MS121, Test 1(a), 8th. Oct. 2019

Name: _____	Student No.: _____

?. Let P, Q and R be propositions defined as follows:
P: Bus 1 is late. Q: Bus 2 is late. R: I get to work.
The compound proposition 'If bus 1 is late or bus 2 is late, then I do not get to work.' can be expressed as

(A) R $\Rightarrow$ [(**not** P) **or** (**not** Q)] , (B) R $\Rightarrow$ [(**not** P) **or** Q],
(C) R $\Rightarrow$ [(**not** P) **and** (**not** Q)], (D) P $\Rightarrow$ [(**not** Q) **and** (**not** R)]
Answer: C

The compound statement is (P **or** Q) $\Rightarrow$ (**not**R). Using A $\Rightarrow$ B $\equiv$ (**not** B) $\Rightarrow$ (**not**A), this is equivalent to **not**(**not**R) $\Rightarrow$ **not** (P **or** Q) which in turn is equivalent to R $\Rightarrow$ [(**not** P) **and** (**not** Q)].

?. The negation of P $\Rightarrow$ Q is equivalent to

(A) P **and** Q, (B) P **and** (**not** Q), (C) (**not** P) **and** Q, (D) (**not** P) **and** (**not** Q)
Answer: B

We know P $\Rightarrow$ Q is equivalent to (**not**P) **or** Q so its negation is equivalent to **not** [(**not**P) **or** Q] or P **and** (**not**Q).

?. The negation of the statement 'All tests are not difficult.' is the following:

(A) All tests are not difficult. (B) All tests are difficult.
(C) Some tests are not difficult. (D) Some tests are difficult.
Answer: D

Let 't' be a test and P(t) the predicate 'Test t is difficult'. Then the original statement can be expressed as $\forall\ t,$ **not**(P(t)) and its negation is $\exists\ t,$ **not** (**not**(P(t))) or $\exists\ t, P(t)$. This is the statement 'At least one test is difficult'.

?. A sequence of numbers $x_1, x_2, \ldots, x_n, \ldots$ is defined inductively by
$x_1 = 5$ and $x_{k+1} = 2x_k - 3$ for $k \geq 1$.
The numbers x_4 and x_5 take the following values respectively:

(A) 19 and 34, (B) 18 and 35, (C) 19 and 35, (D) 18 and 33.
Answer: C

$x_2 = 2x_1 - 3 = 7$, $x_3 = 2x_2 - 3 = 11$, $x_4 = 2x_3 - 3 = 19$, $x_5 = 2x_4 - 3 = 35$.