Definition: A relation R on a set A is called an equivalence relation if R is reflexive, symmetric and transitive.

Example: Let A be a set of first year DCU students and define the relation R on A by $(a, b) \in R$ if b is in the same programme as a. R is an equivalence relation.

Note: Both the digraph of R and the matrix of R have special properties. We'll look at them in a smaller example.

Example: Suppose $A = \{1, 2, 3, 4, 5, 6\}$ and let $A_1 = \{1, 2, 5\}$, $A_2 = \{4, 6\}$ and $A_3 = \{3\}$. Then

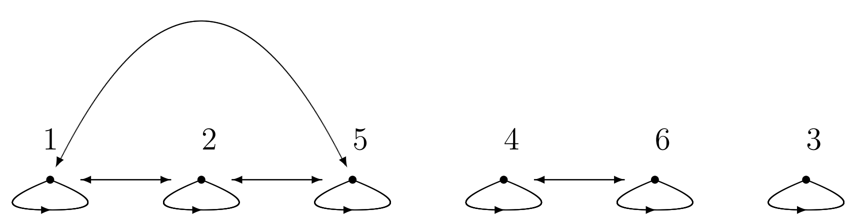
$$A_1 \cap A_2 = \emptyset, \quad A_1 \cap A_3 = \emptyset, \quad A_2 \cap A_3 = \emptyset, \quad A_1 \cup A_2 \cup A_3 = A.$$

Put a relation R on A by saying $(x, y) \in R$ if x and y are in the same subset A_1, A_2 or A_3. So

$$R = \{(1,1), (1,2), (1,5), (2,1), (2,2), (2,5), (3,3),$$

$$(4,4), (4,6), (5,1), (5,2), (5,5), (6,4), (6,6)\}$$

(This is similar to the last example but with more manageable numbers. You can think of A_1 as the students in the $CA1$ programme, A_2 as the students in the $BS1$ programme and A_3 as the students in the $AL1$ programme. The intersection and union properties correspond to the fact that each student is in precisely one programme.)
R is an equivalence relation: Each element is in the same subset as itself; if b is in the same subset as a then a is in the same subset as b; if b is in the same subset as a and c is in the same subset as b all three are in the same subset and c is in the same subset as a. Here is the digraph:



Here is the matrix:

	1	2	5	3	4	6
1	T	T	T	F	F	F
2	T	T	T	F	F	F
5	T	T	T	F	F	F
3	F	F	F	T	F	F
4	F	F	F	F	T	T
6	F	F	F	F	T	T

Definition: If A is a non-empty set, a partition of A is a collection of non-empty subsets, $A_1, A_2, \ldots, A_n$ satisfying
1. $A = A_1 \cup A_2 \cup \ldots \cup A_n$ and
2. $A_i \cap A_j = \emptyset$ whenever $i \neq j$.
The sets A_i are called the blocks of the partition.

Example: If A and B are sets, the collection $\{A \cap (\sim B), A \cap B, (\sim A) \cap B\}$ forms a partition of $A \cup B$.

Example: For $A = \mathbb{Z}$, $n = 2$, A_1 the set of odd integers and A_2 the set of even integers, we get a partition of $\mathbb{Z}$. $A_1 \cap A_2 = \emptyset$ and $A_1 \cup A_2 = \mathbb{Z}$.

Example: Let A be a set of first year DCU students taking CA, BS or AL, A_1 be the set of CA1 students in A, A_2 be the set of BS1 students in A and A_3 be the set of AL1 students in A. Then the collection of subsets A_1, A_2, A_3 defines a partition of A.