MS121, Test 1(a), 8th. Oct. 2019

Name: ______________________	Student No.: ______________________

?. Let P, Q and R be propositions defined as follows:
P: Bus 1 is late.   Q: Bus 2 is late.   R: I get to work.
The compound proposition 'If bus 1 is late or bus 2 is late, then I do not get to work.' can be expressed as

(A) R $\Rightarrow$ [(**not** P) **or** (**not** Q)] ,   (B) R $\Rightarrow$ [(**not** P) **or** Q],
(C) R $\Rightarrow$ [(**not** P) **and** (**not** Q)],   (D) P $\Rightarrow$ [(**not** Q) **and** (**not** R)]
Answer: $\boxed{\text{C}}$

The compound statement is (P **or** Q) $\Rightarrow$ (**not**R). Using A $\Rightarrow$ B $\equiv$ (**not** B) $\Rightarrow$ (**not**A), this is equivalent to **not**(**not**R) $\Rightarrow$ **not** (P **or** Q) which in turn is equivalent to R $\Rightarrow$ [(**not** P) **and** (**not** Q)].

?. The negation of P $\Rightarrow$ Q is equivalent to

(A) P **and** Q, (B) P **and** (**not** Q), (C) (**not** P) **and** Q, (D) (**not** P) **and** (**not** Q)
Answer: $\boxed{\text{B}}$

We know P $\Rightarrow$ Q is equivalent to (**not**P) **or** Q so its negation is equivalent to **not** [(**not**P) **or** Q] or P **and** (**not**Q).

?. The negation of the statement 'All tests are not difficult.' is the following:

(A) All tests are not difficult.   (B) All tests are difficult.
(C) Some tests are not difficult.   (D) Some tests are difficult.
Answer: $\boxed{\text{D}}$

Let 't' be a test and P(t) the predicate 'Test t is difficult'. Then the original statement can be expressed as $\forall\ t,$ **not**(P(t)) and its negation is $\exists\ t,$ **not** (**not**(P(t))) or $\exists\ t, P(t)$. This is the statement 'At least one test is difficult'.

?. A sequence of numbers $x_1, x_2, \ldots, x_n, \ldots$ is defined inductively by
$x_1 = 5$ and $x_{k+1} = 2x_k - 3$ for $k \geq 1$.
The numbers x_4 and x_5 take the following values respectively:

(A) 19 and 34,   (B) 18 and 35,   (C) 19 and 35,   (D) 18 and 33.
Answer: $\boxed{\text{C}}$

$x_2 = 2x_1 - 3 = 7$, $x_3 = 2x_2 - 3 = 11$, $x_4 = 2x_3 - 3 = 19$, $x_5 = 2x_4 - 3 = 35$.

MS121, Test 1(a), 8th. Oct. 2019

Name: ____________________________	Student No.: ____________________________

?. Let P, Q and R be propositions defined as follows:
P: Bus 1 is late. Q: Bus 2 is late. R: I get to work.
The compound proposition 'If bus 1 is late or bus 2 is late, then I do not get to work.' can be expressed as

(A) R $\Rightarrow$ [(**not** P) **or** (**not** Q)] , (B) R $\Rightarrow$ [(**not** P) **or** Q],
(C) R $\Rightarrow$ [(**not** P) **and** (**not** Q)], (D) P $\Rightarrow$ [(**not** Q) **and** (**not** R)]
Answer: $\boxed{\text{C}}$

The compound statement is (P **or** Q) $\Rightarrow$ (**not**R). Using A $\Rightarrow$ B $\equiv$ (**not** B) $\Rightarrow$ (**not**A), this is equivalent to **not**(**not**R) $\Rightarrow$ **not** (P **or** Q) which in turn is equivalent to R $\Rightarrow$ [(**not** P) **and** (**not** Q)].

?. The negation of P $\Rightarrow$ Q is equivalent to

(A) P **and** Q, (B) P **and** (**not** Q), (C) (**not** P) **and** Q, (D) (**not** P) **and** (**not** Q)
Answer: $\boxed{\text{B}}$

We know P $\Rightarrow$ Q is equivalent to (**not**P) **or** Q so its negation is equivalent to **not** [(**not**P) **or** Q] or P **and** (**not**Q).

?. The negation of the statement 'All tests are not difficult.' is the following:

(A) All tests are not difficult. (B) All tests are difficult.
(C) Some tests are not difficult. (D) Some tests are difficult.
Answer: $\boxed{\text{D}}$

Let 't' be a test and P(t) the predicate 'Test t is difficult'. Then the original statement can be expressed as $\forall\ t,$ **not**(P(t)) and its negation is $\exists\ t,$ **not** (**not**(P(t))) or $\exists\ t, P(t)$. This is the statement 'At least one test is difficult'.

?. A sequence of numbers $x_1, x_2, \ldots, x_n, \ldots$ is defined inductively by
$x_1 = 5$ and $x_{k+1} = 2x_k - 3$ for $k \geq 1$.
The numbers x_4 and x_5 take the following values respectively:

(A) 19 and 34, (B) 18 and 35, (C) 19 and 35, (D) 18 and 33.
Answer: $\boxed{\text{C}}$

$x_2 = 2x_1 - 3 = 7$, $x_3 = 2x_2 - 3 = 11$, $x_4 = 2x_3 - 3 = 19$, $x_5 = 2x_4 - 3 = 35$.

