

# STAT234: Lecture 1 - Basics of Data Analysis !!

Kushal K. Dey

# Requisites

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Office hour time (7 sessions) + Problem session (4 hrs) each week:  
Timings to be decided later

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Examples: (Rather questions ! )

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- ▶ How much money did you spend on lunch?

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- ▶ How much money did you spend on lunch?
- ▶ How What grade did you get in STAT 234?

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- ▶ How much money did you spend on lunch?
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# Backdrop

Beginning of Statistics?

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1532 → First weekly data on deaths in London (Sir W. Petty)

1539 → Data collection on marriages, baptism and death in France

1654 → Correspondence with gambling and probability (Fermat and Pascal)

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Topics covered in this course will focus on the period between 1890s-1960s.

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## Beginning of Statistics?

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Topics covered in this course will focus on the period between 1890s-1960s.

Recently there is a lot of interest in Data Science and Big Data Analysis, which are essentially applying statistics on data of large size (many GBs or TBs). For example, Facebook, Twitter and Google generates massive data of user activity for billions of users daily!

# Lets look at some Data!!

As a toy exercise, lets analyze twitter feed of.....

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Load the data

```
#library(devtools); install_github('kkdey/TrumpTwitterFeed')
library(TrumpTwitterFeed)
data("trump.data.frame")
dim(trump.data.frame)
```

```
[1] 1336    6
```

# Snapshot of the Data-1

```
head(trump.data.frame[,1:5], 3)
```

	tweet_month	tweet_year	tweet_day	retweets	favorites
1	2016-Mar	2016	26	7625	24147
2	2016-Mar	2016	26	6412	19867
3	2016-Mar	2016	26	4773	15029

```
tail(trump.data.frame[,1:5], 3)
```

	tweet_month	tweet_year	tweet_day	retweets	favorites
1334	2015-Oct	2015	13	732	1640
1335	2015-Oct	2015	13	974	2254
1336	2015-Oct	2015	13	4578	8393

## Snapshot of the Data- 2

```
glimpse(trump.data.frame)
```

```
Observations: 1,336
```

```
Variables: 6
```

```
$ tweet_month (fctr) 2016-Mar, 2016-Mar, 2016-Mar, 2016...  
$ tweet_year  (fctr) 2016, 2016, 2016, 2016, 2016, 2016...  
$ tweet_day   (fctr) 26, 26, 26, 26, 26, 26, 26, 26, 26...  
$ retweets    (dbl) 7625, 6412, 4773, 7079, 5143, 5374,...  
$ favorites    (dbl) 24147, 19867, 15029, 20798, 15922, ...  
$ tweet_text   (fctr) Remember, I am the only candidate ...
```

```
summary(trump.data.frame)
```

tweet_month	tweet_year	tweet_day	retweets
2015-Oct: 93	2015:434	03 : 93	Min. : 322
2015-Nov:138	2016:902	28 : 89	1st Qu.: 1270
2015-Dec:203		12 : 75	Median : 2356
2016-Jan:226		24 : 70	Mean : 3113
2016-Feb:324		23 : 69	3rd Qu.: 4130
2016-Mar:352		15 : 68	Max. : 25524
		(Other):872	

```
favorites  
Min. : 713  
1st Qu.: 3729  
Median : 6905
```



## Sorting number of retweets

```
sorted_retweet_counts <- sort(trump.data.frame$retweets)
```

```
tail(sorted_retweet_counts)
```

```
[1] 18287 18303 18638 19252 25323 25524
```

```
head(sorted_retweet_counts)
```

```
[1] 322 375 412 418 439 455
```

## Sorting number of retweets

```
quantile(~retweets, data=trump.data.frame)
```

0%	25%	50%	75%	100%
322	1270	2356	4130	25524

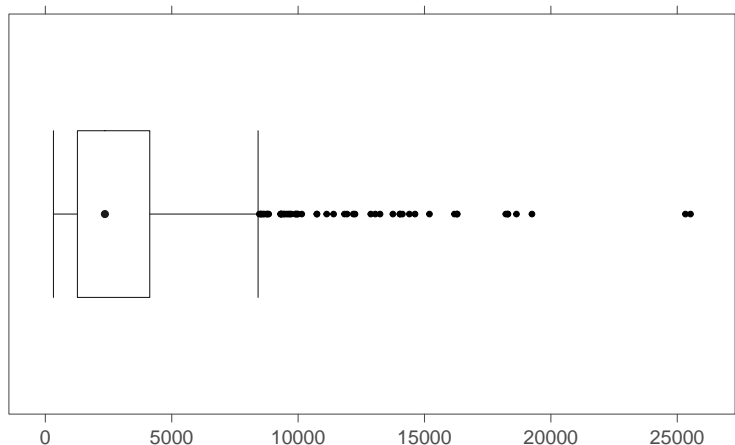
IQR or inter-quartile range is the difference between the 75 th quantile and the 25 tquantile.

```
IQR(~retweets, data=trump.data.frame)
```

```
[1] 2860
```

## Box plot of retweets

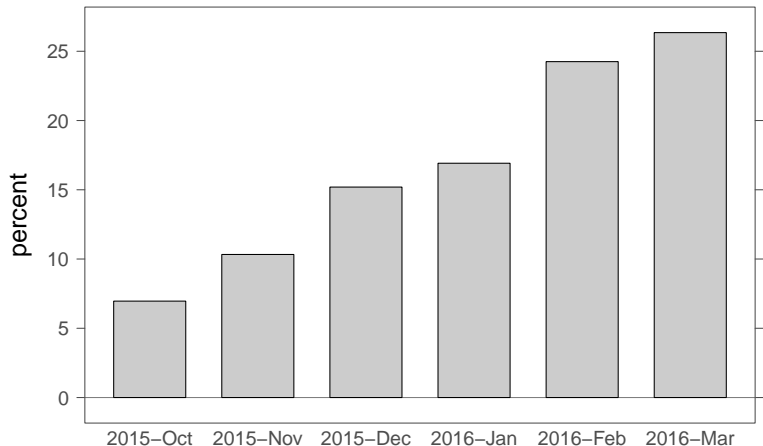
```
bwplot(~ retweets, data=trump.data.frame,  
       xlab="Retweets box plot distribution")
```



Retweets box plot distribution

## Bar graph

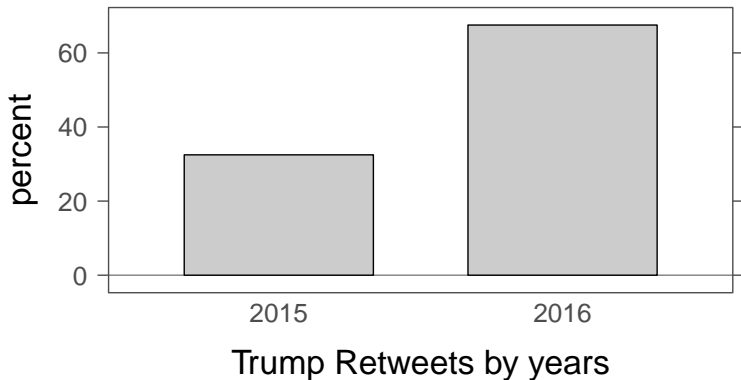
```
bargraph(retweets ~ tweet_month, data=trump.data.frame,  
         type="percent", xlab="Trump Retweets by month", cex=0.5)
```



