Logical Equivalence Example

The logical equivalence ((**not** Q) $\Rightarrow$ (**not** P)) $\equiv$ (P $\Rightarrow$ Q) went a little too quickly for some people. So let's slow it down somewhat. Firstly it may help to write implication using two other simple propositions. Let's use R and S:

R	S	R $\Rightarrow$ S
T	T	T
T	F	F
F	T	T
F	F	T

Now try to build the truth table for (**not** Q) $\Rightarrow$ (**not** P). The first part is as it was for other examples in lecture.

P	Q	**not** P	**not** Q	((**not** Q) $\Rightarrow$ (**not** P))
T	T	F	F	
T	F	F	T	
F	T	T	F	
F	F	T	T	

To complete the last column we are doing an R $\Rightarrow$ S construction, where R is column 4 and S is column 3. For row 1, R has value F and S has value F so R $\Rightarrow$ S should have value T from the R $\Rightarrow$ S truth table.

P	Q	**not** P	**not** Q	((**not** Q) $\Rightarrow$ (**not** P))
T	T	F	F	T
T	F	F	T	
F	T	T	F	
F	F	T	T	

For row 2, R has value T and S has value F so R $\Rightarrow$ S should have value F from the R $\Rightarrow$ S truth table.

P	Q	**not** P	**not** Q	((**not** Q) $\Rightarrow$ (**not** P))
T	T	F	F	T
T	F	F	T	F
F	T	T	F	
F	F	T	T	

For row 3, R has value F and S has value T so R $\Rightarrow$ S should have value T from the R $\Rightarrow$ S truth table.

P	Q	**not** P	**not** Q	((**not** Q) $\Rightarrow$ (**not** P))
T	T	F	F	T
T	F	F	T	F
F	T	T	F	T
F	F	T	T	

For row 4, R has value T and S has value T so R $\Rightarrow$ S should have value T from the R $\Rightarrow$ S truth table.

P	Q	**not** P	**not** Q	((**not** Q) $\Rightarrow$ (**not** P))
T	T	F	F	T
T	F	F	T	F
F	T	T	F	T
F	F	T	T	T

Now the last column has value F only when P has value T and Q has value F. Thus the compound statement in the last column is logically equivalent to P $\Rightarrow$ Q.