Garden Sprinkler Experiment

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I. Introduction

Watering a garden or lawn by hand is not an easy task. If the garden is large, it may take up a lot of time. In order to promote garden watering, garden sprinklers appeared. Nowadays, there are a lot of benefits we can get from installing a garden sprinkler, and the automatic sprinkler system also is the best investment for the garden. It can bring healthy and beautiful lawns. While saving money, it also saves a lot of time, allowing people to devote more time to activities they really like.

- Convenience: We can set a sprinkler system that will automatically water the garden at fixed points and water the lawn instead of watering by ourselves, which could save valuable time and have more leisure time. Regular and quantitative watering is also more convenient to keep the lawn healthy and green, without drying up.
- Aesthetics and safety: There's nothing attractive about a garden hose stretching across the lawn. The hose is also a tripping hazard for children and pets playing in the yard. In contrast, the garden sprinkler is much more Aesthetics and safe than the hose which makes it a more pleasing option.
- Water the optimal amount: Advanced garden sprinkler systems feature weather and soil moisture sensors to deliver the right amount of water right when it's needed, and watering more evenly than ourselves. In addition, the automatic irrigation system can be programmed to discharge a more precise amount of water in the target area, thereby promoting water conservation (saving money).

When defining if a garden sprinkler is at its high quality, its low consumption of water and the wide spray range are significant criterias. There are eight factors affecting the quality of the garden sprinkler which are vertical nozzle angle, tangential nozzle angle, nozzle profile, diameter sprinkler head, static friction moment, dynamic friction moment, entrance pressure, and diameter flow line. The goals of our project are to identify the factors those are significantly affect the water consumption and spray range and figure out the optimal factors' setting to minimize the water consumption and maximize the spray range. To achieve the goals, we will design the experiment , analysis the experimental results and draw the conclusions.

Research Question:
- What is the relevant factors that drive the water consumption and spray range ?.
- What is the optimal factors' settings that maximize the spray range and minimize the water consumption?

Part I: Design of Experiment

Question 1. Propose a cost-efficient experimental design. Motivate your decision in statistical and practical terms.

In this experiment, we aim to find the relevant factors and determine the best combination of garden sprinklers to minimize water consumption and maximize the spray range. We first consider eight factors. If we want a full factorial design, we need a total of 2^8(256) runs to complete all combinations. However, the maximum number of tests for the entire experiment process allowed by the budget is 20 (N). In this case, a full factorial design will not work. Therefore, as a substitute, we choose the regular part analysis factor design. The smallest regular partial factor of 8 factors that we can use is a 16-run 2^(8-4) fractional factorial design. Therefore, we finally decided to perform a 2^(8-4) fractional factorial design.

```
## Design:  8-4.1
##    16  runs,  8  factors,
##    Resolution  IV
##    Generating columns:  7 11 13 14
##    WLP (3plus):  0 14 0 0 0 ,  0  clear 2fis
```

```
##    alpha beta     Aq   d   mt   mf pin dzul
## 1      0    0 2e-06 0.1 0.01 0.01   1    5
## 2     90    0 2e-06 0.1 0.02 0.02   2    5
## 3      0   90 2e-06 0.1 0.02 0.02   1   10
## 4     90   90 2e-06 0.1 0.01 0.01   2   10
## 5      0    0 4e-06 0.1 0.02 0.01   2   10
## 6     90    0 4e-06 0.1 0.01 0.02   1   10
## 7      0   90 4e-06 0.1 0.01 0.02   2    5
## 8     90   90 4e-06 0.1 0.02 0.01   1    5
## 9      0    0 2e-06 0.2 0.01 0.02   2   10
## 10    90    0 2e-06 0.2 0.02 0.01   1   10
## 11     0   90 2e-06 0.2 0.02 0.01   2    5
## 12    90   90 2e-06 0.2 0.01 0.02   1    5
## 13     0    0 4e-06 0.2 0.02 0.02   1    5
## 14    90    0 4e-06 0.2 0.01 0.01   2    5
## 15     0   90 4e-06 0.2 0.01 0.01   1   10
## 16    90   90 4e-06 0.2 0.02 0.02   2   10
```

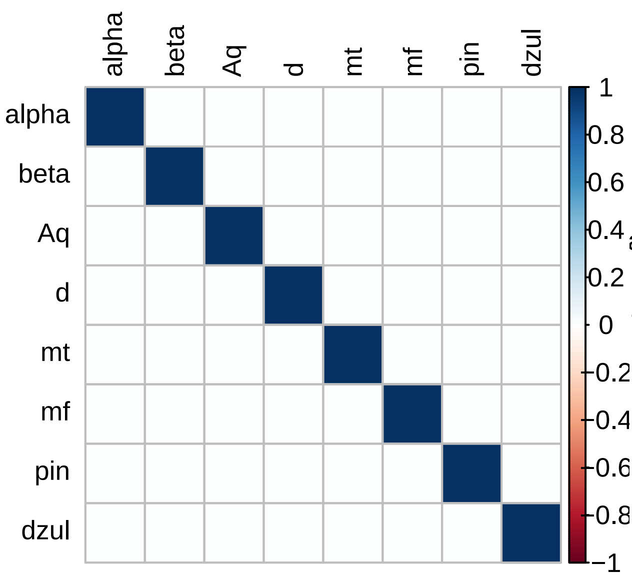
```
##    alpha beta Aq  d mt mf pin dzul
## 1     -1   -1 -1 -1 -1 -1  -1   -1
## 2      1   -1 -1 -1  1  1   1   -1
## 3     -1    1 -1 -1  1  1  -1    1
## 4      1    1 -1 -1 -1 -1   1    1
## 5     -1   -1  1 -1  1 -1   1    1
## 6      1   -1  1 -1 -1  1  -1    1
## 7     -1    1  1 -1 -1  1   1   -1
## 8      1    1  1 -1  1 -1  -1   -1
## 9     -1   -1 -1  1 -1  1   1    1
## 10     1   -1 -1  1  1 -1  -1    1
## 11    -1    1 -1  1  1 -1   1   -1
## 12     1    1 -1  1 -1  1  -1   -1
## 13    -1   -1  1  1  1  1  -1   -1
## 14     1   -1  1  1 -1 -1   1   -1
## 15    -1    1  1  1 -1 -1  -1    1
## 16     1    1  1  1  1  1   1    1
```

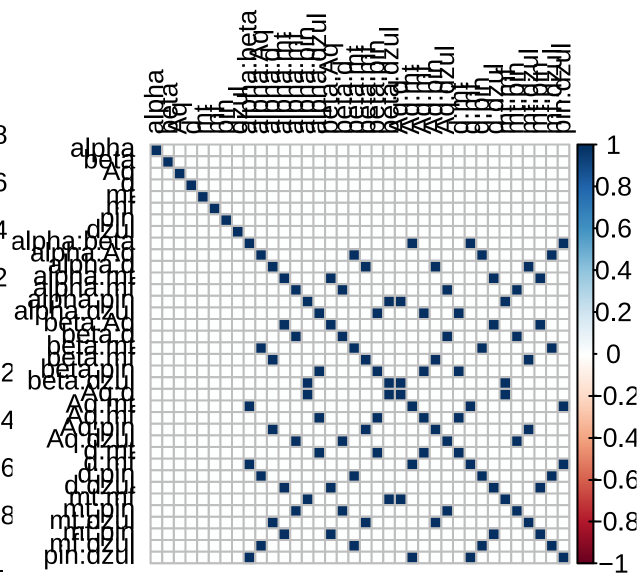
Question 2. What is the performance of your design for studying the main effects of the factors only? Can your design estimate all two-factor interactions? Why or why not?

We can visualize the aliasing in this design using a color map on correlations.

```
## $generators
## [1] "E=ABC" "F=ABD" "G=ACD" "H=BCD"
```

```
## $legend
## [1] "A=alpha" "B=beta"  "C=Aq"    "D=d"     "E=mt"    "F=mf"    "G=pin"
## [8] "H=dzul"
##
## $main
## character(0)
##
## $fi2
## [1] "AB=CE=DF=GH" "AC=BE=DG=FH" "AD=BF=CG=EH" "AE=BC=DH=FG" "AF=BD=CH=EG"
## [6] "AG=BH=CD=EF" "AH=BG=CF=DE"
```





If we assume that the two-factor interactions are negligible, the aliasing structure of the design for the model involving only the main effects is excellent. There is no aliasing among the main effects.That is, the performance of the design for studying the main effects of the factors only is pretty good.