Trump Retweets by month

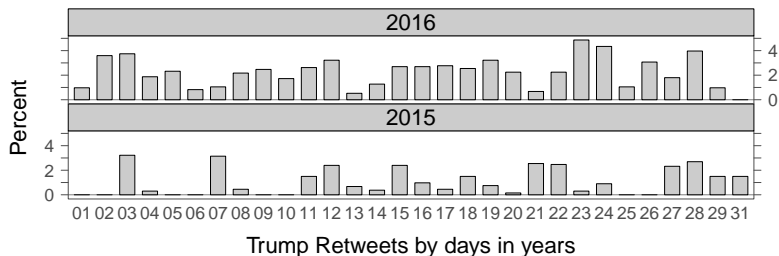
## Bar graph

```
bargraph(retweets ~ tweet_year,  
         data=trump.data.frame, type="percent",  
         xlab="Trump Retweets by years", cex=0.5)
```



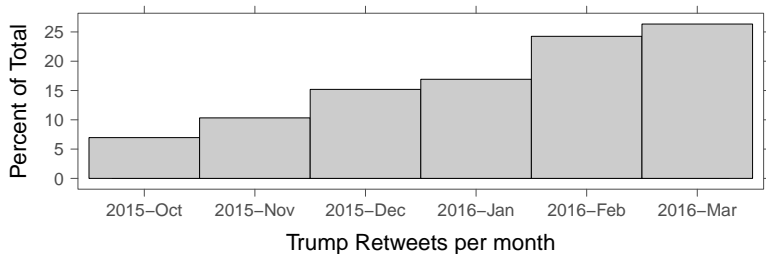
## Bar graph

```
bargraph(retweets~ tweet_day | tweet_year,  
          data=trump.data.frame, type="percent",  
          xlab="Trump Retweets by days in years",  
          ylab="Percent", layout=c(1,2))
```



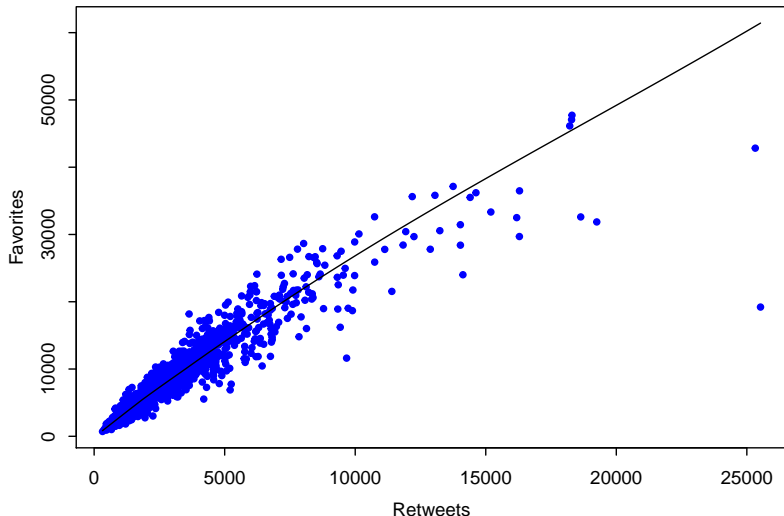
# Histogram

```
histogram(retweets ~ tweet_month, data=trump.data.frame,  
          type="percent", xlab="Trump Retweets per month")
```



# Scatter Plot

```
scatter.smooth(trump.data.frame$retweets,  
               trump.data.frame$favorites, lwd=1, pch=20,  
               col="blue", xlab="Retweets", ylab="Favorites")
```





# The Average is the Balancing Point

Consider the data  $x_1 = 9, x_2 = 3, x_3 = 15, x_4 = 1$

- ▶ What is the average of these values?

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- ▶ The average is the “balancing point” of the data, the “center of mass” (assigning each data value the same mass =  $1/4$ )

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- ▶ The average is the “balancing point” of the data, the “center of mass” (assigning each data value the same mass =  $1/4$ )

Talk a moment with your neighbor. See if you can come up an equation to express this “balancing point” property of the average.

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Talk a moment with your neighbor. See if you can come up an equation to express this “balancing point” property of the average.

**Proof:** Show that for **any** sample of size  $n$ , 
$$\sum_{i=1}^n (x_i - \bar{x}) = 0$$

# How to Prove the Math Stuff I

- ▶ A proof is a “paragraph” of mathematical “sentences”,
- ▶ written in order to make logical sense to the reader.  
...just like you do in the Core all the time!
- ▶ It's your personal argument as to why a claim must be true.
- ▶ Justify each step (“sentence”) using statistics  
(and using results already proven in the course).

# How to Prove the Math Stuff II

OK. Our first proof is to confirm an equation.

**Proof:** Show that for **any** sample of size  $n$ ,  $\sum_{i=1}^n (x_i - \bar{x}) = 0$

Start on the left side:  $\sum_{i=1}^n (x_i - \bar{x})$

= rewrite

= and rewrite

= and rewrite again

= until arriving at the right side = 0

**In groups:** Write down a first step.



# Our First Proof!

Four common starting points.

Three are great, but one is incorrect. Which one? Why?

$$1. \sum_{i=1}^n (x_i - \bar{x}) = (x_1 - \bar{x}) + (x_2 - \bar{x}) + \cdots + (x_n - \bar{x})$$

$$2. \sum_{i=1}^n (x_i - \bar{x}) = \sum_{i=1}^n \left[ x_i - \frac{1}{n} \sum_{j=1}^n x_j \right]$$

$$3. \sum_{i=1}^n (x_i - \bar{x}) = 0$$

$$4. \sum_{i=1}^n (x_i - \bar{x}) = \left[ \sum_{i=1}^n x_i \right] - \left[ \sum_{i=1}^n \bar{x} \right]$$

Is “ $\Sigma$ ” confusing you? Read Chapter 0 (Math Supplement).

# Our First Proof!

Starting with the first option:

$$\sum_{i=1}^n (x_i - \bar{x}) = (x_1 - \bar{x}) + (x_2 - \bar{x}) + \cdots + (x_n - \bar{x})$$

=

=

=

=

# Our First Proof!

Starting with the first option:

$$\begin{aligned}\sum_{i=1}^n (x_i - \bar{x}) &= (x_1 - \bar{x}) + (x_2 - \bar{x}) + \cdots + (x_n - \bar{x}) \\ &= (x_1 + x_2 + \cdots + x_n) - \underbrace{(\bar{x} + \bar{x} + \cdots + \bar{x})}_{n \text{ times}} \\ &= \\ &= \\ &= \end{aligned}$$

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$$\begin{aligned}\sum_{i=1}^n (x_i - \bar{x}) &= (x_1 - \bar{x}) + (x_2 - \bar{x}) + \cdots + (x_n - \bar{x}) \\&= (x_1 + x_2 + \cdots + x_n) - \underbrace{(\bar{x} + \bar{x} + \cdots + \bar{x})}_{n \text{ times}} \\&= \left[ \sum_{i=1}^n x_i \right] - n\bar{x} = \left[ \frac{n}{n} \sum_{i=1}^n x_i \right] - n\bar{x} \\&= n\bar{x} - n\bar{x} \quad \text{since } \bar{x} = \frac{1}{n} \sum_{i=1}^n x_i \quad (\text{Justification required!}) \\&= 0\end{aligned}$$

Let's agree that  $b - b = 0$  for any real number  $b$ . :)



# Measuring Spread of Data Distribution I

The average deviation  $\frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})$  **always** = 0!

