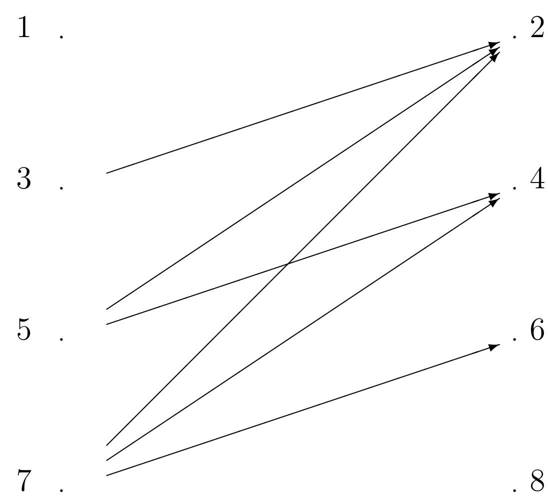
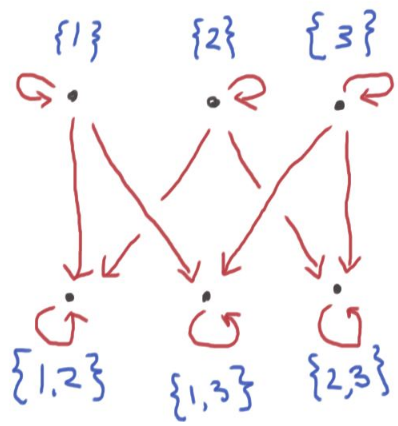
Definition: If R is a binary relation between sets A and B the digraph of the relation is a graph with vertex set $A \cup B$ and an edge joining a to b if $(a, b) \in R$.

Example: For the last example, with $A = \{1, 3, 5, 7\}$ and $B = \{2, 4, 6, 8\}$ and R_1 the relation $x > y$, the digraph is

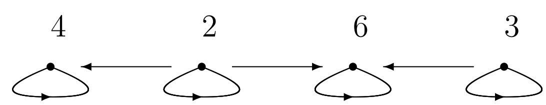


Note: For the case $A = B$, we just use A as the vertex set of the digraph. This is because $A \cup A = A$.

Example: The set of 1 or 2 element subsets of $\{1,2,3\}$ with the relation $A \subseteq B$.



Example: If $A = \{2,3,4,6\}$ and R is the relation given by 'is a divisor of' then the corresponding digraph is



Note: Recall how we visualised the Cartesian product of A and B by an array. This required ordering the elements of A and B by $A = \{a_1, a_2, \ldots, a_n\}$ and $B = \{b_1, b_2, \ldots, b_m\}$. This array allows us to specify a relation R by placing a T or F in the appropriate entry of the array.

Definition: If R is a relation between A and B and A and B are ordered by $A = \{a_1, a_2, \ldots, a_n\}$ and $B = \{b_1, b_2, \ldots, b_m\}$, then the matrix of R is the array whose (i,j)th entry is T if $(a_i, b_j) \in R$ and F if $(a_i, b_j) \notin R$.

Example: For $A = \{1,3,5,7\}$ and $B = \{2,4,6,8\}$ the relation $x > y$ is given by the matrix

$$\begin{pmatrix} F & F & F & F \\ T & F & F & F \\ T & T & F & F \\ T & T & T & F \end{pmatrix}$$

Example: If $A = B$ is the set of all pages on the web and R is the relation 'web page A links to web page B' a variation on the corresponding matrix is used to compute the pagerank of each page on the web. This is done regularly by Google in what is called the largest matrix calculation in the world.

Note: Sometimes a relation R between A and B is simply specified by writing the predicate statement aRb whenever $(a, b) \in R$. (Recall that a predicate statement is one involving variables.)

Example: We could just write
$3 > 2$, $5 > 2$, $5 > 4$,$7 > 2$, $7 > 4$, $7 > 6$
to specify the relation $a > b$ between $A = \{1, 3, 5, 7\}$ and $B = \{2, 4, 6, 8\}$.

Properties of a relation on a single set.

To illustrate important properties that a relation on a single set might have we will consider the following examples of relations.

