Example: What is the smallest n which has the property that among any n numbers chosen from

$$1, 2, 3, 4, 5, 6, 7, 8, 9, 10$$

there will be a pair which sums to 11?

This example is more subtle. Here the set A is the set of numbers in a set of size n, the set B is the set of pairs of numbers which sum to 11

$$B = \{(1, 10), (2, 9), (3, 8), (4, 7), (5, 6)\}$$

and the function $f : A \to B$ takes each number to the pair that contains it.

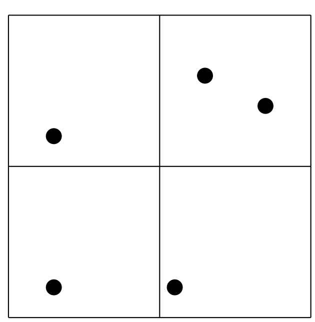
$$1 \mapsto (1, 10), 2 \mapsto (2, 9), 3 \mapsto (3, 8), 4 \mapsto (4, 7), 5 \mapsto (5, 6),$$

$$6 \mapsto (5, 6), 7 \mapsto (4, 7), 8 \mapsto (3, 8), 9 \mapsto (2, 9), 10 \mapsto (1, 10)$$

Since we want $|A| = n > 5 = |B|$, f will not be injective and two numbers will sum to 11 provided $n \geq 6$.

Example: Five points are chosen on the unit square. Show that two are within a distance of $\sqrt{2}/2$ of each other.

Split the square into four squares of sidelength $1/2$. Here the set A is the set of five points, the set B is the set of four small squares and the function $f : A \to B$ takes each point to one of the small squares containing it. (It may lie on the boundary of two or more small squares.) Since $|A| = 5 > 4 = |B|$, f will not be injective and two points lie in the same small square and are within a distance of $\sqrt{2}/2$ of each other.



Example: Each of the 9 squares on a 3×3 grid contains one of the integers $-1, 0$ or 1. Show that among the 8 resulting sums (three rows, three columns and two diagonals) there will always be two that add to the same number.

This example is again more subtle. Here the set A is the set of 8 triples from such a grid, the set B is the set of possible sums of three numbers from the set $\{-1, 0, 1\}$

$$B = \{-3, -2, -1, 0, 1, 2, 3\}$$

and the function $f : A \to B$ takes each triple to its sum. Since $|A| = 8 > 7 = |B|$, f will not be injective and two triples will sum to the same number.