Need a different measure for “typical size of deviations” (spread)

There are many measures of spread:

- ▶ mean squared deviation (*MSD* or “variance”),
- ▶ mean absolute deviation (*MAD*),
- ▶ standard deviation (*SD*) = root *MSD* = *RMSD* =  $\sqrt{MSD}$ ,
- ▶ interquartile range (*IQR*= range of middle 50% of data)
- ▶ range,
- ▶ ...and more (not covered in this course).

## Measuring Spread of Data Distribution II

Let's consider two common loss functions (measures of spread)

- ▶ The mean of absolute deviations:

$$MAD(w) = \frac{1}{n} \sum_{i=1}^n |x_i - w|$$

- ▶ The mean of squared deviations:

$$MSD(w) = \frac{1}{n} \sum_{i=1}^n (x_i - w)^2$$

What value of  $w$  should we choose using  $MAD$ ? Using  $MSD$ ?

It seems reasonable that  $w$  should be in the “center” of the data for each measure. But which value in the middle would be best?

One optimality criteria: Choose  $w$  that minimizes  $MAD$  or  $MSD$ .

## A more objective qualification

- ▶ Let's say  $w$  is a good candidate for the center. Then in some sense  $x_1 - w$ ,  $x_2 - w$ ,  $x_n - w$  should be small in a collective fashion.
- ▶ How about we combine these quantities?
- ▶

$$MSD(w) = \frac{1}{n} \{ (x_1 - w)^2 + (x_2 - w)^2 + \cdots + (x_n - w)^2 \} = \frac{1}{n} \sum_{i=1}^n (x_i - w)^2$$

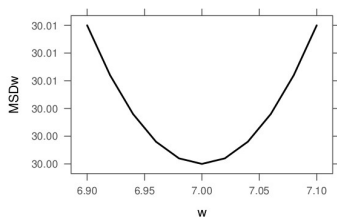
## Behavior of MSD function

Let's take the very simple dataset comprising of only 4 points 1, 3, 15 and 9.

	w	MSD <sub>w</sub>
[1,]	1	66
[2,]	2	55
[3,]	3	46
[4,]	4	39
[5,]	5	34
[6,]	6	31
[7,]	7	30
[8,]	8	31
[9,]	9	34
[10,]	10	39
[11,]	11	46
[12,]	12	55
[13,]	13	66
[14,]	14	79
[15,]	15	94

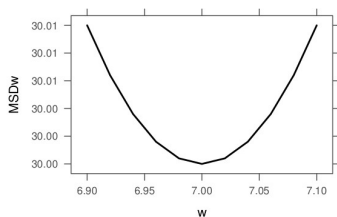
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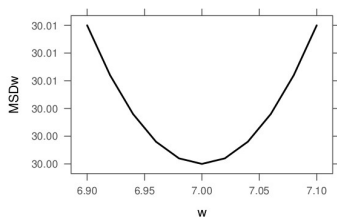
Let's take the very simple dataset comprising of only 4 points 1, 3, 15 and 9.



7 is the mean !!

## Behavior of MSD function

Let's take the very simple dataset comprising of only 4 points 1, 3, 15 and 9.



7 is the mean !! Can we prove it analytically as well?

We call MSD as this function evaluated at  $\bar{x}$ , i.e

$$MSD = L_1(\bar{x}) = \frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2$$

# Loss function

- ▶ MSD function:

$$L_1(w) = \frac{1}{n} \sum_{i=1}^n (x_i - w)^2$$

- ▶ MAD function:

$$L_2(w) = \frac{1}{n} \sum_{i=1}^n |x_i - w|$$

- ▶ MAD stands for Mean Absolute Deviation
- ▶ Minimize MSD function  $\rightarrow$  Mean



# Loss function

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$$L_2(w) = \frac{1}{n} \sum_{i=1}^n |x_i - w|$$

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- ▶ Minimize MAD function  $\rightarrow$  Median

# Loss function

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- ▶ MAD stands for Mean Absolute Deviation
- ▶ Minimize MSD function  $\rightarrow$  Mean
- ▶ Minimize MAD function  $\rightarrow$  Median
- ▶ Median is another measure of central tendency

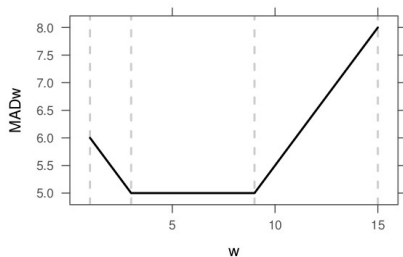
## Behavior of MAD

Let's take the same dataset again 1, 3, 15 and 9.

	w	MAD <sub>w</sub>
[1,]	1	6.0
[2,]	2	5.5
[3,]	3	5.0
[4,]	4	5.0
[5,]	5	5.0
[6,]	6	5.0
[7,]	7	5.0
[8,]	8	5.0
[9,]	9	5.0
[10,]	10	5.5
[11,]	11	6.0
[12,]	12	6.5
[13,]	13	7.0
[14,]	14	7.5
[15,]	15	8.0

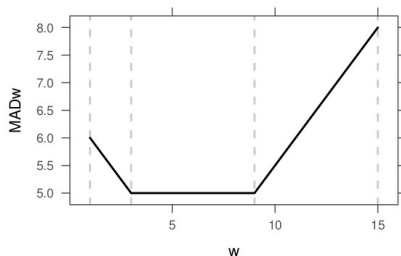
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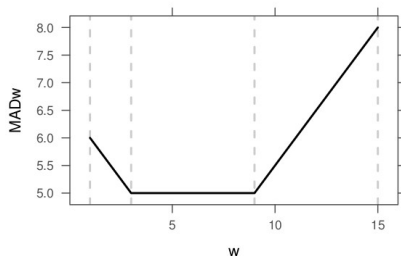
Let's take the same dataset again 1, 3, 15 and 9.



Minimum attained at any point between 3 to 9.

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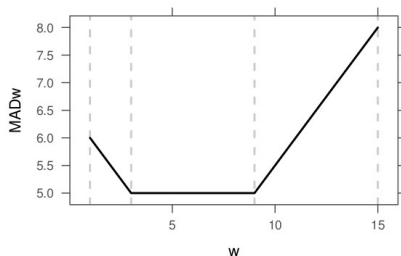
Let's take the same dataset again 1, 3, 15 and 9.



