Adverse Selection Example

IEOR 290: Healthcare Analytics

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Problem

Suppose a restaurant would like to purchase marinated steak from a meat distributor. The restaurant's utility for the steaks is given by $S(q) = 15\sqrt{q}$. The fixed costs for the distributors are \$35, and if the distributor is inefficient (efficient), then its marginal costs are \$0.35 (\$0.25). Assume that the restaurant believes that there is a 30% chance that the distributor is efficient.

Solution

a. What are the first-best production levels?

Solution: Note that equating marginal utility to marginal costs for the inefficient distributor gives:

$$S'(q_1^I) = \theta^I \Rightarrow q_1^I : \frac{15}{2\sqrt{q_1^I}} = 0.35 \Rightarrow q_1^I = 459$$

Similarly, equating marginal utility to marginal costs for the efficient distributor gives:

$$S'(q_1^E) = \theta^E \Rightarrow q_1^E : \frac{15}{2\sqrt{q_1^E}} = 0.25 \Rightarrow q_1^E = 900$$

Solution (Cont.)

b. What are the contracts to implement the first-best production levels?

Solution: These contracts allow for zero information rent, meaning that the inefficient distributor should be offered the contract:

$$(q_1^I = 459,\ t_1^I = \theta_1^I q_1^I + F = 0.35 \cdot 459 + 35 = 195.65)$$

and the efficient distributor should be offered the contract:

$$(q_1^E = 900,\ t_1^E = \theta_1^E q_1^E + F = 0.25 \cdot 900 + 35 = 260)$$

Solution (Cont.)

c. How much profit would the meat distributor make if the restaurant offers a menu of contracts $\{(q_1^I, t_1^I), (q_1^E, t_1^E)\}$?

Solution: If the meat distributor is inefficient,

$$\text{profit} = t_1^I - \theta_1^I q_1^I - F = 0$$

$$(t_1^E - \theta_1^I q_1^E - F = (\theta_1^E q_1^E + F) - (\theta_1^I q_1^E + F) = (\theta^E - \theta^I) q_1^E < 0)$$

If the meat distributor is efficient,

$$\text{profit} = t_1^I - \theta^E q_1^I - F = 195.65 - 0.25 \cdot 459 - 35 = 45.9$$

$$(t_1^E - \theta_1^E q_1^E - F = 0)$$

Solution (Cont.)

d. What are the second-best production levels?

Solution: The production level for the efficient agent remains unchanged:

$$q_2^E = q_1^E = 900$$

The production level for the inefficient agent decreases to:

$$q_2^I : S'(q_2^I) = \theta^I + \frac{\nu}{1-\nu}(\theta^I - \theta^E)$$

$$\Rightarrow q_2^I : \frac{15}{2\sqrt{q_2^I}} = 0.35 + \frac{0.3}{1-0.3}(0.35 - 0.25)$$

$$\Rightarrow q_2^I = 364$$

Solution (Cont.)

e. What is the menu of contracts for the second-best production levels?

Solution: The transfer for the efficient agent is:

$$t_2^E = \theta^E q_2^E + (\theta_I - \theta_E) q_2^I + F = 0.25 \cdot 900 + 0.1 \cdot 364 + 35 = 296.4$$

and the transfer for the inefficient agent is:

$$t_2^I = \theta_2^I q_2^I + F = 0.35 \cdot 364 + 35 = 162.4$$

Hence, the menu of contracts are

$$\{(q_2^I = 364, t_2^I = 162.4), (q_2^E = 900, t_2^E = 296.4)\}$$

Solution (Cont.)

f. What is the information rent of an efficient distributor for the menu of contracts for the second-best production levels? Is this higher or lower than the profit gained for the menu of contracts for the first-best production levels?

Solution: information rent $= (\theta^I - \theta^E)q_2^I = 0.1 \cdot 364 = 36.4 < 45.9$

$$t_2^I - \theta^I q_2^I - F = 0$$

$$t_2^E - \theta^E q_2^E - F = 0$$

$$t_2^I - \theta^E q_2^I - F = (\theta^I q_2^I + F) - (\theta^E q_2^I + F) = (\theta^I - \theta^E)q_2^I \quad (\text{note } \theta^I > \theta^E)$$

Hence, the meat distributor gathers less profit / information rent with the second-best contract.