

MS121, Test 2, 30th. Oct. 2018 Average: 2.5

?. If X , Y and Z are sets then $X \cup (\sim Y) \cup Z$ does not contain

(A) $X \cap Y$, (B) $Y \cap Z$, (C) $(\sim X) \cap Y$, (D) $(\sim X) \cap (\sim Y)$

Answer: C: The set $(\sim X) \cap Y$ will contain $(\sim X) \cap Y \cap (\sim Z)$ which is the complement of $X \cup (\sim Y) \cup Z$.

?. Suppose X , Y and Z are sets, $|X \cup Y \cup Z| = 12$, $|X| = 4$, $|Y| = 8$, $|Z| = 7$, $|X \cap Y| = 3$, $|X \cap Z| = 3$ and $|Y \cap Z| = 3$. How many elements belong to X but do not belong to Y or Z ?

(A) 0, (B) 1, (C) 2, (D) 3

Answer: A: The inclusion-exclusion formula gives

$$\begin{aligned} 12 &= |X \cup Y \cup Z| = |X| + |Y| + |Z| - |X \cap Y| - |X \cap Z| - |Y \cap Z| + |X \cap Y \cap Z| \\ &= 4 + 8 + 7 - 3 - 3 - 3 + |X \cap Y \cap Z| \end{aligned}$$

Deduce that $|X \cap Y \cap Z| = 2$, $|X \cap Y \cap (\sim Z)| = 1$, $|X \cap (\sim Y) \cap Z| = 1$ so that $|X \cap (\sim Y) \cap (\sim Z)| = 4 - 2 - 1 - 1 = 0$.

?. Suppose $R = \{(1, 1), (1, 3), (2, 4), (3, 1), (3, 3), (4, 2), (4, 4)\}$ is a relation on the set $X = \{1, 2, 3, 4\}$. Then R is

(A) Reflexive (B) Symmetric (C) Antisymmetric (D) Transitive

Answer: B: $(2, 2) \notin R$ so not reflexive. $(1, 3), (3, 1) \in R$ so not antisymmetric. $(2, 4), (4, 2) \in R$ but $(2, 2) \notin R$ so not transitive.

?. Suppose $R = \{(1, 3), (2, 4), (3, 1), (4, 5), (5, 6), (6, 2)\}$ is a relation on the set $X = \{1, 2, 3, 4, 5, 6\}$. Which pair is not in the transitive closure of R ?

(A) $(2, 6)$ (B) $(2, 2)$ (C) $(2, 1)$ (D) $(2, 5)$

Answer: C: $(2, 4), (4, 5) \in R$ so $(2, 5)$ in closure. $(2, 4), (4, 5), (5, 6) \in R$ so $(2, 6)$ in closure. $(2, 4), (4, 5), (5, 6), (6, 2) \in R$ so $(2, 2)$ in closure.