Minimum attained at any point between 3 to 9. All the points in  $[3,9]$  qualify as the median.

## Behavior of MAD

Let's take the same dataset again 1, 3, 15 and 9.



Minimum attained at any point between 3 to 9. All the points in  $[3,9]$  qualify as the median. To keep it definite we will take  $(3+9)/2=6$  as our median here.

## Measures of Center: Median

**Definition** First order the data points  $x_1, x_2, \dots, x_n$  in ascending order (including repetitions). Then **sample median**  $\tilde{x}$  of the *sample*  $x_1, x_2, \dots, x_n$  is the single middle value of the *ordered set* if  $n$  is odd and the average of two middle values if  $n$  is even.

Let's take an example.



# Measures of Center: Median

**Example** Consider the following data set:

6.3, 10.2, 3.8, 7.9, 8.0, 5.5, 6.8

- ▶ Number of observations? Odd or even?

# Measures of Center: Median

**Example** Consider the following data set:

6.3, 10.2, 3.8, 7.9, 8.0, 5.5, 6.8

- ▶ Number of observations? Odd or even?

Ans. 7. Odd.

# Measures of Center: Median

**Example** Consider the following data set:

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- ▶ Number of observations? Odd or even?

Ans. 7. Odd.

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Ans. Middle position is 4. Median is 6.8

## Measures of Center: Median

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- ▶ What are the 2 middle positions? What is the median?  
Ans. Middle positions are 4 and 5. Median is  $\frac{6.8+7.3}{2} = 7.05$ .

# Measures of Center: Median

So we can formulate the sample median  $\tilde{x}$  as:

- ▶ The  $(\frac{n+1}{2})$ th ordered valued in the ordered list obtained from the sample when  $n$  is odd.
- ▶ The average of  $(\frac{n}{2})$ th and  $(\frac{n}{2} + 1)$ th ordered values in the ordered list when  $n$  is even.

## Formulas for Sample Average, Variance, SD

$$\text{sample average} = \bar{x} = \text{"x-bar"} = \frac{1}{n} \sum_{i=1}^n x_i$$

$$\text{sample variance} = s^2 = \text{"s-squared"} = \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2$$

$$\begin{aligned} \text{sample standard deviation} = s = \sqrt{s^2} &= \sqrt{\frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2} \\ &= \text{"typical" distance from the average} \end{aligned}$$

Why divide by  $(n - 1)$  instead of  $n$  for sample variance and SD?

## Why divide by $(n - 1)$ for sample variance and SD? I

**Variance** has a particular meaning in statistics:  
**mean squared distance from the average**

$$MSD_n(\bar{x}) = \frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2$$

Why collect data (**statistics**)?

To learn about the population (**parameters**).

$$\text{population mean} = \mu = \text{"myoo"} = \frac{1}{N} \sum_{i=1}^N x_i$$

$$\text{popn variance} = \sigma^2 = \text{"sigma squared"} = \frac{1}{N} \sum_{i=1}^N (x_i - \mu)^2$$

## Why divide by $(n - 1)$ for sample variance and SD? II

$$\text{truth} = \text{popn variance} = \sigma^2 = MSD_N(\mu) = \frac{1}{N} \sum_{i=1}^N (x_i - \mu)^2$$

If we know the true popn mean  $(\mu)$  and had a sample of  $n$ , use

$$\text{estimate} = \hat{\sigma}_{\mu}^2 = MSD_n(\mu) = \frac{1}{n} \sum_{i=1}^n (x_i - \mu)^2 \quad (1)$$

But, we almost never know  $\mu$ ! That's why we sample!

$$\text{realistic estimate} = \hat{\sigma}_{\bar{x}}^2 = MSD_n(\bar{x}) = \frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2 \quad (2)$$

The problem:  $(2) \leq (1)$ . Why? ...and why is this a problem?  
How does dividing by  $(n - 1)$  for (2) help? solve the problem?

## Why divide by $(n - 1)$ for sample variance and SD? III

OK. So, we should divide by a number smaller than  $n$ .

But, why  $(n - 1)$  in particular?

**Claim:** Just  $(n - 1)$  observations and  $\bar{x}$  are sufficient to determine the one remaining observation.

**Proof:** We know  $n\bar{x} = x_1 + x_2 + \cdots + x_n$ , since  $\bar{x} = \frac{1}{n} \sum x_i$   
So,  $x_n = n\bar{x} - (x_1 + x_2 + \cdots + x_{n-1})$ .

In a sense,  $\sum (x_i - \bar{x})^2$  adds up  $(n - 1)$  “independent” values.

We say that the sum  $\sum (x_i - \bar{x})^2$  has  $(n - 1)$  **degrees of freedom** .

So, the sample average squared deviation (variance) is defined as

$$s^2 = \text{“s-squared”} = \frac{1}{n - 1} \sum_{i=1}^n (x_i - \bar{x})^2$$

# Linear Transformation of Data I

Sometimes we want to analyze data in different units

Temperature unit in USA : degree Fahrenheit

Temperature unit in India: degree Celsius

$$\text{Temperature: Celsius} = \frac{5}{9}(\text{Fahrenheit} - 32)$$

$$\text{Temperature: Celsius} = -\left(\frac{160}{9}\right) + \left(\frac{5}{9}\right) \text{ Fahrenheit}$$



## Linear Transformation of Data II

High temperature in Chicago last 5 days of December

```
Fahrenheit <- c(39, 39, 29, 28, 31)  
OLD <- options(digits=3)
```

```
Fahrenheit
```

```
[1] 39 39 29 28 31
```

```
mean(Fahrenheit)
```

```
[1] 33.2
```

## Linear Transformation of Data III

```
Celsius = -(160/9) + (5/9)*Fahrenheit
```

```
rbind(Fahrenheit, Celsius)
```

	[,1]	[,2]	[,3]	[,4]	[,5]
Fahrenheit	39.00	39.00	29.00	28.00	31.000
Celsius	3.89	3.89	-1.67	-2.22	-0.556

```
mean(Celsius)
```

```
[1] 0.667
```

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```
-(160/9) + (5/9) * mean(Fahrenheit)
```

```
[1] 0.667
```

## Linear Transformation of Data IV

**Claim:** If data  $x_1, x_2, \dots, x_n$   
are linearly transformed to  $y_i = a + bx_i$

Then,  $\bar{y} = a + b\bar{x}$ .

**Proof:**

A proof appears in Section 1.4 (Math Supplement).

# Linear Transformation of Data V

