Text Categorization: Discriminative Classifiers

Part 1

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Overview

- What is text categorization?
- Why text categorization?
- How to do text categorization?
  - Generative probabilistic models
  - **Discriminative approaches**
- How to evaluate categorization results?

Anatomy of Naïve Bayes Classifier

Two categories: θ_1 and θ_2

$$\text{score}(d) = \log \frac{p(\theta_1 | d)}{p(\theta_2 | d)} = \log \frac{p(\theta_1)\prod_{w \in V} p(w|\theta_1)^{c(w,d)}}{p(\theta_2)\prod_{w \in V} p(w|\theta_2)^{c(w,d)}}$$

$$= \boxed{\log \frac{p(\theta_1)}{p(\theta_2)}} + \sum_{w \in V} \underline{c(w,d)} \boxed{\log \frac{p(w|\theta_1)}{p(w|\theta_2)}}$$

Category bias (β_0) doesn't depend on d!

Sum over all words (features $\{x_i\}$)

Feature value: x_i=c(w,d)

Weight on each word (feature) β_i

Generalize

$$d = (x_1, x_2, \ldots, x_M), \quad x_i \in \Re$$

$$\text{score}(d) = \beta_0 + \sum_{i=1}^{M} x_i \beta_i \qquad \beta_i \in \Re$$

= Logistic Regression!

Discriminative Classifier 1: Logistic Regression

Binary Response Variable: Y $\in$ {0,1}

Predictors: $X = (x_1, x_2, \ldots, x_M), \; x_i \in \Re$

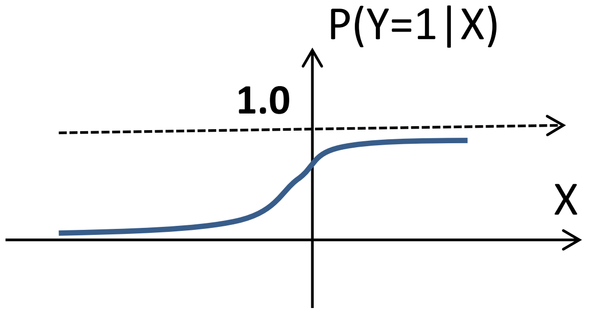
$$Y = \begin{cases} 1 & \text{category}(d) = \theta_1 \\ 0 & \text{category}(d) = \theta_2 \end{cases}$$

Modeling p(Y|X) directly

Allow many other features than words!

$$\log \frac{p(\theta_1 \mid d)}{p(\theta_2 \mid d)} = \log \frac{p(Y=1 \mid X)}{p(Y=0 \mid X)} = \log \frac{p(Y=1 \mid X)}{1 - p(Y=1 \mid X)} = \beta_0 + \sum_{i=1}^{M} x_i \beta_i \qquad \beta_i \in \Re$$

$$p(Y=1 \mid X) = \frac{e^{\beta_0 + \sum_{i=1}^{M} x_i \beta_i}}{e^{\beta_0 + \sum_{i=1}^{M} x_i \beta_i} + 1}$$



Estimation of Parameters

- Training Data: T={(Xi, Yi)}, i=1,2, …, |T|
- Parameters: $\vec{\beta} = (\beta_0, \beta_1, \ldots, \beta_M)$
- Conditional likelihood: $p(T \mid \vec{\beta}) = \prod_{i=1}^{|T|} p(Y = Y_i \mid X = X_i, \vec{\beta})$

Y_i =1

$$p(Y = 1 \mid X) = \frac{e^{\beta_0 + \sum_{i=1}^{M} x_i \beta_i}}{e^{\beta_0 + \sum_{i=1}^{M} x_i \beta_i} + 1}$$

Y_i =0

$$p(Y = 0 \mid X) = \frac{1}{e^{\beta_0 + \sum_{i=1}^{M} x_i \beta_i} + 1}$$

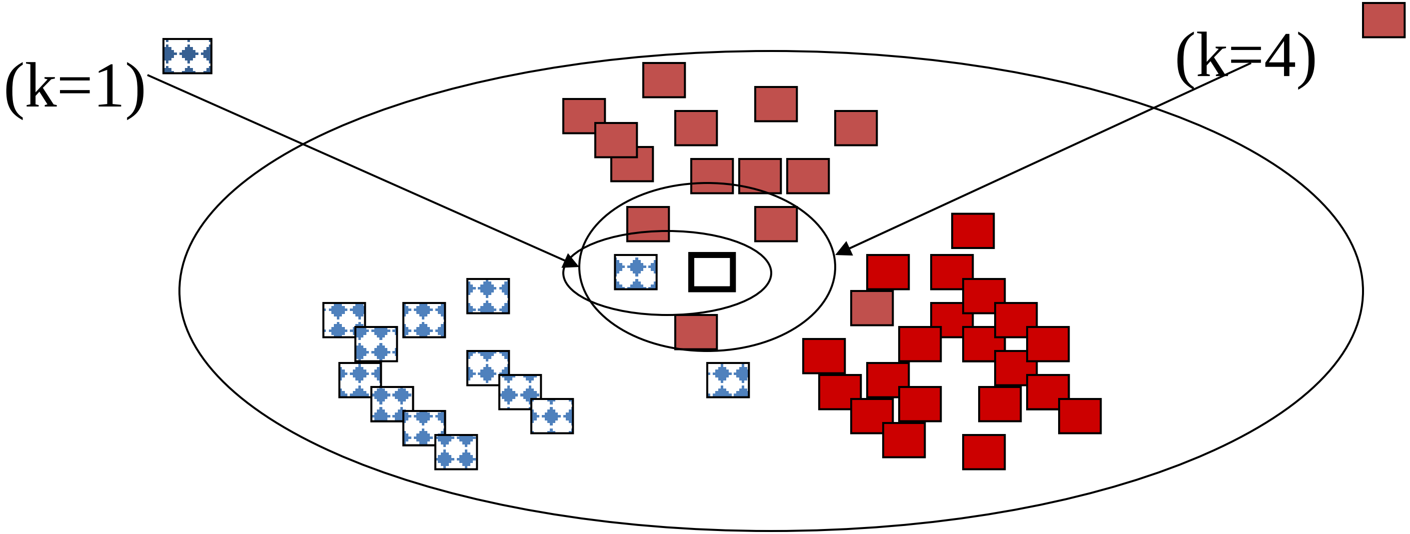
- Maximum Likelihood estimate $\vec{\beta}^* = \arg\max_{\vec{\beta}} p(T \mid \vec{\beta})$

Can be computed in many ways (e.g., Newton's method)

Discriminative Classifier 2: K-Nearest Neighbors (K-NN)

- Find k examples in the training set that are most similar to the text object to be classified (“neighbor” documents)
- Assign the category that is most common in these neighbor text objects (neighbors vote for the category)
- Can be improved by considering the distance of a neighbor (a closer neighbor has more influence)
- Can be regarded as a way to directly estimate the conditional probability of label given data instance, i.e., $p(Y|X)$
- Need a similarity function to measure similarity of two text objects

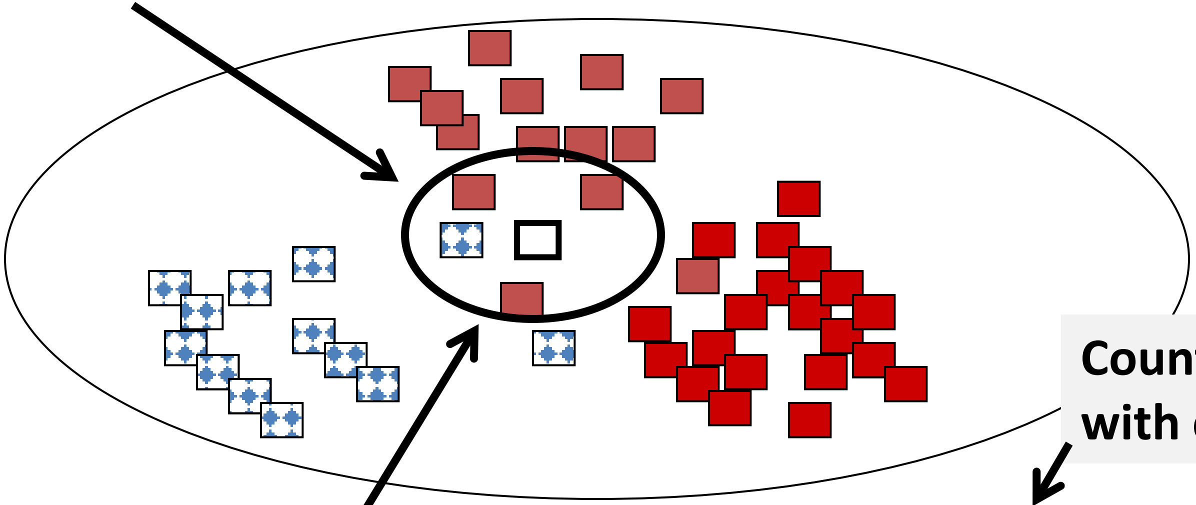
Illustration of K-NN Classifier



K-NN as an Estimate of p(Y|X)

