Webwork

For semester 1, part of the MS121 Continuous assessment is webwork homework.

So do it.

Set 1 due on Thursday week (17th Oct. 2019)

"I cannot get in!"

website	http://webwork.dcu.ie/webwork2/MS121/
username	joseph.bloggs7@mail.dcu.ie (your DCU e-mail address)
password	19202122 (your DCU student ID)

Change the password once you are in.

Test 1

This will take place in tutorial in week 3. Go to the correct tutorial for you based on the first letter of your surname (A-G, H-M, N-Z). Here is the solution to last year's test:

MS121, Test 1, 9th. Oct. 2018

Name: ____________________________	Student No.: ____________________________

?. Let P and Q be propositions defined as follows:
P: I studied hard. Q: I passed my exam.
The compound proposition 'If I studied hard, then I passed my exam.' can be expressed as

(a) Q $\Rightarrow$ P, (b) Q $\Rightarrow$ (**not** P), (c) (**not** Q) $\Rightarrow$ P, (d) (**not** Q) $\Rightarrow$ (**not** P)

Answer: $\boxed{\text{d}}$ The proposition can be expressed as P $\Rightarrow$ Q which we saw is equivalent to (d).

?. Suppose that R and S are propositions, R has value T and S has value F. Then the two compound statements $[R \Rightarrow (\textbf{not } S)]$ and $[S \Rightarrow (\textbf{not } R)]$ take the following values respectively.

(a) T and T, (b) T and F, (c) F and T, (d) F and F

Answer: $\boxed{\text{a}}$ **not** S has value T so $[R \Rightarrow (\textbf{not } S)]$ is already true. Similarly S has value F so $[S \Rightarrow (\textbf{not } R)]$ is already true.

?. The negation of the statement 'McGregor won all his fights.' is the following:

(a) McGregor won all his fights. (b) McGregor won some of his fights.
(c) McGregor failed to win any of his fights. (d) McGregor failed to win at least one of his fights.

Answer: $\boxed{\text{d}}$ If P_i is the statement 'McGregor won his ith fight' then
not ($\forall i$: P_i) is equivalent to $\exists i$: (**not** P_1)

?. A sequence of numbers $x_1, x_2, \ldots, x_n, \ldots$ is defined inductively by
$x_1 = 1$ and $x_{k+1} = x_k/(x_k + k)$ for $k > 1$.
The numbers x_4 and x_5 take the following values respectively.

(a) 1/16 and 1/65, (b) 1/16 and 1/64, (c) 1/15 and 1/61, (d) 1/15 and 1/60

Answer: $\boxed{\text{a}}$

$x_2 = (x_1)/(x_1 + 1) = 1/(1 + 1) = 1/2$
$x_3 = (x_2)/(x_2 + 2) = (1/2)/((1/2) + 2) = 1/5$
$x_4 = (x_3)/(x_3 + 3) = (1/5)/((1/5) + 3) = 1/16$
$x_5 = (x_4)/(x_4 + 4) = (1/16)/((1/16) + 4) = 1/65$

Example: For every integer n bigger than 1, $n^2 > n + 1$.

Proof: Base case: $n = 2$ (the first integer bigger than 1), LHS is $2^2 = 4$ while RHS is 3.

For the inductive step we will need some facts about inequalities:
(a) If $a < b$ then $a + c < b + c$ for any number c.
(b) If $a < b$ and $c > 0$ then $ca < cb$.

Inductive step: Assume $P(k)$, that is,

$$k^2 > k + 1.$$

and try to deduce $P(k + 1)$, that is

$$(k + 1)^2 > k + 1 + 1 = k + 2.$$

Bearing in mind that $k > 1$ we argue

$$\begin{aligned}
(k+1)^2 &= k^2 + 2k + 1 \\
&> (k+1) + 2k + 1 \text{ (by } P(k) using(a)) \\
&> k + 1 + 2(1) + 1 \text{ (since } k > 1 using(b)) \\
&> k + 2
\end{aligned}$$

Example: All cars have the same colour.
See `https://en.wikipedia.org/wiki/All_horses_are_the_same_color`

This famous false proof by induction goes like this. Let P(n) be the proposition

In any set of n cars all the cars have the same colour

Thus we have to prove P(n) for all positive integers n and induction is the proof method we try. For the base case P(1)

In any set of 1 cars all the cars have the same colour

the statement is true so we just need to prove P(k) $\Rightarrow$ P($k + 1$). Assume P(k) is true and a set of $k + 1$ cars is given. Line them up in a row:

$c_1, c_2, \ldots, c_k, c_{k+1}$

By P(k), the first k cars

$(c_1, c_2, \ldots, c_k), c_{k+1}$

have the same colour, colour 1 say. Also by P(k), the last k cars

$c_1, (c_2, \ldots, c_k, c_{k+1})$

have the same colour, colour 2 say. But the cars in the overlap

c_1, (c_2, ..., c_k), c_{k+1}
are the same cars, so colour 1 and colour 2 are the same.

The proof is false since the argument given for $\mathrm{P}(k) \Rightarrow \mathrm{P}(k+1)$ makes the unspoken (false) assumption that there is an 'overlap' so that $k > 1$. In fact the argument for $\mathrm{P}(1) \Rightarrow \mathrm{P}(2)$ is false.