```
sd(Celsius)
```

```
[1] 3
```

```
(5/9) * sd(Fahrenheit)
```

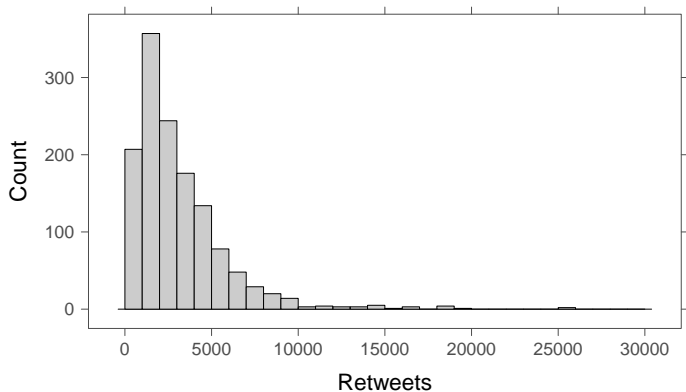
```
[1] 3
```

**Claim:** If data  $x_1, x_2, \dots, x_n$   
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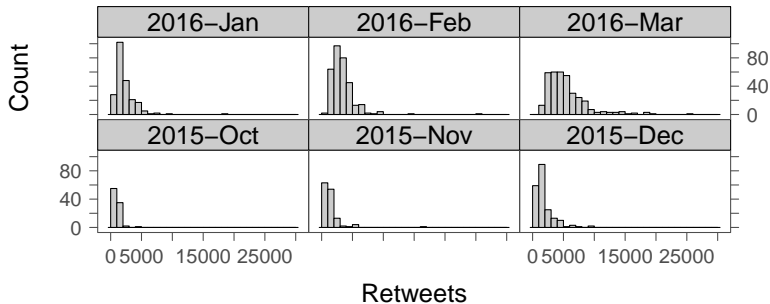
Then,  $SD(y) = s_y = |b|s_x = |b|SD(x)$ .

**Proof:** On your own for HW #2.

# The distribution of retweets |

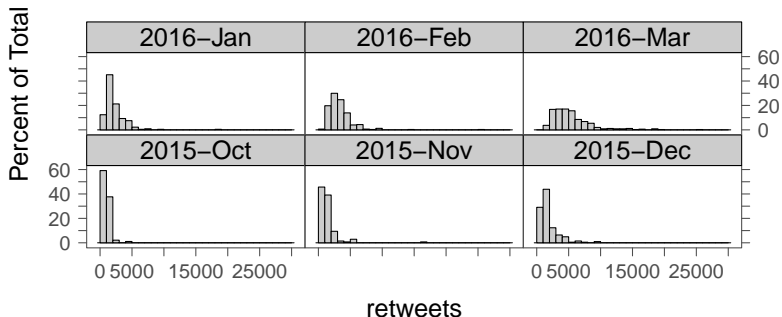


# The distribution of retweets II

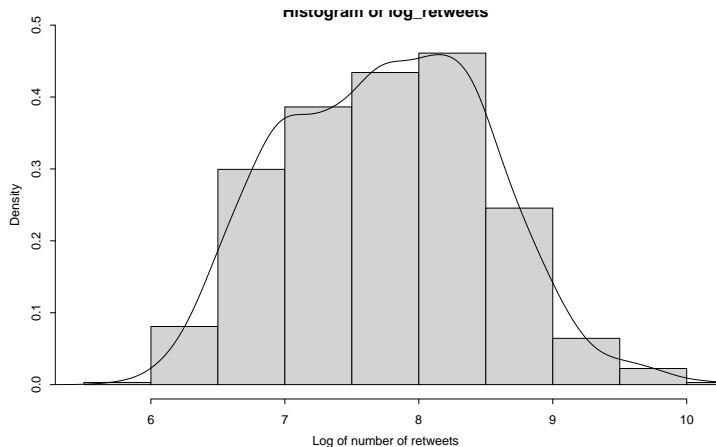


# The distribution of retweets III

Let's make the comparison based on percentages, not counts

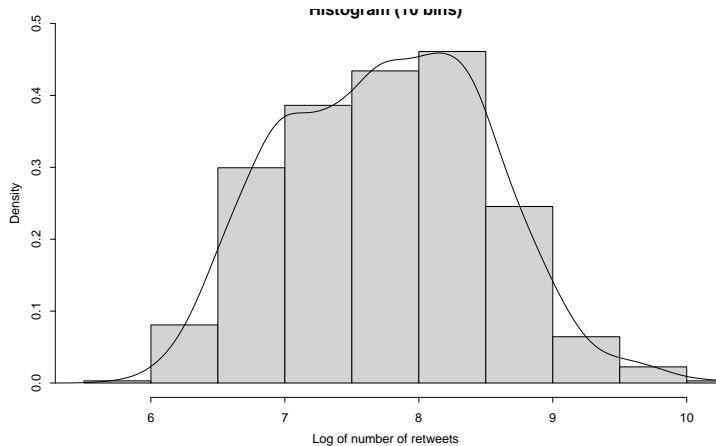


# The distribution of logarithm of retweets |

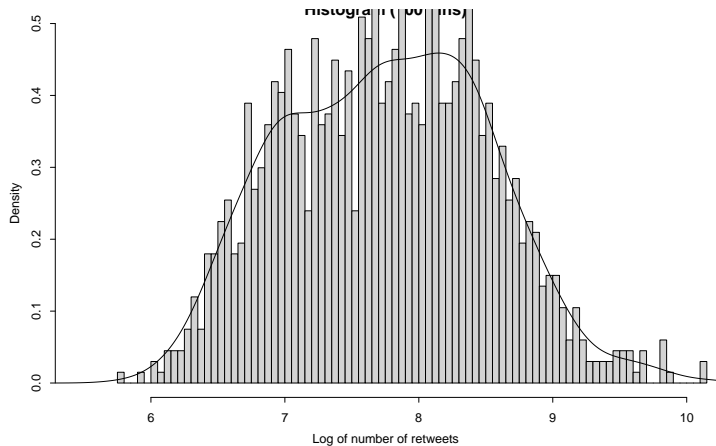




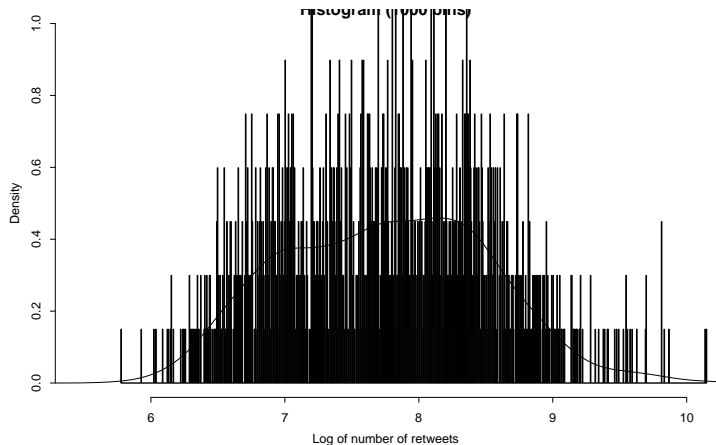
## The distribution of logarithm of retweets II



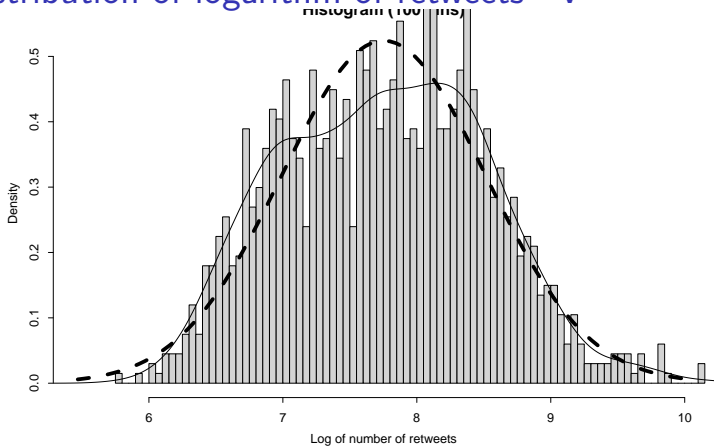
# The distribution of logarithm of retweets III



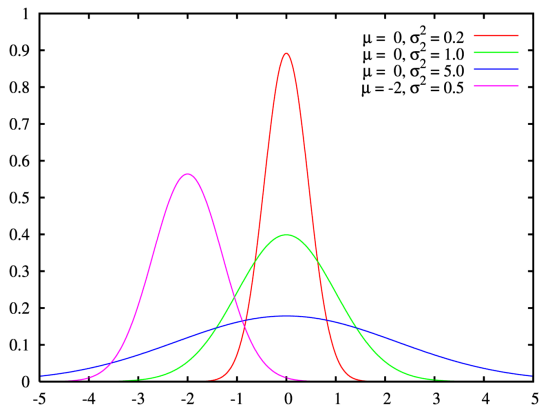
# The distribution of logarithm of retweets IV



# The distribution of logarithm of retweets V



# Normal Distribution



# Facts about Normal Distribution

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- ▶ Two parameters. Mean ( $\mu$ ) and standard deviation ( $\sigma$ )
- ▶  $\mu$  and  $\sigma$  are population parameters.
- ▶  $\mu = 0$  and  $\sigma = 1$  refers to the standard normal distribution.
- ▶ Centred around  $\mu$ .
- ▶ For standard normal

$$P(X > 1) = P(X < -1)$$

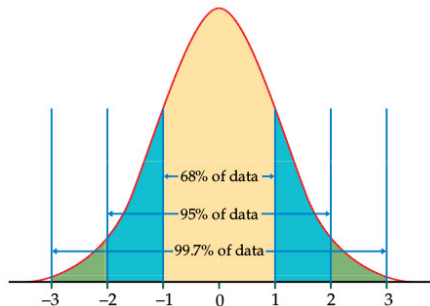
Why?

- ▶ The formula for  $N(\mu, \sigma)$  is (don't be scared - just FYI!)

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