Assume $p(\theta_i|d)$ is <u>locally smooth</u>, i.e., the same for all the d's in this region R

$p(\theta_i|d) = p(\theta_i|R)$



Count of d's in R with category θ_i

$$p(\theta_i \mid R) = \frac{c(\theta_i, R)}{|R|}$$

Total # of docs in R

Estimate $p(\theta_i|R)$ based on the known categories in the region

Text Categorization: Discriminative Classifiers

Part 2

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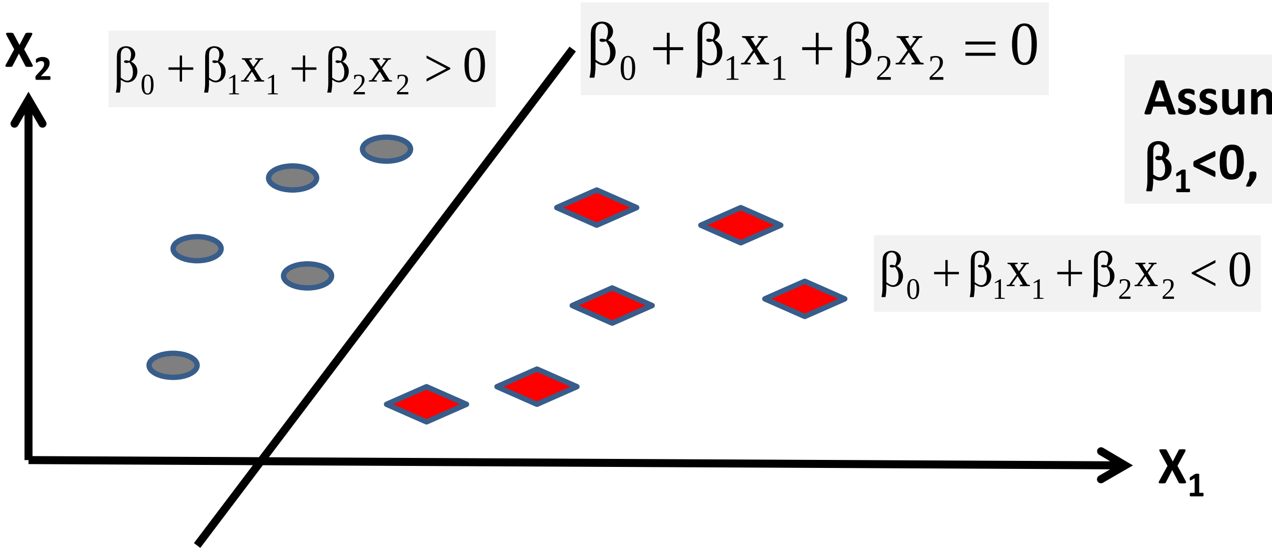
Discriminative Classifier 3: Support Vector Machine (SVM)

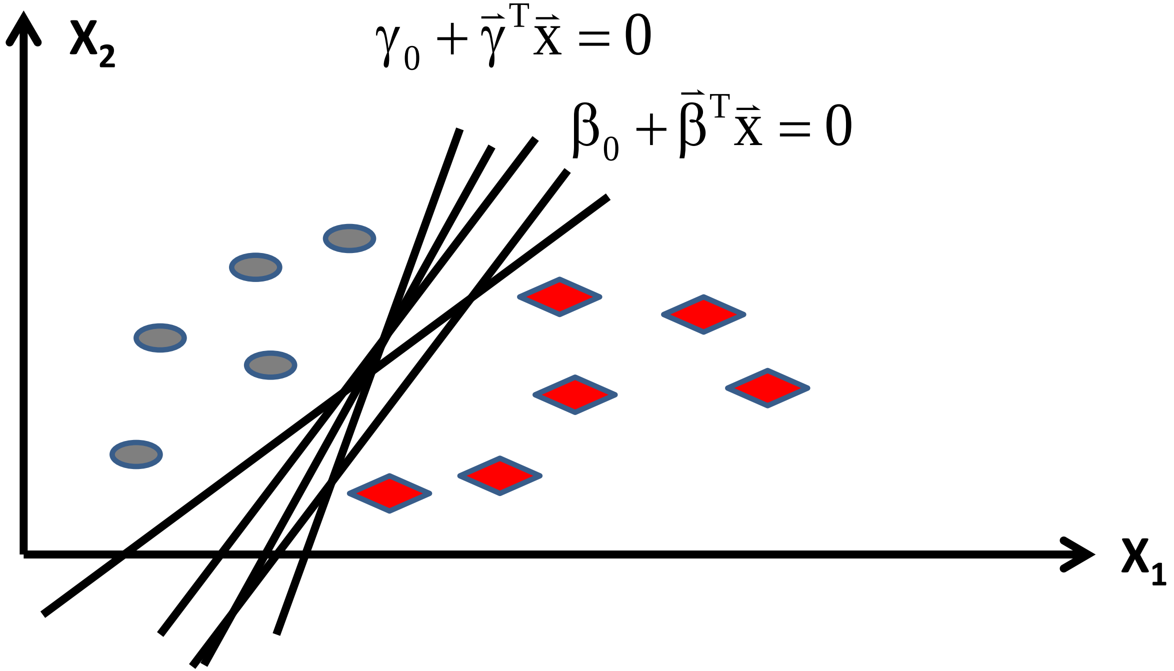
$$f(X) \geq 0 \Rightarrow X \text{ is in category } \theta_1$$
$$f(X) < 0 \Rightarrow X \text{ is in category } \theta_2$$

- Consider two categories: $\{\theta_1, \theta_2\}$
- Use a linear separator $f(X) = \beta_0 + \sum_{i=1}^{M} x_i \beta_i \quad \beta_i \in \Re$



