MS121, Test 4, 11th. Dec. 2018

Name: _____	Student No.: _____

?. Grocery items are distributed among 3 shopping bags. What is the minimum number of grocery items required to guarantee that at least one bag has at least 4 items?

(A) 4, (B) 10, (C) 9, (D) 12.

Answer: B: This is the extended pigeonhole principle. Here A is a set of grocery items, B is a set of bags, $f : A \to B$ takes a grocery item to the bag it ends up in. Since $|B| = 3$ at least one bag will have more than $k = 3$ items if $|A| \geq k|B| + 1$.

?. A committee of 4 must be chosen from a group of 4 men and 4 women but must contain at least 1 man and at least 1 woman. In how many ways can this be done?

(A) 70, (B) 69, (C) 240, (D) 68.

Answer: D: The total number of committees is $\begin{pmatrix} 8 \\ 4 \end{pmatrix} = 70$. Subtract the two possible committees with all men or all women. (If you pick the one man and one women first you will end up overcounting. The committee is unordered so there should not be a first man and a first woman.)

?. 6 people are to be split up into three teams of size 3, 2 and 1. In how many ways can this be done?

(A) 720, (B) 360, (C) 60, (D) 6.

Answer: C: This is the multinomial coefficient $\begin{pmatrix} 6 \\ 3, 2, 1 \end{pmatrix} = \frac{6!}{3!2!1!}$

?. A fair coin is tossed 5 times. What is the probability of H's coming up more often than T's?

(A) 1/2, (B) 17/32, (C) 15/32, (D) 5/8.

Answer: A: Since 5 is odd, H's come up more often than T's half the time and T's come up more often than H's the other half. Alternatively, the answer is the probability of 5 or 4 or 3 H's and therefore $(1/32)(1 + 5 + 10)$.