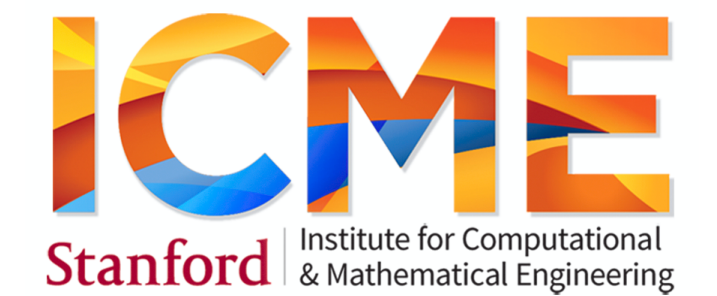




# A Formal Methods Approach Towards Deep Learning Interpretability

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## Summary

We tackle the problem of **node classification** where the goal is to classify nodes in a graph by leveraging nodes' features and graph structure. We focus on **semi-supervised** settings where only a subset of nodes is labelled and we aim at **transferring knowledge** across labelled and unlabelled nodes.

**Problem:** node classification on a new population, not connected to the labelled nodes

**Solution:** hallucinate edges between the labelled nodes and the unlabelled nodes, to reinforce the information flow

**Results:** our method achieves +3.6% and +3.4% gain in accuracy over standard baselines on cora and citeseer datasets.

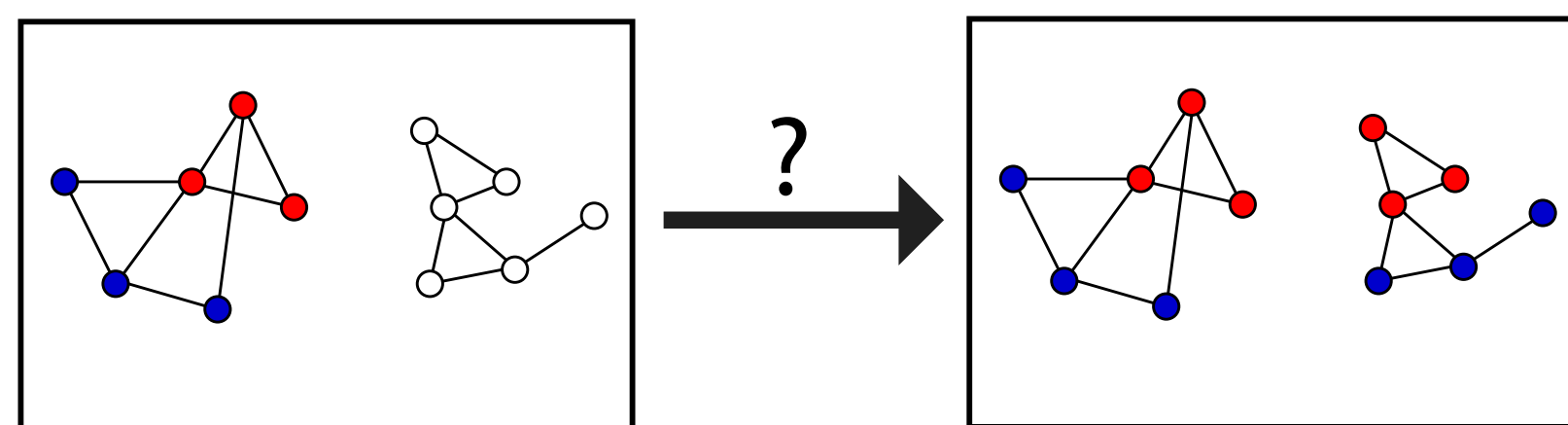
## Problem Setup

Input:

adjacency matrices  $A_L$  and  $A_U$  for the two sets of nodes;

features vectors  $X_L$  and  $X_U$ ;