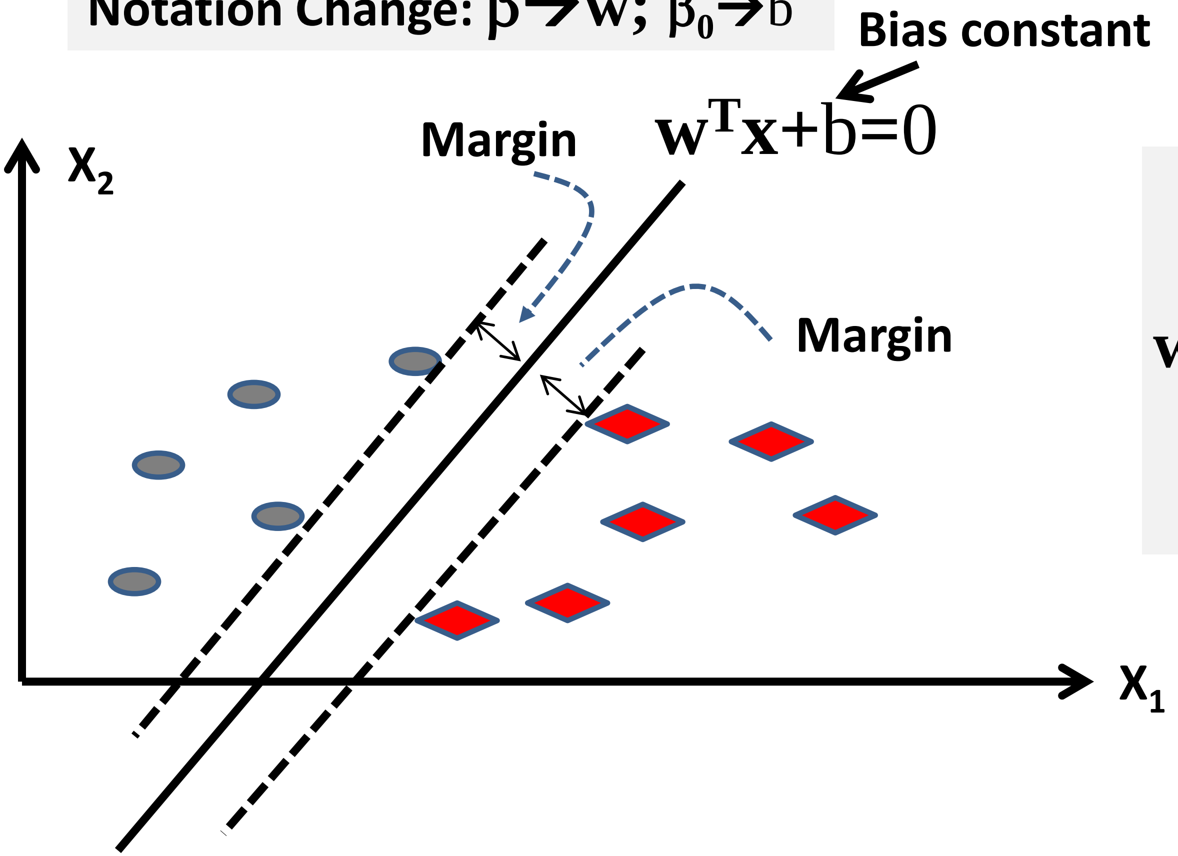
Assume $\beta_1 < 0$, $\beta_2 > 0$

Which Linear Separator Is the Best?



Best Separator = Maximize the Margin

Notation Change: $\beta \rightarrow w$; $\beta_0 \rightarrow b$



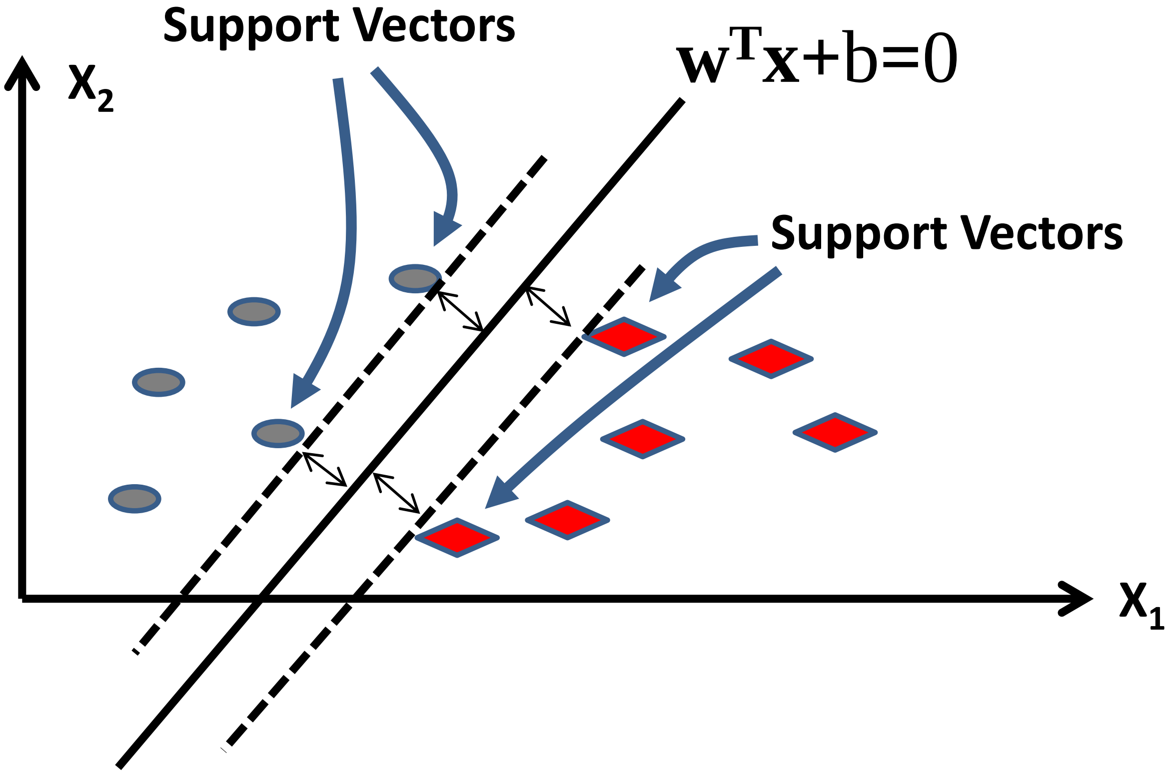
Feature Weights

$$\mathbf{w} = \begin{pmatrix} w_1 \\ w_2 \\ \dots \\ w_M \end{pmatrix}$$

Feature Vector (e.g., word counts)

$$\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ \dots \\ x_M \end{pmatrix}$$

Only the Support Vectors Matter



Linear SVM

Classifier: $\mathbf{f(x)=w^Tx}+b$

Parameters: $\mathbf{w}$, b

$f(X) \geq 0 \Rightarrow X$ is in category θ_1

$f(X) < 0 \Rightarrow X$ is in category θ_2

Training Data: $T=\{(\mathbf{x}_i, \mathbf{y}_i)\}$, $i=1, \ldots, |T|$. $\mathbf{x}_i$ is a feature vector; $\mathbf{y}_i \in \{-1, 1\}$

Goal 1: Correct labeling on training data:

If $\mathbf{y}_i=1$ ➔ $\mathbf{w^Tx_i}+b \geq 1$

If $\mathbf{y}_i=-1$ ➔ $\mathbf{w^Tx_i}+b \leq -1$

Constraint

$$\boxed{\forall \mathbf{i}, \mathbf{y_i}(\mathbf{w^Tx_i}+b)\geq 1}$$

Objective

$$\boxed{\textbf{Minimize } \Phi(\mathbf{w})=\mathbf{w^Tw}}$$

Goal 2: Maximize margin

Large margin $\Leftrightarrow$ **Small** $\mathbf{w^Tw}$

The optimization problem is quadratic programming with linear constraints

Linear SVM with Soft Margin

Classifier: $\mathbf{f(x)}=\mathbf{w}^T\mathbf{x}+b>0$?

Parameters: $\mathbf{w}$, b

Training Data: $T=\{(\mathbf{x}_i, \mathbf{y}_i)\}$, $i=1, \ldots, |T|$.

Added to allow training errors

Find w, b, and ξ_i **to minimize** $\Phi(\mathbf{w})=\mathbf{w}^T\mathbf{w}\boxed{+C\Sigma_{\mathbf{i}\in[\mathbf{1},|\mathbf{T}|]}\xi_i}$

Subject to $\forall \mathbf{i}\in[\mathbf{1},|\mathbf{T}|]$, $\mathbf{y_i}(\mathbf{w}^T\mathbf{x_i}+b)\geq 1\boxed{-\xi_i, \quad \xi_i\geq 0}$

C>0 is a parameter to control the trade-off between minimizing the errors and maximizing the margin

The optimization problem is still quadratic programming with linear constraints

Summary of Text Categorization Methods

- Many methods are available, but no clear winner
  - All require effective feature representation (need domain knowledge)
  - It is useful to compare/combine multiple methods for a particular problem
- Most techniques rely on supervised machine learning and thus can be applied to **any** text categorization problem!
  - Humans annotate training data and design features
  - Computer optimizes the combination of features
  - Good performance requires 1) effective features and 2) plenty of training data
  - Performance is generally (much) more affected by the effectiveness of features than by the choice of a specific classifier

Summary of Text Categorization Methods (cont.)