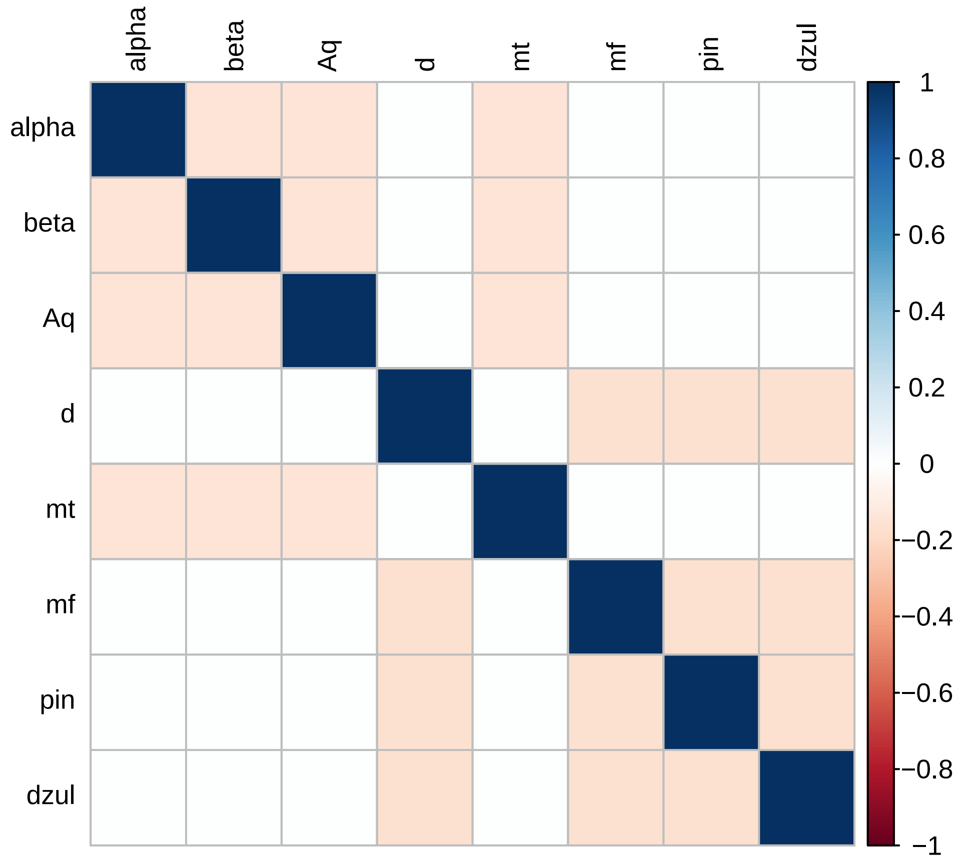
If we consider a model including the main effects only, then we can study the variance inflation factors for the estimates of the coefficients in this model.

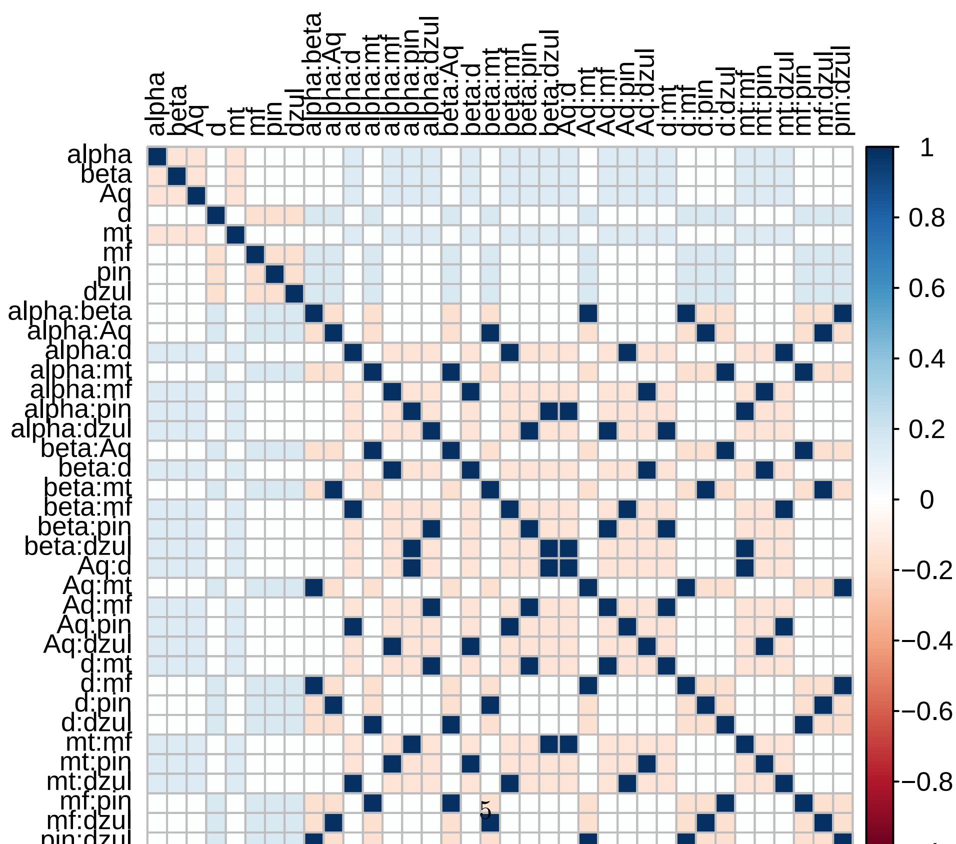
```
## Variance the estimates when sigma^2 = 1

## (Intercept)       alpha        beta          Aq           d          mt
##      0.0625      0.0625      0.0625      0.0625      0.0625      0.0625
##          mf         pin        dzul
##      0.0625      0.0625      0.0625
```

```
##
##  Variance inflation factors

## (Intercept)       alpha        beta          Aq           d          mt
##           1           1           1           1           1           1
##          mf         pin        dzul
##           1           1           1
```

However,our design cannot estimate all two-factor interactions because there are some pairs of interaction that have a large correlation rate with other pairs, meaning they are aliasing with each other.

Question 3. The production engineers are concerned about having some failed tests in the experiment, given by sprinklers which cannot spray water. If you remove two randomly chosen test combinations, what is the performance of the resulting design?





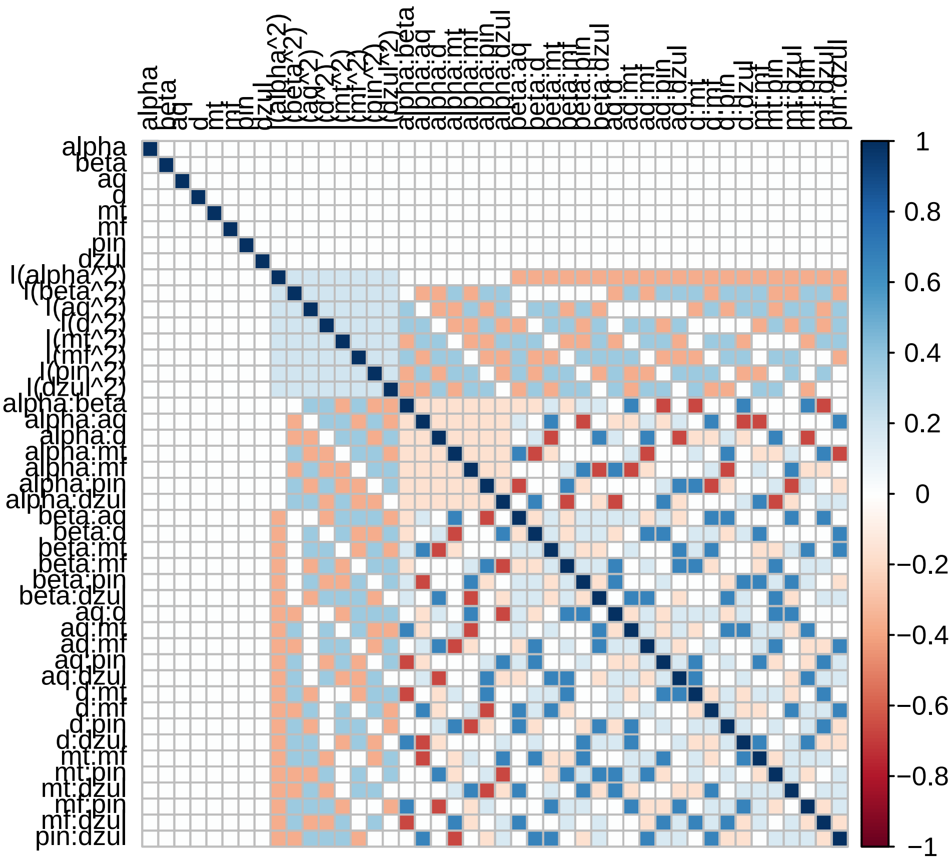
```
## Variance the estimates when sigma^2 = 1

## (Intercept)       alpha        beta          Aq           d          mt
##  0.08333333  0.07812500  0.07812500  0.07812500  0.08333333  0.07812500
##          mf         pin        dzul
##  0.08333333  0.08333333  0.08333333


##
##  Variance inflation factors

## (Intercept)       alpha        beta          Aq           d          mt
##    1.166667    1.093750    1.093750    1.093750    1.166667    1.093750
##          mf         pin        dzul
##    1.166667    1.166667    1.166667
```

The main effects are confounded with other main effects as well as interactions.The resulting design has worse performance than the previous one.

