Syntagmatic Relation Discovery: Entropy

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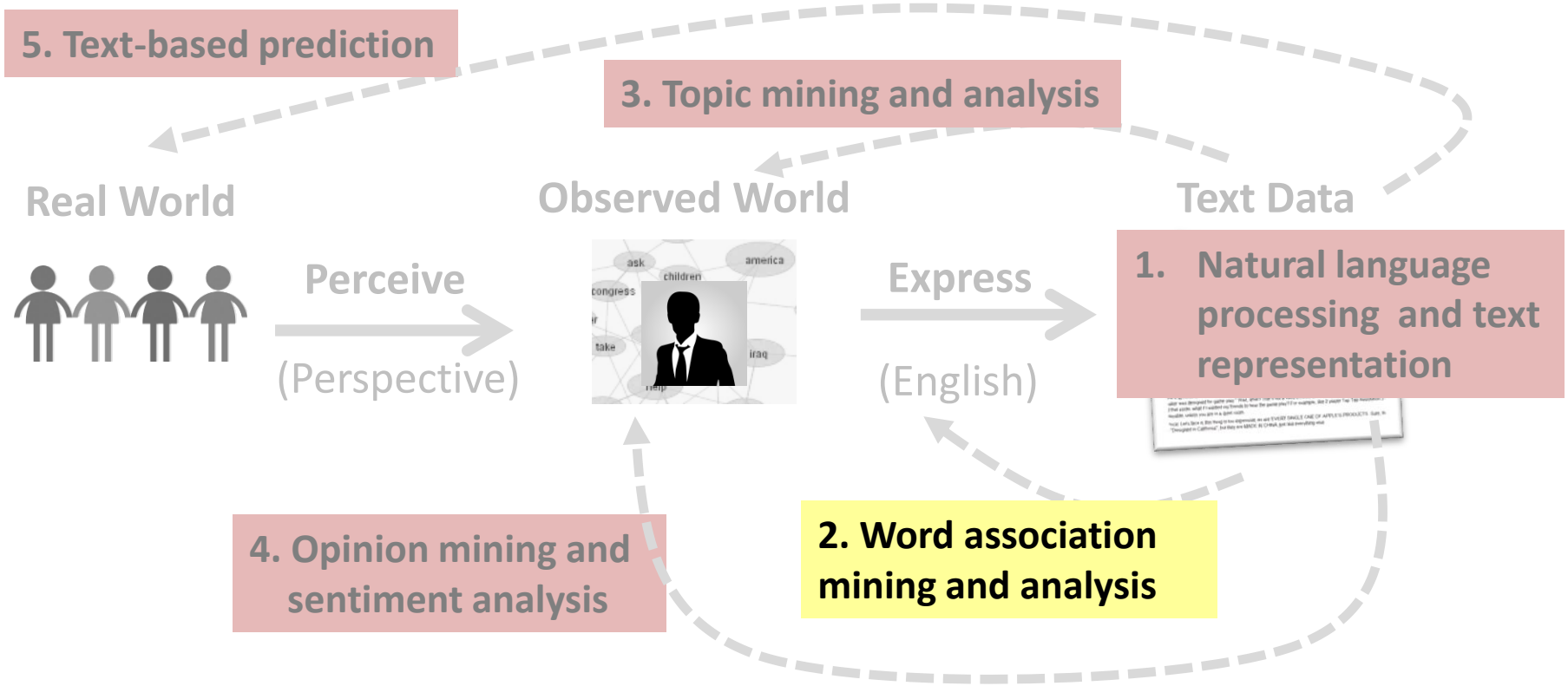
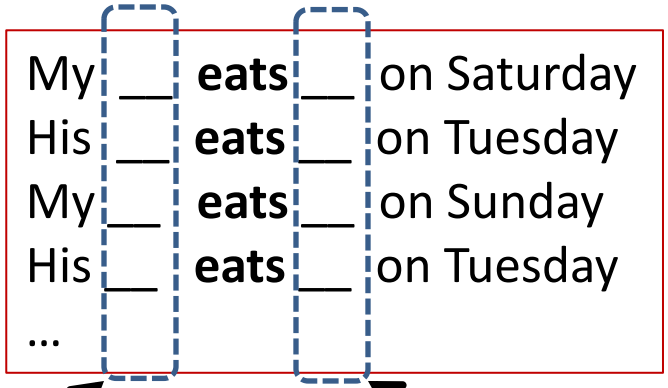


Syntagmatic Relation = Correlated Occurrences

Whenever "**eats**" occurs, what **other words** also tend to occur?

My cat **eats** fish on Saturday
His cat **eats** turkey on Tuesday
My dog **eats** meat on Sunday
His dog **eats** turkey on Tuesday
…



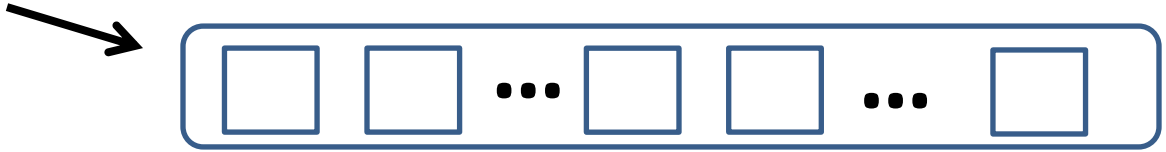
What words tend to occur to the **left** of "**eats**"**?**

What words are to the **right?**

Word Prediction: Intuition

Prediction Question: Is word **W** present (or absent) in this segment?

Text Segment (any unit, e.g., sentence, paragraph, document)



Are some words easier to predict than others?