- How to design effective features? (application-specific)
  - Analyze the categorization problem and exploit domain knowledge
  - Perform error analysis to obtain insights
  - Leverage machine learning techniques (e.g., feature selection, dimension reduction, deep learning)
- How to obtain "enough" training examples?
  - Low-quality ("pseudo") training examples may be leveraged
  - Exploit unlabeled data (using semi-supervised learning techniques)
  - Domain adaptation/transfer learning ("borrow" training examples from a related domain/problem)

Suggested Reading

Manning, Chris D., Prabhakar Raghavan, and Hinrich Schütze. *Introduction to Information Retrieval*. Cambridge: Cambridge University Press, 2007. (Chapters 13-15)

Text Categorization: Evaluation

Part 1

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Overview

- What is text categorization?
- Why text categorization?
- How to do text categorization?
  - Generative probabilistic models
  - Discriminative approaches
- **How to evaluate categorization results?**

General Evaluation Methodology

- Have humans to create a test collection where every document is tagged with the desired categories ("ground truth")
- Generate categorization results using a system on the test collection
- Compare the system categorization decisions with the human-made categorization decisions and quantify their similarity (or equivalently difference)
  - The higher the similarity is, the better the results are
  - Similarity can be measured from different perspectives to understand the quality of results in detail (e.g., which category performs better?)
  - In general, different categorization mistakes may have a different cost that inevitably depends on specific applications, but it is okay not to consider such a cost variation for **relative comparison of methods**

Classification Accuracy (Percentage of Correct Decisions)

	$\mathbf{c_1}$	$\mathbf{c_2}$	$\mathbf{c_3}$	**…**	$\mathbf{c_k}$
$\mathbf{d_1}$	y(+)	y(-)	n(+)		n(+)
$\mathbf{d_2}$	y(-)	n(+)	y(+)		n(+)
$\mathbf{d_3}$	n(+)	n(+)	y(+)		n(+)
…					
$\mathbf{d_N}$	…		…		

+/- human answer

(+= correct; **-** =incorrect)

y/n system result

(y=yes; n=no)

$$\textbf{Classification Accuracy} = \frac{\textbf{Total number of correct decisions}}{\textbf{Total number of decisions made}}$$

$$= \frac{\mathrm{count}(y(+)) + \mathrm{count}(n(-))}{kN}$$

Problems with Classification Accuracy

- Some decision errors are more serious than others
  - It may be more important to get the decisions right on some documents than others
  - It may be more important to get the decisions right on some categories than others
  - E.g., spam filtering: missing a legitimate email costs more than letting a spam go
- Problem with imbalanced test set
  - Skewed test set: 98% in category 1; 2% in category 2
  - Strong baseline: put all instances in category 1 ➔ 98% accuracy!

Per-Document Evaluation

	c_1	c_2	c_3	…	c_k
d_1	y(+)	y(-)	n(+)		n(+)
d_2	y(-)	n(+)	y(+)		n(+)
d_3	n(+)	n(+)	y(+)		n(+)

How good are the decisions on d_i?

When the system says "yes," how many are correct?

Precision $= \dfrac{TP}{TP + FP}$

	System ("y")	System ("n")
Human (+)	True Positives TP	False Negatives FN
Human (-)	False Positives FP	True Negatives TN

Recall $= \dfrac{TP}{TP + FN}$

Does the doc have all the categories it should have?

Per-Category Evaluation

	c_1	c_2	c_3	…	c_k
d_1	y(+)	y(-)	n(+)		n(+)
d_2	y(-)	n(+)	y(+)		n(+)
d_3	n(+)	n(+)	y(+)		n(-)

How good are the decisions on c_i?

	System ("y")	System ("n")
Human (+)	True Positives TP	False Negatives FN
Human (-)	False Positives FP	True Negatives TN

When the system says "yes," how many are correct?

$$\textbf{Precision} = \frac{TP}{TP + FP}$$

$$\textbf{Recall} = \frac{TP}{TP + FN}$$

Has the category been assigned to all the docs of this category?

Combine Precision and Recall: F-Measure

$$F_\beta = \frac{1}{\frac{\beta^2}{\beta^2+1}\frac{1}{R} + \frac{1}{\beta^2+1}\frac{1}{P}} = \frac{(\beta^2+1)P*R}{\beta^2 P + R}$$

$$F_1 = \frac{2PR}{P+R}$$

P: precision

R: recall

β: parameter (often set to 1)

Why not 0.5*P+0.5*R?

What is R if the system says "y" for all category-doc pairs?

Text Categorization: Evaluation

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(Macro) Average Over All the Categories

	c_1	c_2	c_3	…	c_k
d_1	y(+)	y(-)	n(+)		n(+)
d_2	y(-)	n(+)	y(+)		n(+)
d_3	n(+)	n(+)	y(+)		n(+)
…					
d_N	…	…			

Aggregate

Precision	**p1**	**p2**	**p3**	**…**	**pk**	→ **Overall Precision**
Recall	**r1**	**r2**	**r3**	**…**	**rk**	→ **Overall Recall**
F-Measure	**f1**	**f2**	**f3**	**…**	**fk**	→ **Overall F score**

(Macro) Average Over All the Documents

	$\mathbf{c_1}$	$\mathbf{c_2}$	$\mathbf{c_3}$	…	$\mathbf{c_k}$	**Precision**	**Recall**	**F-Measure**
$\mathbf{d_1}$	y(+)	y(-)	n(+)		n(+)	**p1**	**r1**	**f1**
$\mathbf{d_2}$	y(-)	n(+)	y(+)		n(+)	**p2**	**r2**	**f2**
$\mathbf{d_3}$	n(+)	n(+)	y(+)		n(+)			
…						**…**	**…**	**…**
$\mathbf{d_N}$						**pN**	**rN**	**fN**

Aggregate

Overall Precision

Overall Recall

Overall F score

Micro-Averaging of Precision and Recall

$$\begin{array}{c|ccccc|}
 & \mathbf{c_1} & \mathbf{c_2} & \mathbf{c_3} & \ldots & \mathbf{c_k} \\
\hline
\mathbf{d_1} & y(+) & y(-) & n(+) & & n(+) \\
\mathbf{d_2} & y(-) & n(+) & y(+) & & n(+) \\
\mathbf{d_3} & n(+) & n(+) & y(+) & & n(+) \\
\ldots & & & & & \\
\mathbf{d_N} & \ldots & & \ldots & & \\
\hline
\end{array}$$

First pool all decisions, then compute precision and recall

	System ("y")	System ("n")
Human (+)	True Positives (TP)	False Negatives (FN)
Human (-)	False Positives(FP)	True Negatives(TN)

$$\textbf{Precision} = \frac{\mathrm{TP}}{\mathrm{TP}+\mathrm{FP}}$$

$$\textbf{Recall} = \frac{\mathrm{TP}}{\mathrm{TP}+\mathrm{FN}}$$

Sometimes Ranking Is More Appropriate

- The categorization results are often passed to a human for
  - further editing (e.g., correcting system mistakes on news categories)
  - prioritizing a task (e.g., routing an email to the right person for processing)
- In such cases, we can evaluate the results as a ranked list if the system can give scores for the decisions
  - E.g., discovery of spam emails (➔ rank emails for the “spam” category)
  - Often more appropriate to frame the problem as a ranking problem instead of a categorization problem (e.g., ranking documents in a search engine)

Summary of Categorization Evaluation

- Evaluation is always very important, so get it right!
- Measures must reflect the **intended use** of the results for a particular application (e.g., spam filtering vs. news categorization)
  - Consider: How will the results be further processed (by a user)?
  - Ideally associate a different cost with each different decision error
- Commonly used measures for **relative** comparison of different methods:
  - Accuracy, precision, recall, F score
  - Variations: per-document, per-category, micro vs. macro averaging
- Sometimes **ranking** may be more appropriate

Suggested Reading

- Manning, Chris D., Prabhakar Raghavan, and Hinrich Schütze. *Introduction to Information Retrieval*. Cambridge: Cambridge University Press, 2007. (Chapters 13-15)
- Yang, Yiming. 1999. An Evaluation of Statistical Approaches to Text Categorization. *Inf. Retr.* 1, 1-2 (May 1999), 69-90. DOI=10.1023/A:1009982220290

