STAT234: Lecture 5 - Hypothesis Testing

Kushal K. Dey

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- Generalization to continuous distribution
- Bernoulli, Binomial and Normal Distribution

Where is the Statistics in all this?

How can I use these concepts?

A Romantic Experiment !

Most people are right-handed and even the right eye is dominant for most people. Molecular biologists have suggested that late-stage human embryos tend to turn their heads to the right. German bio-psychologist Onur Güntürkün conjectured that this tendency to turn to the right manifests itself in other ways as well, so he studied kissing couples to see if both people tended to lean to their right more often than to their left (and if so, how strong the tendency is). He and his researchers observed couples from age 13 to 70 in public places such as airports, train stations, beaches, and parks in the United States, Germany, and Turkey. They were careful not to include couples who were holding objects such as luggage that might have affected which direction they turned. In total, 124 kissing pairs were observed with 80 couples leaning right (*Nature*, 2003).

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Researchers however want to know about p, so they sample 124 couples.

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We do not know p !!

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What is 80?

$$\sum_{i=1}^{n} X_i = 80$$

$$\hat{p} = \frac{1}{n}\sum_{i=1}^{n} X_i = \frac{80}{124} = \frac{5}{8}$$

$\hat{p}$ is called the sampling proportion- here proportion of kissing right observed.

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$$p = 0.5$$

Now, if the null hypothesis was true,

$$\begin{aligned} X &= 1 \;\; \textit{with prob}\; 0.5 && (4) \\ &= 0 \;\; \textit{with prob}\; 0.5 && (5) \\ & && (6) \end{aligned}$$

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and we have observed $X_1, X_2, \cdots, X_{124}$.

What is the distribution of $\sum_{i=1}^{124} X_i$?

How to test the hypothesis?

$$\sum_{i=1}^{124} X_i \sim Bin\,(124, 0.5)$$

Can you compute $Pr[\sum_{i=1}^{124} X_i = 80]$ under this probability table.

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$$Pr\left[\sum_{i=1}^{124} X_i = 80\right] = \binom{124}{80} 0.5^{80} 0.5^{(124-80)}$$

We can also compute

$$Pr\left[\sum_{i=1}^{124} X_i >= 80\right] = \sum_{y=80}^{124} \binom{124}{y} 0.5^{y} 0.5^{(124-y)}$$

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$$E(X) = 124 \times 0.5 = 62$$

So, 44 is as far from 62 as 80 is.

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What if $\sum_{i=1}^{124} X_i$ is very small.

$$E(X) = 124 \times 0.5 = 62$$

So, 44 is as far from 62 as 80 is.

So, probability of observing something as extreme as observed value should be

$$pvalue : Pr\left[\sum_{i=1}^{124} X_i >= 80\right] + Pr\left[\sum_{i=1}^{124} X_i <= 44\right]$$