Question 4. The production engineers took an introductory course in experimental design. Using a commercial software, they came up with the experimental plan shown in Table 2. How does your full design compare with this one?

alternative exprimental design

```
##    alpha beta aq  d mt mf pin dzul
## 1     -1    1 -1  1  1  0  -1   -1
## 2      0    0  0  0  0  0   0    0
## 3     -1    1  1  0 -1 -1   1   -1
## 4     -1   -1  1 -1  1  1   0   -1
## 5      1   -1 -1  0  1  1  -1    1
## 6     -1    0 -1 -1  1 -1   1    1
## 7      1    1 -1 -1  0  1   1   -1
## 8     -1   -1  1  1  0 -1  -1    1
## 9     -1   -1 -1  1 -1  1   1    0
## 10     1   -1  1 -1 -1  0   1    1
## 11     1    1  1 -1  1 -1  -1    0
## 12     1    1 -1  1 -1 -1   0    1
## 13    -1    1  0 -1 -1  1  -1    1
## 14     1   -1  0  1  1 -1   1   -1
## 15     0   -1 -1 -1 -1 -1  -1   -1
## 16     0    1  1  1  1  1   1    1
## 17     1    0  1  1 -1  1  -1   -1
```




```r
A <- c(-1,0,-1,-1,1,-1,1,-1,-1,1,1,1,-1,1,0,0,1)
B <- c(1,0,1,-1,-1,0,1,-1,-1,-1,1,1,1,-1,-1,1,0)
C <- c(-1,0,1,1,-1,-1,-1,1,-1,1,1,-1,0,0,-1,1,1)
D <- c(1,0,0,-1,0,-1,-1,1,1,-1,-1,1,-1,1,-1,1,1)
E <- c(1,0,-1,1,1,1,0,0,-1,-1,1,-1,-1,1,-1,1,-1)
F. <- c(0,0,-1,1,1,-1,1,-1,1,0,-1,-1,1,-1,-1,1,1)
G <- c(-1,0,1,0,-1,1,1,-1,1,1,-1,0,-1,1,-1,1,-1)
H <- c(-1,0,-1,-1,1,1,-1,1,0,1,0,1,1,-1,-1,1,-1)

data5 <- data.frame("Alpha"=A, "Beta"=B, "Aq"=C, "d"=D, "mt"=E, "mf"=F., "pin"=G, "dzul"=H)
data5
```

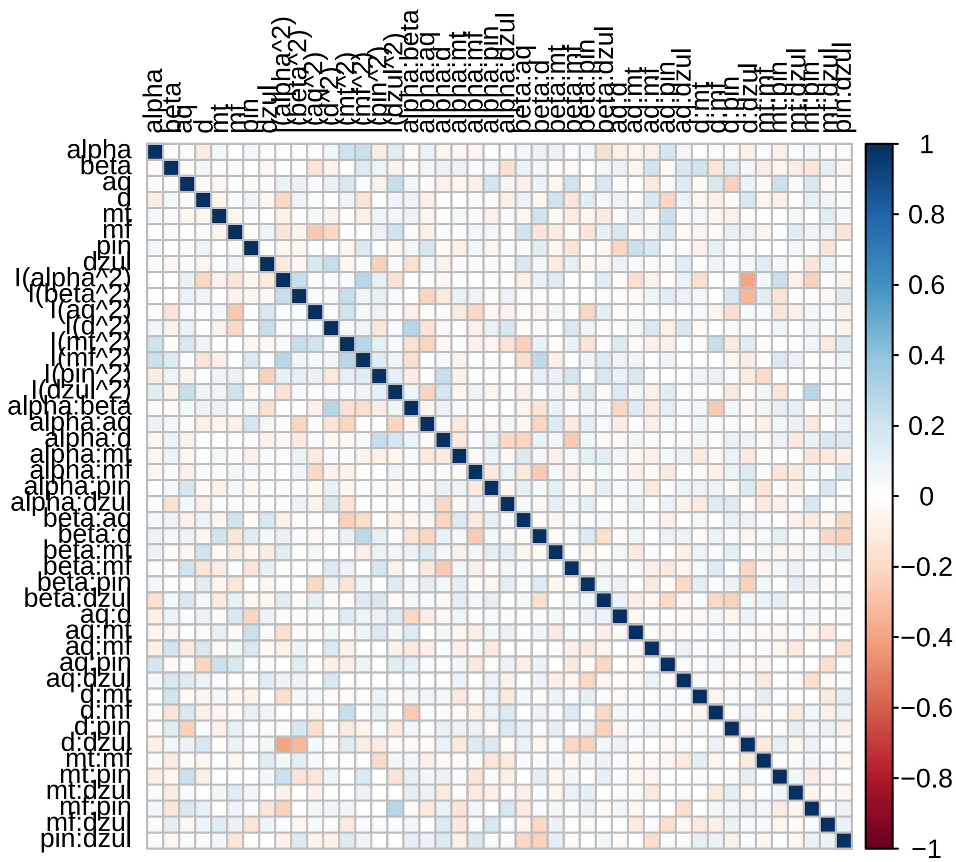
```
##    Alpha Beta Aq  d mt mf pin dzul
## 1     -1    1 -1  1  1  0  -1   -1
## 2      0    0  0  0  0  0   0    0
## 3     -1    1  1  0 -1 -1   1   -1
## 4     -1   -1  1 -1  1  1   0   -1
## 5      1   -1 -1  0  1  1  -1    1
## 6     -1    0 -1 -1  1 -1   1    1
## 7      1    1 -1 -1  0  1   1   -1
## 8     -1   -1  1  1  0 -1  -1    1
## 9     -1   -1 -1  1 -1  1   1    0
## 10     1   -1  1 -1 -1  0   1    1
## 11     1    1  1 -1  1 -1  -1    0
## 12     1    1 -1  1 -1 -1   0    1
## 13    -1    1  0 -1 -1  1  -1    1
```

```
## 14      1    -1  0  1  1 -1    1    -1
## 15      0    -1 -1 -1 -1 -1   -1    -1
## 16      0     1  1  1  1  1    1     1
## 17      1     0  1  1 -1  1   -1    -1
```

full model with factor of 3 levels

```
##       alpha beta aq  d mt mf pin dzul
## 19       -1   -1  1 -1 -1 -1  -1   -1
## 75        1   -1  1  1 -1 -1  -1   -1
## 169      -1    1 -1 -1  1 -1  -1   -1
## 225       1    1 -1  1  1 -1  -1   -1
## 235      -1   -1  1  1  1 -1  -1   -1
## 458       0    1  1  0  1  0  -1   -1
## 504       1    1  0 -1 -1  1  -1   -1
## 550      -1   -1  0  1 -1  1  -1   -1
## 678       1   -1 -1  0  1  1  -1   -1
## 1521      1    1 -1  1 -1 -1   1   -1
## 1582     -1    1  0  0  0 -1   1   -1
## 1644      1    0  1 -1  1 -1   1   -1
## 1837     -1   -1 -1  1  0  0   1   -1
## 1949      0    0 -1 -1 -1  1   1   -1
## 1992      1   -1  1  0 -1  1   1   -1
## 2023     -1    1  1  1 -1  1   1   -1
## 2125     -1   -1  1 -1  1  1   1   -1
## 2178      1    1  0  1  1  1   1   -1
## 2524     -1    0  0 -1  0  0  -1    0
## 2707     -1    1 -1  0 -1  1  -1    0
## 2943      1    1  1 -1 -1 -1   0    0
## 3700     -1   -1 -1  1 -1 -1   1    0
## 3810      1   -1 -1 -1  1 -1   1    0
## 3868     -1    1 -1  1  1 -1   1    0
## 4377      1   -1 -1 -1 -1 -1  -1    1
## 4453     -1    1  1  1 -1 -1  -1    1
## 4557      1   -1  1 -1  1 -1  -1    1
## 4835      0   -1 -1  1  1  0  -1    1
## 4885     -1    1  1 -1 -1  1  -1    1
## 4938      1    0  1  1 -1  1  -1    1
## 4942     -1   -1 -1 -1  0  1  -1    1
## 5031      1    1 -1 -1  1  1  -1    1
## 5101     -1    1  1  1  1  1  -1    1
## 5243      0    0 -1  1  0 -1   0    1
## 5419     -1   -1  1  1 -1  0   0    1
## 5763      1   -1  0 -1  1  1   0    1
## 5839     -1    1 -1 -1 -1 -1   1    1
## 5853      1   -1  1 -1 -1 -1   1    1
## 6013     -1   -1  1 -1  1 -1   1    1
## 6075      1    1  1  1  1 -1   1    1
## 6375      1   -1 -1  1 -1  1   1    1
## 6389      0    1  0  1 -1  1   1    1
## 6426      1    1  1 -1  0  1   1    1
## 6511     -1    0 -1  0  1  1   1    1
## 6554      0   -1  1  1  1  1   1    1
```

```
## 'data.frame':    45 obs. of  8 variables:
##  $ alpha: num  -1 1 -1 1 -1 0 1 -1 1 1 ...
##  $ beta : num  -1 -1 1 1 -1 1 1 -1 -1 1 ...
##  $ aq   : num  1 1 -1 -1 1 1 0 0 -1 -1 ...
##  $ d    : num  -1 1 -1 1 1 0 -1 1 0 1 ...
##  $ mt   : num  -1 -1 1 1 1 1 -1 -1 1 -1 ...
##  $ mf   : num  -1 -1 -1 -1 -1 0 1 1 1 -1 ...
##  $ pin  : num  -1 -1 -1 -1 -1 -1 -1 -1 -1 1 ...
##  $ dzul : num  -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 ...
```



The full design has better performance than the alternative experimental design.