Opinion Mining and Sentiment Analysis: Motivation

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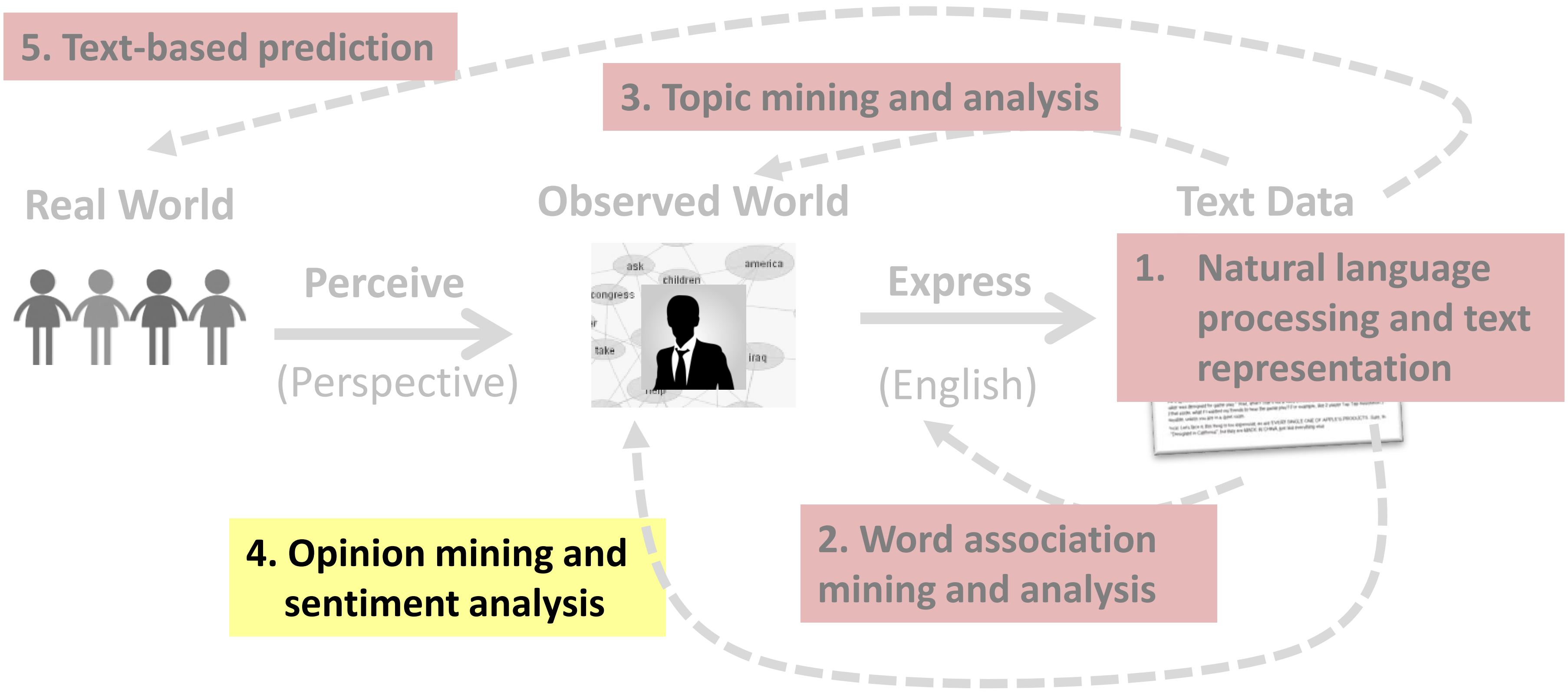
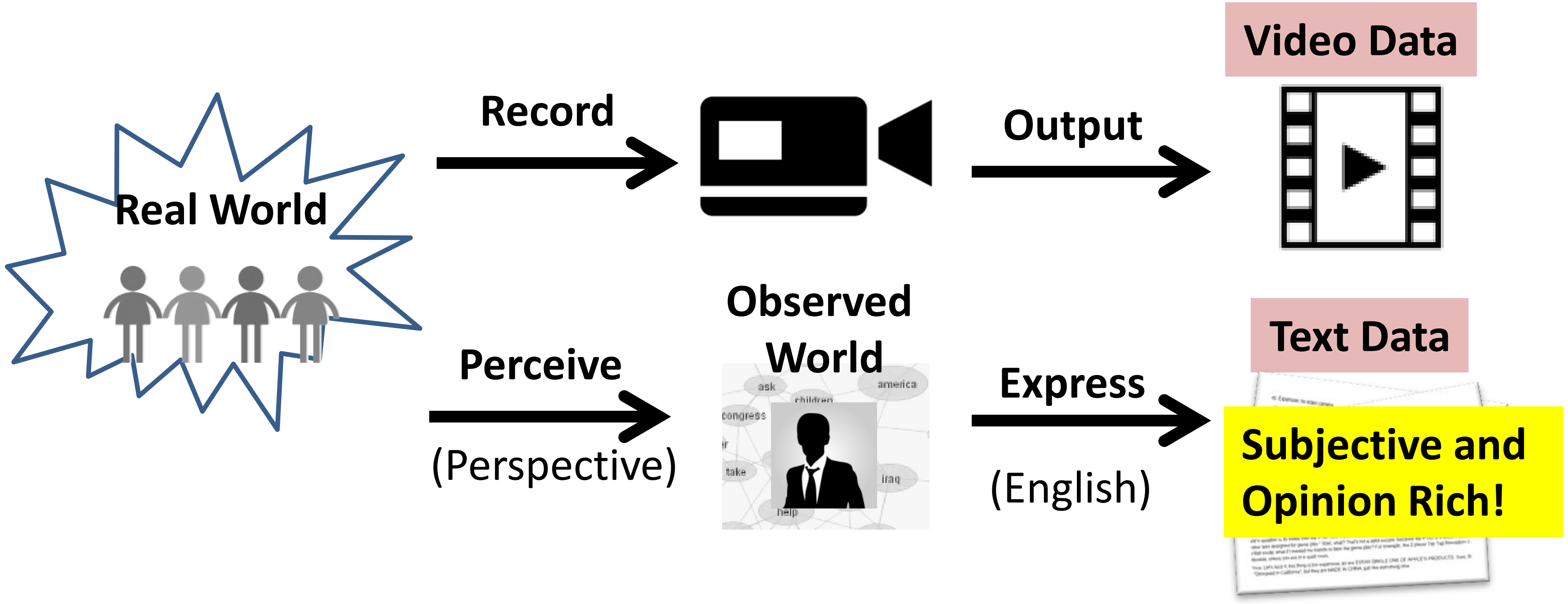
Opinion Mining and Sentiment Analysis: Motivation



Objective vs. Subjective Sensors



How can we mine and analyze opinion buried in text?

What Is an Opinion?

Objective statement or **Factual** statement **(can be proved right/wrong)**

Opinion ≈ a **subjective** statement describing what a **person** **believes** or **thinks** about **something**

Opinion Target

Opinion Holder

Depends on **culture, background, and context**

Opinion Representation

- Basic Opinion Representation
  – Opinion **holder**: Whose opinion is this?
  – Opinion **target**: What is this opinion about?
  – Opinion **content**: What exactly is the opinion?
- Enriched Opinion Representation
  – Opinion **context**: Under what situation (e.g., time, location) was the opinion expressed?
  – Opinion **sentiment**: What does the opinion tell us about the opinion holder's feeling (e.g., positive vs. negative)?

A Product Review (Explicit Holder and Target)

- Basic Opinion Representation
  - Opinion **holder**: Whose opinion is this?
  - Opinion **target**: What is this opinion about?
  - Opinion **content**: What exactly is the opinion?
- Enriched Opinion Representation
  - Opinion **context**: Under what situation (e.g., time, location) was the opinion expressed?
  - Opinion **sentiment**: What does the opinion tell us about the opinion holder's feeling (e.g., positive vs. negative)?

Reviewer X

Product: iPhone 6

Review Text

Year = 2015

Positive

Relatively Easy to Mine and Analyze

A Sentence in News (Implicit Holder and Target)

Target **Holder**

"… In an effort to get residents to wake up and pay attention to **Hurricane Sandy**, the **governor of Connecticut** just **said** that **Sandy** might be as **bad** as the **worst hurricane** ever to hit **New England**--the **hurricane of 1938**…. "

