Problem Sheet 1

MS121 Semester 2 IT Mathematics

Exercise 1.
Express the following sets as intervals:

(a) $\{x \in \mathbb{R} | \ -5 \leq x, \ x < 20\}$,

(b) $\{x \in \mathbb{R} | \ x > 7\}$,

(c) $\{x \in \mathbb{R} | \ x \leq 27\}$.

$\oslash$

Solution 1.
(a) $[-5, 20)$,

(b) $(7, \infty)$, (∞ is not a real number, so it is never included)

(c) $(-\infty, 27]$.

$\diamondsuit$

Exercise 2.
Draw or sketch the graphs of the following functions. Sometimes it helps to first make a short table with well chosen values of x and $f(x)$.

(a) $f(x) = (x-2)^2 - 1$,

(b) g, the straight line through the points $(0, 2)$ and $(3, -2)$,

(c) $\sin(x)$.

$\oslash$

Solution 2.
(a) $f(x) = (x-2)^2 + 1$ is a parabola with minimum at $(2, -1)$ and zeroes at $(1, 0)$ and $(3, 0)$.

(b) g is given by $g(x) = 2 - \frac{4}{3}x$, so the graph is $y = 2 - \frac{4}{3}x$.

(c) $\sin(x)$ is a standard oscillating function, with $\sin(0) = 0$.

$\diamondsuit$

Exercise 3.
The functions $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ are given by the formulae

$$f(x) = 2 - x, \qquad g(x) = 4x - x^3.$$

Express each of the following functions as a formula in terms of x:

(a) $f + g$,

(b) $g - f$,

(c) $f \cdot g$,

(d) $f \circ g$,

(e) $g \circ f$,

(f) g/f.

$\oslash$

Solution 3.

(a) $(f + g)(x) = f(x) + g(x) = (2 - x) + (4x - x^3) = 2 + 3x - x^3$,

(b) $(g - f)(x) = g(x) - f(x) = (4x - x^3) - (2 - x) = -2 + 5x - x^3$,

(c) $f \cdot g(x) = f(x) \cdot g(x) = (2 - x)(4x - x^3) = 8x - 4x^2 - 2x^3 + x^4$,

(d) $f \circ g(x) = f(g(x)) = 2 - g(x) = 2 - 4x + x^3$,

(e) $g \circ f(x) = g(f(x)) = 4f(x) - f(x)^3 = 4(2 - x) - (2 - x)^3 = 8x - 6x^2 + x^3$,

(f) $g/f(x) = \frac{g(x)}{f(x)} = \frac{4x-x^3}{2-x}$. This may be simplified $\frac{x(2-x)(2+x)}{2-x} = x(2 + x) = 2x + x^2$ (on $x \neq 2$).

$\diamondsuit$

Exercise 4.
Find the natural domain of the following functions. Express your answers in terms of intervals.

(a) $f(x) = 2x^2$,

(b) $f(x) = x^8$,

(c) $f(x) = x^9$,

(d) $f(x) = \frac{1}{x-7}$,

(e) $f(x) = |x - 4|$,

(f) $f(x) = \sqrt{x^2 - 4}$.

(g) $f(x) = \frac{\sqrt{2-x}}{x^2-1}$,

(h) $f(x) = \frac{\sqrt{4-\sqrt{x}}}{\sqrt{x^2+1}}$.

For the functions in parts $(a) - (f)$, can you also give the range? $\oslash$

Solution 4.

(a) Domain$(f) = \mathbb{R}$ and Range$(f) = [0, \infty)$,

(b) Domain$(f) = \mathbb{R}$ and Range$(f) = [0, \infty)$,

(c) Domain$(f) = \mathbb{R}$ and Range$(f) = \mathbb{R}$,

(d) Domain$(f) = (-\infty, 7) \cup (7, \infty)$ and Range$(f) = (-\infty, 0) \cup (0, \infty)$,

(e) Domain$(f) = \mathbb{R}$ and Range$(f) = [0, \infty)$,

(f) Domain$(f) = (-\infty, -2] \cup [2, \infty)$ and Range$(f) = [0, \infty)$.

(g) Domain$(f) = (-\infty, -1) \cup (-1, 1) \cup (1, 2]$, because the numerator only makes sense when $x \leq 2$ and the quotient makes no sense when $x = -1$ or $x = 1$.

(h) Domain$(f) = [0, 16]$, because the numerator only makes sense when $x \geq 0$ and $\sqrt{x} \leq 4$, i.e. $0 \leq x \leq 16$. The quotient is no problem, because $\sqrt{x^2+1} \geq 1 > 0$ for all $x \in \mathbb{R}$.

$\diamondsuit$