STAT390: Homework 8

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Question 1

```
mean <- mean(sample_data)
se <- sqrt((3 - (3*(mean))^2) / length(sample_data))
se
```

```
## [1] 0.02564568
```

Homework 8

1 a) Take expected value of estimator

$$E\left[\frac{3}{n}\sum_{i=1}^{n} x_i\right] = \theta$$

$$E[x] = \frac{1}{2}\int_{-1}^{1} x + \theta x^2 \rightarrow \frac{1}{2}\left(\frac{1}{2}x^2 + \frac{\theta}{3}x^3\right)\Big|_{-1}^{1}$$

$$\frac{1}{4}x^2 + \frac{\theta}{6}x^3\Big|_{-1}^{1}$$

$$\left(\frac{1}{4} + \frac{\theta}{6}\right) - \left(\frac{1}{4} - \frac{\theta}{6}\right) = \frac{2\theta}{6} = \frac{\theta}{3}$$

$$E\left[\frac{3}{n}\sum_{i=1}^{n} x_i\right] = \frac{3}{n}\sum_{i=1}^{n} E[x_i] = \frac{3}{n}\sum_{i=1}^{n} E[x_i] = \frac{3}{n}\cdot\frac{\theta n}{3}$$

$$\boxed{E\left[\frac{3}{n}\sum_{i=1}^{n} x_i\right] = \theta}$$

b) $E[X] = \frac{1}{2}\int_{-1}^{1} x + \theta x^2 \rightarrow \frac{1}{2}\left(\frac{1}{2}x^2 + \frac{\theta}{3}x^3\right)\Big|_{-1}^{1}$

$$\frac{1}{4}x^2 + \frac{\theta}{6}x^3\Big|_{-1}^{1}$$

$$\left(\frac{1}{4} + \frac{\theta}{6}\right) - \left(\frac{1}{4} - \frac{\theta}{6}\right) = \frac{2\theta}{6} = \frac{\theta}{3}$$

$$\frac{\theta}{3} = \bar{X}_n = \frac{1}{n}\sum_{i=1}^{n} x_i$$

$$\boxed{\hat{\theta} = \frac{3}{n}\sum_{i=1}^{n} x_i}$$

c) $SE(\hat{\theta}_{MME}) = \sqrt{Var(\hat{\theta}_{MME})}$

$$Var(\hat{\theta}) = Var\left(\frac{3}{n}\sum_{i=1}^{n} x_i\right) = \frac{9}{n^2}\sum_{i=1}^{n} Var(x_i)$$

$$Var(x_i) = E[x_i^2] - E[x_i]^2$$

$$= \frac{1}{2}\int_{-1}^{1} x^2 + \theta x^3 \, dx - \frac{\theta^2}{9}$$

$$\rightarrow \frac{1}{6}x^3 + \frac{\theta}{8}x^4\Big|_{-1}^{1} = \left(\frac{1}{6} + \frac{\theta}{8}\right) - \left(-\frac{1}{6} + \frac{\theta}{8}\right)$$

$$= \frac{2}{6} - \frac{\theta^2}{9} = \frac{3-\theta^2}{9}$$

$$\frac{9}{n^2} \sum_{i=1}^{n} \frac{3-\theta^2}{9} = \frac{9}{n^2} \frac{n(3-\theta^2)}{9}$$

