MS121, Test 2a, 29th. Oct. 2019

Name: ____________________	Student No.: ____________________

?. Which of the following statements about two sets X and Y is not logically equivalent to the others?

(A) $X \subseteq (\sim Y)$ (B) $Y \subseteq (\sim X)$ (C) $(\sim X) \cap (\sim Y) = \emptyset$ (D) $X \cap Y = \emptyset$

Answer: $\boxed{\text{C}}$

The other three are logically equivalent. (D) says there are no elements in both X and Y which is equivalent to saying all of X's elements are not in Y (A) or all of Y's elements are not in X (B).

?. Suppose X, Y and Z are sets, $|X \cup Y \cup Z| = 15$, $|X| = 3$, $|Y| = 9$, $|Z| = 9$, $|X \cap Y| = 2$, $|X \cap Z| = 0$ and $|Y \cap Z| = 4$. How many elements belong to Y but do not belong to X or Z?

(A) 1, (B) 2, (C) 3, (D) 4

Answer: $\boxed{\text{C}}$

Since $|X \cap Z| = 0$, we know $|X \cap Y \cap Z| = 0$ and $|X \cap (\sim Y) \cap Z| = 0$. From the usual Venn diagram of 3 sets we get $|X \cap Y \cap (\sim Z)| = 2$ and $|(\sim X) \cap Y \cap Z| = 4$. From that we deduce $|(\sim X) \cap Y \cap (\sim Z)| = 3$.

?. Suppose $R = \{(1,1),(2,3),(3,3),(3,4),(4,4)\}$ is a relation on the set $S = \{1,2,3,4\}$. Then R is

(A) Reflexive (B) Symmetric (C) Antisymmetric (D) Transitive

Answer: $\boxed{\text{C}}$

There are only two pairs (x,y) with $x \neq y$, namely $(2,3)$ and $(3,4)$ and for each of these R does not contain (y,x). This makes R antisymmetric. (A) fails since there is no $(2,2)$. (B) fails since $(2,3) \in R$ but $(3,2) \notin R$. (D) fails since $(2,3) \in R$ and $(3,4) \in R$ but $(2,4) \notin R$.

?. Suppose $R = \{(1,1),(1,3),(1,4),(2,2),(3,3),(3,4),(4,1),(4,4)\}$ is a relation on the set $S = \{1,2,3,4\}$. Then R will be an equivalence relation when we add the two elements

(A) $(3,1)$ and $(2,3)$, (B) $(3,1)$ and $(4,3)$, (C) $(4,3)$ and $(2,4)$, (D) $(2,3)$ and $(2,4)$,

Answer: $\boxed{\text{B}}$

Just to satisfy the symmetry property we need these pairs. All the other possibilities involve some $(2,x)$ with no $(x,2)$ so that symmetry will not hold.