Part II: Analysis of the Results

Data:

```
##    alpha beta Aq  d mt mf pin dzul      range
## 1     -1   -1 -1 -1 -1 -1  -1   -1 0.19236764
## 2      1   -1 -1 -1  1  1   1   -1 0.00000000
## 3     -1    1 -1 -1  1  1  -1    1 0.23450150
## 4      1    1 -1 -1 -1 -1   1    1 0.01280741
## 5     -1   -1  1 -1  1 -1   1    1 0.32894043
## 6      1   -1  1 -1 -1  1  -1    1 0.00000000
## 7     -1    1  1 -1 -1  1   1   -1 0.38104848
## 8      1    1  1 -1  1 -1  -1   -1 0.02997635
## 9     -1   -1 -1  1 -1  1   1    1 0.31772243
## 10     1   -1 -1  1  1 -1  -1    1 0.03356131
## 11    -1    1 -1  1  1 -1   1   -1 0.31974521
## 12     1    1 -1  1 -1  1  -1   -1 0.00000000
## 13    -1   -1  1  1  1  1  -1   -1 0.22686769
## 14     1   -1  1  1 -1 -1   1   -1 0.02132011
## 15    -1    1  1  1 -1 -1  -1    1 0.32358838
## 16     1    1  1  1  1  1   1    1 0.06262631
```

```
##    alpha beta Aq  d mt mf pin dzul consumption
## 1     -1   -1 -1 -1 -1 -1  -1   -1    3.290304
## 2      1   -1 -1 -1  1  1   1   -1    4.619100
## 3     -1    1 -1 -1  1  1  -1    1    3.658242
## 4      1    1 -1 -1 -1 -1   1    1    4.787440
## 5     -1   -1  1 -1  1 -1   1    1    9.675282
## 6      1   -1  1 -1 -1  1  -1    1    6.610304
## 7     -1    1  1 -1 -1  1   1   -1    8.708282
## 8      1    1  1 -1  1 -1  -1   -1    6.025671
## 9     -1   -1 -1  1 -1  1   1    1    4.942020
## 10     1   -1 -1  1  1 -1  -1    1    3.447793
## 11    -1    1 -1  1  1 -1   1   -1    4.640643
## 12     1    1 -1  1 -1  1  -1   -1    3.335403
## 13    -1   -1  1  1  1  1  -1   -1    6.102466
## 14     1   -1  1  1 -1 -1   1   -1    8.633511
## 15    -1    1  1  1 -1 -1  -1    1    6.795141
## 16     1    1  1  1  1  1   1    1    9.780997
```

Question 5. Collect data using your recommended design in Question 1. Conduct a detailed data analysis.

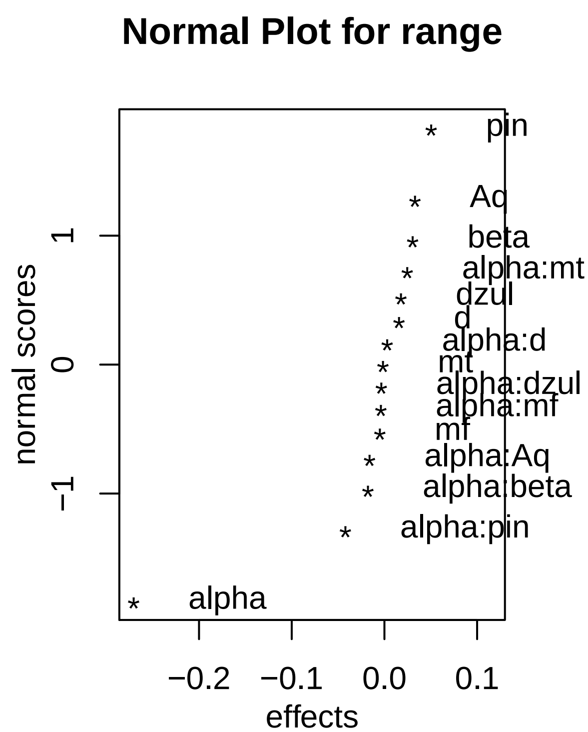
For spray range:

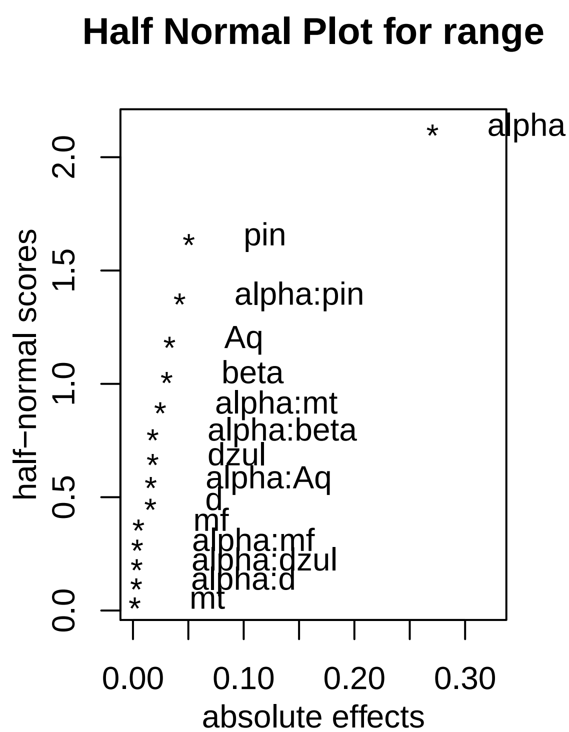
Full Model :

```
##
##  alpha:beta = Aq:mt = d:mf = pin:dzul
##  alpha:Aq = beta:mt = d:pin = mf:dzul
##  alpha:d = beta:mf = Aq:pin = mt:dzul
##  alpha:mt = beta:Aq = d:dzul = mf:pin
```

```
##  alpha:mf = beta:d = Aq:dzul = mt:pin
##  alpha:pin = beta:dzul = Aq:d = mt:mf
##  alpha:dzul = beta:pin = Aq:mf = d:mt

##
## Call:
## lm.default(formula = range ~ (.)^2, data = data1)
##
## Residuals:
## ALL 16 residuals are 0: no residual degrees of freedom!
##
## Coefficients: (21 not defined because of singularities)
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)  0.1553171         NA      NA       NA
## alpha       -0.1352806         NA      NA       NA
## beta         0.0152196         NA      NA       NA
## Aq           0.0164789         NA      NA       NA
## d            0.0078619         NA      NA       NA
## mt          -0.0007897         NA      NA       NA
## mf          -0.0024713         NA      NA       NA
## pin          0.0252092         NA      NA       NA
## dzul         0.0089014         NA      NA       NA
## alpha:beta  -0.0089035         NA      NA       NA
## alpha:Aq    -0.0080346         NA      NA       NA
## alpha:d      0.0014786         NA      NA       NA
## alpha:mt     0.0122943         NA      NA       NA
## alpha:mf    -0.0019086         NA      NA       NA
## alpha:pin   -0.0210572         NA      NA       NA
## alpha:dzul  -0.0016891         NA      NA       NA
## beta:Aq             NA         NA      NA       NA
## beta:d              NA         NA      NA       NA
## beta:mt             NA         NA      NA       NA
## beta:mf             NA         NA      NA       NA
## beta:pin            NA         NA      NA       NA
## beta:dzul           NA         NA      NA       NA
## Aq:d                NA         NA      NA       NA
## Aq:mt               NA         NA      NA       NA
## Aq:mf               NA         NA      NA       NA
## Aq:pin              NA         NA      NA       NA
## Aq:dzul             NA         NA      NA       NA
## d:mt                NA         NA      NA       NA
## d:mf                NA         NA      NA       NA
## d:pin               NA         NA      NA       NA
## d:dzul              NA         NA      NA       NA
## mt:mf               NA         NA      NA       NA
## mt:pin              NA         NA      NA       NA
## mt:dzul             NA         NA      NA       NA
## mf:pin              NA         NA      NA       NA
## mf:dzul             NA         NA      NA       NA
## pin:dzul            NA         NA      NA       NA
##
## Residual standard error: NaN on 0 degrees of freedom
## Multiple R-squared:      1,  Adjusted R-squared:    NaN
## F-statistic:   NaN on 15 and 0 DF,  p-value: NA
```

```
## Estimated effects:

##        alpha         beta           Aq            d           mt           mf
## -0.270561284  0.030439257  0.032957783  0.015723701 -0.001579458 -0.004942555
##          pin         dzul   alpha:beta     alpha:Aq      alpha:d     alpha:mt
##  0.050418438  0.017802786 -0.017807093 -0.016069270  0.002957288  0.024588569
##     alpha:mf    alpha:pin   alpha:dzul
## -0.003817164 -0.042114396 -0.003378145
```