Negative **Context**

Target

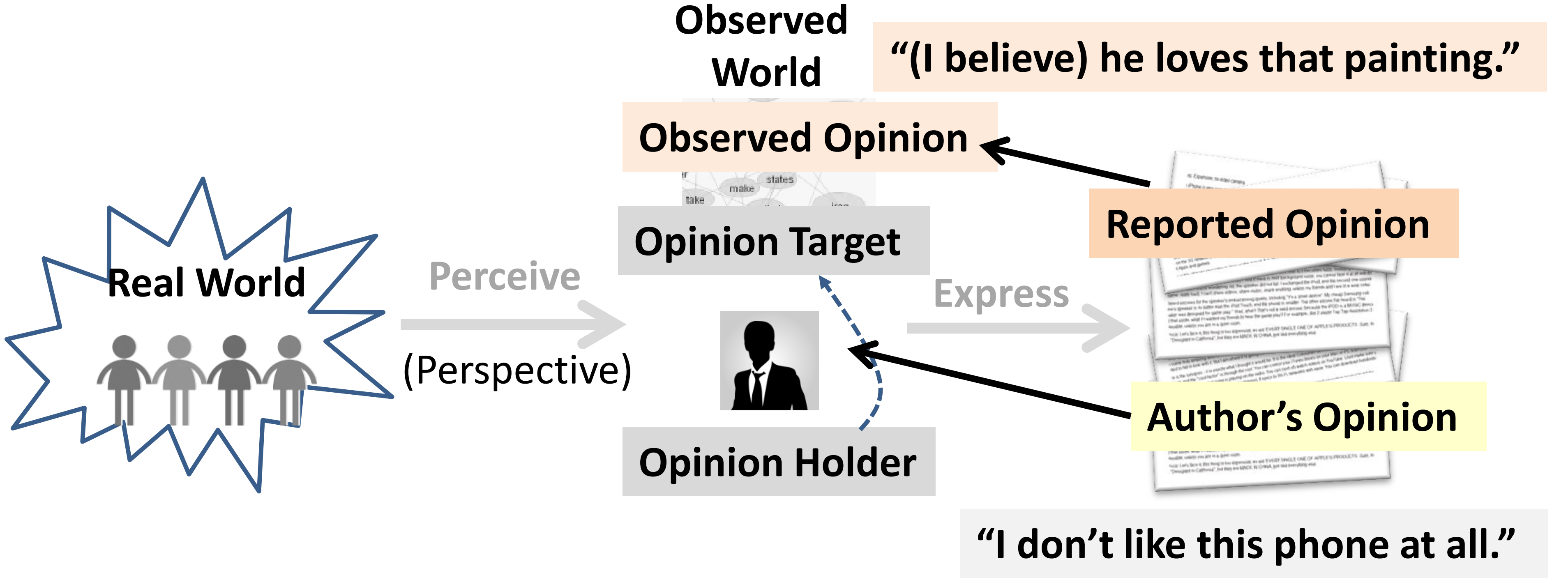
Harder to Mine and Analyze: Need deeper NLP

Source: Blodget, H. (2012, October 28). Hurricane Sandy is being compared to the worst hurricane ever to hit New England. *Business Insider*. *Business Insider*. Retrieved from http://www.businessinsider.com/hurricane-sandy-vs-hurricane-of-1938-2012-10.

Variations of Opinions

- **Opinion holder**: Individual vs. group
- **Opinion target**: One entity, a group of entities, one attribute of an entity, someone else's opinion, etc.
- **Opinion content**:
  - Surface variation: one sentence/phrase, a paragraph, a whole article
  - Sentiment/emotion variation: positive vs. negative, happy vs. sad, etc.
- **Opinion context**
  - Simple context: Different time, location, etc.
  - Complex context: Potentially includes the entire discourse context of an opinion

Different Kinds of Opinions in Text Data



Indirect/Inferred Opinion: "This phone ran out of battery in just 1 hour."

The Task of Opinion Mining

Text Data

A Set of Opinion Representations

Opinion Holder

Opinion Target

Opinion Content

Opinion Context

Opinion Sentiment

Often some elements of the representation are already known

Simplest Opinion Mining task(s)?

Why Opinion Mining?

- **Decision Support**
  - Help consumers choose a product or service
  - Help voters decide whom to vote for
  - Help policy makers design new policy
- **Understand People**
  - Help understand people's preferences to better serve them (e.g., optimize a product search engine; optimize recommender systems)
  - Help with advertising (targeted advertising)
- **"Voluntary Survey" (humans as sensors; aggregated opinions)**
  - Business intelligence
  - Market research
  - Data-driven social science research
  - Gain advantage in **any** prediction (text-based prediction)

Opinion Mining and Sentiment Analysis: Sentiment Classification

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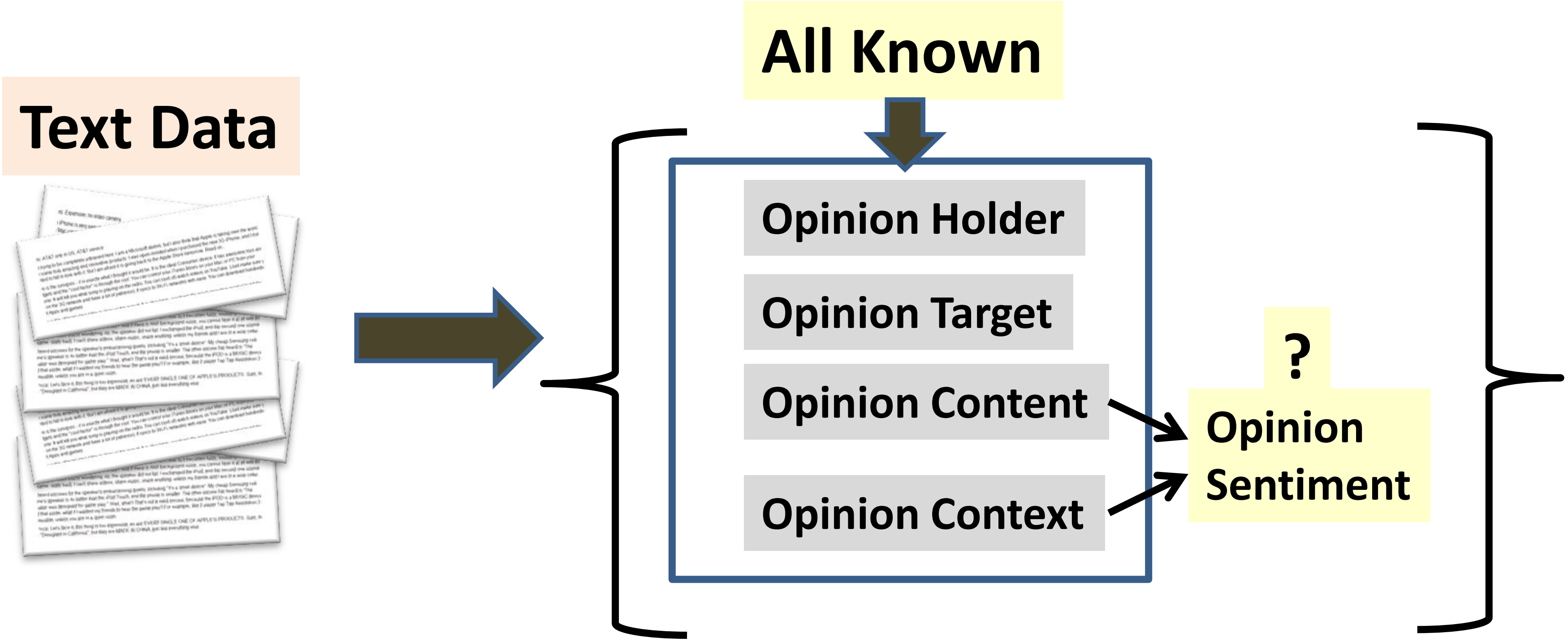
Sentiment Classification



Sentiment Classification: Task Definition

- Input: An opinionated text object
- Output: A sentiment tag/label
  - Polarity analysis: e.g., categories = {positive, negative, neutral}, or categories ={5, 4, 3, 2, 1}
  - Emotion analysis (beyond polarity): e.g., categories ={happy, sad, fearful, angry, surprised, disgusted}
- A special case of text categorization! ➔ Any text categorization method can be used to do sentiment classification
- Further improvement comes from
  - More sophisticated features appropriate for sentiment tagging
  - Consideration of the order of the categories (e.g., ordinal regression)

Commonly Used Text Features

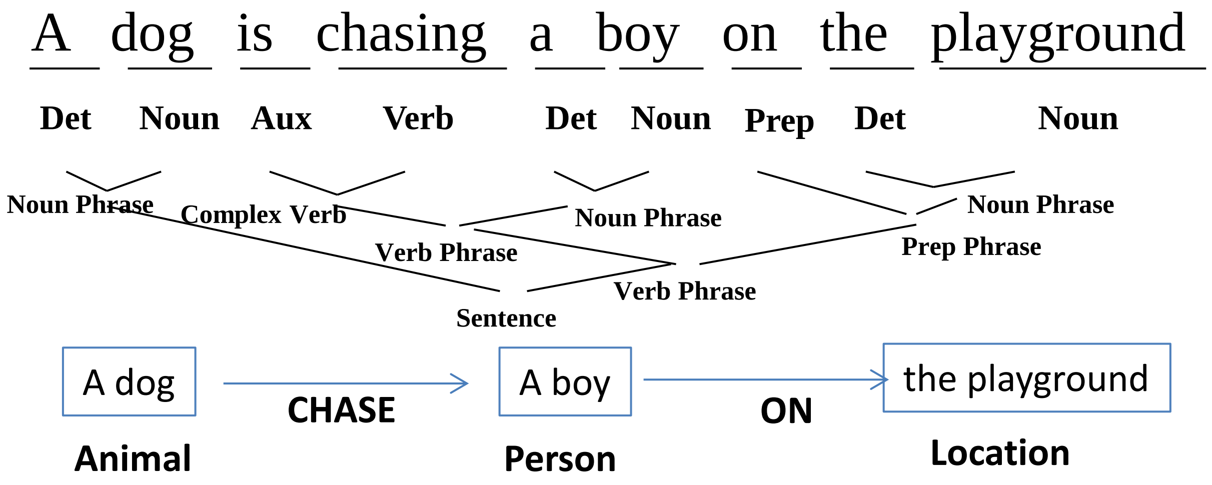
- Character n-grams: can be mixed with different n's
  - General and robust to spelling/recognition errors, but less discriminative than words
- Word n-grams: can be mixed with different n's
  - Unigrams are often very effective, but not for sentiment analysis (e.g. , "it's not good" or "it's not as good as")
  - Long n-grams are discriminative, but may cause overfitting
- POS tag n-grams: mixed n-gram with words and POS tags
  - E.g., "ADJECTIVE NOUN" or "great NOUN"

Commonly Used Text Features (cont.)