labels  $y_L$

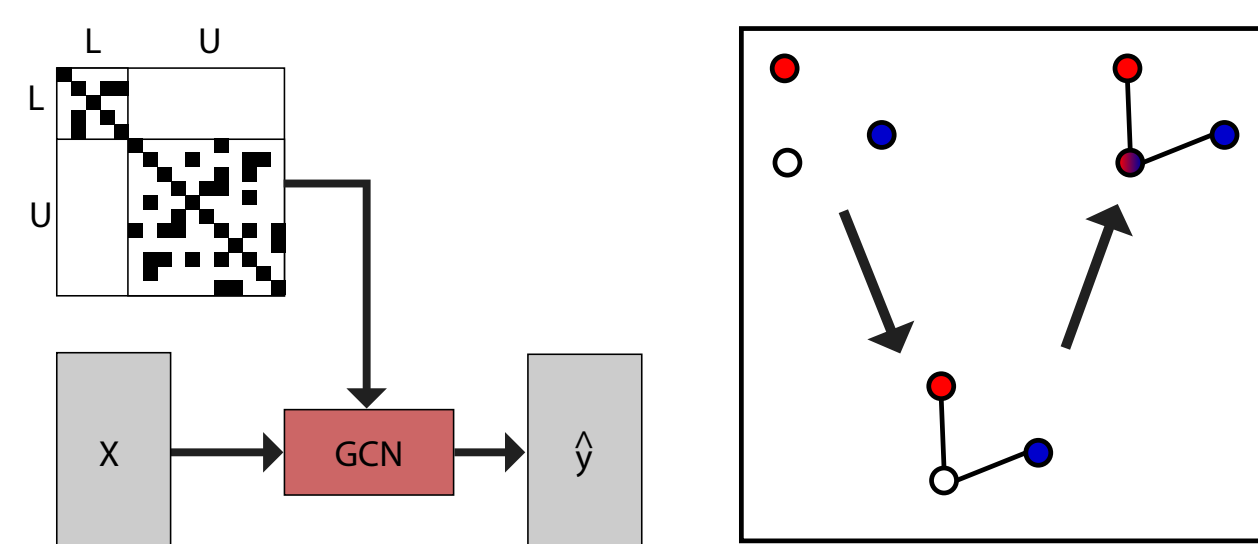


Output:

predictions for the unlabelled nodes  $\hat{y}_L$

## Baseline model

GCN [1] for node classification:  $\hat{y} = h(Ah(AXW_1)W_2)$

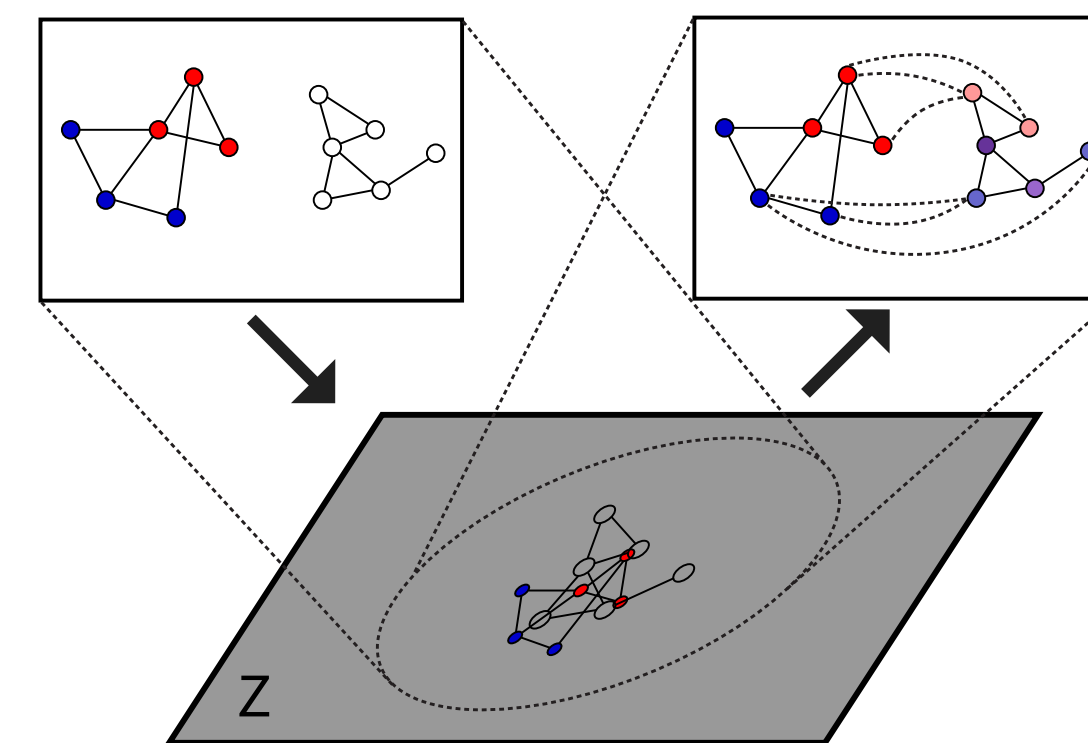


## Approach - Hallucigraph

learn low-dimensional node embeddings to encode node similarity (VGAE [2])

hallucinate edges and complete the adjacency matrix (edges)

