Text Clustering: Generative Probabilistic Models

Part 3

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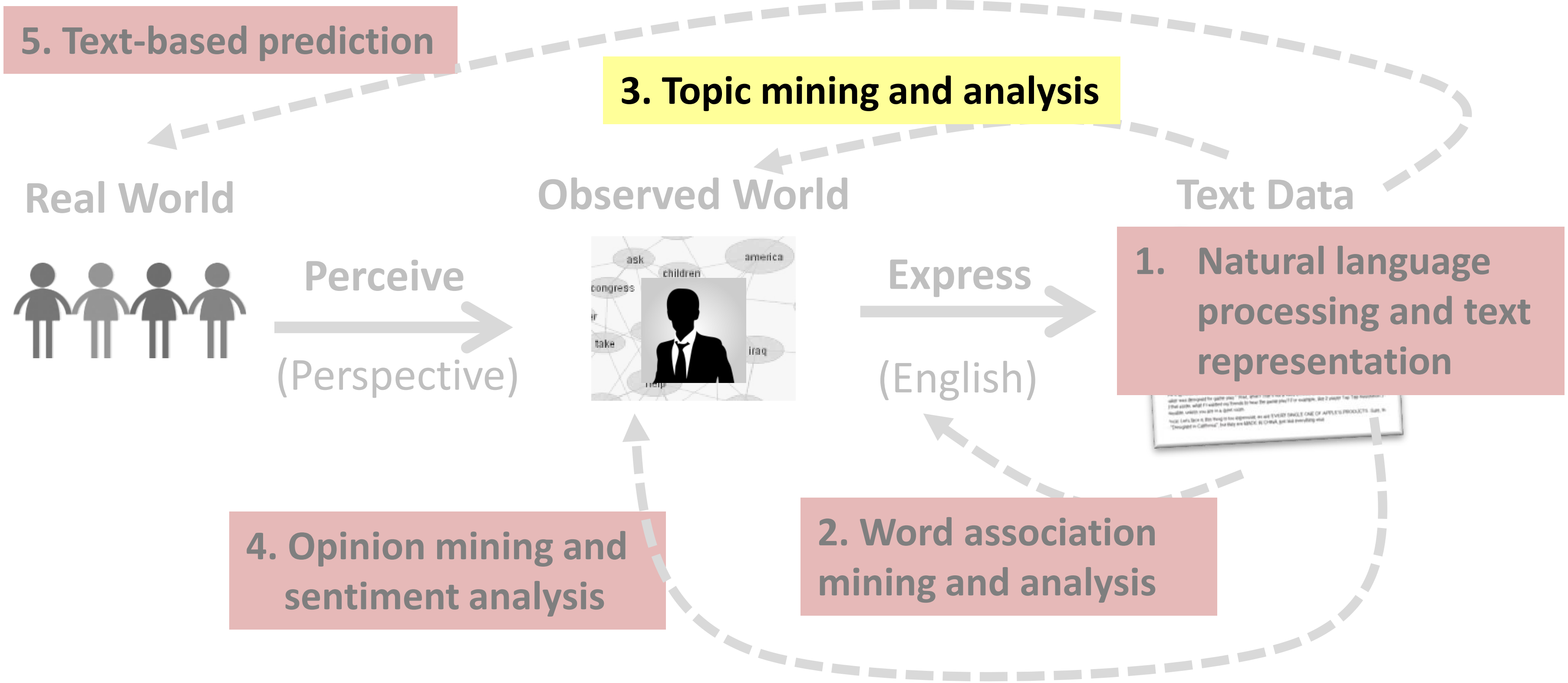
Text Clustering: Generative Probabilistic Models (Part 3)



5. Text-based prediction

3. Topic mining and analysis

Real World

Observed World

Text Data

Perceive

(Perspective)

Express

(English)

1. Natural language processing and text representation

4. Opinion mining and sentiment analysis

2. Word association mining and analysis

How Can We Compute the ML Estimate?

- Data: a collection of documents C={d_1, …, d_N}
- Model: mixture of k unigram LMs: Λ=({θ_i}; {p(θ_i)}), i$\in$[1,k]
  - To generate a document, first **choose a $\boldsymbol{\theta_i}$** according to p(θ_i) and then generate **all** words in the document using p(w|θ_i)
- Likelihood:

$$p(d \mid \Lambda) = \sum_{i=1}^{k} [p(\theta_i) \prod_{w \in V} p(w \mid \theta_i)^{c(w,d)}]$$

$$p(C \mid \Lambda) = \prod_{j=1}^{N} p(d_j \mid \Lambda)$$

- Maximum Likelihood estimate

$$\Lambda^* = \arg\max_{\Lambda} p(C \mid \Lambda)$$

EM Algorithm for Document Clustering

- Initialization: Randomly set **Λ=({θ_i}; {p(θ_i)}), i$\in$[1,k]**
- **Repeat until likelihood p(C|Λ) converges**
  - **E-Step: Infer which distribution has been used to generate document d: hidden variable Z_d $\in$[1, k]**

$$p^{(n)}(Z_d = i \mid d) \propto p^{(n)}(\theta_i) \prod_{w \in V} p^{(n)}(w \mid \theta_i)^{c(w,d)}$$