MS121, Test 2, 30th. Oct. 2018 Average: 2.5

?. If X, Y and Z are sets then $X \cup (\sim Y) \cup Z$ does not contain

(A) $X \cap Y$, (B) $Y \cap Z$, (C) $(\sim X) \cap Y$, (D) $(\sim X) \cap (\sim Y)$

Answer: $\boxed{\text{C}}$: The set $(\sim X) \cap Y$ will contain $(\sim X) \cap Y \cap (\sim Z)$ which is the complement of $X \cup (\sim Y) \cup Z$.

?. Suppose X, Y and Z are sets, $|X \cup Y \cup Z| = 12$, $|X| = 4$, $|Y| = 8$, $|Z| = 7$, $|X \cap Y| = 3$, $|X \cap Z| = 3$ and $|Y \cap Z| = 3$. How many elements belong to X but do not belong to Y or Z?

(A) 0, (B) 1, (C) 2, (D) 3

Answer: $\boxed{\text{A}}$: The inclusion-exclusion formula gives

$$12 = |X \cup Y \cup Z| = |X| + |Y| + |Z| - |X \cap Y| - |X \cap Z| - |Y \cap Z| + |X \cap Y \cap Z|$$

$$= 4 + 8 + 7 - 3 - 3 - 3 + |X \cap Y \cap Z|$$

Deduce that $|X \cap Y \cap Z| = 2$, $|X \cap Y \cap (\sim Z)| = 1$, $|X \cap (\sim Y) \cap Z| = 1$ so that $|X \cap (\sim Y) \cap (\sim Z)| = 4 - 2 - 1 - 1 = 0$.

?. Suppose $R = \{(1, 1), (1, 3), (2, 4), (3, 1), (3, 3), (4, 2), (4, 4)\}$ is a relation on the set $X = \{1, 2, 3, 4\}$. Then R is

(A) Reflexive (B) Symmetric (C) Antisymmetric (D) Transitive

Answer: $\boxed{\text{B}}$: $(2, 2) \notin R$ so not reflexive. $(1, 3), (3, 1) \in R$ so not antisymmetric. $(2, 4), (4, 2) \in R$ but $(2, 2) \notin R$ so not transitive.

?. Suppose $R = \{(1, 3), (2, 4), (3, 1), (4, 5), (5, 6), (6, 2)\}$ is a relation on the set $X = \{1, 2, 3, 4, 5, 6\}$. Which pair is not in the transitive closure of R?

(A) $(2, 6)$ (B) $(2, 2)$ (C) $(2, 1)$ (D) $(2, 5)$

Answer: $\boxed{\text{C}}$: $(2, 4), (4, 5) \in R$ so $(2, 5)$ in closure. $(2, 4), (4, 5), (5, 6) \in R$ so $(2, 6)$ in closure. $(2, 4), (4, 5), (5, 6), (6, 2) \in R$ so $(2, 2)$ in closure.

MS121, Test 1(b), 9th. Oct. 2019

Name: ___________________________	Student No.: ___________________________

?. Let P, Q and R be propositions defined as follows:
P: Bus 1 is late. Q: Bus 2 is late. R: I get to work.
The compound proposition 'If bus 1 is late and bus 2 is late, then I do not get to work.' can be expressed as

(A) R $\Rightarrow$ [(**not** P) **or** (**not** Q)] , (B) R $\Rightarrow$ [(**not** P) **or** Q],
(C) R $\Rightarrow$ [(**not** P) **and** (**not** Q)], (D) P $\Rightarrow$ [(**not** Q) **and** (**not** R)]
Answer: $\boxed{\text{A}}$

The compound statement is (P **and** Q) $\Rightarrow$ (**not**R). Using A $\Rightarrow$ B $\equiv$ (**not** B) $\Rightarrow$ (**not**A), this is equivalent to **not**(**not**R) $\Rightarrow$ **not** (P **and** Q) which in turn is equivalent to R $\Rightarrow$ [(**not** P) **or** (**not** Q)].

?. The negation of Q $\Rightarrow$ P is equivalent to

(A) P **and** Q, (B) P **and** (**not** Q), (C) (**not** P) **and** Q, (D) (**not** P) **and** (**not** Q)
Answer: $\boxed{\text{C}}$

We know Q $\Rightarrow$ P is equivalent to (**not**Q) **or** P so its negation is equivalent to **not** [(**not**Q) **or** P] or Q **and** (**not**P).

?. The negation of the statement 'All tests are difficult.' is the following:

(A) All tests are not difficult. (B) All tests are difficult.
(C) Some tests are not difficult. (D) Some tests are difficult.
Answer: $\boxed{\text{C}}$

Let 't' be a test and P(t) the predicate 'Test t is difficult'. Then the original statement can be expressed as $\forall\ t, \mathrm{P}(t)$ and its negation is $\exists\ t, \mathbf{not}(\mathrm{P}(t))$. This is the statement 'At least one test is not difficult'.

?. A sequence of numbers $x_1, x_2, \ldots, x_n, \ldots$ is defined inductively by $x_1 = 4$ and $x_{k+1} = 3x_k - 2$ for $k \geq 1$.
The numbers x_3 and x_4 take the following values respectively:

(A) 28 and 83, (B) 27 and 82, (C) 28 and 82, (D) 27 and 79.
Answer: $\boxed{\text{C}}$

$x_2 = 3x_1 - 2 = 10$, $x_3 = 3x_2 - 2 = 28$, $x_4 = 3x_3 - 2 = 82$.