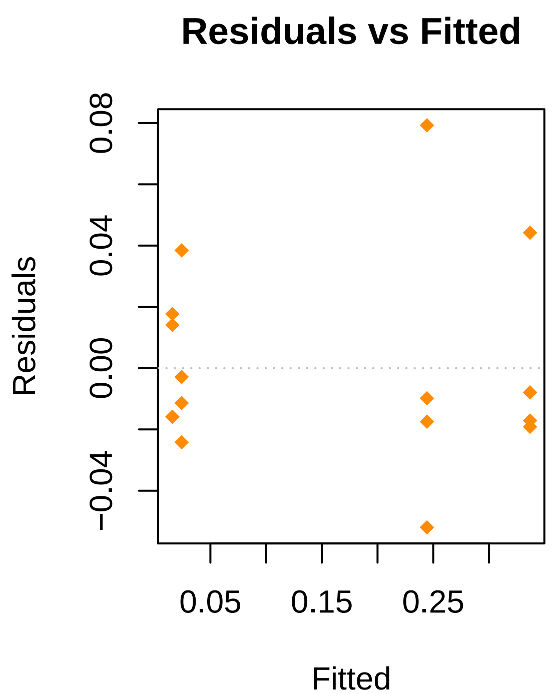
The plots above suggest that alpha,pin and alpha:pin are active(significant), thus we are going to refine our model using alpha,pin and alpha:pin.

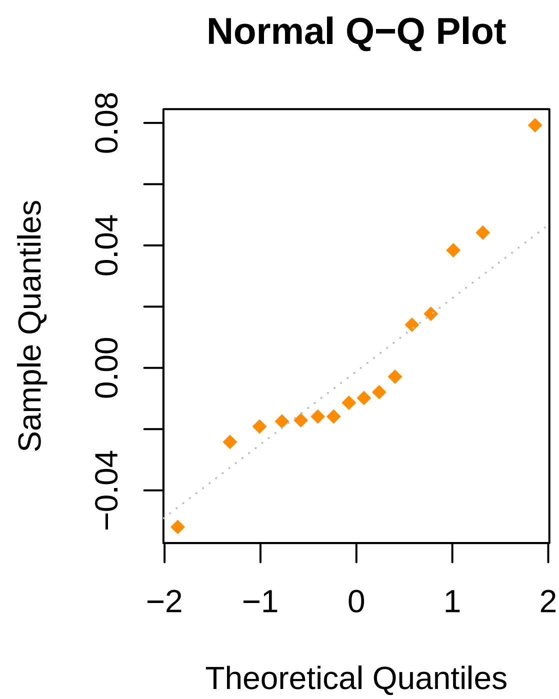
Refine Model:

```
##
## Call:
## lm.default(formula = range ~ alpha + pin + alpha:pin, data = data1)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.05196 -0.01721 -0.01060  0.01499  0.07926
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  0.155317   0.008919  17.414 6.98e-10 ***
```

```
## alpha       -0.135281   0.008919 -15.167 3.42e-09 ***
## pin          0.025209   0.008919   2.826   0.0153 *
## alpha:pin   -0.021057   0.008919  -2.361   0.0360 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.03568 on 12 degrees of freedom
## Multiple R-squared:  0.9531, Adjusted R-squared:  0.9413
## F-statistic:  81.2 on 3 and 12 DF,  p-value: 3.076e-08
```

Model Adequacy Checking





From the **Normal Q-Q** plot, most of the points are close to the dashed line so the residual is generally distributed as normal. The **normality assumption** is satisfied.

From the **Residuals vs Fitted** plot,there is no pattern(relationship) found(i.e. residuals is distributed randomly and independently around zero), so the **constant-variance assumption** is satisfied.

There is nothing unusual about the residual plots.We conclude that the assumptions for analysis of variance are satisfied.

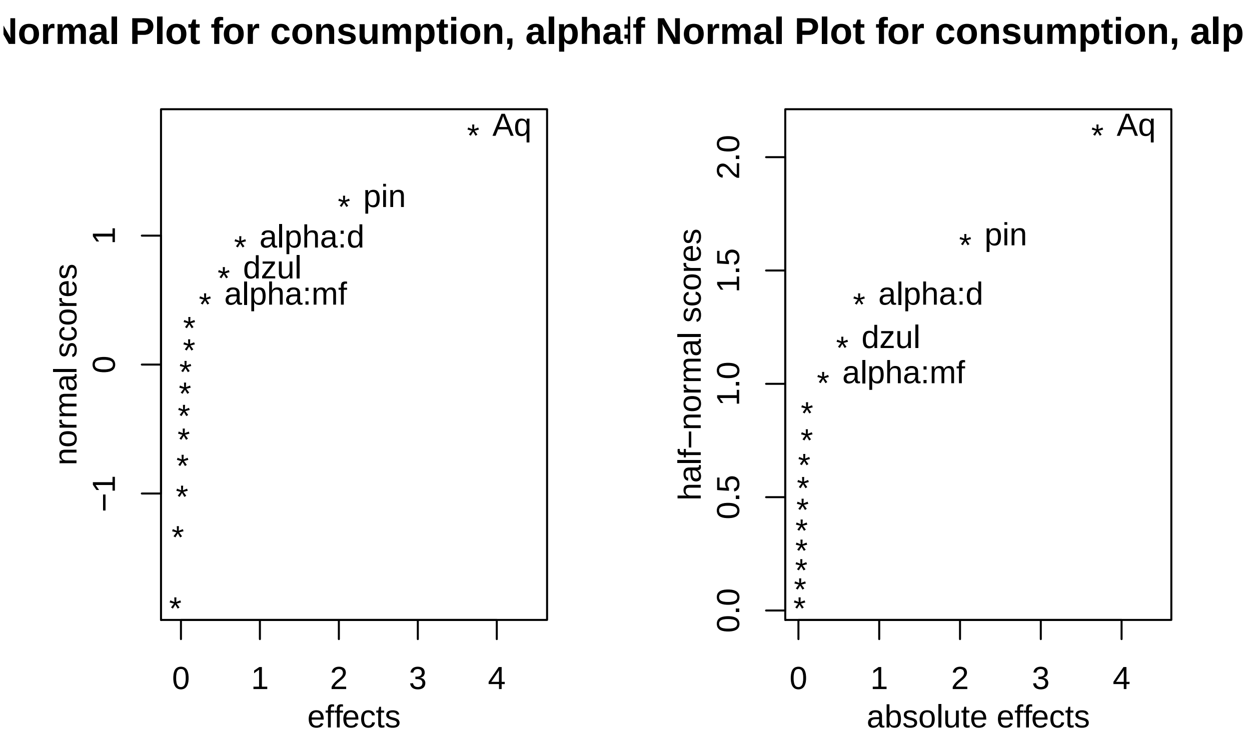
For consumption:

```
##
## Call:
## lm.default(formula = consumption ~ (.)^2, data = data2)
```

```
##
## Residuals:
## ALL 16 residuals are 0: no residual degrees of freedom!
##
## Coefficients: (21 not defined because of singularities)
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  5.940788         NA      NA       NA
## alpha       -0.035760         NA      NA       NA
## beta         0.025690         NA      NA       NA
## Aq           1.850669         NA      NA       NA
## d            0.018959         NA      NA       NA
## mt           0.052987         NA      NA       NA
## mf           0.028814         NA      NA       NA
## pin          1.032622         NA      NA       NA
## dzul         0.271365         NA      NA       NA
## alpha:beta   0.051660         NA      NA       NA
## alpha:Aq     0.006924         NA      NA       NA
## alpha:d      0.375439         NA      NA       NA
## alpha:mt     0.010376         NA      NA       NA
## alpha:mf     0.152609         NA      NA       NA
## alpha:pin    0.017613         NA      NA       NA
## alpha:dzul  -0.019759         NA      NA       NA
## beta:Aq            NA         NA      NA       NA
## beta:d             NA         NA      NA       NA
## beta:mt            NA         NA      NA       NA
## beta:mf            NA         NA      NA       NA
## beta:pin           NA         NA      NA       NA
## beta:dzul          NA         NA      NA       NA
## Aq:d               NA         NA      NA       NA
## Aq:mt              NA         NA      NA       NA
## Aq:mf              NA         NA      NA       NA
## Aq:pin             NA         NA      NA       NA
## Aq:dzul            NA         NA      NA       NA
## d:mt               NA         NA      NA       NA
## d:mf               NA         NA      NA       NA
## d:pin              NA         NA      NA       NA
## d:dzul             NA         NA      NA       NA
## mt:mf              NA         NA      NA       NA
## mt:pin             NA         NA      NA       NA
## mt:dzul            NA         NA      NA       NA
## mf:pin             NA         NA      NA       NA
## mf:dzul            NA         NA      NA       NA
## pin:dzul           NA         NA      NA       NA
##
## Residual standard error: NaN on 0 degrees of freedom
## Multiple R-squared:      1,  Adjusted R-squared:    NaN
## F-statistic:   NaN on 15 and 0 DF,  p-value: NA
```

```
##
##  alpha:beta = Aq:mt = d:mf = pin:dzul
##  alpha:Aq = beta:mt = d:pin = mf:dzul
##  alpha:d = beta:mf = Aq:pin = mt:dzul
##  alpha:mt = beta:Aq = d:dzul = mf:pin
##  alpha:mf = beta:d = Aq:dzul = mt:pin
```

```
##  alpha:pin = beta:dzul = Aq:d = mt:mf
##  alpha:dzul = beta:pin = Aq:mf = d:mt

## Estimated effects:

##       alpha        beta          Aq           d          mt          mf
## -0.07152010  0.05137985  3.70133848  0.03791822  0.10597360  0.05762863
##         pin        dzul  alpha:beta    alpha:Aq     alpha:d    alpha:mt
##  2.06524385  0.54272968  0.10332074  0.01384806  0.75087868  0.02075188
##    alpha:mf   alpha:pin  alpha:dzul
##  0.30521870  0.03522520 -0.03951761
```