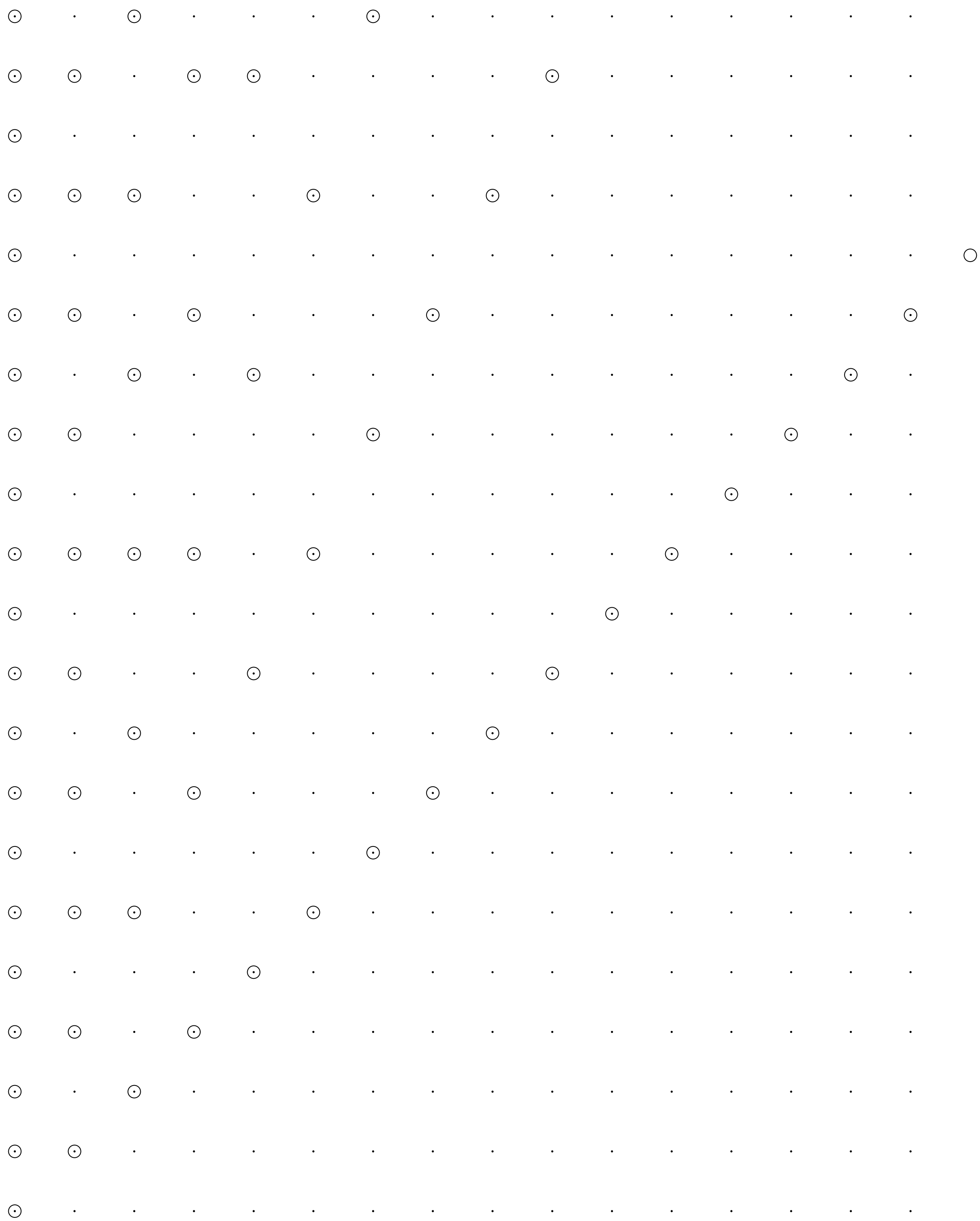
Example 1: If $A = \{1, 2, 3, 4\}$ and $R = \{(2, 1), (2, 3), (3, 2), (4, 4)\}$.

Example 2: If A is the set of all pages on the web and R is the relation 'web page X links to web page Y'.

Example 3: If $A = \mathbb{N} = \{1, 2, 3, \ldots\}$ and R is the relation given by 'is a divisor of'. The subset R of $A \times A$ is shown in a figure below. $A \times A$ is an infinite grid with lower left corner at $(1, 1)$ and the elemeents of R are circled.