MS121, Test 1(c), 11th. Oct. 2019

Name: ______________________	Student No.: ______________________

?. Let P, Q and R be propositions defined as follows:
P: Bus 1 is late. Q: Bus 2 is late. R: It is rush hour.
The compound proposition 'If it is rush hour then bus 1 is late or bus 2 is late.' can be expressed as

(A) [(**not** P) **or** (**not** Q)] $\Rightarrow$ **not** R , (B) [(**not** P) **or** (**not** Q)] $\Rightarrow$ R,
(C) [(**not** P) **and** (**not** Q)] $\Rightarrow$ **not** R, (D) [(**not** P) **and** (**not** Q)] $\Rightarrow$ R
Answer: $\boxed{\text{C}}$

The statement is of form R $\Rightarrow$ [P **or** Q] so is equivalent to **not** [P **or** Q] $\Rightarrow$ **not** R which in turn is equivalent to [(**not** P) **and** (**not** Q)] $\Rightarrow$ **not** R.

?. The negation of P $\Rightarrow$ **not** Q is equivalent to

(A) P **and** Q, (B) P **and** (**not** Q), (C) (**not** P) **and** Q, (D) (**not** P) **and** (**not** Q)
Answer: $\boxed{\text{A}}$

X $\Rightarrow$ Y is equivalent to (**not** X) **or** Y so the negation of X $\Rightarrow$ Y is X **and** (**not**Y). Here the negation of P $\Rightarrow$ (**not** Q) is P **and** Q.

?. The negation of the statement 'At least one test is not difficult.' is the following:

(A) All tests are not difficult. (B) All tests are difficult.
(C) Some tests are not difficult. (D) Some tests are difficult.
Answer: $\boxed{\text{B}}$

If t is a test and P(t) is the statement 'test t is difficult' then 'At least one test is not difficult.' is the statement $\exists t$, **not** P(t). Its negation is $\forall t$, P(t) or 'For all tests t, t is difficult'.

?. A sequence of numbers $x_1, x_2, \ldots, x_n, \ldots$ is defined inductively by
$x_1 = 1$ and $x_{k+1} = 2x_k + k$ for $k \geq 1$.
The numbers x_3 and x_4 take the following values respectively:

(A) 8 and 18, (B) 8 and 19, (C) 9 and 19, (D) 9 and 21.
Answer: $\boxed{\text{B}}$

$x_2 = (2)x_1 + 1 = (2)(1) + 1 = 3, \quad x_3 = (2)x_2 + 2 = (2)(3) + 2 = 8,$
$x_4 = (2)x_3 + 3 = (2)(8) + 3 = 19.$

MS121, Test 1(d), 18th. Oct. 2019

Name: ____________________	Student No.: ____________________

?. Let P, Q and R be propositions defined as follows:
P: We win game 1. Q: We win game 2. R: We qualify for the next round.
The compound proposition 'If we win game 1 and we win game, then we qualify for the next round.' can be expressed as

(A) **not** R $\Rightarrow$ [(**not** P) **and** (**not** Q)] , (B) **not** R $\Rightarrow$ [(**not** P) **or** (**not** Q)],
(C) [**not** R $\Rightarrow$ (**not** P)] **and** (**not** Q) , (D) [**not** R $\Rightarrow$ (**not** P)] **or** (**not** Q)
Answer: $\boxed{\text{B}}$

The statement is of form [P **and** Q] $\Rightarrow$ R so is equivalent to **not** R $\Rightarrow$ **not**[P **and** Q] which in turn is equivalent to **not** R $\Rightarrow$ [(**not** P) **or** (**not** Q)].

?. The negation of (**not** P) $\Rightarrow$ (**not** Q) is equivalent to

(A) P **and** Q, (B) P **and** (**not** Q), (C) (**not** P) **and** Q, (D) (**not** P) **and** (**not** Q)
Answer: $\boxed{\text{C}}$

X $\Rightarrow$ Y is equivalent to (**not** X) **or** Y so the negation of X $\Rightarrow$ Y is X **and** (**not**Y).
Here the negation of (**not** P) $\Rightarrow$ (**not** Q) is (**not** P) **and** Q.

?. The negation of the statement 'Some modules are interesting.' is the following:

(A) Some modules are interesting. (B) All modules are interesting
(C) Some modules are not interesting (D) All modules are not interesting
Answer: $\boxed{\text{D}}$

If m is a module and P(m) is the statement 'module m is interesting' then 'Some modules are interesting.' is the statement $\exists m$, P(m). Its negation is $\forall m$, **not**P(m) or 'For all modules m, m is not interesting'.

?. A sequence of numbers $x_1, x_2, \ldots, x_n, \ldots$ is defined inductively by
$x_1 = 1$ and $x_{k+1} = kx_k + 2$ for $k \geq 1$.
The numbers x_4 and x_5 take the following values respectively:

(A) 26 and 104, (B) 26 and 106, (C) 24 and 106, (D) 24 and 98.
Answer: $\boxed{\text{B}}$

$x_2 = (1)x_1 + 2 = (1)(1) + 2 = 3,\quad x_3 = (2)x_2 + 2 = (2)(3) + 2 = 8,$
$x_4 = (3)x_3 + 2 = (3)(8) + 2 = 26,\quad x_5 = (4)x_4 + 2 = (4)(26) + 2 = 106.$

MS121, Test 3a, 21st. Nov. 2018

Name: ______________________	Student No.: ______________________

?. If D is the set of divisors of 36 with partial order 'is a divisor of', which one of the following is an immediate predecessor of 36?

