MS121 Discrete Mathematics, Tutorial 7

1. Suppose $A = \{a, b, c, d\}$ and $B = \{p, q, r\}$. Which of the following relations between A and B are functions? Explain.

$$R_1 = \{(a, q), (b, q), (c, q), (d, q)\}$$

$$R_2 = \{(a, q), (b, r), (c, q)\}$$

$$R_3 = \{(a, q), (b, q), (c, q), (d, q), (c, s)\}$$

$$R_4 = \{(a, q), (b, r), (c, p), (d, q)\}$$

2. Sketch the graphs of the following functions and for each determine
(a) its range, (b) is it injective? (c) is it surjective? Explain.

$$f : \mathbb{N} \to \mathbb{N} : n \mapsto n^2 + 1$$

$$g : \mathbb{R} \to \mathbb{R} : x \mapsto |x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

3. The floor function assigns to a real number x the largest integer less than or equal to x. This integer is denoted $\lfloor x \rfloor$.
(i) Let $A = \{-1, 0, 1, 2\}$ and $f : A \to \mathbb{Z}$ be given by $f(x) = \lfloor (x^2 + 1)/3 \rfloor$. Find the range of f.
(ii) Show that the function $g : \mathbb{Z} \to \mathbb{Z} : x \mapsto \lfloor x/5 \rfloor$ is onto but not one-to-one.

4. Let $A = \{a, b, c, d\}$, $B = P(A)$ be the power set of A and let $h : B \to \mathbb{Z}$ be given by $h(C) = |C|$ (cardinality). What is the range of h? Is h injective? Explain.