MS121, Test 1(c), 11th. Oct. 2019

Name: _____	Student No.: _____

?. Let P, Q and R be propositions defined as follows:
P: Bus 1 is late. Q: Bus 2 is late. R: It is rush hour.
The compound proposition 'If it is rush hour then bus 1 is late or bus 2 is late.' can be expressed as

(A) [(**not** P) **or** (**not** Q)] $\Rightarrow$ **not** R , (B) [(**not** P) **or** (**not** Q)] $\Rightarrow$ R,
(C) [(**not** P) **and** (**not** Q)] $\Rightarrow$ **not** R, (D) [(**not** P) **and** (**not** Q)] $\Rightarrow$ R
Answer: C

The statement is of form R $\Rightarrow$ [P **or** Q] so is equivalent to **not** [P **or** Q] $\Rightarrow$ **not** R which in turn is equivalent to [(**not** P) **and** (**not** Q)] $\Rightarrow$ **not** R.

?. The negation of P $\Rightarrow$ **not** Q is equivalent to

(A) P **and** Q, (B) P **and** (**not** Q), (C) (**not** P) **and** Q, (D) (**not** P) **and** (**not** Q)
Answer: A

X $\Rightarrow$ Y is equivalent to (**not** X) **or** Y so the negation of X $\Rightarrow$ Y is X **and** (**not**Y). Here the negation of P $\Rightarrow$ (**not** Q) is P **and** Q.

?. The negation of the statement 'At least one test is not difficult.' is the following:

(A) All tests are not difficult. (B) All tests are difficult.
(C) Some tests are not difficult. (D) Some tests are difficult.
Answer: B

If t is a test and P(t) is the statement 'test t is difficult' then 'At least one test is not difficult.' is the statement $\exists t$, **not** P(t). Its negation is $\forall t$, P(t) or 'For all tests t, t is difficult'.

?. A sequence of numbers $x_1, x_2, \ldots, x_n, \ldots$ is defined inductively by
$x_1 = 1$ and $x_{k+1} = 2x_k + k$ for $k \geq 1$.
The numbers x_3 and x_4 take the following values respectively:

(A) 8 and 18, (B) 8 and 19, (C) 9 and 19, (D) 9 and 21.
Answer: B

$x_2 = (2)x_1 + 1 = (2)(1) + 1 = 3$, $x_3 = (2)x_2 + 2 = (2)(3) + 2 = 8$,
$x_4 = (2)x_3 + 3 = (2)(8) + 3 = 19$.