MS121, Test 2b, 30th. Oct. 2019

Name: ____________________________	Student No.: ____________________________

?. Which of the following statements about two sets X and Y is not logically equivalent to the others?

(A) $(\sim X) \subseteq (\sim Y)$ (B) $Y \subseteq X$ (C) $(\sim X) \cap Y = \emptyset$ (D) $X \cap Y = \emptyset$

Answer: $\boxed{\text{D}}$

The other three are logically equivalent. (C) says there are no elements in both $\sim X$ and Y which is equivalent to saying all of $\sim X$'s elements are not in Y (A) or all of Y's elements are not in $\sim X$ (B).

?. Suppose X, Y and Z are sets, $|X \cup Y \cup Z| = 10$, $|X| = 4$, $|Y| = 4$, $|Z| = 6$, $|X \cap Y| = 0$, $|X \cap Z| = 2$ and $|Y \cap Z| = 2$. How many elements belong to Z but do not belong to X or Y?

(A) 1, (B) 2, (C) 3, (D) 4

Answer: $\boxed{\text{B}}$

Since $|X \cap Y| = 0$, we know $|X \cap Y \cap Z| = 0$ and $|X \cap Y \cap (\sim Z)| = 0$. From the usual Venn diagram of 3 sets we get $|X \cap (\sim Y) \cap Z| = 2$ and $|(\sim X) \cap Y \cap Z| = 2$. From that we deduce $|(\sim X) \cap (\sim Y) \cap Z| = 2$.

?. Suppose $R = \{(1,3),(2,2),(2,4),(3,1),(3,3),(4,2),(4,4)\}$ is a relation on the set $S = \{1,2,3,4\}$. Then R is

(A) Reflexive (B) Symmetric (C) Antisymmetric (D) Transitive

Answer: $\boxed{\text{B}}$

The pairs (x,y) with $x \neq y$, each have a (y,x), namely $(1,3)$ and $(3,1)$, $(2,4)$ and $(4,2)$. This makes R symmetric. (A) fails since there is no $(1,1)$. (C) fails since $(1,3) \in R$ and $(3,1) \in R$. (D) fails since $(1,3) \in R$ and $(3,1) \in R$ but $(1,1) \notin R$.

?. Suppose $R = \{(1,1),(1,2),(2,1),(2,4),(3,3),(4,1),(4,2),(4,4)\}$ is a relation on the set $S = \{1,2,3,4\}$. Then R will be an equivalence relation when we add the two elements

(A) $(2,2)$ and $(1,4)$, (B) $(3,2)$ and $(1,4)$, (C) $(3,2)$ and $(2,2)$, (D) $(3,4)$ and $(2,2)$,

Answer: $\boxed{\text{A}}$

Just to satisfy the symmetry and reflexivity properties we need these pairs. All the other possibilities involve some $(3,x)$ with no $(x,3)$ so that symmetry will not hold.