# Properties of Normal distribution

For a Normal Distribution ( mean  $\mu = 0$  and sd  $\sigma = 1$ )

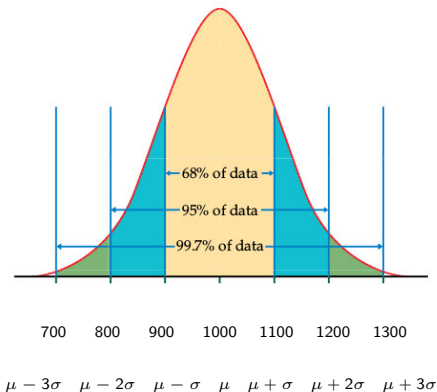


This is called the 68% – 95% – 99.7% rule. This often simplifies our calculations.

What happens for a general  $\mu$  and  $\sigma$ ?

# Properties of Normal distribution

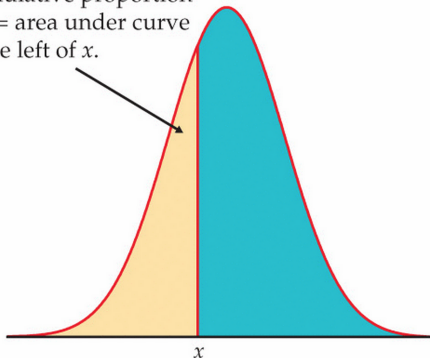
For a Normal Distribution (  $\mu = 1000$  and  $\sigma = 100$  )



# Cumulative Proportions and Standard Normal Table

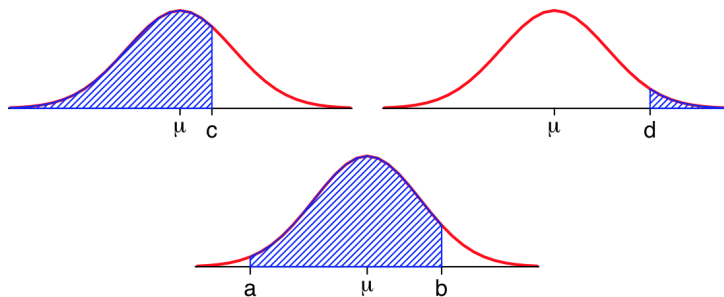
- ▶ **Cumulative proportions:** The proportion of observations in a distribution that lie at or below a given value  $x$ .
- ▶ For  $N(0, 1)$ , use the standard normal table (Table A in textbook) to calculate cumulative proportions for  $x$ .

Cumulative proportion  
at  $x$  = area under curve  
to the left of  $x$ .



# Cumulative Proportions and Standard Normal Table

For  $N(0,1)$ , we can calculate all of the following areas using cumulative proportions:



# Normal Distribution

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## Standardization and z-score

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- ▶ Example? Assume that, time spent on the calls ( The data from Lecture 1 ) follows approximately a normal distribution with mean  $\mu = 1000$  and  $\sigma = 100$ .
- ▶ Suppose on particular observation is 870,

$$z - \text{score} = \frac{670 - 1000}{100} = -1.3$$

- ▶ We will use this z-score later to make some conclusions.



# The normal density model in R I

The "normal density" model.

A good fit for these data distributions?

A numerical look

What percent of area under standard normal density is above/below 1?

## The normal density model in R II

