STAT390: Homework 2

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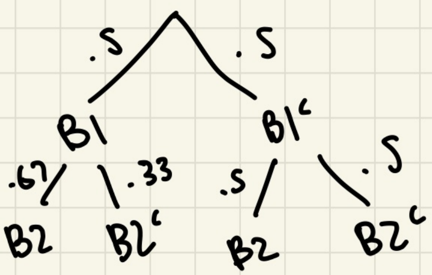
Homework 2

① $P(B2 \mid B1) + P(B2 \mid B1^c) = P(B2)$

$P(B1)P(B2 \mid B1) + P(B1^c)P(B2 \mid B1^c)$



$.5(.67) + .5(.5) = \boxed{.585 = 58.5\%}$

② $P(A) = 2/3$ $P(B) = 2/5$ $P(A \cup B) = 4/5$

R_1 infected R_2 infected R_1 or R_2 infected

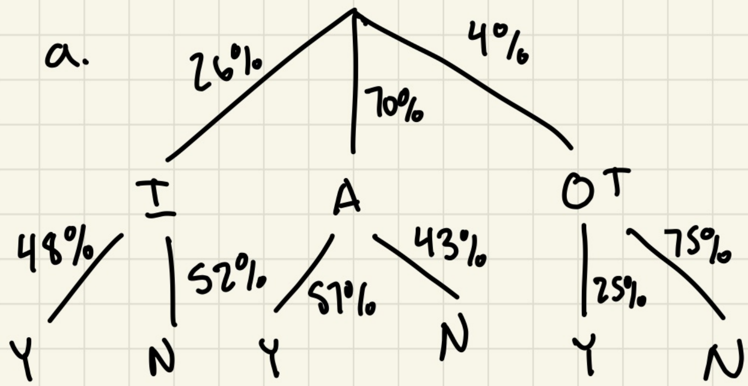
$$P(B \mid A) = \frac{P(B \cap A)}{P(A)}$$

$$P(B \mid A) = \frac{P(A) + P(B) - P(A \cup B)}{P(A)}$$

$$\frac{\frac{2}{3} + \frac{2}{5} - \frac{4}{5}}{2/3} = \frac{\frac{2}{3} - \frac{2}{5}}{2/3} = \frac{\frac{10}{15} - \frac{6}{15}}{\frac{10}{15}} = \frac{\frac{4}{15}}{\frac{10}{15}} = \frac{4}{10} = \boxed{\frac{2}{5}}$$

③

a.



ios = I
android = A
other = OT

virus = Y
no virus = N

$$P(Y) = P(Y|I)P(I) + P(Y|A)P(A) + P(Y|OT)P(OT)$$

$$.48 \cdot .26 + .57 \cdot .70 + .25 \cdot .04$$

b. $\boxed{P(Y) = .5338 = 53.38\%}$

c. $$P(A|Y) = \frac{P(A \cap Y)}{P(Y)}$$

$$= \frac{P(A)\,P(Y|A)}{P(Y)} = \frac{.70 \cdot .57}{.5338} = \boxed{.7475}$$

d. $$P(I^c | N) = \frac{P(I^c \cap N)}{P(N)}$$

$$P(N|A)P(A) + P(N|OT)P(OT)$$

$$.43 \cdot .70 + .75 \cdot .04$$

$$= .331$$

$$P((A \cup OT) \cap N)$$

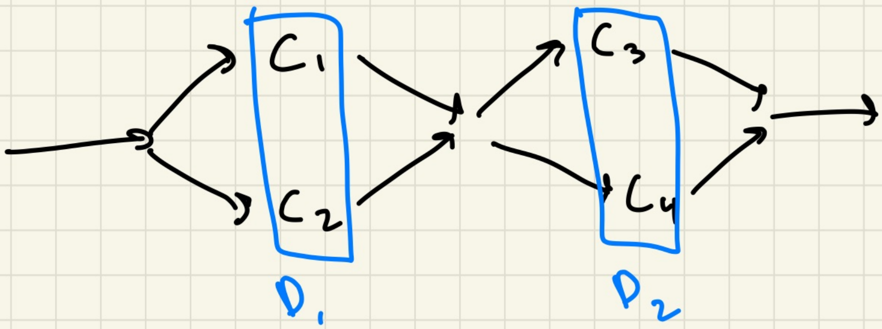
$$= P((A \cap N) \cup (OT \cap N))$$

$$= P(A \cap N) + P(OT \cap N)$$

$$P(N) = 1 - P(Y)$$

$$\frac{.331}{1 - .5338} = \boxed{.71}$$

④



a.