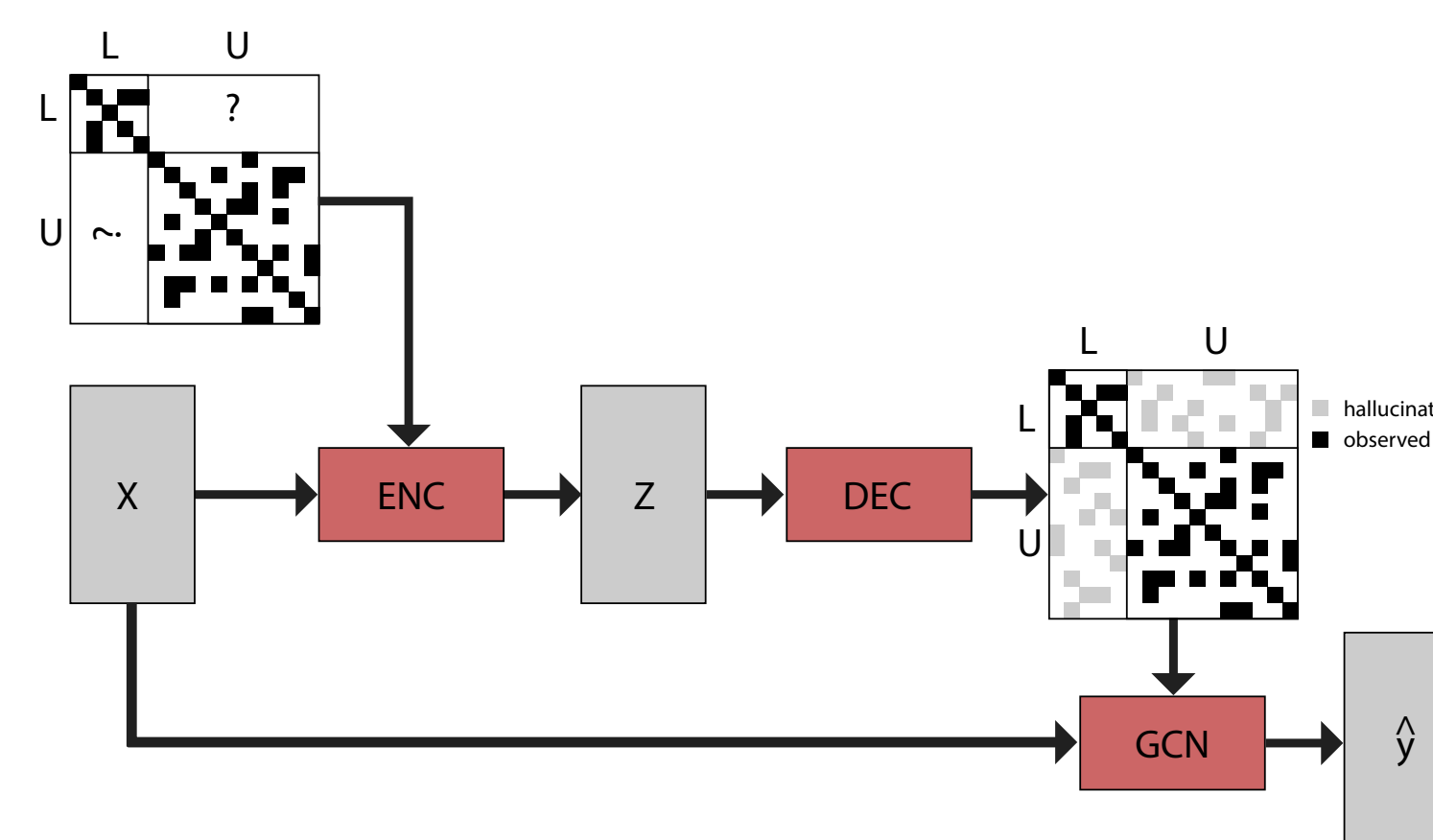
run a GCN with the completed adjacency to predict node labels



$$Z \sim \mathcal{N}(\mu_Z, \sigma_Z^2) \text{ and } \tilde{A} = \sigma(ZZ^T).$$

with  $\mu_Z, \sigma_Z = \text{GCN}(A, X)$

$$\mathcal{L}_{LP} = -\mathbb{E}_{Z \sim q(Z|A, X)} [A_{ij} \log \tilde{A}_{ij} + (1 - A_{ij}) \log (1 - \tilde{A}_{ij})] + \text{KL}(q(Z|A, X) || p(Z)).$$



