Untyped Lambda Calculus Problems

*Due one week – solve **only 5** out of 10*

1. Rewrite these **Boolean** expressions as Lambda expressions
   - α.β + α.γ + β.γ
   - *xor* (*or* α β) (*and not* α γ)
2. Define > and < for two [Church encoded] numerical arguments
3. Define positive and negative **integers** using pairs of **natural** numbers
   - Define *addition* and *subtraction*
4. Define the *division* of positive integers recursively
5. Define **the factorial function** n! = n·(n−1)⋯1 recursively
6. Define **rational** numbers as pairs of integers
   - Define *addition*, *multiplication*
7. Define **complex** numbers provided some encoding for **real** numbers exists
   - Define *addition* and *division*
8. Define *subtraction* using **Peano numbers**
9. Define **the successor function** using only A, B and ID

   A = λ x y . y

   B = λ x y z . x z (y z)

   ID = λ x . x
10. Write a **quine**
    - a quine is an expression that eventually reduces to itself