(A) 9 (B) 18 , (C) 6 , (D) 4.

Answer: $\boxed{\text{B}}$: Using the notation $x \mid y$ for x is a divisor of y, we have $9 \mid 18 \mid 36$, $6 \mid 18 \mid 36$ and $4 \mid 12 \mid 36$, while $18 \mid 36$ with no divisor between 18 and 36.

?. Suppose $A = \{a, b, c, d\}$, $B = \{x, y, z\}$ and R is the relation between A and B given by $R = \{(a, x), (b, x), (b, y), (c, y), (d, z)\}$. R is not a function. Removing which of the following pairs from R results in a function?

(A) (a, x), (B) (b, y), (C) (c, y), (D) (d, z).

Answer: $\boxed{\text{B}}$: Removing either of (a, x), (c, y) or (d, z) will still leave two pairs with b as first element (and no pairs with a particular element of A as first element).

?. Suppose $S = \{0, 1, 2, 3, 4, 5\}$, $T = \{0, 2, 4\}$ and $f : S \to T$ is given by $f(k) = r$ where r is the remainder when $4k$ is divided by 6. Then f is

(A) Injective but not surjective (B) Surjective but not injective (C) Bijective (D) Neither injective nor surjective.

Answer: $\boxed{\text{B}}$: $f(0) = 4(0)\text{mod}6 = 0$, $f(1) = 4(1)\text{mod}6 = 4$, $f(2) = 4(2)\text{mod}6 = 2$, $f(3) = 4(3)\text{mod}6 = 0$, $f(4) = 4(4)\text{mod}6 = 4$, $f(5) = 4(5)\text{mod}6 = 2$. Since $f(0) = f(3)$, f is not injective. f is surjective since $f(0) = 0$, $f(2) = 2$ and $f(1) = 4$..

?. The inverse of $f(x) = (3x + 2)/(5x - 3)$ is

(A) $g(y) = (3y + 2)/(6y - 3)$, (B) $g(y) = (3y + 2)/(5y - 3)$,
(C) $g(y) = (3y + 2)/(5y - 2)$, (D) $g(y) = (3y + 2)/(6y - 2)$,

Answer: $\boxed{\text{B}}$: $y = (3x + 2)/(5x - 3)$ implies $y(5x - 3) = 3x + 2$, which in turn implies $5xy - 3y = 3x + 2$ and hence $5xy - 3x = 3y + 2$ or $x(5y - 3) = 3y + 2$.

MS121, Test 4, 11th. Dec. 2018

Name: ______________________________	Student No.: ______________________________

?. Grocery items are distributed among 3 shopping bags. What is the minimum number of grocery items required to guarantee that at least one bag has at least 4 items?

(A) 4, (B) 10, (C) 9, (D) 12.

Answer: $\boxed{\text{B}}$: This is the extended pigeonhole principle. Here A is a set of grocery items, B is a set of bags, $f : A \to B$ takes a grocery item to the bag it ends up in. Since $|B| = 3$ at least one bag will have more than $k = 3$ items if $|A| \geq k|B| + 1$.

?. A committee of 4 must be chosen from a group of 4 men and 4 women but must contain at least 1 man and at least 1 woman. In how many ways can this be done?

(A) 70, (B) 69, (C) 240, (D) 68.

Answer: $\boxed{\text{D}}$: The total number of committees is $\left(\begin{array}{c} 8 \\ 4 \end{array} \right) = 70$. Subtract the two possible committees with all men or all women. (If you pick the one man and one women first you will end up overcounting. The committee is unordered so there should not be a first man and a first woman.)

?. 6 people are to be split up into three teams of size 3, 2 and 1. In how many ways can this be done?

(A) 720, (B) 360, (C) 60, (D) 6.

Answer: $\boxed{\text{C}}$: This is the multinomial coefficient $\left(\begin{array}{c} 6 \\ 3, 2, 1 \end{array} \right) = \frac{6!}{3!2!1!}$

?. A fair coin is tossed 5 times. What is the probability of H's coming up more often than T's?

(A) 1/2, (B) 17/32, (C) 15/32, (D) 5/8.

Answer: $\boxed{\text{A}}$: Since 5 is odd, H's come up more often than T's half the time and T's come up more often than H's the other half. Alternatively, the answer is the probability of 5 or 4 or 3 H's and therefore $(1/32)(1 + 5 + 10)$.

MS121, Test 2a, 29th. Oct. 2019

Name: ________________________	Student No.: ________________________

?. Which of the following statements about two sets X and Y is not logically equivalent to the others?

(A) $X \subseteq (\sim Y)$ (B) $Y \subseteq (\sim X)$ (C) $(\sim X) \cap (\sim Y) = \emptyset$ (D) $X \cap Y = \emptyset$

Answer: $\boxed{\text{C}}$

The other three are logically equivalent. (D) says there are no elements in both X and Y which is equivalent to saying all of X's elements are not in Y (A) or all of Y's elements are not in X (B).

?. Suppose X, Y and Z are sets, $|X \cup Y \cup Z| = 15$, $|X| = 3$, $|Y| = 9$, $|Z| = 9$, $|X \cap Y| = 2$, $|X \cap Z| = 0$ and $|Y \cap Z| = 4$. How many elements belong to Y but do not belong to X or Z?