$$\sum_{i=1}^{k} p^{(n)}(Z_d = i \mid d) = 1$$

  - **M-Step: Re-estimation of all parameters**

$$p^{(n+1)}(\theta_i) \propto \sum_{j=1}^{N} p^{(n)}(Z_{d_j} = i \mid d_j)$$

$$\sum_{i=1}^{k} p^{(n+1)}(\theta_i) = 1$$

$$p^{(n+1)}(w \mid \theta_i) \propto \sum_{j=1}^{N} c(w, d_j) p^{(n)}(Z_{d_j} = 1 \mid d_j)$$

$$\sum_{w \in V} p^{(n+1)}(w \mid \theta_i) = 1, \quad \forall i \in [1, k]$$

An Example of 2 Clusters

Random Initialization

$p(\theta_1) = p(\theta_2) = 0.5$

	$p(w\|\theta_1)$	$p(w\|\theta_2)$
text	0.5	0.1
mining	0.2	0.1
medical	0.2	0.75
health	0.1	0.05

E-step **Document d**

Hidden variables:

$Z_d \in \{1, 2\}$

	c(w,d)
text	2
mining	2
medical	0
health	0

$$p(Z_d = 1 | d) = \frac{p(\theta_1) p(\text{"text"}|\theta_1)^2 p(\text{"mining"}|\theta_1)^2}{p(\theta_1) p(\text{"text"}|\theta_1)^2 p(\text{"mining"}|\theta_1)^2 + p(\theta_2) p(\text{"text"}|\theta_2)^2 p(\text{"mining"}|\theta_2)^2}$$

$$= \frac{0.5 * 0.5^2 * 0.2^2}{0.5 * 0.5^2 * 0.2^2 + 0.5 * 0.1^2 * 0.1^2} = \frac{100}{101}$$

$$p(Z_d = 2 | d) = ?$$

Normalization to Avoid Underflow

	$p(w\|\theta_1)$	$p(w\|\theta_2)$	$p(w\|\bar{\theta})$
text	0.5	0.1	(0.5+0.1)/2
mining	0.2	0.1	(0.2+0.1)/2
medical	0.2	0.75	(0.2+0.75)/2
health	0.1	0.05	(0.1+0.05)/2

Average of $p(w|\theta_i)$
as a possible normalizer

$$p(Z_d=1|d)=\frac{\dfrac{p(\theta_1)p(\text{"text"}|\theta_1)^2p(\text{"min ing"}|\theta_1)^2}{\boxed{p(\text{"text"}|\bar{\theta})^2p(\text{"min ing"}|\bar{\theta})^2}}}{\dfrac{p(\theta_1)p(\text{"text"}|\theta_1)^2p(\text{"min ing"}|\theta_1)^2}{\boxed{p(\text{"text"}|\bar{\theta})^2p(\text{"min ing"}|\bar{\theta})^2}}+\dfrac{p(\theta_2)p(\text{"text"}|\theta_2)^2p(\text{"min ing"}|\theta_2)^2}{\boxed{p(\text{"text"}|\bar{\theta})^2p(\text{"min ing"}|\bar{\theta})^2}}}$$

An Example of 2 Clusters (cont.)

From E-Step

	$P(Z_d=1\|d)$
d1	0.9
d2	0.1
d3	0.8

	c("text")	c("mining")
d1	2	3
d2	1	2
d3	4	3

M-Step

$p(\theta_1)$=? $p(\theta_2)$= ?

$$p(\theta_1) = \frac{p(Z_{d_1}=1|d_1)+p(Z_{d_2}=1|d_2)+p(Z_{d_3}=1|d_3)}{3}$$

$$= \frac{0.9+0.1+0.8}{3} = 0.6$$

	$p(w\|\theta_1)$	$p(w\|\theta_2)$
text	?	?
mining	?	?
medical	?	?
health	?	?

$$p(\text{"text"}|\theta_1) \propto c(\text{"text"},d_1) * p(Z_{d_1}=1|d_1)+\ldots+c(\text{"text"},d_3) * p(Z_{d_3}=1|d_3)$$

$$= 2*0.9+1*0.1+4*0.8$$

$$p(\text{"min ing"}|\theta_1) \propto 3*0.9+2*0.1+3*0.8$$

$$p(\text{"text"}|\theta_1)+p(\text{"min ing"}|\theta_1)+p(\text{"medical"}|\theta_1)+p(\text{"health"}|\theta_1)=1$$

Summary of Generative Model for Clustering

- A slight variation of topic model can be used for clustering documents
  - Each **cluster** is represented by a **unigram LM** **$p(w|\theta_i)$** ➔ **Term cluster**
  - A document is generated by first choosing a unigram LM and then generating **ALL words** in the document using this **single LM**
  - Estimated model parameters give both a topic characterization of each cluster and a probabilistic assignment of a document into each cluster
  - "Hard" clusters can be obtained by forcing a document into the cluster corresponding to the unigram LM most likely used to generate the document
- EM algorithm can be used to compute the ML estimate
  - Normalization is often needed to avoid underflow