complement set.

Table 2: Iterations performed by each cut-set constraints.

Iteration 1	Z	1677														
Subtour A	A	N	H	K	A											
Subtour B	B	L	C	B												
Subtour C	D	I	F	D												
Subtour D	E	O	E													
Subtour E	G	P	M	G												
Iteration 2	Z	2186														
Subtour A	A	N	H	A												
Subtour B	B	L	C	B												
Subtour C	D	I	F	D												
Subtour D	E	O	K	E												
Subtour E	G	P	M	G												
Iteration 3	Z	2188														
Subtour A	A	N	H	C	B	L	E	O	K	A						
Subtour B	D	I	F	D												
Subtour C	G	P	M	G												
Iteration 4	Z	2188														
Subtour A	A	N	H	C	B	L	E	O	K	A						
Subtour B	D	G	D													
Subtour C	F	I	F													
Subtour D	M	P	M													
Iteration 5	Z	2200														
Subtour A	A	N	H	C	B	L	E	O	K	A						
Subtour B	F	I	G	D	F											
Subtour C	M	P	M													

Iteration 6	Z	2206														
Subtour A	A	N	H	C	B	L	E	O	K	A						
Subtour B	F	M	P	G	D	I	F									
Iteration 7	Z	2317														
Subtour A	A	N	H	C	B	L	E	O	F	K	A					
Subtour B	G	P	M	I	D	G										
Iteration 8	Z	2338														
Subtour A	A	N	H	C	B	L	E	O	F	I	G	P	M	D	K	A

We finally see an optimal sequence with no sub-tours is the 8th iteration with a minimum distance of 2338 miles with 34 simplex iterations.

Table 3: Summary table for the Objective function value and No. of Iterations

Stage	**Objective function value**	**No of Simplex iterations**
0	1674	15
1	1677	17
2	2186	27
3	2188	24
4	2188	22
5	2200	24
6	2206	23
7	2317	26
8	2338	34

Visualizations

Figure 6: One of the optimal routes found by the Branch and Cut formulation

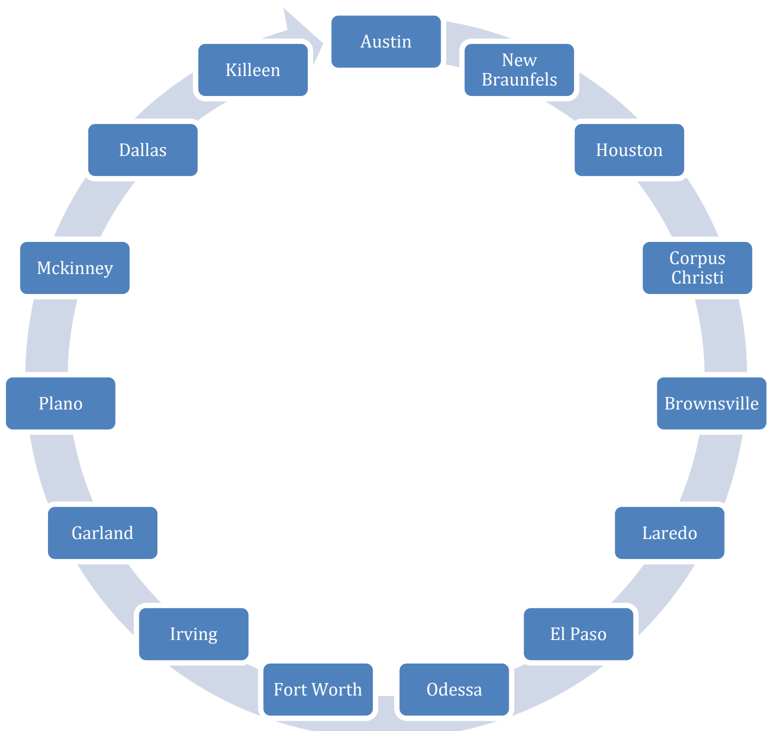
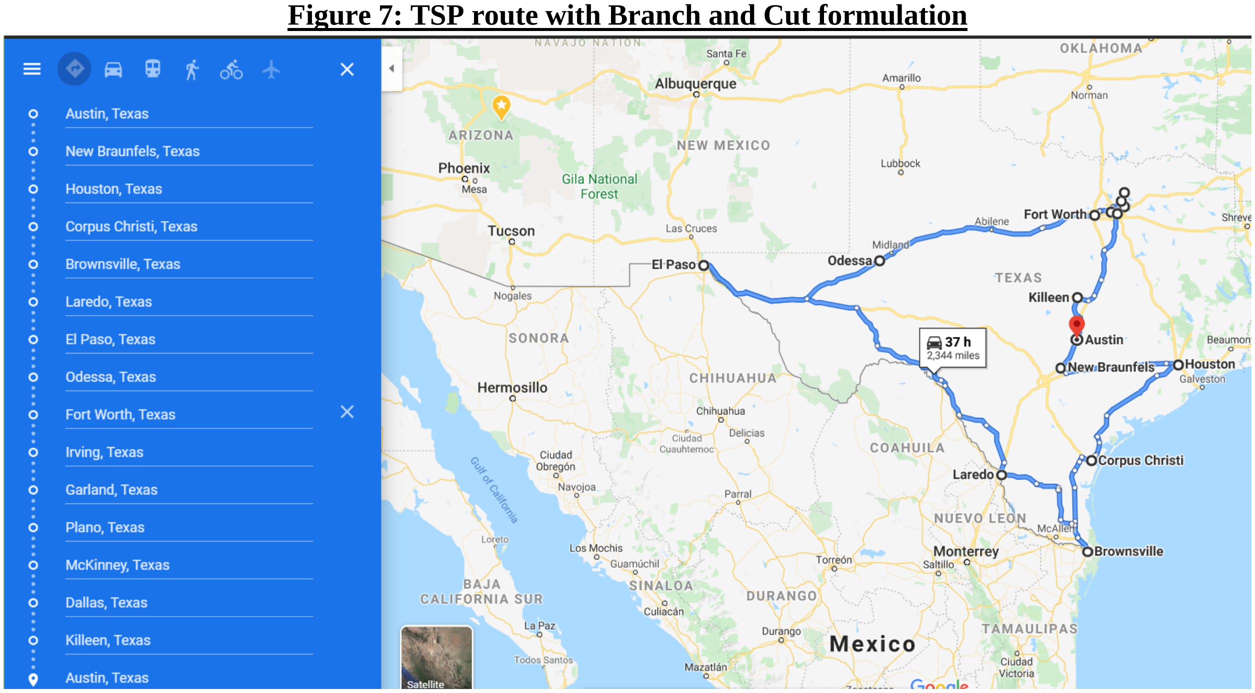


Figure 7: TSP route with Branch and Cut formulation



Recommendations

In this case, both the solutions take the same amount of time, although MTZ gives us a lower total distance traveled.
Taking into consideration the wear and tear of the vehicle and a higher chance of breaking down for the longer distance, we recommend the MTZ route instead of the Branch and Cut route.
Hence we suggest to go with the MTZ *route*: