MS121, Test 4c, 13th. Dec. 2019

Name: ______________________	Student No.: ______________________

?. The smallest number of playing cards required from the same deck of 52 in order to ensure that at least 4 come from the same suit ($\heartsuit$, $\spadesuit$, $\diamondsuit$, $\clubsuit$) is

(A) 13 (B) 12 , (C) 4 , (D) 16

Answer: $\boxed{\text{A}}$: This is the extended pigeonhole principle. Here A is a set of cards, B is the set of suits, $f : A \to B$ takes a card to the suit it is from. Since $|B| = 4$ at least one suit will have more than $k = 3$ cards if $|A| \geq k|B| + 1$.

?. The number of 3 element subsets of $\{1, 2, 3, 4, 5, 6, 7, 8\}$ containing at least one of $\{1, 2, 3\}$ is

(A) 63 (B) 64 (C) 45 (D) 46

Answer: $\boxed{\text{D}}$: Use the subtraction rule. The total number of subsets minus the number without 1, 2 or 3 is $\binom{8}{3} - \binom{5}{3} = 56 - 10.$

?. 8 identical presents are distributed in 4 numbered sacks such as $(1, 2, 3, 2)$. The number of different ways this can be done is

(A) $\binom{12}{3}$ (B) $\binom{11}{3}$ (C) $\binom{12}{4}$ (D) $\binom{11}{4}$

Answer: $\boxed{\text{B}}$: The number is the number of 8-selections from 4. The answer is $\binom{8+4-1}{4-1}$. In terms of stars and bars the example $(1, 2, 3, 2)$ would be written $*|**|***|**$ and a general distrubution is equivalent to a choice of 3 places for the bars in a string of length 11.

?. A four-sided tetrahedaral die is tossed three times. The probability that at least one number is repeated is

(A) 5/8 (B) 39/64 (C) 1/2 (D) 33/64

Answer: $\boxed{\text{A}}$: Easier to compute the probability that the three numbers shown are different. This is $(1/64)(4)(3)(2) = 3/8$. So the probability we want is $5/8$.