MS121, Test 2c, 1st. Nov. 2019

Name: ____________________	Student No.: ____________________

?. Which of the following statements about two sets X and Y is not logically equivalent to the others?

(A) $(\sim Y) \subseteq (\sim X)$ (B) $X \subseteq Y$ (C) $(\sim X) \cap Y = \emptyset$ (D) $X \cap (\sim Y) = \emptyset$

Answer: $\boxed{\text{C}}$

The other three are logically equivalent. (D) says there are no elements in both $\sim Y$ and X which is equivalent to saying all of $\sim Y$'s elements are in $\sim X$ (A) or all of X's elements are in Y (B).

?. Suppose X, Y and Z are sets, $|X \cup Y \cup Z| = 12$, $|X| = 8$, $|Y| = 5$, $|Z| = 3$, $|X \cap Y| = 3$, $|X \cap Z| = 1$ and $|Y \cap Z| = 0$. How many elements belong to X but do not belong to Y or Z?

(A) 1, (B) 2, (C) 3, (D) 4

Answer: $\boxed{\text{D}}$

Since $|Y \cap Z| = 0$, we know $|X \cap Y \cap Z| = 0$ and $|(\sim X) \cap Y \cap Z| = 0$. From the usual Venn diagram of 3 sets we get $|X \cap Y \cap (\sim Z)| = 3$ and $|X \cap (\sim Y) \cap Z| = 1$. From that we deduce $|X \cap (\sim Y) \cap (\sim Z)| = 4$.

?. Suppose $R = \{(1,3), (2,2), (2,4), (3,3), (4,2), (4,4)\}$ is a relation on the set $S = \{1, 2, 3, 4\}$. Then R is

(A) Reflexive (B) Symmetric (C) Antisymmetric (D) Transitive

Answer: $\boxed{\text{D}}$

(A) fails since there is no $(1,1)$. (B) fails since $(1,3) \in R$ but $(3,1) \notin R$. (C) fails since $(2,4) \in R$ and $(4,2) \in R$. (D) holds by checking all cases.

?. Suppose $R = \{(1,1), (1,3), (1,4), (2,2), (3,3), (3,4), (4,1), (4,4)\}$ is a relation on the set $S = \{1, 2, 3, 4\}$. Then R will be an equivalence relation when we add the two elements

(A) $(3,1)$ and $(3,2)$, (B) $(4,2)$ and $(4,3)$, (C) $(3,1)$ and $(4,3)$, (D) $(1,2)$ and $(2,1)$,

Answer: $\boxed{\text{C}}$

Just to satisfy the symmetry property we need these pairs. All the other possibilities leave out at least one of these two pairs.