MS121 Discrete Mathematics, Tutorial 2

1. (a) Use a direct proof to show that if n and m are integers and 3 is a factor of both n and m, then 3 is a factor of any number of the form $nx+my$ where x and y are integers.

(b) Let m and n be integers. Prove, using the contrapositive, that if $m+n$ is an even integer, then m and n are either both even or both odd.

2. Prove the following by mathematical induction for all integers $n \geq 1$.
(a)

$$1 + 2 + \ldots + n = \frac{n(n+1)}{2}$$

(b)

$$1^2 + 2^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

Use the formula in part (a) to evaluate the sum: $1 + 2 + 3 + \ldots + 100$. Use the formula in part (b) to evaluate the sum: $1 + 4 + 9 + \ldots + 100$.

3. A sequence of numbers $x_1, x_2, \ldots, x_n, \ldots$ is defined recursively as follows:

$$x_1 = 5 \text{ and } \; x_{n+1} = 2x_n - 3 \text{ for } n \geq 1.$$

(i) Evaluate x_2, x_3, x_4 and x_5.
(ii) Use mathematical induction to prove that $x_n = 2^n + 3$.