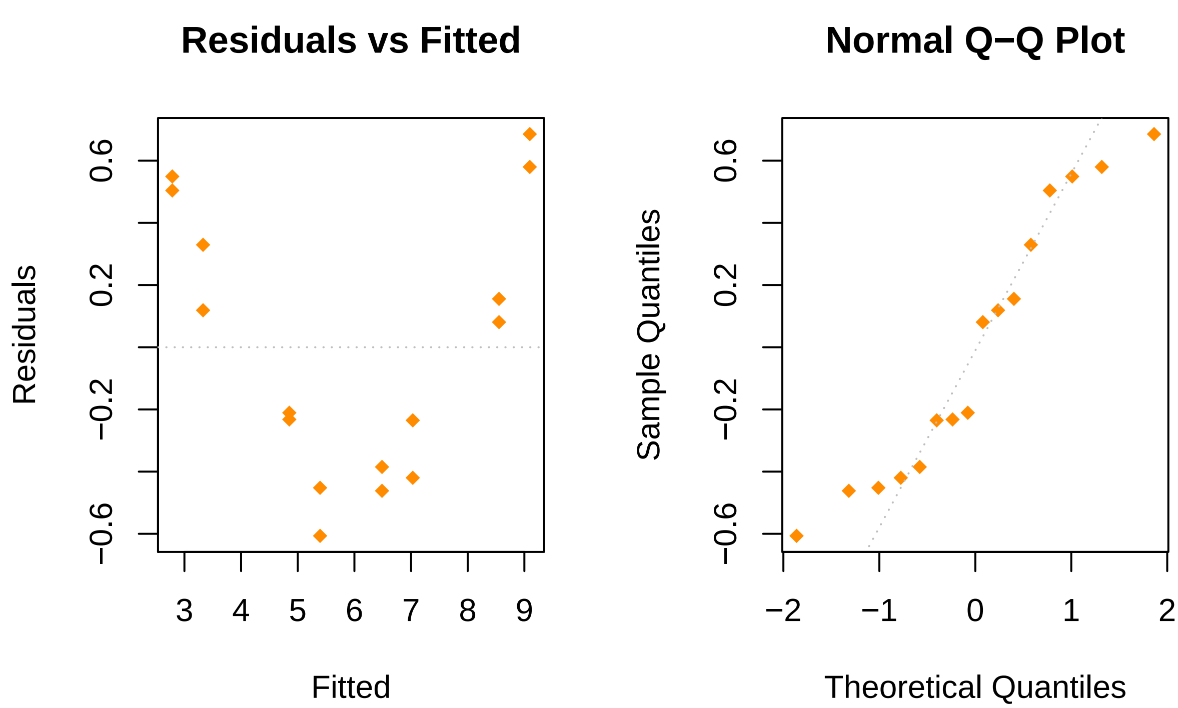
The plots above suggest Aq,pin, dzul,alpha:d,alpha:mf are active(significant).However, since alpha, d and mf are inactive(insignificant), by hierarchical rule, we are going refine our model using Aq,pin, dzul only

Refine Model

```
##
## Call:
## lm.default(formula = consumption ~ Aq + pin + dzul, data = data2)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.60666 -0.39373 -0.06497  0.37308  0.68555
```

```
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   5.9408     0.1203  49.377 3.12e-15 ***
## Aq            1.8507     0.1203  15.382 2.91e-09 ***
## pin           1.0326     0.1203   8.583 1.82e-06 ***
## dzul          0.2714     0.1203   2.255   0.0436 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.4813 on 12 degrees of freedom
## Multiple R-squared:  0.9633, Adjusted R-squared:  0.9542
## F-statistic: 105.1 on 3 and 12 DF,  p-value: 7.003e-09
```

Model Adequacy Checking



As we can see from the **Residuals vs Fitted** plot, there seems to appear a bowl curve, so the constant-variance assumption might be violated.

To be safe, we will try to use log transformation to improve the model.

Log-transform the **consumption** :

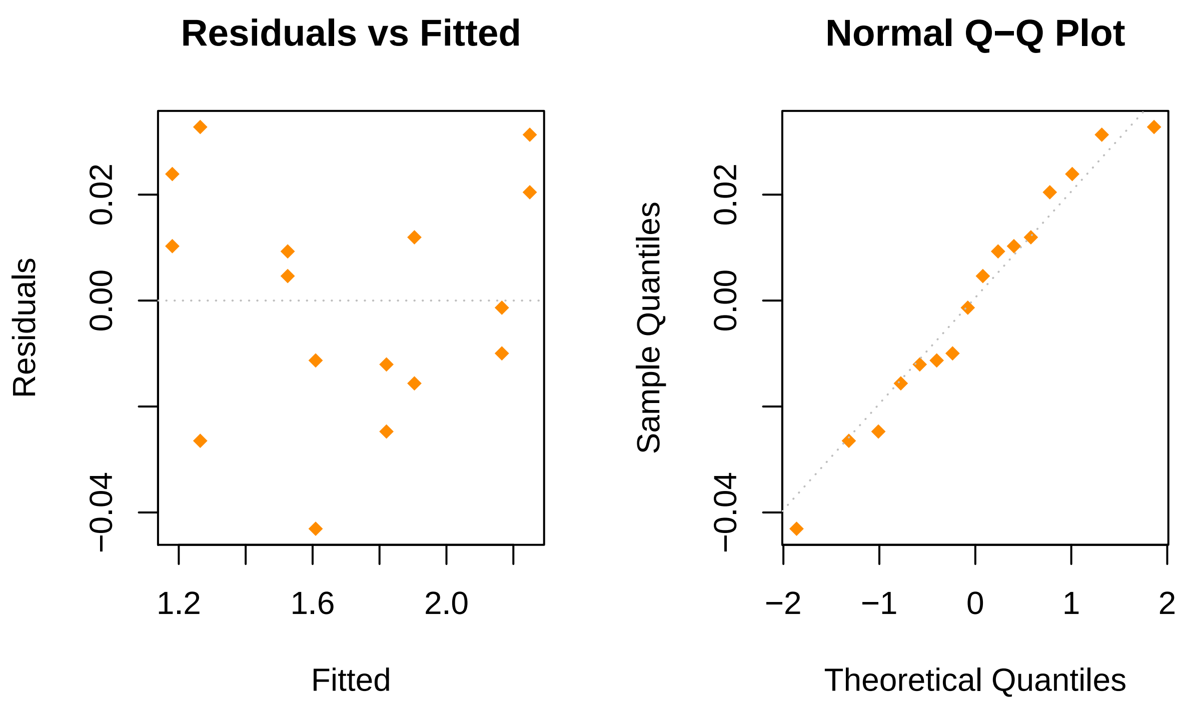
```
##    alpha beta Aq  d mt mf pin dzul consumption
## 1     -1   -1 -1 -1 -1 -1  -1   -1    1.190980
## 2      1   -1 -1 -1  1  1   1   -1    1.530200
```

```
## 3      -1    1 -1 -1  1  1  -1    1    1.296983
## 4       1    1 -1 -1 -1 -1   1    1    1.565996
## 5      -1   -1  1 -1  1 -1   1    1    2.269574
## 6       1   -1  1 -1 -1  1  -1    1    1.888630
## 7      -1    1  1 -1 -1  1   1   -1    2.164275
## 8       1    1  1 -1  1 -1  -1   -1    1.796029
## 9      -1   -1 -1  1 -1  1   1    1    1.597774
## 10      1   -1 -1  1  1 -1  -1    1    1.237734
## 11     -1    1 -1  1  1 -1   1   -1    1.534853
## 12      1    1 -1  1 -1  1  -1   -1    1.204594
## 13     -1   -1  1  1  1  1  -1   -1    1.808693
## 14      1   -1  1  1 -1 -1   1   -1    2.155651
## 15     -1    1  1  1 -1 -1  -1    1    1.916208
## 16      1    1  1  1  1  1   1    1    2.280441
```

