MS121, Test 4b, 11th. Dec. 2019

Name: ____________________	Student No.: ____________________

?. The smallest number of university staff required in order to ensure that at least 3 come from the same faculty (Science, Business, Arts, Engineering) is

(A) 3 (B) 9 , (C) 8 , (D) 12

Answer: $\boxed{\text{B}}$: This is the extended pigeonhole principle. Here A is a set of staff, B is the set of faculties, $f : A \to B$ takes a staff member to the faculty they are from. Since $|B| = 4$ at least one province will have more than $k = 2$ people if $|A| \geq k|B| + 1$.

?. The number of 4 element subsets of $\{1, 2, 3, 4, 5, 6, 7\}$ containing at least one of $\{1, 2\}$ is

(A) 40 (B) 30 (C) 31 (D) 35

Answer: $\boxed{\text{B}}$: Use the subtraction rule. The total number of subsets minus the number without 1 or 2 is $\begin{pmatrix} 7 \\ 4 \end{pmatrix} - \begin{pmatrix} 5 \\ 4 \end{pmatrix} = 35 - 5$.

?. 7 identical pages are distributed in 3 numbered piles such as $(4, 0, 3)$. The number of different ways this can be done is

(A) $\begin{pmatrix} 10 \\ 2 \end{pmatrix}$ (B) $\begin{pmatrix} 7 \\ 3 \end{pmatrix}$ (C) $\begin{pmatrix} 10 \\ 3 \end{pmatrix}$ (D) $\begin{pmatrix} 9 \\ 2 \end{pmatrix}$

Answer: $\boxed{\text{D}}$: The number is the number of 7-selections from 3. The answer is $\begin{pmatrix} 7+3-1 \\ 3-1 \end{pmatrix}$. In terms of stars and bars the example $(4, 0, 3)$ would be written $* * * * || * **$ and a general distrubution is equivalent to a choice of 2 places for the bars in a string of length 9.

?. A fair coin is tossed six times. The probability that H shows more often than T is

(A) 21/64 (B) 11/32 (C) 23/64 (D) 1/2

Answer: $\boxed{\text{B}}$: The sample space is the set of strings of length 6 in H and T. This gives 64 equally likely outcomes. The numbers of strings with 6 H's is 1, with 5 H's is 6 and with 4 H's is 15. The probability of one of these is $(1/64)(1 + 6 + 15)$.