

MS121, Test 4a, 10th. Dec. 2019

Name: _____	Student No.: _____
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?. The smallest number of Irish people required in order to ensure that at least 4 come from the same province (Connacht, Leinster, Munster or Ulster) is

- (A) 4 (B) 12 , (C) 13 , (D) 16

Answer: C: This is the extended pigeonhole principle. Here A is a set of Irish people, B is the set of provinces, $f : A \rightarrow B$ takes a person to the province they are from. Since $|B| = 4$ at least one province will have more than $k = 3$ people if $|A| \geq k|B| + 1$.

?. The number of 3 element subsets of $\{1, 2, 3, 4, 5, 6, 7\}$ containing at least one even number is

- (A) 60 (B) 30 (C) 31 (D) 35

Answer: C: Use the subtraction rule. The total number of subsets minus the number with all odd numbers is $\binom{7}{3} - \binom{4}{3} = 35 - 4$.

?. 4 pieces of fruit are picked from a bowl containing 5 apples, 5 oranges and 5 pears. How many different distributions of fruit (e.g. $(2 \times \text{Ap})(1 \times \text{Or})(1 \times \text{Pe})$) are possible?

- (A) $\binom{15}{4}$ (B) $\binom{6}{3}$ (C) $\binom{6}{2}$ (D) $\binom{7}{2}$

Answer: C: The number is the number of 4-selections from 3. The answer is $\binom{4+3-1}{3-1}$. In terms of stars and bars the example $(2 \times \text{Ap})(1 \times \text{Or})(1 \times \text{Pe})$ would be written $**|*|*$ and a general distrubution is equivalent to a choice of 2 places for the bars in a string of length 6.

?. A fair (six-sided) die is tossed twice. The probability that the two numbers shown are different is

- (A) $1/2$ (B) $1/6$ (C) $5/36$ (D) $7/36$

Answer: ?: Easier to compute the probability that the two numbers shown are the same. This is $(1/36)(6)(1) = 1/6$. So the probability we want is $5/6$.