MS121, Test 1(d), 18th. Oct. 2019

Name: ____________________________	Student No.: ____________________________

?. Let P, Q and R be propositions defined as follows:
P: We win game 1. Q: We win game 2. R: We qualify for the next round.
The compound proposition 'If we win game 1 and we win game, then we qualify for the next round.' can be expressed as

(A) **not** R $\Rightarrow$ [(**not** P) **and** (**not** Q)] , (B) **not** R $\Rightarrow$ [(**not** P) **or** (**not** Q)],
(C) [**not** R $\Rightarrow$ (**not** P)] **and** (**not** Q) , (D) [**not** R $\Rightarrow$ (**not** P)] **or** (**not** Q)
Answer: $\boxed{\text{B}}$

The statement is of form [P **and** Q] $\Rightarrow$ R so is equivalent to **not** R $\Rightarrow$ **not**[P **and** Q] which in turn is equivalent to **not** R $\Rightarrow$ [(**not** P) **or** (**not** Q)].

?. The negation of (**not** P) $\Rightarrow$ (**not** Q) is equivalent to

(A) P **and** Q, (B) P **and** (**not** Q), (C) (**not** P) **and** Q, (D) (**not** P) **and** (**not** Q)
Answer: $\boxed{\text{C}}$

X $\Rightarrow$ Y is equivalent to (**not** X) **or** Y so the negation of X $\Rightarrow$ Y is X **and** (**not**Y).
Here the negation of (**not** P) $\Rightarrow$ (**not** Q) is (**not** P) **and** Q.

?. The negation of the statement 'Some modules are interesting.' is the following:

(A) Some modules are interesting. (B) All modules are interesting
(C) Some modules are not interesting (D) All modules are not interesting
Answer: $\boxed{\text{D}}$

If m is a module and P(m) is the statement 'module m is interesting' then 'Some modules are interesting.' is the statement $\exists m$, P(m). Its negation is $\forall m$, **not**P(m) or 'For all modules m, m is not interesting'.

?. A sequence of numbers $x_1, x_2, \ldots, x_n, \ldots$ is defined inductively by
$x_1 = 1$ and $x_{k+1} = kx_k + 2$ for $k \geq 1$.
The numbers x_4 and x_5 take the following values respectively:

(A) 26 and 104, (B) 26 and 106, (C) 24 and 106, (D) 24 and 98.
Answer: $\boxed{\text{B}}$

$x_2 = (1)x_1 + 2 = (1)(1) + 2 = 3,\quad x_3 = (2)x_2 + 2 = (2)(3) + 2 = 8,$
$x_4 = (3)x_3 + 2 = (3)(8) + 2 = 26,\quad x_5 = (4)x_4 + 2 = (4)(26) + 2 = 106.$