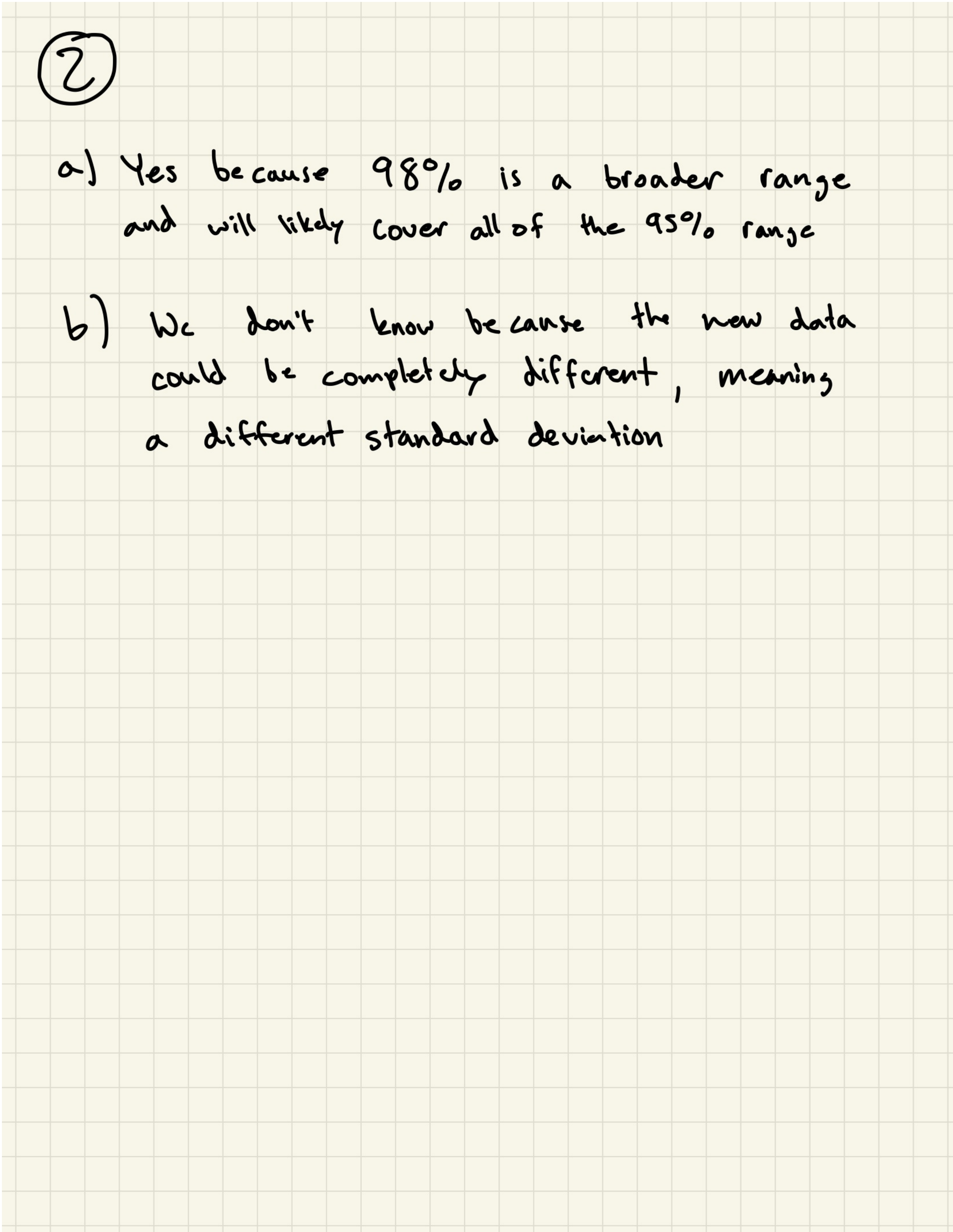
$$\text{Var}(\hat{\theta}) = \frac{3-\theta^2}{n}$$

$$\boxed{SE(\hat{\theta}) = \sqrt{\frac{3-\theta^2}{n}}}$$

$$\hat{SE}(\hat{\theta}) = \sqrt{\frac{3-\hat{\theta}^2}{n}} = \sqrt{\frac{3-(3\bar{X}_n)^2}{n}}$$

$$\boxed{\hat{SE}(\hat{\theta}_{MME}) = 0.02565}$$ Calculated on R

Figure 1: Homework 8 Page 3



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a) Yes because 98% is a broader range and will likely cover all of the 95% range

b) We don't know because the new data could be completely different, meaning a different standard deviation

Figure 2: Homework 8 Page 4

③ $n = 10$ $\quad \bar{X}_n = 198.5$ $\quad \sigma = 1.65$

a) $$\bar{x} \pm t_{n-1,\alpha} \frac{\sigma}{\sqrt{n}}$$

$$198.5 \pm t_{9,.025} \frac{1.65}{\sqrt{10}} \qquad t_{9,.025} = 2.262$$

$$198.5 \pm \frac{2.262(1.65)}{\sqrt{10}}$$

$$\boxed{95\%\ CI = (197.3197,\ 199.6803)}$$

b) $$\bar{X} \pm z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} \qquad w = .45$$

$$\frac{2 z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}}{2} \le \frac{.45}{2}$$

$$\frac{z_{\frac{\alpha}{2}} \sigma}{.225} \le \sqrt{n} \qquad z_{\frac{\alpha}{2}} = z_{\frac{.10}{2}} = z_{.05} = 1.645$$

$$\frac{1.645(1.65)}{.225} \le \sqrt{n} \rightarrow \boxed{\left(\frac{1.645(1.65)}{.225}\right)^2 \le n = 145.5}$$