1) W = “meat” **2) W=“the”** **3) W=“unicorn”**

Word Prediction: Formal Definition

Binary Random Variable : $X_w \in \{0, 1\}$

$$X_w = \begin{cases} 1 & w \text{ is present} \\ 0 & w \text{ is absent} \end{cases}$$

$$p(X_w = 1) + p(X_w = 0) = 1$$

The more random X_w is, the more difficult the prediction would be.

How does one quantitatively measure the "randomness" of a random variable like Xw?

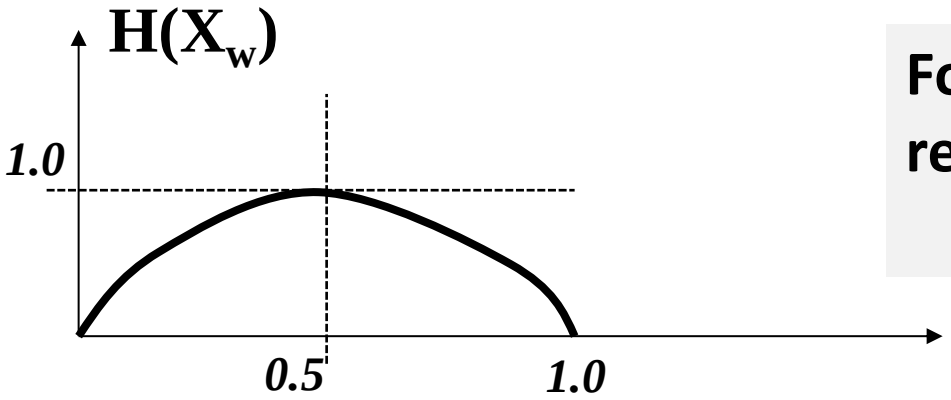
Entropy H(X) Measures Randomness of X

$$H(X_w) = \sum_{v \in \{0,1\}} -p(X_w = v)\log_2 p(X_w = v)$$

$$= -p(X_w = 0)\log_2 p(X_w = 0) - p(X_w = 1)\log_2 p(X_w = 1)$$

$$X_w = \begin{cases} 1 & w \text{ is present} \\ 0 & w \text{ is absent} \end{cases}$$

Define $0 \log_2 0 = 0$



For what X_w , does $H(X_w)$ reach maximum/minimum?

E.g., $P(X_w=1)=1$? $P(X_w=1)=0.5$?

or equivalently P(Xw=0) (Why?)

Entropy H(X): Coin Tossing

$$H(X_{coin}) = -p(X_{coin}=0)\log_2 p(X_{coin}=0) - p(X_{coin}=1)\log_2 p(X_{coin}=1)$$

X_{coin}: tossing a coin

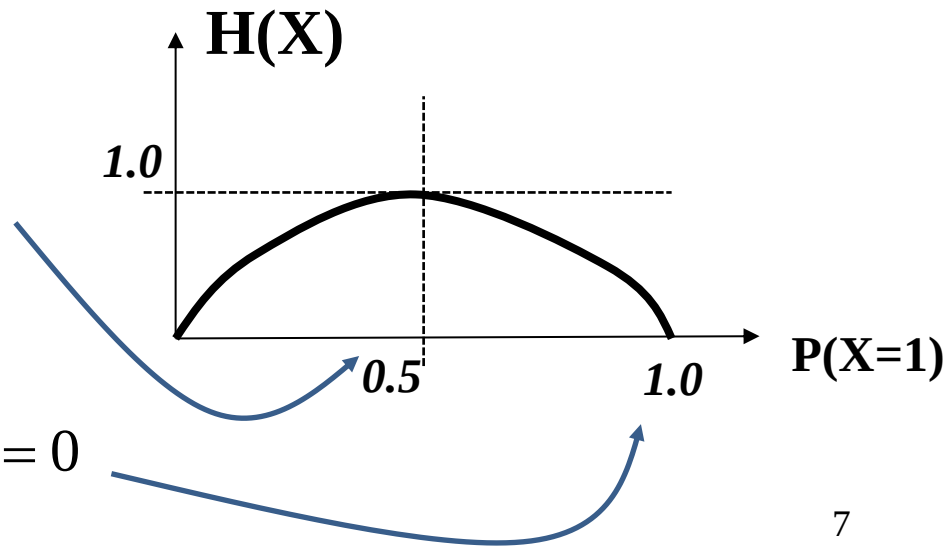
$$X_{coin} = \begin{cases} 1 & \text{Head} \\ 0 & \text{Tail} \end{cases}$$

Fair coin: p(X=1)=p(X=0)=1/2

$$H(X) = -\frac{1}{2}\log_2\frac{1}{2} - \frac{1}{2}\log_2\frac{1}{2} = 1$$

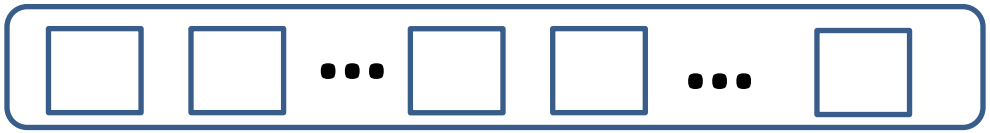
Completely biased: p(X=1)=1

$$H(X) = -0*\log_2 0 - 1*\log_2 1 = 0$$



Entropy for Word Prediction

Is word **W** present (or absent) in this segment?



1) W = "meat" 2) W = "the" 3) W = "unicorn"

Which is **high/low**? **$H(X_{meat})$, $H(X_{the})$, or $H(X_{unicorn})$?**

$H(X_{the}) \approx 0$ ➔ no uncertainty since $p(X_{the}=1) \approx 1$

High entropy words are harder to predict!