

## MS121 Discrete Mathematics, Tutorial 8

1. Suppose  $f(x) = (-x + 3)/(2x + 1)$ . What is the natural domain of  $f$ ? What is the range of  $f$ ? Show that  $f(x)$  is bijective as a function from its natural domain to its range and compute the inverse function.
2. Let  $h = g \circ f \circ g$  where  $f : \mathbb{R} \rightarrow \mathbb{Z}$  is the floor function and  $g : \mathbb{R} \rightarrow \mathbb{R} : x \mapsto -x$ . Compute  $h(3.4)$ ,  $h(7)$ ,  $h(-1.3)$ . Describe what  $h$  is doing to a general real number  $x$ .
3. (a) Suppose  $f : A \rightarrow B$  and  $g : B \rightarrow C$  are both one-to-one. Show that the composition  $g \circ f : A \rightarrow C$  is also one-to-one.  
(b) Suppose  $f : A \rightarrow B$  and  $g : B \rightarrow C$  are both onto. Show that the composition  $g \circ f : A \rightarrow C$  is also onto.
4. Show that in any set of 4 integers at least two have the same remainder when divided by 3.
5. Show that in any set of 4 integers chosen from  $\{1, 3, 5, 7, 9, 11\}$ , at least one pair of numbers sums to 12.