(A) 1, (B) 2, (C) 3, (D) 4

Answer: $\boxed{\text{C}}$

Since $|X \cap Z| = 0$, we know $|X \cap Y \cap Z| = 0$ and $|X \cap (\sim Y) \cap Z| = 0$. From the usual Venn diagram of 3 sets we get $|X \cap Y \cap (\sim Z)| = 2$ and $|(\sim X) \cap Y \cap Z| = 4$. From that we deduce $|(\sim X) \cap Y \cap (\sim Z)| = 3$.

?. Suppose $R = \{(1,1),(2,3),(3,3),(3,4),(4,4)\}$ is a relation on the set $S = \{1,2,3,4\}$. Then R is

(A) Reflexive (B) Symmetric (C) Antisymmetric (D) Transitive

Answer: $\boxed{\text{C}}$

There are only two pairs (x,y) with $x \neq y$, namely $(2,3)$ and $(3,4)$ and for each of these R does not contain (y,x). This makes R antisymmetric. (A) fails since there is no $(2,2)$. (B) fails since $(2,3) \in R$ but $(3,2) \notin R$. (D) fails since $(2,3) \in R$ and $(3,4) \in R$ but $(2,4) \notin R$.

?. Suppose $R = \{(1,1),(1,3),(1,4),(2,2),(3,3),(3,4),(4,1),(4,4)\}$ is a relation on the set $S = \{1,2,3,4\}$. Then R will be an equivalence relation when we add the two elements

(A) $(3,1)$ and $(2,3)$, (B) $(3,1)$ and $(4,3)$, (C) $(4,3)$ and $(2,4)$, (D) $(2,3)$ and $(2,4)$,

Answer: $\boxed{\text{B}}$

Just to satisfy the symmetry property we need these pairs. All the other possibilities involve some $(2,x)$ with no $(x,2)$ so that symmetry will not hold.

MS121, Test 2b, 30th. Oct. 2019

Name: ____________________________	Student No.: ____________________________

?. Which of the following statements about two sets X and Y is not logically equivalent to the others?

(A) $(\sim X) \subseteq (\sim Y)$ (B) $Y \subseteq X$ (C) $(\sim X) \cap Y = \emptyset$ (D) $X \cap Y = \emptyset$

Answer: $\boxed{\text{D}}$

The other three are logically equivalent. (C) says there are no elements in both $\sim X$ and Y which is equivalent to saying all of $\sim X$'s elements are not in Y (A) or all of Y's elements are not in $\sim X$ (B).

?. Suppose X, Y and Z are sets, $|X \cup Y \cup Z| = 10$, $|X| = 4$, $|Y| = 4$, $|Z| = 6$, $|X \cap Y| = 0$, $|X \cap Z| = 2$ and $|Y \cap Z| = 2$. How many elements belong to Z but do not belong to X or Y?

(A) 1, (B) 2, (C) 3, (D) 4

Answer: $\boxed{\text{B}}$

Since $|X \cap Y| = 0$, we know $|X \cap Y \cap Z| = 0$ and $|X \cap Y \cap (\sim Z)| = 0$. From the usual Venn diagram of 3 sets we get $|X \cap (\sim Y) \cap Z| = 2$ and $|(\sim X) \cap Y \cap Z| = 2$. From that we deduce $|(\sim X) \cap (\sim Y) \cap Z| = 2$.

?. Suppose $R = \{(1,3), (2,2), (2,4), (3,1), (3,3), (4,2), (4,4)\}$ is a relation on the set $S = \{1,2,3,4\}$. Then R is

(A) Reflexive (B) Symmetric (C) Antisymmetric (D) Transitive

Answer: $\boxed{\text{B}}$

The pairs (x, y) with $x \neq y$, each have a (y, x), namely $(1,3)$ and $(3,1)$, $(2,4)$ and $(4,2)$. This makes R symmetric. (A) fails since there is no $(1,1)$. (C) fails since $(1,3) \in R$ and $(3,1) \in R$. (D) fails since $(1,3) \in R$ and $(3,1) \in R$ but $(1,1) \notin R$.

?. Suppose $R = \{(1,1), (1,2), (2,1), (2,4), (3,3), (4,1), (4,2), (4,4)\}$ is a relation on the set $S = \{1,2,3,4\}$. Then R will be an equivalence relation when we add the two elements

(A) $(2,2)$ and $(1,4)$, (B) $(3,2)$ and $(1,4)$, (C) $(3,2)$ and $(2,2)$, (D) $(3,4)$ and $(2,2)$,

Answer: $\boxed{\text{A}}$

Just to satisfy the symmetry and reflexivity properties we need these pairs. All the other possibilities involve some $(3, x)$ with no $(x, 3)$ so that symmetry will not hold.

MS121, Test 2c, 1st. Nov. 2019

Name: ______________________	Student No.: ______________________

?. Which of the following statements about two sets X and Y is not logically equivalent to the others?

(A) $(\sim Y) \subseteq (\sim X)$ (B) $X \subseteq Y$ (C) $(\sim X) \cap Y = \emptyset$ (D) $X \cap (\sim Y) = \emptyset$

Answer: $\boxed{\text{C}}$

The other three are logically equivalent. (D) says there are no elements in both $\sim Y$ and X which is equivalent to saying all of $\sim Y$'s elements are in $\sim X$ (A) or all of X's elements are in Y (B).