```
pnorm(-1, m=0, s=1)
```

```
[1] 0.1587
```

```
pnorm(-1)
```

```
[1] 0.1587
```

```
1 - pnorm(1)
```

```
[1] 0.1587
```

## The normal density model in R III

What percent of area under *any* normal density is above/below 1 sd from mean?

```
mtweets <- mean(log_retweets)
mtweets
```

```
[1] 7.753
```

```
stweets <- sd(log_retweets)
stweets
```

```
[1] 0.7613
```

```
1 - pnorm(mtweets + stweets, m=mtweets, s=stweets)
```

```
[1] 0.1587
```

```
pnorm(mtweets - stweets, m=mtweets, s=stweets)
```

```
[1] 0.1587
```

## The normal density model in R IV

What percent of the observed data are right/left of 1 sd from mean?

```
sum(log_retweets >= mtweets + stweets, na.rm=TRUE) / n
```

```
[1] 0.1639
```

```
sum(log_retweets <= mtweets - stweets, na.rm=TRUE) / n
```

```
[1] 0.1886
```

## The normal density model in R V

What percent of the model/data are 2 sd to the right mean?

```
1 - pnorm(2)
```

```
[1] 0.02275
```

```
sum(log_retweets >= mtweets + 2*stweets, na.rm=TRUE) / n
```

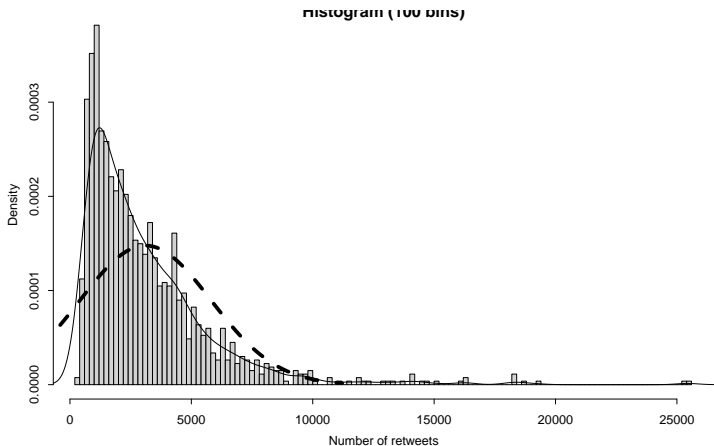
```
[1] 0.02096
```

What percent of the model/data are 2 sd to the left of mean?

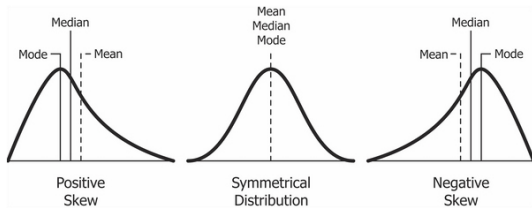
```
pnorm(-2)
```

```
sum(log_retweets <= mtweets - 2*stweets, na.rm=TRUE) / n
```

# When things are not normal!!



# Skewness



## A Step back: Quantiles

- ▶ The  $k$ th quantile of a set of values divides them so that  $100 * k$  of the values lie below and  $100 * (1 - k)$  of the values lie above.



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## A Step back: Quantiles

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## A Step back: Quantiles

Lets look at the quantiles of a set of values - 3.4, 2.3, 6.7, 2.1, 5.0

## A Step back: Quantiles

Lets look at the quantiles of a set of values - 3.4, 2.3, 6.7, 2.1, 5.0

First sort the values in order

2.1, 2.3 3.4 5.0 6.7

Then the quantiles for this data is given as follows

Sample fraction	0	0.25	0.50	0.75	1
Quantiles	2.1	2.3	3.4	5.0	6.7

# Quantiles in R

0%	25%	50%	75%	100%
2.1	2.3	3.4	5.0	6.7

Consider the Trump twitter feed data

```
favstats(retweets | tweet_year, data=trump.data.frame)
```

	tweet_year	min	Q1	median	Q3	max	mean	sd	n
1	2015	322	818.2	1108	1674	16289	1527	1397	434
2	2016	581	2065.2	3239	4753	25524	3876	2841	902
	missing								
1		0							
2		0							

## IQR, Boxplot and outliers

- ▶ Another measure is  $IQR = Q_3 - Q_1$



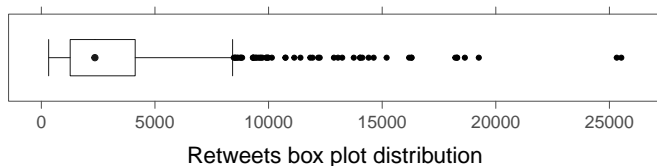
## IQR, Boxplot and outliers

- ▶ Another measure is  $IQR = Q_3 - Q_1$
- ▶  $1.5 \times IQR$  rule : If an observation falls more than  $1.5 \times IQR$  above the third quartile or below the first quartile, call it a *suspected* outlier (Caution: Not always!!).

# IQR, Boxplot and outliers

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```
bwplot( retweets, data=trump.data.frame, xlab="Retweets box  
plot distribution")
```



# Normal Quantile Plot

How can I tell whether my data is sufficiently close to normal?

- ▶ Given data  $x = (x_1, x_2, \dots, x_n)$ , compute

$$y_i = \frac{x_i - \bar{x}}{s}$$

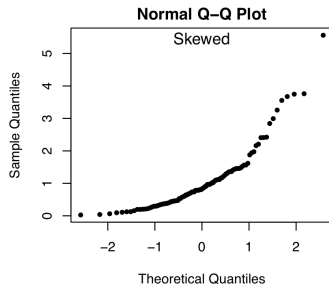
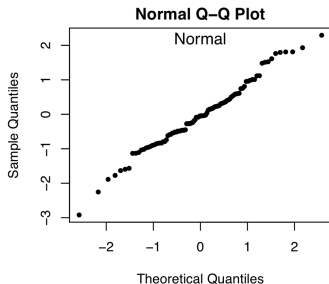
where  $\bar{x} = \text{mean}(x)$  and  $s = \text{sd}(x)$ .

- ▶ Arrange the  $y$  data in increasing order:

$$y_{[1]} \leq y_{[2]} \leq \dots \leq y_{[n]}$$

- ▶ Find the z-scores for all the percentiles  $(\frac{1}{n}, \frac{2}{n}, \dots, \frac{n}{n})$  from standard normal table
- ▶ Plot  $y_{[i]}$  values on the vertical axis against z-scores on the horizontal axis from  $i = 1, \dots, n$ .

# How Normal Quantile plot looks!

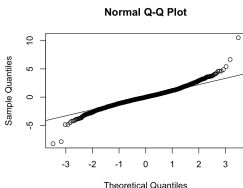


If the data are approximately normal, the Q-Q plot will be **close to a straight line**.