p-value

```
upper_tail_p <- pbinom(80,124,0.5,lower.tail=FALSE) + dbinom(80,
lower_tail_p <- pbinom(44,124,0.5,lower.tail=TRUE)
pvalue <- upper_tail_p + lower_tail_p
pvalue
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[1] 0.001565
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By convention in statistics, we usually say if *pvalue* is less than 0.05, then the null hypothesis is rejected.

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So, what do we do here?

We reject the null or we believe the researcher.

Normal Approximation

But we know that for large n, if

$$\sum_{i=1}^{n} X_i \sim Bin(n, p)$$

then one can use a normal approximation

$$\sum_{i=1}^{n} X_i \sim N\left(np, np(1-p)\right)$$

Here if we assume $n = 124$, which is a fair assumption,

$$\sum_{i=1}^{n} X_i \sim N(124 \times 0.5, 124 \times 0.5 \times 0.5)$$

$$\sum_{i=1}^{n} X_i \sim N(62, 31)$$

p-value Normal

under normal approximation, Continuity correction

$$p - value : Pr\left[\sum_{i=1}^{124} X_i > 79.5\right] + Pr\left[\sum_{i=1}^{124} X_i < 44.5\right]$$

Assume $Y = \sum_{i=1}^{124} X_i$,

$$Pr\left[Y > 79.5\right] := P(\frac{Y-62}{\sqrt{32}} > \frac{79.5-62}{\sqrt{31}})$$

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$$Pr\left[Y < 44.5\right] := P(\frac{Y - 62}{\sqrt{32}} < \frac{44.5 - 62}{\sqrt{31}}) = P(z < -3.14) = 0.0008$$

$$pvalue : 0.0008 + 0.0008 = 0.0016$$

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`http://www.rossmanchance.com/applets/OneProp/OneProp.htm`

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This is where confidence intervals come in

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How confident am I about it? Can I give a measure

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From the previous classes,

$$\hat{p} = \frac{1}{124} \sum_{i=1}^{124} X_i \sim N\left(p, \frac{p(1-p)}{n}\right)$$

The observed value of $\hat{p} = 0.64$.

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But $\hat{p}$ is actually a random variable $\frac{1}{124}\sum_{i=1}^{124} X_i$

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$$\sqrt{n}\frac{\hat{p}-p}{\sqrt{p(1-p)}} \sim N(0,1)$$

$$Pr\left[-\sqrt{n}\frac{0.06}{\sqrt{p(1-p)}} < \sqrt{n}\frac{(\hat{p}-p)}{\sqrt{p(1-p)}} < \sqrt{n}\frac{0.04}{\sqrt{p(1-p)}}\right]$$

$$Pr(Z < \sqrt{n}\frac{0.04}{\sqrt{p(1-p)}}) - Pr(Z < -\sqrt{n}\frac{0.06}{\sqrt{p(1-p)}})$$

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An approximation would be

$$Pr\left[-\sqrt{n}\frac{0.06}{\sqrt{\hat{p}(1-\hat{p})}} < Z < \sqrt{n}\frac{0.04}{\sqrt{\hat{p}(1-\hat{p})}}\right]$$

$$Pr\left[-1.414214 < Z < 0.942809\right] \approx 0.7484612$$

This means that

$$Pr\left[-0.06 < \hat{p} - p < 0.04\right] \approx 0.7484612$$

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The interval $[\hat{p} - 0.04, \hat{p} + 0.06]$, because $\hat{p}$ is random.

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The interval $[\hat{p} - 0.04, \hat{p} + 0.06]$, because $\hat{p}$ is random.

Confidence Interval

Here the observed realization is $\hat{p} = 0.64$. So, for the given sample, the confidence interval is equal to

$$CI\ for\ p : [0.64 - 0.04, 0.64 + 0.06] = [0.60, 0.70]$$

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- If we repeatedly draw samples and calculate confidence intervals using this procedure, 75 % of these intervals $(\hat{p} - 0.04, \hat{p} + 0.06)$ will cover the true p.

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What is the 95% confidence interval here?

$$CIfern : [0.556, 0.723]$$

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Suppose we repeat the following procedure multiple times:

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95% of the intervals thus constructed will cover the true (unknown) population mean/parameter p.

A conversation between you and your boss

- Boss: How confident are you that this interval (0.56, 0.72) contains the true p?

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Applet

http://www.rossmanchance.com/applets/ConfSim.html

Is the CI unique?

if we follow normal distribution table, you can check

$$Pr\left[-1.15 < Z < 1.15\right] = 0.75$$

This would lead to p belonging to the interval

$$p \in \left[\hat{p} - 1.15\frac{\sqrt{\hat{p}(1-\hat{p})}}{\sqrt{n}}, \hat{p} + 1.15\frac{\sqrt{\hat{p}(1-\hat{p})}}{\sqrt{n}}\right]$$

around 75% times. This reduces to the CI $(0.59, 0.689)$ as a 75% confidence interval for p given that $\hat{p} = 0.64$.
But we already saw another 75% confidence interval $(0.6, 0.7)$, which is not symmetric about 0.64.

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In general, CI :- estimate $\pm$ margin of error

If the population distribution is normal, the interval is *exact*.
Otherwise, it is *approximately correct for large n*.

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For a given confidence level $(1-\alpha)$, how do we find z^*?

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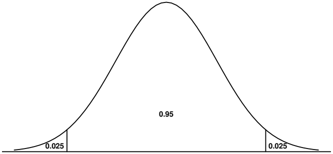

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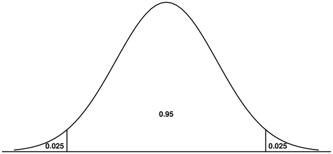

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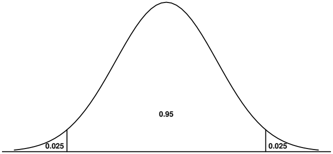

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Common z^* values:

Confidence Level	90%	95%	99%
z^*	1.645	1.96	2.576