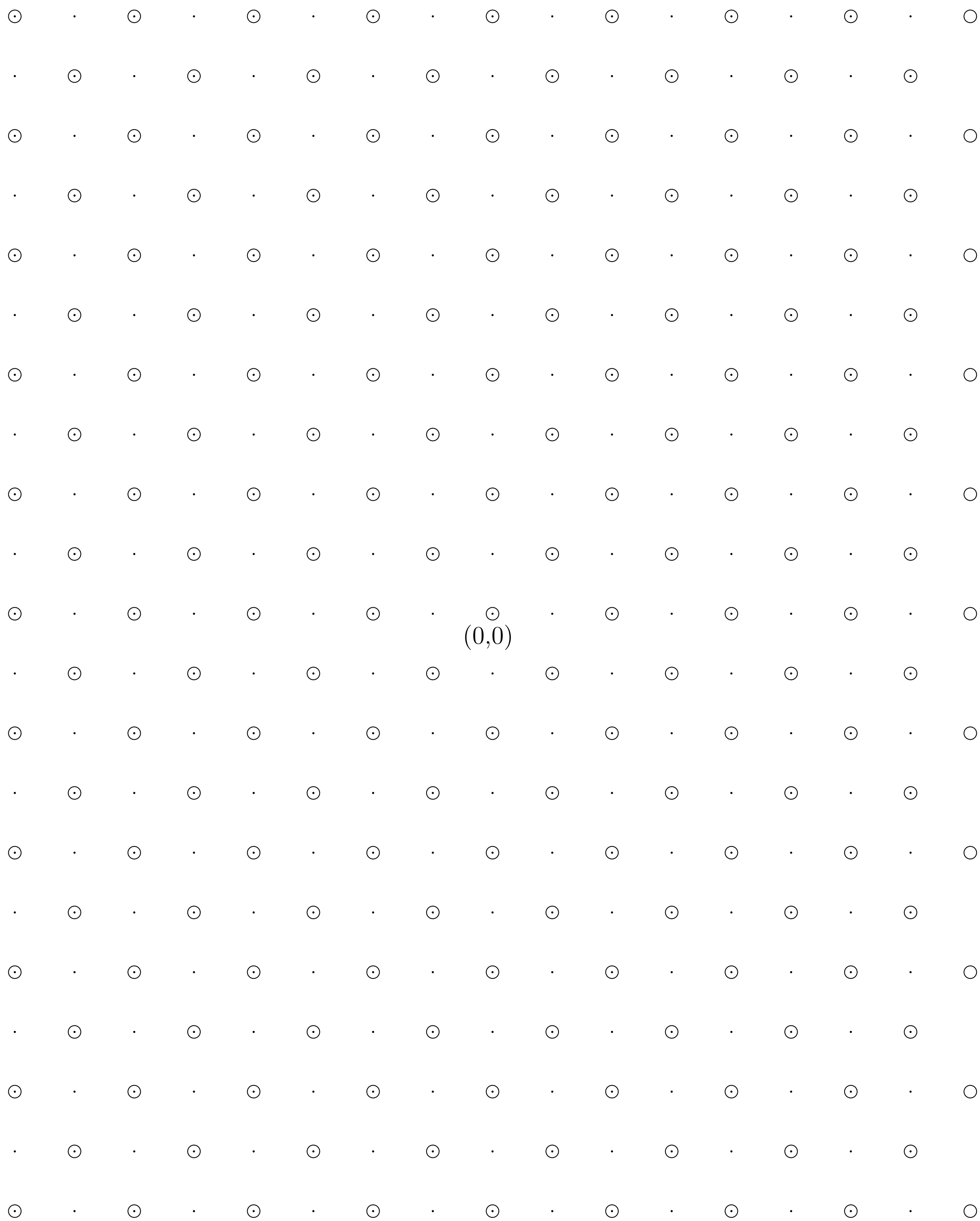
Example 4: If $A = \mathbb{Z}$ and R is the relation given by $(x, y) \in R$ if and only if $y - x$ is even. The subset R of $A \times A$ is shown in a figure below. $A \times A$ is an infinite grid with no corners this time and the elements of R are circled.

Example 5: Let A be the set of first year students in DCU and define the relation R on A by $(a, b) \in R$ if b is in the same programme as a.



(1,1)

The relation ‘a divides b’ inside $\{1, 2, 3, 4, \ldots\} \times \{1, 2, 3, 4, \ldots\}$.



The relation '$y - x$ is even' inside $\mathbb{Z} \times \mathbb{Z}$.