- Word classes
  - Syntactic (= POS tags)
  - Semantic Concept: e.g., thesaurus/ontology, recognized entities
  - Empirical word clusters (e.g., cluster of paradigmatically or syntagmatically related words)
- Frequent patterns in text (e.g., frequent word set; collocations)
  - More specific/discriminative than words
  - May generalize better than pure n-grams
- Parse tree-based (e.g., frequent subtrees, paths)
  - Even more discriminative, but need to avoid overfitting
- Pattern discovery algorithms are very useful for feature construction

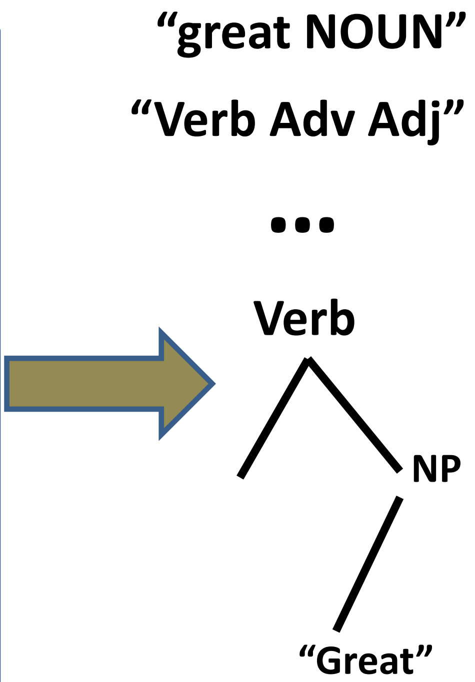
NLP Enriches Text Representation with Complex Features

A dog is chasing a boy on the playground



Dog(d1). Boy(b1). Playground(p1). Chasing(d1,b1,p1).

Speech Act = REQUEST



Feature Construction for Text Categorization

- Feature design affects categorization accuracy significantly
- A combination of machine learning, error analysis, and domain knowledge is most effective
  - Domain knowledge → seed features, feature space
  - Machine learning → feature selection, feature learning
  - Error analysis → feature validation
- NLP enriches text representation ➔ enriches feature space (more likely overfitting!)
- Optimizing the tradeoff between **exhaustivity** and **specificity** is a major goal

high coverage (frequent)

discriminative (infrequent)

Sentiment Analysis: Ordinal Logistic Regression

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Motivation: Rating Prediction

- Input: An opinionated text document **d**
- Output: Discrete rating **r** $\in${**1, 2, …, k**}
- Using regular text categorization techniques
  - Doesn't consider the order and dependency of the categories
  - The features distinguishing r=2 from r=1 may be the same as those distinguishing r=k from r=k-1 (e.g., positive words generally suggest a higher rating)
- Solution: Add order to a classifier (e.g., ordinal logistic regression)

Logistic Regression for Binary Sentiment Classification

Binary Response Variable: Y $\in$\{0,1\} **Predictors:** $X = (x_1, x_2, \ldots, x_M), \; x_i \in \Re$

$$Y = \begin{cases} 1 & X \text{ is POSITIVE} \\ 0 & X \text{ is NEGATIVE} \end{cases}$$

$$\log \frac{p(Y=1|X)}{p(Y=0|X)} = \log \frac{p(Y=1|X)}{1 - p(Y=1|X)} = \beta_0 + \sum_{i=1}^{M} x_i \beta_i \quad \beta_i \in \Re$$

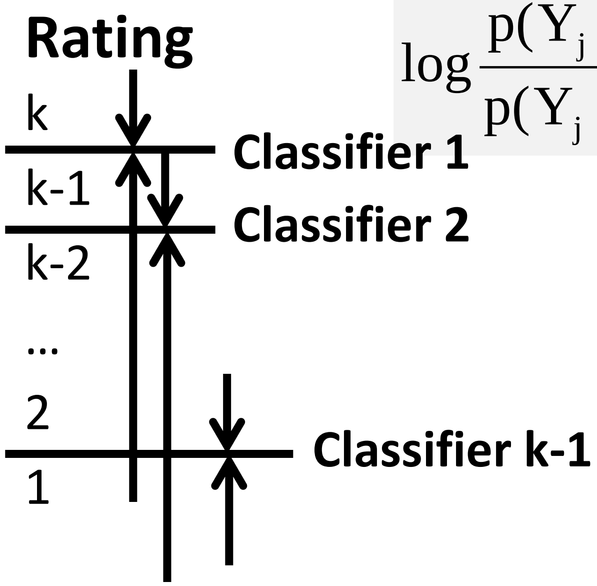
$$p(Y=1|X) = \frac{e^{\beta_0 + \sum_{i=1}^{M} x_i \beta_i}}{e^{\beta_0 + \sum_{i=1}^{M} x_i \beta_i} + 1}$$

Logistic Regression for Multi-Level Ratings

$$Y_j = \begin{cases} 1 & \text{rating is } j \text{ or above} \\ 0 & \text{rating is lower than } j \end{cases}$$

Predictors: $X = (x_1, x_2, \ldots, x_M),\ \ x_i \in \Re$

Rating: $r \in \{1, 2, \ldots, k\}$



$$\log \frac{p(Y_j = 1 \mid X)}{p(Y_j = 0 \mid X)} = \log \frac{p(r \ge j \mid X)}{1 - p(r \ge j \mid X)} = \alpha_j + \sum_{i=1}^{M} x_i \beta_{ji} \qquad \beta_{ji} \in \Re$$

$$p(r \ge j \mid X) = \frac{e^{\alpha_j + \sum_{i=1}^{M} x_i \beta_{ji}}}{e^{\alpha_j + \sum_{i=1}^{M} x_i \beta_{ji}} + 1}$$

Rating Prediction with Multiple Logistic Regression Classifiers

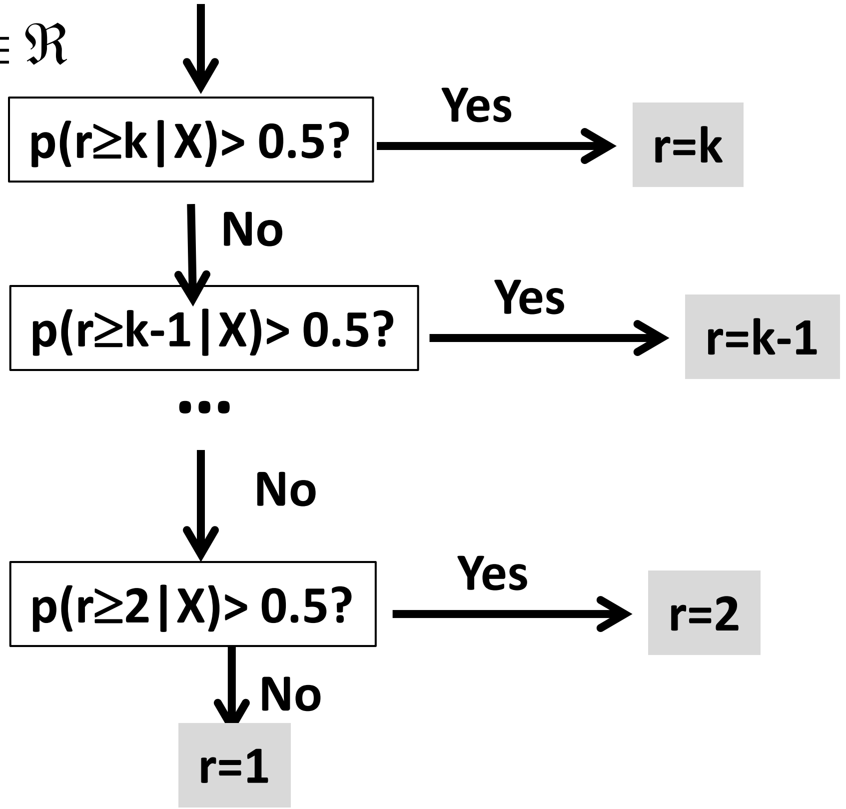
Text Object: $X = (x_1, x_2, \ldots, x_M),\ \ x_i \in \Re$