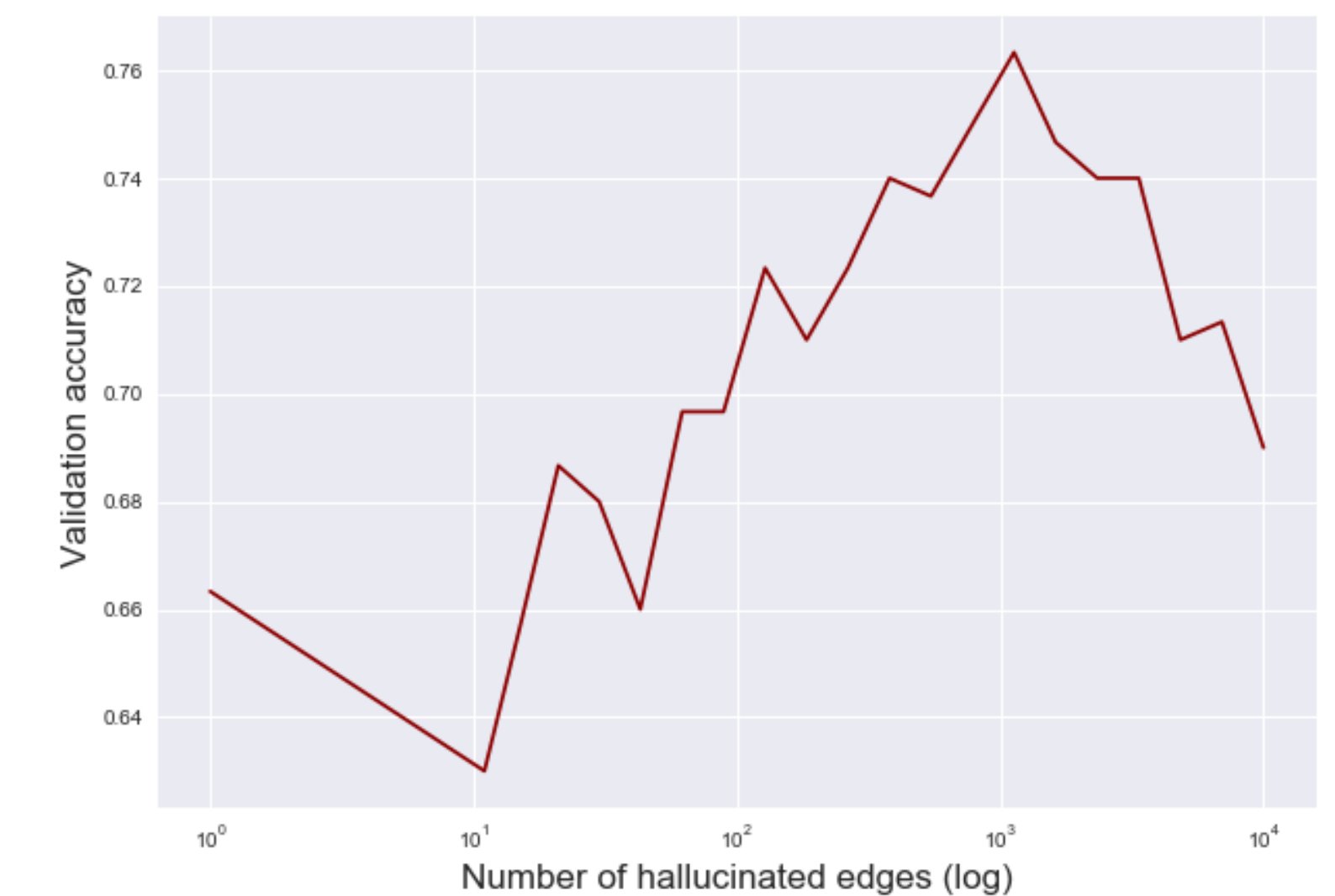
produces  $\hat{A}$ :

topK (K hyper-parameter)

sampling using gumbel softmax [4] trick (allows gradients to flow)

$$\hat{y} = \text{GCN}(\hat{A}, X)$$

## Results



Model	cora	citeseer
MLP	56.4%	56.8 %
GCN	71.3%	65.8%
Hallucigraph	<b>74.9%</b>	<b>69.2%</b>

Accuracy results for node classification task on three publication datasets where we removed all  $LU$  edges; using plain nodes features without edges (MLP); using the edges within labelled nodes and within unlabelled nodes (GCN); using Hallucigraph.

As shown in [1], the GCN improves on standalone MLP by leveraging the connections between nodes

By adding "hallucinated" edges, we improve the connectivity structure, and we obtain more predictive power

## References

- [1] Thomas N Kipf and Max Welling. Semi-supervised classification with graph convolutional networks. arXiv preprint arXiv:1609.02907, 2016.
- [2] Thomas N Kipf and Max Welling. Variational graph auto-encoders. arXiv preprint arXiv:1611.07308, 2016.
- [3] Aditya Grover, Aaron Zweig, and Stefano Ermon. Graphite: Iterative generative modeling of graphs. arXiv preprint arXiv:1803.10459, 2018
- [4] Eric Jang, Shixiang Gu, and Ben Poole. Categorical reparameterization with gumbel-softmax. arXiv preprint arXiv:1611.01144, 2016