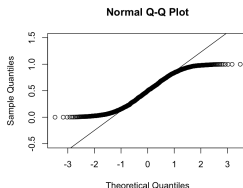
# Normal Quantile Plot

**Systematic deviations** from a straight line indicate a non-normal distribution

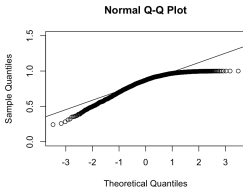
Heavy tails at both end



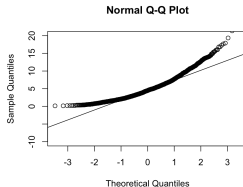
Light tails at both end



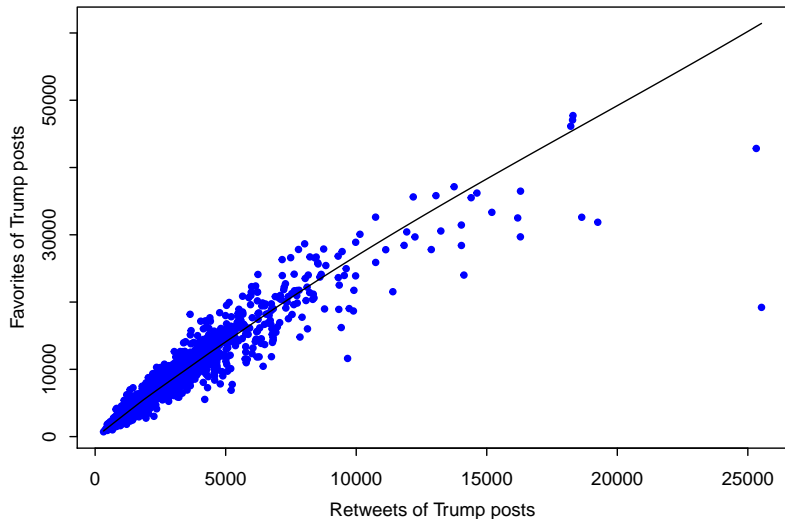
Left skewed



Right skewed



# Association between Variables



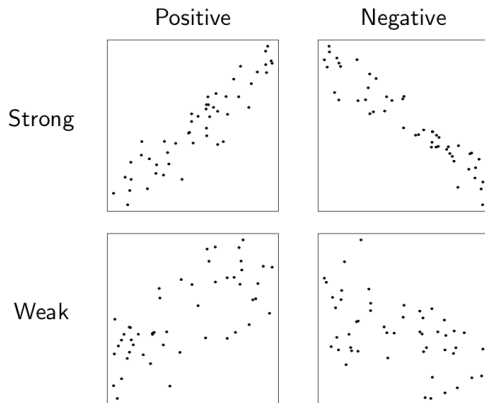
# Describing Scatterplots

You can describe patterns in a scatterplot in three aspects:

- ▶ Form: Linear, curved...
- ▶ Direction: Positive, negative
- ▶ Strength: Strong, weak

# Scatterplot

Examples of linear relationships:





## Applying quantile plots to log Retweets data I

```
mtweets <- mean(log(trump.data.frame$retweets+1))
stweets <- sd(log(trump.data.frame$retweets+1))
stdtweets = (log(trump.data.frame$retweets+1) - mtweets) / stweets
head(sort(stdtweets),3)

[1] -2.594 -2.395 -2.271

tail(sort(stdtweets),3)

[1] 2.775 3.135 3.145
```

## Applying quantile plots to log Retweets data II

```
p = c(0.01, 0.025, 0.16, 0.25, 0.50, 0.75, 0.84, 0.975, 0.99)
modelQuantile = qnorm(p)
modelQuantile
```

```
[1] -2.3263 -1.9600 -0.9945 -0.6745  0.0000  0.6745  0.9945
[8]  1.9600  2.3263
```

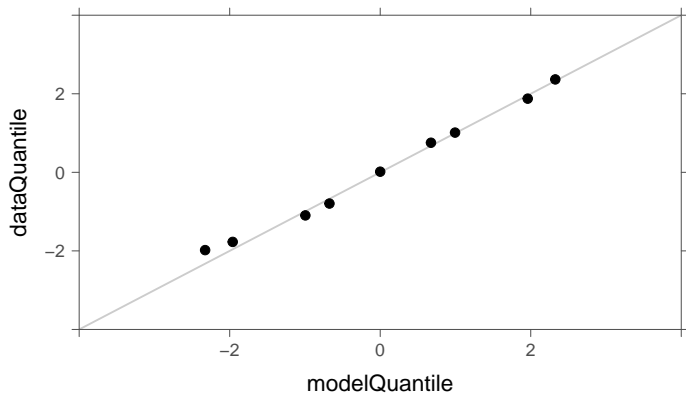
```
dataQuantile = quantile(stdtweets, p, na.rm=TRUE)
dataQuantile
```

```
      1%      2.5%      16%      25%      50%      75%
-1.98042 -1.77134 -1.09683 -0.79466  0.01657  0.75343
      84%      97.5%      99%
 1.01146  1.87581  2.36527
```

```
rbind(dataQuantile, modelQuantile)
```

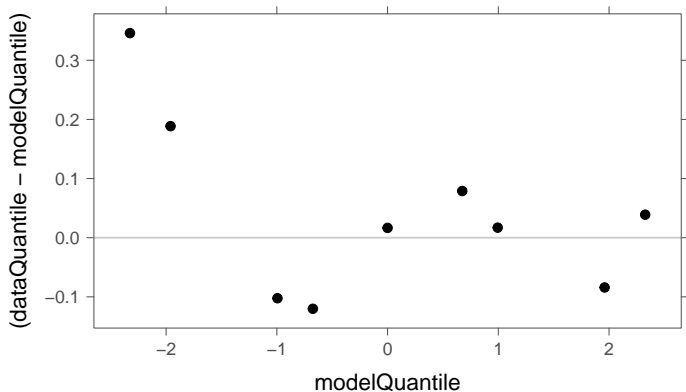
```
      1%      2.5%      16%      25%      50%      75%
dataQuantile -1.980 -1.771 -1.0968 -0.7947 0.01657 0.7534
modelQuantile -2.326 -1.960 -0.9945 -0.6745 0.00000 0.6745
      84%      97.5%      99%
dataQuantile  1.0115 1.876 2.365
modelQuantile 0.9945 1.960 2.326
```

## Quantile plots I



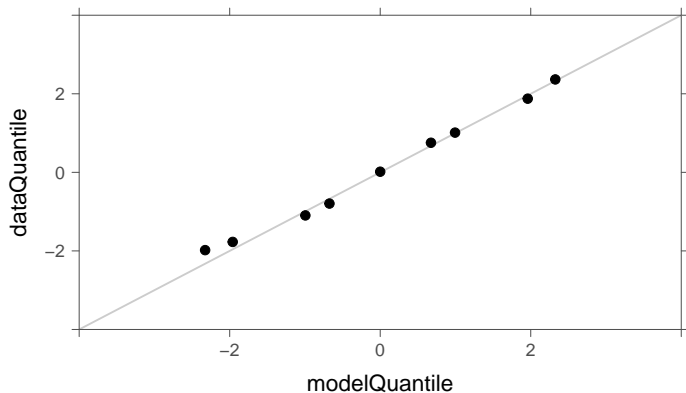
## Quantile plots II

I wish the "normal probability plot" was actually plotted like this (much easier to read)

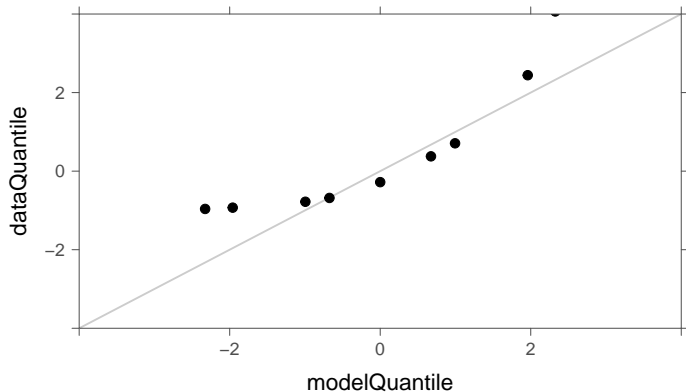


## Quantile plots III

But here is the style of plot traditionally called the "normal probability plot" or "normal quantile plot"



## Quantile plot for Retweets Data I



Questions?