MS121, Test 1(b), 9th. Oct. 2019

Name: _____	Student No.: _____

?. Let P, Q and R be propositions defined as follows:
P: Bus 1 is late. Q: Bus 2 is late. R: I get to work.
The compound proposition 'If bus 1 is late and bus 2 is late, then I do not get to work.' can be expressed as

(A) R $\Rightarrow$ [(**not** P) **or** (**not** Q)] , (B) R $\Rightarrow$ [(**not** P) **or** Q],
(C) R $\Rightarrow$ [(**not** P) **and** (**not** Q)], (D) P $\Rightarrow$ [(**not** Q) **and** (**not** R)]
Answer: A

The compound statement is (P **and** Q) $\Rightarrow$ (**not**R). Using A $\Rightarrow$ B $\equiv$ (**not** B) $\Rightarrow$ (**not**A), this is equivalent to **not**(**not**R) $\Rightarrow$ **not** (P **and** Q) which in turn is equivalent to R $\Rightarrow$ [(**not** P) **or** (**not** Q)].

?. The negation of Q $\Rightarrow$ P is equivalent to

(A) P **and** Q, (B) P **and** (**not** Q), (C) (**not** P) **and** Q, (D) (**not** P) **and** (**not** Q)
Answer: C

We know Q $\Rightarrow$ P is equivalent to (**not**Q) **or** P so its negation is equivalent to **not** [(**not**Q) **or** P] or Q **and** (**not**P).

?. The negation of the statement 'All tests are difficult.' is the following:

(A) All tests are not difficult. (B) All tests are difficult.
(C) Some tests are not difficult. (D) Some tests are difficult.
Answer: C

Let 't' be a test and P(t) the predicate 'Test t is difficult'. Then the original statement can be expressed as $\forall\ t, \mathrm{P}(t)$ and its negation is $\exists\ t, \mathbf{not}(\mathrm{P}(t))$. This is the statement 'At least one test is not difficult'.

?. A sequence of numbers $x_1, x_2, \ldots, x_n, \ldots$ is defined inductively by $x_1 = 4$ and $x_{k+1} = 3x_k - 2$ for $k \geq 1$.
The numbers x_3 and x_4 take the following values respectively:

(A) 28 and 83, (B) 27 and 82, (C) 28 and 82, (D) 27 and 79.
Answer: C

$x_2 = 3x_1 - 2 = 10$, $x_3 = 3x_2 - 2 = 28$, $x_4 = 3x_3 - 2 = 82$.