?. Suppose X, Y and Z are sets, $|X \cup Y \cup Z| = 12$, $|X| = 8$, $|Y| = 5$, $|Z| = 3$, $|X \cap Y| = 3$, $|X \cap Z| = 1$ and $|Y \cap Z| = 0$. How many elements belong to X but do not belong to Y or Z?

(A) 1, (B) 2, (C) 3, (D) 4

Answer: $\boxed{\text{D}}$

Since $|Y \cap Z| = 0$, we know $|X \cap Y \cap Z| = 0$ and $|(\sim X) \cap Y \cap Z| = 0$. From the usual Venn diagram of 3 sets we get $|X \cap Y \cap (\sim Z)| = 3$ and $|X \cap (\sim Y) \cap Z| = 1$. From that we deduce $|X \cap (\sim Y) \cap (\sim Z)| = 4$.

?. Suppose $R = \{(1,3), (2,2), (2,4), (3,3), (4,2), (4,4)\}$ is a relation on the set $S = \{1, 2, 3, 4\}$. Then R is

(A) Reflexive (B) Symmetric (C) Antisymmetric (D) Transitive

Answer: $\boxed{\text{D}}$

(A) fails since there is no $(1,1)$. (B) fails since $(1,3) \in R$ but $(3,1) \notin R$. (C) fails since $(2,4) \in R$ and $(4,2) \in R$. (D) holds by checking all cases.

?. Suppose $R = \{(1,1), (1,3), (1,4), (2,2), (3,3), (3,4), (4,1), (4,4)\}$ is a relation on the set $S = \{1, 2, 3, 4\}$. Then R will be an equivalence relation when we add the two elements

(A) $(3,1)$ and $(3,2)$, (B) $(4,2)$ and $(4,3)$, (C) $(3,1)$ and $(4,3)$, (D) $(1,2)$ and $(2,1)$,

Answer: $\boxed{\text{C}}$

Just to satisfy the symmetry property we need these pairs. All the other possibilities leave out at least one of these two pairs.

MS121, Test 3a, 19th. Oct. 2019

Name: ______________________	Student No.: ______________________

?. If P is the set of divisors of 90 with partial order 'is a divisor of', which one of the following is **not** an immediate successor of 3?

(A) 6, (B) 9, (C) 30, (D) 15.

Answer: $\boxed{\text{C}}$: In the partial order we can fit both 6 and 15 between 3 and 30.

?. Suppose $X = \{x, y, z\}$, $Y = \{a, b\}$ and $Z = \{p, q, r\}$ while
$R = \{(x, a), (y, a), (y, b), (z, b)\}$ is a relation between X and Y and
$S = \{(a, p), (a, q), (b, q), (b, r)\}$ is a relation between Y and Z.
Which one of the following pairs is **not** in $S \circ R$?

(A) (x, p), (B) (y, p), (C) (z, p), (D) (z, q).

Answer: $\boxed{\text{C}}$: We have xRa and aSp so answer is not A. We have yRa and aSp so answer is not B. We have zRb and bSq so answer is not D.

?. Suppose $S = T = \{0, 1, 2, 3, 4, 5, 6\}$ and $f : S \to T$ is given by $f(k) = r$ where r is the remainder when $3k$ is divided by 7. Then f is

(A) Injective but not surjective, (B) Surjective but not injective ,
(C) Bijective, (D) Neither injective nor surjective.

Answer: $\boxed{\text{C}}$: $f(0) = 3(0) \bmod 7 = 0 \bmod 7 = 0$, $f(1) = 3(1) \bmod 7 = 3 \bmod 7 = 3$, $f(2) = 3(2) \bmod 7 = 6 \bmod 7 = 6$, $f(3) = 3(3) \bmod 7 = 9 \bmod 7 = 2$, $(9 = (1)7 + 2)$, $f(4) = 3(4) \bmod 7 = 12 \bmod 7 = 5$, $(12 = (1)7 + 5)$, $f(5) = 3(5) \bmod 7 = 15 \bmod 7 = 1$, $(15 = (2)7 + 1)$, $f(6) = 3(6) \bmod 7 = 18 \bmod 7 = 4$, $(18 = (2)7 + 4)$.

?. The inverse of $f(x) = (2x + 3)/(-x - 2)$ is

(A) $g(y) = (3y + 2)/(-y - 2)$, (B) $g(y) = (2y + 3)/(-2y - 1)$, (C) $g(y) = (3y + 2)/(-2y - 1)$, (D) $g(y) = (2y + 3)/(-y - 2)$.

Answer: $\boxed{\text{D}}$: If $y = (2x + 3)/(-x - 2)$ then $-xy - 2y = 2x + 3$ so that $-xy - 2x = 2y + 3$ and $x = (2y + 3)/(-y - 2)$.

MS121, Test 3b, 20th. Nov. 2019

Name: ______________________	Student No.: ______________________

?. If P is the set of divisors of 150 with partial order 'is a divisor of', which one of the following is **not** an immediate successor of 5?

(A) 10, (B) 15, (C) 30, (D) 25.

Answer: $\boxed{\text{C}}$: In the partial order we can fit both 10 and 15 between 5 and 30.

?. Suppose $X = \{x, y, z\}$, $Y = \{a, b\}$ and $Z = \{p, q, r\}$ while
$R = \{(x, a), (y, b), (z, a)\}$ is a relation between X and Y and
$S = \{(a, q), (b, p), (b, r)\}$ is a relation between Y and Z.
Which one of the following pairs is **not** in $S \circ R$?

(A) (x, q), (B) (y, p), (C) (z, r), (D) (z, q).

