MS121 Discrete Mathematics, Tutorial 1

1. Let P, Q and R be propositions defined as follows:
P: I am hungry.
Q: The fridge is empty.
R: It is Saturday.
Write each of the following propositions as logical expressions involving P, Q and R.
(a) I am hungry and the fridge is not empty.
(b) It is Saturday or I am hungry.
(c) If it is Saturday, then the fridge is empty.
(d) If I am not hungry, then the fridge is not empty.

2. Let P, Q and R be propositions defined in question 1. Write each of the following compound propositions as English sentences.
(a) (**not** P) **and** Q. (b) (**not** R) **or** (**not** P).
(c) (**not** Q)$\Rightarrow$(**not** P). (d) P $\Rightarrow$ R.

3. Use a truth table to show that (**not** P)$\Rightarrow$(**not** Q) and P **or** (**not** Q) are logically equivalent for any propositions P and Q.

4. A compound proposition which is always true is called a tautology. By constructing truth tables, decide which of the following are tautologies:
(a) P **or** (**not** P) (b) P **or** (**not** Q)
(c) [(P $\Rightarrow$ Q) **and** (Q $\Rightarrow$ R)]$\Rightarrow$(P $\Rightarrow$ R)

5. Let x stand for dog and P(x) be the predicate: x barks. Write each of the following propositions in symbolic form (using quantifiers $\forall$ and $\exists$):
a) All dogs bark. b) There is a dog which does not bark. c) No dog barks.
Express the negation of (b) in symbolic form and express the negation of (c) both in symbols and in English.

MS121 Discrete Mathematics, Tutorial 1 hints

1. Identify the simple propositions in each compound proposition and replace it by P, Q or R.

2. Replace P, Q or R by the corresponding simple propositions in each case. Replace $\Rightarrow$ by 'if ... then' and drop brackets.

3. Since there are two simple propositions you will need four rows in the table.

4. For (a) and (b) you will need four rows, for (c) you will need eight rows. The final column will just have T's if the compound proposition is a tautolgy.

5. You may need to rephrase to get close to the symbolic version. For example, the sentence 'All dogs bark.' is equivalent to the more awkward 'For every dog, that dog barks.'