$\boxed{\text{We should sample 146 bags}}$

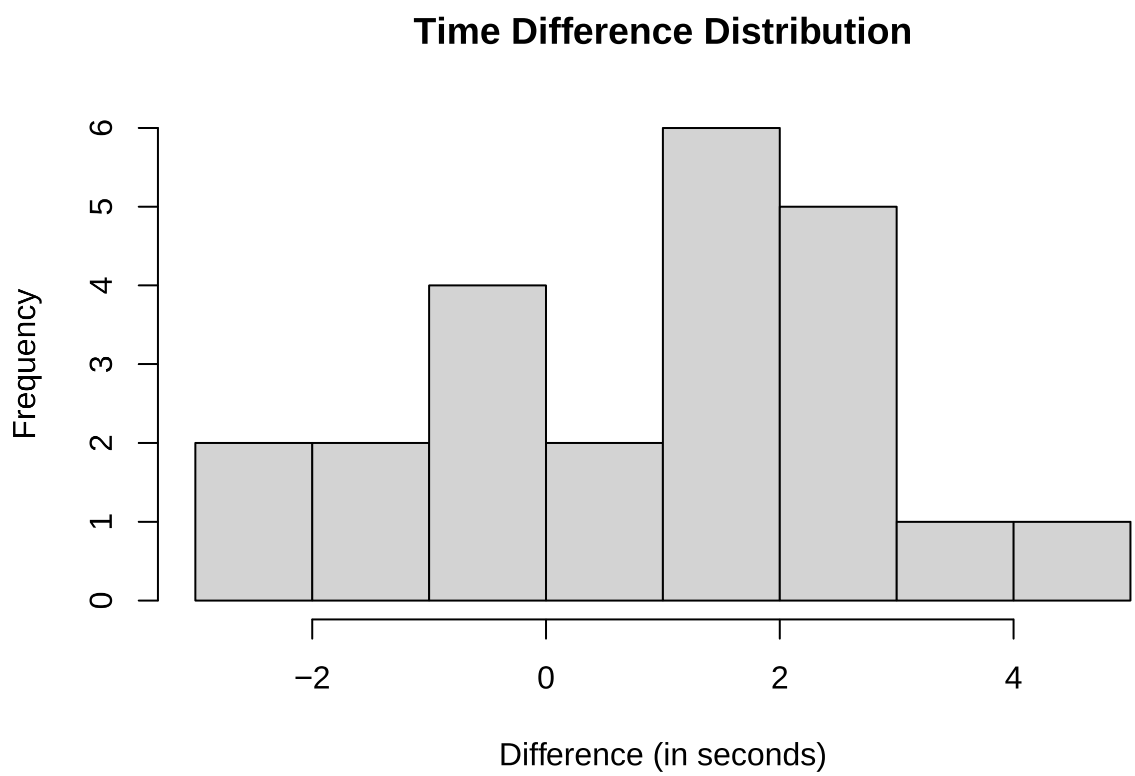
Figure 3: Homework 8 Page 5

c) No because the standard deviation might be different

Figure 4: Homework 8 Page 6

Question 4

```
hist(olympic_data$Diff, main = "Time Difference Distribution", xlab = "Difference (in seconds)")
```




```
time <- olympic_data[,4]
mean(time)
```

```
## [1] 0.8213043
```

```
sd(time)
```

```
## [1] 1.782757
```

```
qt(.95, 22)
```

```
## [1] 1.717144
```

```
qt(.98, 22)
```

```
## [1] 2.182893
```

a) A one-sided CI would be better because we are trying to find if the outer lane has an advantage, not the difference between the times.

b) It is not approximately normally distributed because the histogram is not in a symmetric bell-shaped curve.

4) c) Small sample CI

$$\bar{x}_n \pm t_{n-1,\alpha} \frac{\sigma}{\sqrt{n}}$$

$n = 23$

$\bar{x}_n = .8213$

$\sigma = 1.7828$

$t_{22,.05}$ for 95% $= 1.7171$

$t_{22,.02}$ for 98% $= 2.1829$

For 95%:

$$.8213 - \frac{1.7171(1.7828)}{\sqrt{23}} : [0.183, \infty)$$

For 98%

$$.8213 - \frac{2.1829(1.7828)}{\sqrt{23}} : [0.0098, \infty)$$

d) I'm 95% confident that the outer lane is advantageous because the lower bound difference is greater than zero. Same with 98%

Figure 5: Homework 8 Page 7