Answer: $\boxed{\text{C}}$: We have xRa and aSq so answer is not A. We have yRb and bSp so answer is not B. We have zRa and aSq so answer is not D.

?. Suppose $S = T = \{0, 1, 2, 3, 4, 5, 6\}$ and $f : S \to T$ is given by $f(k) = r$ where r is the remainder when k^2 is divided by 7. Then f is

(A) Injective but not surjective, (B) Surjective but not injective ,
(C) Bijective, (D) Neither injective nor surjective.

Answer: $\boxed{\text{D}}$: The squares of the numbers in $\{0, 1, 2, 3, 4, 5, 6\}$ are $\{0, 1, 4, 9, 16, 25, 36\}$. The remainders mod 7 of these numbers are $\{0, 1, 4, 2, 2, 4, 1\}$.

?. The inverse of $f(x) = (2x + 3)/(-x + 2)$ is

(A) $g(y) = (2y - 3)/(-y + 2)$, (B) $g(y) = (2y - 3)/(y + 2)$, (C) $g(y) = (2y + 3)/(y + 2)$, (D) $g(y) = (2y + 3)/(-y + 2)$.

Answer: $\boxed{\text{B}}$: If $y = (2x + 3)/(-x + 2)$ then $-xy + 2y = 2x + 3$ so that $-xy - 2x = -2y + 3$ and $x = (-2y + 3)/(-y - 2) = (2y - 3)/(y + 2)$.

MS121, Test 3c, 22nd. Nov. 2019

Name: ______________________	Student No.: ______________________

?. If P is the set of divisors of 150 with partial order 'is a divisor of', which one of the following is **not** an immediate predecessor of 30?

(A) 10, (B) 15, (C) 5, (D) 6.

Answer: $\boxed{\text{C}}$: In the partial order we can fit both 10 and 15 between 5 and 30.

?. Suppose $X = \{x, y, z\}$, $Y = \{a, b\}$ and $Z = \{p, q, r\}$ while
$R = \{(x, a), (x, b), (y, b), (z, a)\}$ is a relation between X and Y and
$S = \{(a, p), (a, q), (b, q), (b, r)\}$ is a relation between Y and Z.
Which one of the following pairs is **not** in $S \circ R$?

(A) (x, q), (B) (y, q), (C) (z, r), (D) (z, q).

Answer: $\boxed{\text{C}}$: We have xRa and aSq so answer is not A. We have yRb and bSq so answer is not B. We have zRa and aSq so answer is not D.

?. Suppose $S = \{0, 1, 2, 3, 4, 5, 6, 7\}$, $T = \{0, 2, 4, 6\}$ and $f : S \to T$ is given by $f(k) = r$ where r is the remainder when $6k$ is divided by 8. Then f is

(A) Injective but not surjective, (B) Surjective but not injective ,
(C) Bijective, (D) Neither injective nor surjective.

Answer: $\boxed{\text{B}}$: Multiplying the numbers in $\{0, 1, 2, 3, 4, 5, 6, 7\}$ by 6 gives $\{0, 6, 12, 18, 24, 30, 36, 42\}$. The remainders mod 8 of these numbers are $\{0, 6, 4, 2, 0, 6, 4, 2\}$.

?. The inverse of $f(x) = (2x + 3)/(4x - 2)$ is

(A) $g(y) = (2y + 3)/(4y + 2)$, (B) $g(y) = (2y - 3)/(4y - 2)$, (C) $g(y) = (2y - 3)/(4y + 2)$, (D) $g(y) = (2y + 3)/(4y - 2)$.

Answer: $\boxed{\text{D}}$: If $y = (2x + 3)/(4x - 2)$ then $4xy - 2y = 2x + 3$ so that $4xy - 2x = 2y + 3$ and $x = (2y + 3)/(4y - 2)$.

MS121, Test 4a, 10th. Dec. 2019

Name: ____________________________	Student No.: ____________________________

?. The smallest number of Irish people required in order to ensure that at least 4 come from the same province (Connacht, Leinster, Munster or Ulster) is

(A) 4 (B) 12 , (C) 13 , (D) 16

Answer: $\boxed{\text{C}}$: This is the extended pigeonhole principle. Here A is a set of Irish people, B is the set of provinces, $f : A \to B$ takes a person to the province they are from. Since $|B| = 4$ at least one province will have more than $k = 3$ people if $|A| \geq k|B| + 1$.

?. The number of 3 element subsets of $\{1, 2, 3, 4, 5, 6, 7\}$ containing at least one even number is

(A) 60 (B) 30 (C) 31 (D) 35

Answer: $\boxed{\text{C}}$: Use the subtraction rule. The total number of subsets minus the number with all odd numbers is $\begin{pmatrix} 7 \\ 3 \end{pmatrix} - \begin{pmatrix} 4 \\ 3 \end{pmatrix} = 35 - 4$.

?. 4 pieces of fruit are picked from a bowl containing 5 apples, 5 oranges and 5 pears. How many different distributions of fruit (e.g. (2×Ap)(1×Or)(1×Pe)) are possible?

(A) $\begin{pmatrix} 15 \\ 4 \end{pmatrix}$ (B) $\begin{pmatrix} 6 \\ 3 \end{pmatrix}$ (C) $\begin{pmatrix} 6 \\ 2 \end{pmatrix}$ (D) $\begin{pmatrix} 7 \\ 2 \end{pmatrix}$

Answer: $\boxed{\text{C}}$: The number is the number of 4-selections from 3. The answer is $\begin{pmatrix} 4+3-1 \\ 3-1 \end{pmatrix}$. In terms of stars and bars the example (2×Ap)(1×Or)(1×Pe) would be written $**|*|*$ and a general distrubution is equivalent to a choice of 2 places for the bars in a string of length 6.