Refine Model with log transformation :

```
##
## Call:
## lm.default(formula = consumption ~ Aq + pin + dzul, data = data2)
##
## Residuals:
##       Min        1Q    Median        3Q       Max
## -0.043080 -0.012951  0.001646  0.014074  0.032772
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 1.714913   0.006118 280.309  < 2e-16 ***
## Aq          0.320024   0.006118  52.309 1.57e-15 ***
## pin         0.172432   0.006118  28.185 2.47e-12 ***
## dzul        0.041754   0.006118   6.825 1.84e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.02447 on 12 degrees of freedom
## Multiple R-squared:  0.9967, Adjusted R-squared:  0.9958
## F-statistic:  1192 on 3 and 12 DF,  p-value: 4.09e-15
```

Model Adequacy Checking



From the **Normal Q-Q** plot, most of the points are close to the dashed line so the residual is generally distributed as normal. The **normality assumption** is satisfied.

From the **Residuals vs Fitted** plot,there is no pattern(relationship) found(i.e. residuals is distributed randomly and independently around zero), so the **constant-variance assumption** is satisfied.

There is nothing unusual about the residual plots.We conclude that the assumptions for analysis of variance are satisfied. The model has been improved.

Question 6. What are the most influential factors?

For range :

```
## Analysis of Variance Table
##
## Response: range
##           Df   Sum Sq  Mean Sq  F value    Pr(>F)
## alpha      1 0.292814 0.292814 230.0486 3.423e-09 ***
## pin        1 0.010168 0.010168   7.9885   0.01528 *
## alpha:pin  1 0.007094 0.007094   5.5738   0.03599 *
## Residuals 12 0.015274 0.001273
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Define hypothesis test for main effects as below :
H_0: The main effect[Alpha/pin] is not statistically significant
H_1: The main effect[Alpha/pin] is statistically significant

Define hypothesis test for interaction effects as below :
H_0: The interaction effect[Alpha:pin] is not statistically significant
H_1: The interaction effect[Alpha:pin] is statistically significant

As per the ANOVA table :
The p-value for pin = 0.01528 < 0.1
The p-value for alpha = 3.423e-09 < 0.001
The p-value for pin:alpha = 0.03599 < 0.05

For the response value of **range**,we can confirm that the two-factor interaction **pin:alpha** , and factor **alpha pin** are significant.That is, alpha and pin are the most influential factors for the spray range model.

For consumption:

```
## Analysis of Variance Table
##
## Response: consumption
##           Df  Sum Sq Mean Sq  F value    Pr(>F)
## Aq         1 1.63865 1.63865 2736.247 1.566e-15 ***
## pin        1 0.47573 0.47573  794.376 2.466e-12 ***
## dzul       1 0.02789 0.02789   46.579 1.837e-05 ***
## Residuals 12 0.00719 0.00060
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

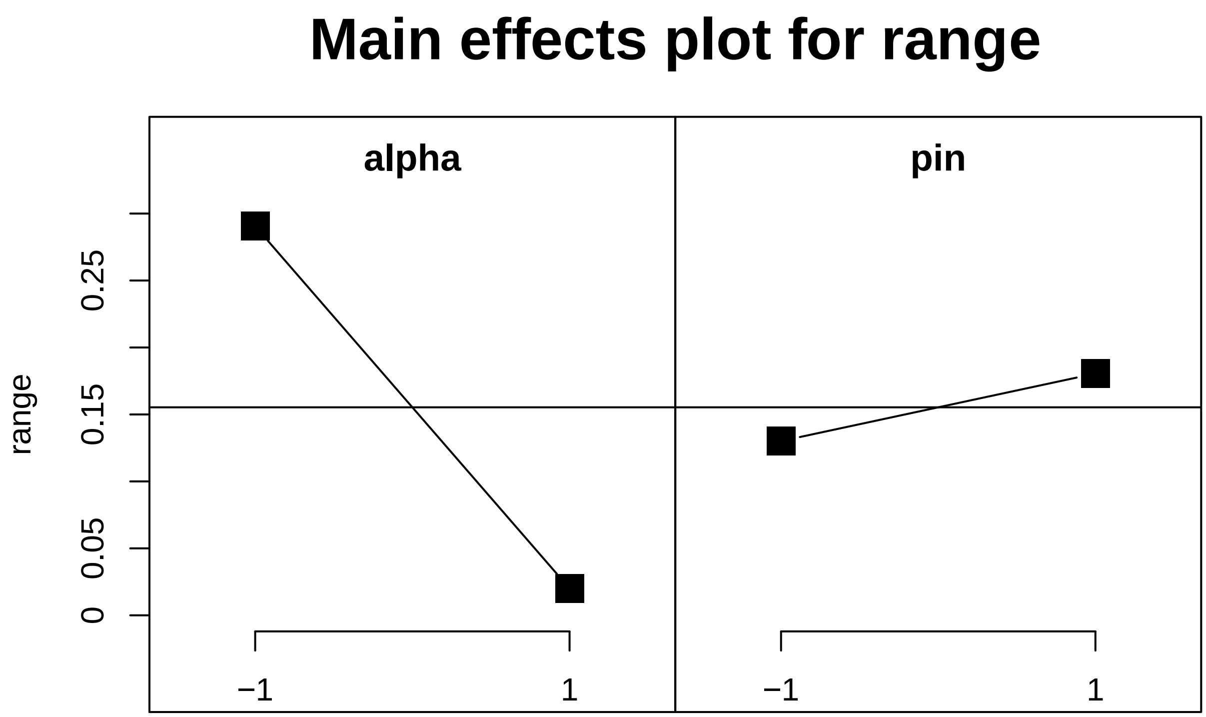
Define hypothesis test for main effects as below : H_0: The main effect[Aq/pin/dzul] is not statistically significant
H_1: The main effect[Aq/pin/dzul] is statistically significant

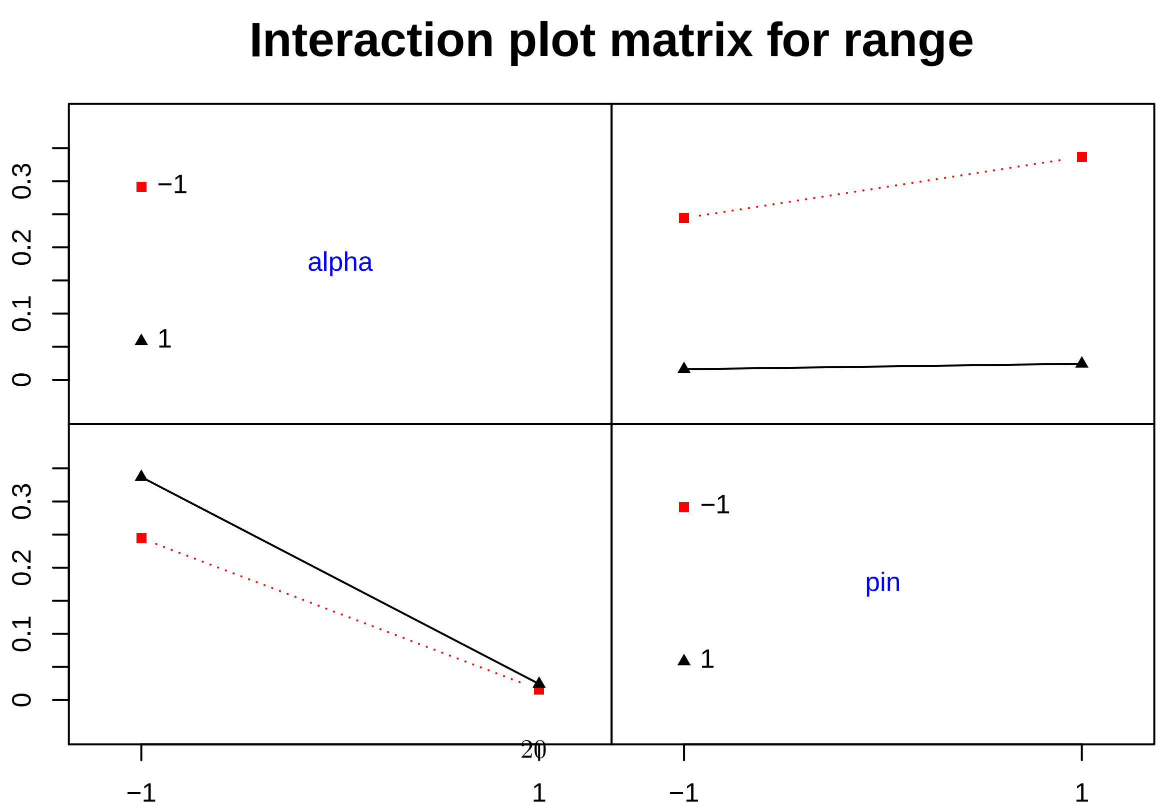
As per the ANOVA table :
The p-value for Aq = 1.566e-15 < 0.001
The p-value for pin = 2.466e-12 < 0.001
The p-value for dzul = 1.837e-05 < 0.001

For the response value of **consumption**, we can confirm that the main effects **Aq pin dzul** are significant. That is, Aq, pin, dzul are the most influential factors for the water consumption model.

