Course: MTH 308

MINI PROJECT 1: PROJECT REPORT

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PROJECT AIM:

Given a simple closed curve in R^2 given by C: [0,1] -> R^2 such that C(t)=(X(t),Y(t)) where 0<= t <= 1 and where X, Y : R → R are infinitely differentiable periodic functions with period 1 and a point (X_0, Y_0).

We had to find a point (X_c, Y_c) on the curve C such that the point is at the minimum distance to (X_0, Y_0) among all the points on the curve.

ALGORITHM USED AND THEORETICAL SUPPORT:

The primary algorithm used to solve this problem is Newton's Method of finding the root of a function. Let $F(t)= (X(t)-X_0)^2+(Y(t)-Y_0)^2$. We need to find the minima of this function. Since this is an infinitely differentiable function, we know that at the minima, $F'(t)=0$. Hence the problem transformed into finding the roots of $F'(t)=2*(X(t)-X_0)*(X'(t))+ 2*(Y(t)-Y_0)*(Y'(t))$. Now we apply Newton's Method to this function to arrive at the zeros of this function.

INITIAL PROBLEMS WITH THIS APPROACH

1. What if Newton's method converges to a root which is actually a local minima and not a global minima?
2. What if Newton's method converges to a root which is actually a local maxima and not a local/global minima?

SOLUTION TO THE ABOVE MENTIONED PROBLEMS

We know that Optimization is often not an exact science and is rather close to coming up with a solution that should work for majority of the problems if not all. With that in mind, the approach taken in this solution is:
We divide the interval from 0 to 1 into 500 equidistant points. Now, we run Newton's Method 500 times considering each of these points as initial guesses. In each case Newton's method must converge to one of the zeros of the function and we compare the F(t) values for all 500 values of 't' we receive and output the 't' which gave the minimum value of F(t).

PROOF OF CORRECTNESS/JUSTIFICATION OF THE ABOVE MENTIONED SOLUTION

Even if some of the initial guesses leads Newton's method to a local minima/maxima, we can be confident that at least one of the 500 initial guesses must converge to the global minima with a very good chance. For a function to fail to produce global minima with this method, we must have that in a difference of 1/500 = 0.002 in the value of the parameter 't', F(t) skips the entire basin of a global minima. The odds of that happening are very low for majority of the infinitely differentiable curves and hence we can be fairly confident that this method leads us to a global minima.

IMPLEMENTATION DETAIL

- We iterate through 0 to 1 via steps of 0.002 and consider each point as the initial guess for the Newton's method.
- In the update step , we need the value of second derivative of F(t). We estimate this by (dXdt(t+h)-dXdt(t))/h for X and similarly for Y.
- The value of h is taken to be 0.001.
- During the update step, care is taken to ensure that t lies between 0 and 1.
- Newton's method is stopped when $F(t_{new})<F(t_{old})$-eps.
- Care has been taken to avoid cancellation error and therefore the convergence criterion has been written in that form.
- Finally the value of converged 't' that gives the minimum value of F(t) is returned.