?. A fair (six-sided) die is tossed twice. The probability that the two numbers shown are different is

(A) 1/2 (B) 1/6 (C) 5/36 (D) 7/36

Answer: $\boxed{?}$: Easier to compute the probability that the two numbers shown are the same. This is $(1/36)(6)(1) = 1/6$. So the probability we want is $5/6$.

MS121, Test 4b, 11th. Dec. 2019

Name: ____________________________	Student No.: ____________________________

?. The smallest number of university staff required in order to ensure that at least 3 come from the same faculty (Science, Business, Arts, Engineering) is

(A) 3 (B) 9 , (C) 8 , (D) 12

Answer: $\boxed{\text{B}}$: This is the extended pigeonhole principle. Here A is a set of staff, B is the set of faculties, $f : A \to B$ takes a staff member to the faculty they are from. Since $|B| = 4$ at least one province will have more than $k = 2$ people if $|A| \geq k|B| + 1$.

?. The number of 4 element subsets of $\{1, 2, 3, 4, 5, 6, 7\}$ containing at least one of $\{1, 2\}$ is

(A) 40 (B) 30 (C) 31 (D) 35

Answer: $\boxed{\text{B}}$: Use the subtraction rule. The total number of subsets minus the number without 1 or 2 is $\begin{pmatrix} 7 \\ 4 \end{pmatrix} - \begin{pmatrix} 5 \\ 4 \end{pmatrix} = 35 - 5$.

?. 7 identical pages are distributed in 3 numbered piles such as $(4, 0, 3)$. The number of different ways this can be done is

(A) $\begin{pmatrix} 10 \\ 2 \end{pmatrix}$ (B) $\begin{pmatrix} 7 \\ 3 \end{pmatrix}$ (C) $\begin{pmatrix} 10 \\ 3 \end{pmatrix}$ (D) $\begin{pmatrix} 9 \\ 2 \end{pmatrix}$

Answer: $\boxed{\text{D}}$: The number is the number of 7-selections from 3. The answer is $\begin{pmatrix} 7+3-1 \\ 3-1 \end{pmatrix}$. In terms of stars and bars the example $(4, 0, 3)$ would be written $* * * * || * **$ and a general distrubution is equivalent to a choice of 2 places for the bars in a string of length 9.

?. A fair coin is tossed six times. The probability that H shows more often than T is

(A) 21/64 (B) 11/32 (C) 23/64 (D) 1/2

Answer: $\boxed{\text{B}}$: The sample space is the set of strings of length 6 in H and T. This gives 64 equally likely outcomes. The numbers of strings with 6 H's is 1, with 5 H's is 6 and with 4 H's is 15. The probability of one of these is $(1/64)(1 + 6 + 15)$.

MS121, Test 4c, 13th. Dec. 2019

Name: ____________________________	Student No.: ____________________________

?. The smallest number of playing cards required from the same deck of 52 in order to ensure that at least 4 come from the same suit ($\heartsuit$, $\spadesuit$, $\diamondsuit$, $\clubsuit$) is

(A) 13 (B) 12 , (C) 4 , (D) 16

Answer: $\boxed{\text{A}}$: This is the extended pigeonhole principle. Here A is a set of cards, B is the set of suits, $f : A \to B$ takes a card to the suit it is from. Since $|B| = 4$ at least one suit will have more than $k = 3$ cards if $|A| \geq k|B| + 1$.

?. The number of 3 element subsets of $\{1, 2, 3, 4, 5, 6, 7, 8\}$ containing at least one of $\{1, 2, 3\}$ is

(A) 63 (B) 64 (C) 45 (D) 46

Answer: $\boxed{\text{D}}$: Use the subtraction rule. The total number of subsets minus the number without 1, 2 or 3 is $\begin{pmatrix} 8 \\ 3 \end{pmatrix} - \begin{pmatrix} 5 \\ 3 \end{pmatrix} = 56 - 10$.

?. 8 identical presents are distributed in 4 numbered sacks such as $(1, 2, 3, 2)$. The number of different ways this can be done is

(A) $\begin{pmatrix} 12 \\ 3 \end{pmatrix}$ (B) $\begin{pmatrix} 11 \\ 3 \end{pmatrix}$ (C) $\begin{pmatrix} 12 \\ 4 \end{pmatrix}$ (D) $\begin{pmatrix} 11 \\ 4 \end{pmatrix}$

Answer: $\boxed{\text{B}}$: The number is the number of 8-selections from 4. The answer is $\begin{pmatrix} 8+4-1 \\ 4-1 \end{pmatrix}$. In terms of stars and bars the example $(1, 2, 3, 2)$ would be written $*|**|***|**$ and a general distrubution is equivalent to a choice of 3 places for the bars in a string of length 11.

?. A four-sided tetrahedaral die is tossed three times. The probability that at least one number is repeated is

(A) 5/8 (B) 39/64 (C) 1/2 (D) 33/64

Answer: $\boxed{\text{A}}$: Easier to compute the probability that the three numbers shown are different. This is $(1/64)(4)(3)(2) = 3/8$. So the probability we want is 5/8.