Question 7. Recommend the settings of the factors that optimize the water consumption and spray range simultaneously.

For range :





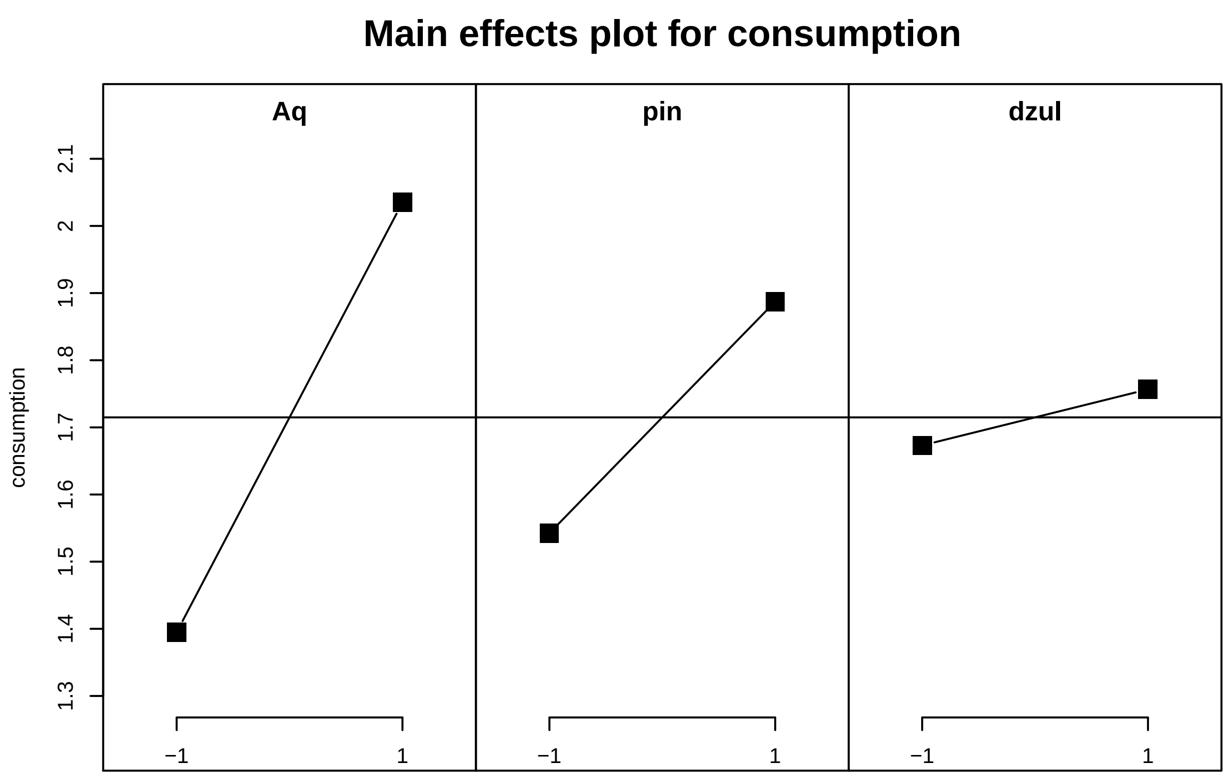
From the main-effects plot and the interaction plot, we recommend alpha = -1 and pin = 1 as the settings of the factors that optimize(maximize) the spray range.

We can also use optim() to confirm our conslusion above.

```
## $par
## [1] -1  1
##
## $value
## [1] -0.325864
##
## $counts
## function gradient
##        8        8
##
## $convergence
## [1] 0
##
## $message
## [1] "CONVERGENCE: NORM OF PROJECTED GRADIENT <= PGTOL"
```

The results of optim() are same to the main-effects plot and the interaction plot,we recommend alpha = -1 and pin = 1 as the settings of the factors that optimize(maximize) the spray range.

For Consumption :



From the main-effects plot, we recommend Aq = -1,pin = -1, dzul = -1 as the settings of the factors that optimize(minimize) the water consumption.

We can also use optim() to confirm our conslusion above.

```
## $par
## [1] -1 -1 -1
##
## $value
## [1] 2.7861
##
## $counts
## function gradient
##        3        3
##
## $convergence
## [1] 0
##
## $message
## [1] "CONVERGENCE: NORM OF PROJECTED GRADIENT <= PGTOL"
```

The results of optim() are same to the main-effects plot and the interaction plot,we recommend Aq = -1,pin = -1, dzul = -1 as the settings of the factors that optimize(minimize) the water consumption.

Overall

As we can see, the optimal setting of the factor **pin** for the range and the consumption is conflicted. To achieve a better overall quality, we would choose the setting of **pin = -1** since it has stronger affect in the consumption rather than the range. Additionally, the p-value of pin is **2.466e-12** which is much smaller than 0.01. It is more statistically significant than the performance in the range.

Therefore, to minimize the water consumption and maximum the spray range simultaneously, the recommended setting of the factors should be **Aq = -1 ; pin = -1 ; dzul = -1 ;alpha = -1**

Question 8. Conduct confirmation experiments using your recommended settings. Are your predictions accurate?

According to the confirmation experiments of section 8.2 from textbook, a simple confirmation experiment is to use the model equation to predict the response at a point of interest in the design space (this should not be one of the runs in the current design) and then actually find/run that treatment combination (perhaps several times), comparing the predicted and observed responses.

For range :

```
##
## Call:
## lm.default(formula = range ~ alpha + pin + alpha:pin, data = data1)
##
## Coefficients:
## (Intercept)        alpha          pin    alpha:pin
##     0.15532     -0.13528      0.02521     -0.02106
```

The final fitted equation for range is :

$$\hat{y} = 0.15532 - 0.13528x_1 + 0.02521x_7 - 0.02106x_1x_7$$

where $\hat{y}$ is the predicted response and x_1 and x_7 denote the coded level of factors alpha and pin respectively.

From Question 7, we know that the optimal setting of x_1 and x_7 for maximizing the spray range are -1 and -1.

Then $\hat{y} = 0.15532 - 0.13528(-1) + 0.02521(-1) - 0.02106(-1)(-1) = 0.24433$

The observed response under the condition that [alpha=pin=-1] are 0.1923676 0.2345015 0.2268677 0.3235884.

```
## [1] 0.1923676 0.2345015 0.2268677 0.3235884
```

We already conduct confirmation experiment for the model of "range"; comparing the predicted response 0.24433 with the observed response [0.1923676 0.2345015 0.2268677 0.3235884], there isn't big difference so we can say the prediction is accurate.

For consumption :

```
##
## Call:
## lm.default(formula = consumption ~ Aq + pin + dzul, data = data2)
##
## Coefficients:
## (Intercept)           Aq          pin         dzul
##     1.71491      0.32002      0.17243      0.04175
```

The final fitted equation for water consumption is :

$$\hat{y} = 1.71491 + 0.32002x_3 + 0.17243x_7 + 0.04175x_8$$

where $\hat{y}$ is the log predicted response and x_3, x_7 and x_8 denote the coded level of factors Aq,pin and dzul respectively.

From Question 7, we know that the optimal setting of x_3, x_7 and x_8 for minimizing the water consumption are -1,-1 and -1.

Then $\hat{y} = 1.71491 + 0.32002(-1) + 0.17243(-1) + 0.04175(-1) = 1.18071$

$e^{\hat{y}} = 3.256686$

The exponential observed response of consumption under the condition that (Aq=pin=dzul=-1) are 3.290304 3.335403

We already conduct confirmation experiment for the model of "consumption"; comparing the predicted value 3.256686 with the observed value [3.290304 3.335403], they are quite close so we can say the prediction is accurate.