Mesh and shape smoothing

Contribution to the CGAL project

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Motivation

Project objective

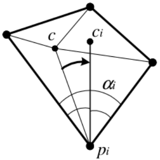
Enhance *Polygon mesh processing* [2, 1] with

- Mesh smoothing [4]: Improve quality of elements.
- Shape smoothing [3]: Improve overall shape.

Both algorithms aim at moving vertices locally without changing the connectivity between them.

Mesh smoothing

Angle bisecting



1. $c_{new} = \frac{1}{k}\sum_{i=1}^{k} c_i$ or

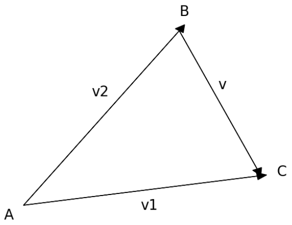
   $$c_{new} = \frac{1}{\sum_{i=1}^{k} 1/\alpha_i^2}\sum_{i=1}^{k} c_i \qquad \text{with weights}$$

2. Location on tangent plane to the vertex

   $$p' = c_{new} + nn^T(p_i - c_{new})$$

3. Projection to original surface with AABB tree

Area equalization



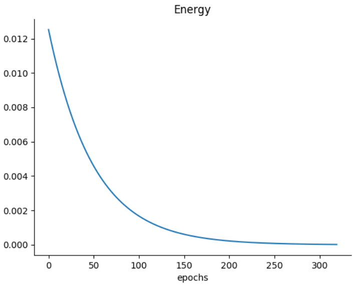
vertex A is being moved

$$S = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ v1_x & v1_y & v1_z \\ v2_x & v2_y & v2_z \end{vmatrix}$$

$$\text{minimize} \quad r = \sum_{i=1}^{k} (S_i(x, y, z) - S_{avg})^2$$

S_{avg}: average of all areas around a vertex

Minimization with gradient descent



$$r = \sum_{i=1}^{k} (S_i(x, y, z) - S_{avg})^2$$

$$\boxed{p_{new} = p - \eta * \nabla r}$$

$$\nabla r = (v_z - v_y) x_A + (v_x - v_z) y_A + (v_y - v_x) z_A$$

convergence based on energy

$$J = \frac{1}{k} (S - S_{avg})^2$$

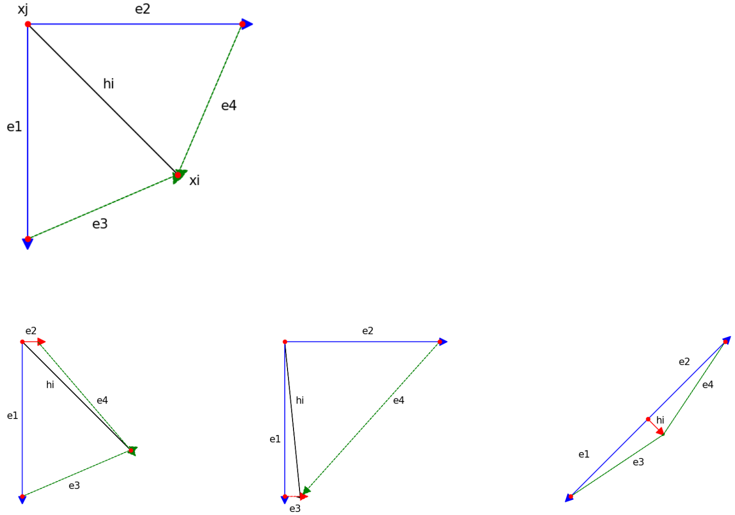
k = number of incident edges

learning rate

$$\eta = \frac{\eta_0}{1 + t_0 * epoch}$$

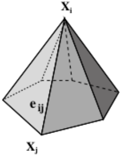
$$\eta_0 = 0.01, t_0 = 0.001$$

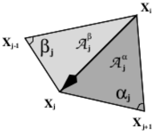
Degenerate cases



Shape smoothing

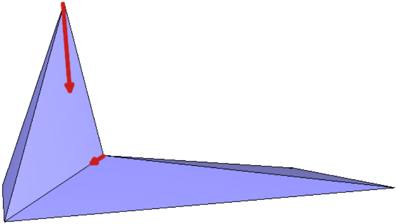
Mean curvature flow





$$\frac{\partial x_i}{\partial t} = -\overline{k_i}\mathbf{n}$$

$$\overline{k_i}\mathbf{n} = \frac{1}{\sum_j (cota_j + cotb_j)} \sum_j \underbrace{(cota_j + cotb_j)}_{\text{weight}}(x_i - x_j)$$



Degenerate cases

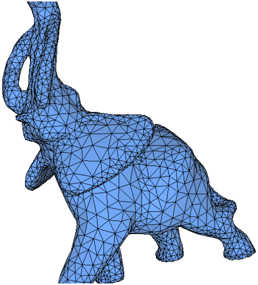
$$cot\ a = \frac{cos\ a}{sin\ a}$$

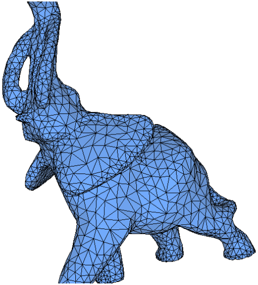
$$\lim_{a \to 0} cot\ a = \lim_{a \to \pi} cot\ a = \infty$$

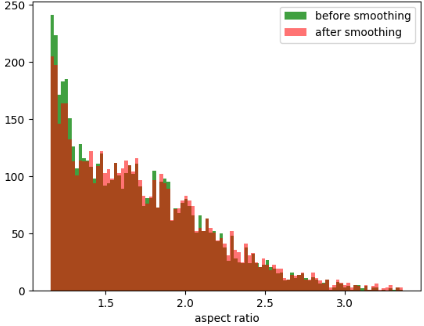
Zero angles carry infinite weight. Degenerate cases are removed before and during shape smoothing.

Results evaluation

Mesh smoothing

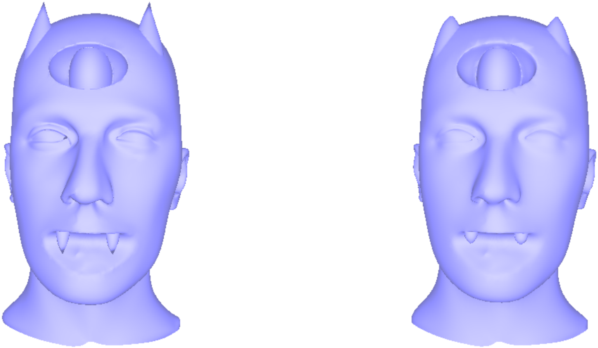






$$\text{aspect ratio} = \frac{\text{largest altitude}}{\text{shortest edge}}$$

Shape smoothing



Code & discussion

Links

- development branch
  `https://github.com/CGAL/cgal-public-dev/tree/gsoc2017-smoothing-kkatrio`
- blog
  `https://kkatrio.github.io/projects/smoothing-gsoc17/`
- documentation
  `https://kkatrio.github.io/CGAL/doc_output/Polygon_mesh_processing/`
- *konst.katrioplas@gmail.com*

References

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Polygon mesh processing.
CRC press, 2010.

Polygon mesh processing user manual.
`https://doc.cgal.org/latest/Polygon_mesh_processing/index.html`.

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Implicit fairing of arbitrary meshes using diffusion and curvature flow.
In *Computer Graphics (Proc. SIGGRAPH '99)*, pages 317–324, 1999.

V. Surazhsky and C. Gotsman.
High quality compatible triangulations.
Engineering with Computers, 20(2):147–156, 2004.