Rating: $r \in \{1, 2, \ldots, k\}$

After training k-1 Logistic Regression Classifiers

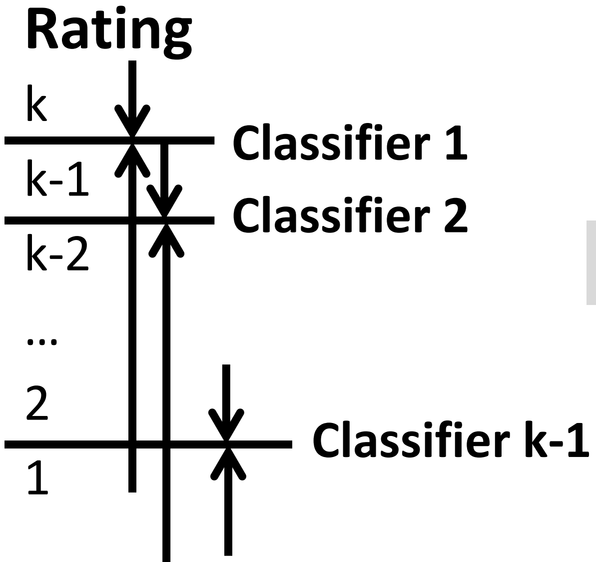
$$p(r \geq j \mid X) = \frac{e^{\alpha_j + \sum_{i=1}^{M} x_i \beta_{ji}}}{e^{\alpha_j + \sum_{i=1}^{M} x_i \beta_{ji}} + 1}$$

j=k, k-1, …, 2



Problems with k-1 Independent Classifiers?

$$\log \frac{p(Y_j = 1 \mid X)}{p(Y_j = 0 \mid X)} = \log \frac{p(r \geq j \mid X)}{1 - p(r \geq j \mid X)} = \alpha_j + \sum_{i=1}^{M} x_i \beta_{ji} \quad \beta_{ji} \in \Re$$



$$p(r \geq j \mid X) = \frac{e^{\alpha_j + \sum_{i=1}^{M} x_i \beta_{ji}}}{e^{\alpha_j + \sum_{i=1}^{M} x_i \beta_{ji}} + 1}$$

How many parameters are there in total? **(k-1)*(M+1)**

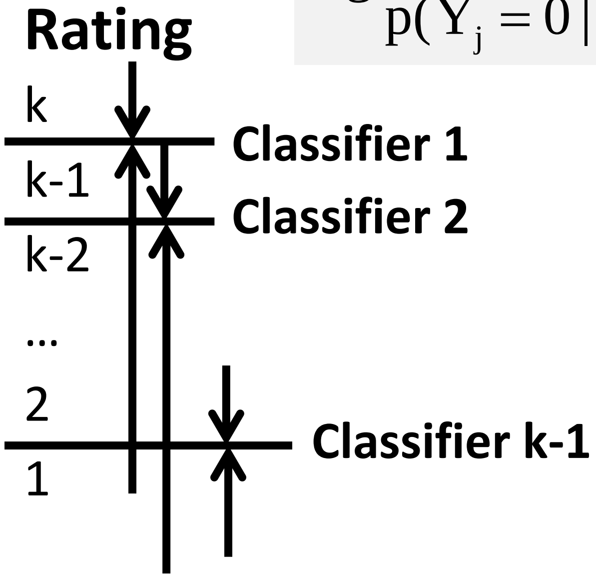
The k-1 classification problems are dependent.
The positive/negative features tend to be similar!

Ordinal Logistic Regression

Key Idea: $\forall i = 1, \ldots, M, \forall j = 3, \ldots, k, \beta_{ji} = \beta_{j-1i}$

➔ **Share training data** ➔ **Reduce # of parameters**

$$\log \frac{p(Y_j = 1 | X)}{p(Y_j = 0 | X)} = \log \frac{p(r \geq j | X)}{1 - p(r \geq j | X)} = \alpha_j + \sum_{i=1}^{M} x_i \beta_i \quad \beta_i \in \Re$$



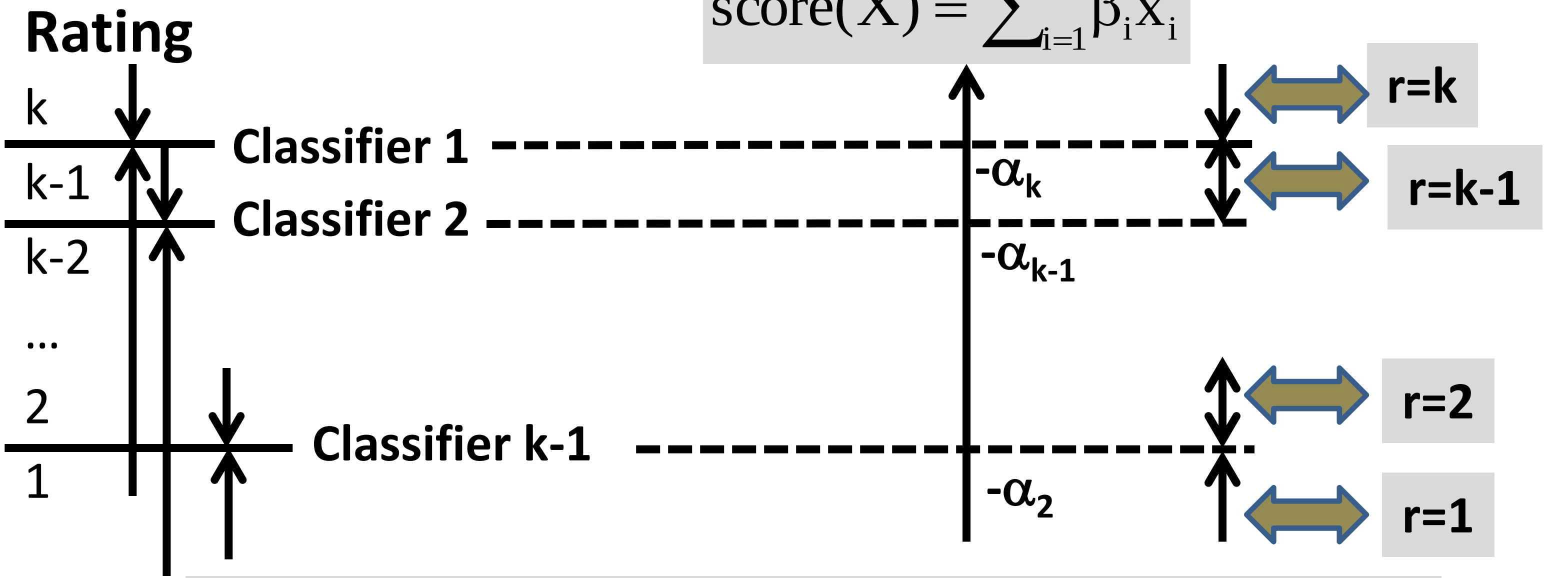
$$p(r \geq j | X) = \frac{e^{\alpha_j + \sum_{i=1}^{M} x_i \beta_i}}{e^{\alpha_j + \sum_{i=1}^{M} x_i \beta_i} + 1}$$

How many parameters are there in total? **M+k-1**

Ordinal Logistic Regression: Rating Prediction

$$p(r \geq j \mid X) \geq 0.5 \Leftrightarrow \frac{e^{\alpha_j + \mathrm{score}(X)}}{e^{\alpha_j + \mathrm{score}(X)} + 1} \geq 0.5 \Leftrightarrow \mathrm{score}(X) \geq -\alpha_j$$

$$\mathrm{score}(X) = \sum_{i=1}^{M} \beta_i x_i$$



r=j $\Leftrightarrow$ score $\in [-\alpha_j, -\alpha_{j+1})$, define $\alpha_1 = \infty$, $\alpha_{k+1} = -\infty$