$$P(D_1) = P(C_1 \cup C_2)$$

$$1 - P(D_1) = 1 - P(C_1^c \cap C_2^c)$$

$$= 1 - P(C_1^c \mid C_2^c)\, P(C_2^c)$$

$$1 - .15 \cdot .15 = .9775$$

$$P(D_1 \cap D_2) = P(D_1 \mid D_2)\, P(D_2)$$

$$.9775 \cdot .9775 = \boxed{.9555}$$

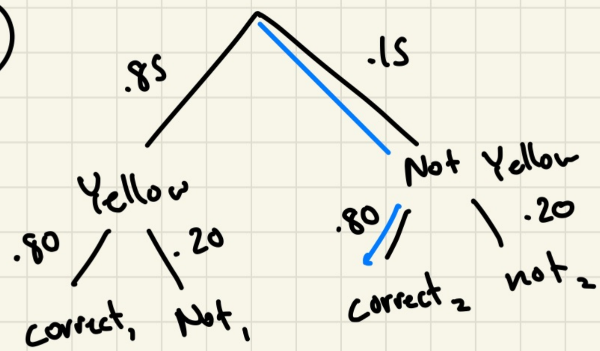
b. $P(E) = P(D_1 \cap D_2)$

$$P(C_2^c \mid E) = \frac{P(C_2^c \cap E)}{P(E)}$$

$$\frac{P(C_1 \cap C_2^c \cap (C_3 \cup C_4))}{P(E)} = \frac{P(C_1) \cdot P(C_2^c) \cdot [1 - P(C_3) \cdot P(C_4)]}{P(E)}$$

$$\frac{.85 \cdot .15 \cdot .9775}{.9555} = \boxed{.1304}$$

5

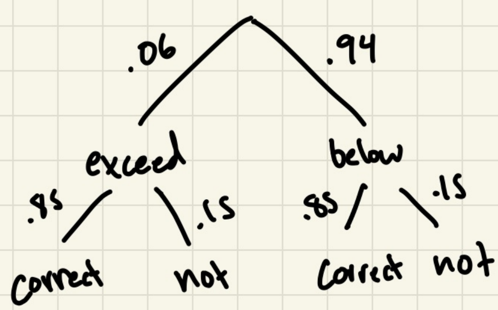


$$P(C_2 \cap NY) = P(C_2 \mid NY)\,P(NY)$$
$$= .80 \cdot .15$$
$$= \boxed{.12}$$

6

a. The probability that the driver's blood alcohol content doesn't exceed legal limit given the analyzer indicates it does exceed it.

b.



$P(B) = .06$

$P(A \mid B) = .85$

$P(A) = ?$

$P(B^c \mid A) = \dfrac{P(B^c \cap A)}{P(A)}$

$$P(A) = .06 \cdot .85 + .94 \cdot .15 = .192$$

$$P(B^c|A) = \frac{P(B^c)\,P(A|B^c)}{P(A)} = \frac{.94 \cdot .15}{.192} = \boxed{.734}$$

(C)

$$P(B|A) = .95$$

$$.95 = \frac{P(B \wedge A)}{P(A)} \qquad .95 = \frac{P(B)\,P(A|B)}{P(A)}$$

$$.95 = \frac{.06 \cdot p}{.06 \cdot p + .94 \cdot (1-p)}$$

$$.95 = \frac{.06p}{.06p + .94 - .94p}$$

$$.95 = \frac{.06p}{.94 - .88p}$$

$$.95(.94 - .88p) = .06p$$

$$.893 - .836p = .06p$$

$$.893 = .896p$$

$$\boxed{.997 = p}$$

Question 7

```
pSP500 <- sp500[,7]
pSP500 <- rev(pSP500)
rSP500 <- diff(log(pSP500))*100
ndays <- length(rSP500)
sum(rSP500 < 0)/ndays
```

```
## [1] 0.4684298
```

is the probability that the log-returns are negative.

```
numCons <- sum(rSP500[1:(ndays-1)]<0 & rSP500[2:ndays]<0)
numPrev <- sum(rSP500[1:(ndays-1)]<0)
numCons/numPrev
```

```
## [1] 0.43257
```

is the probability that it will be down two consecutive days. I would say they are because the probability isn't very high to make us believe that having a down day leads the next day to be down as well.

```
c <- sum(rSP500[1:ndays] >= 1.5)
c/ndays
```

```
## [1] 0.07290922
```

is the probability that the absolute value of the log-returns is at least 1.5%.

Looking for P("following day log-return abs value >= 1.5%" | "abs val log-return of selected day >= 1%").

```
oneP <- sum(rSP500[1:ndays] >= 1)
following <- sum(rSP500[2:ndays] >= 1.5 & rSP500[1:ndays-1] >= 1)
following/oneP
```

```
## [1] 0.08866995
```

is the probability that the following day has an absolute value log-return of at least 1.5% given that the current day is at least 1%.