The multiple shortest path problem with path deconfliction

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Overview

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2. **Mathematical formulation**
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Paper

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Discrete Optimization

The multiple shortest path problem with path deconfliction

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Problem statement – Common scenario

- Problem:
  - Routing multiple agents over a network from sources to terminus nodes
- Solution:
  - Identify agents' respective shortest paths
- Multiple Shortest Path Problem (**MSPP**)

Problem statement – Less common scenario (I)

- Problem:
  - Routing multiple agents over a network from sources to terminus nodes in a contested environment
- Solution (?):
  - Identify agents' respective shortest paths

Question

Is the previous solution still valid?

Problem statement – Less common scenario (II)

Answer

NO

- Respective agents' path *conflict*
- Adversary can observe agents by observing a limited number of location
- Agents' paths deconfliction *is* important

Problem statement – Less common scenario (III)

- Problem:
  - Routing multiple agents over a network from sources to terminus nodes in a contested environment
- Balanced solution:
  - Agents' respective shortest paths
  - Spatial deconfliction of agents' paths
- Multiple Shortest Path Problem with Path Deconfliction (**MSPP-PD**)

Problem statement – Remarks on MSPP-PD (I)

- Balanced solution:
  - Agents' respective shortest paths
  - Spatial deconfliction of agents' paths

Questions

1. When a path conflict arise?
2. How to quantitatively measure it?

Problem statement – Remarks on MSPP-PD (II)

Answers

1. Arise when more than one agent traverse an arc and/**or** node
2. Alternative penalty metrics:
   - Binary
   - Linear
   - Quadratic

1 MSPP-PD problem $\longleftrightarrow$ 6 variants

Problem statement – Remarks on MSPP-PD (III)

- Balanced solution:
  - Agents' respective shortest paths
  - Spatial deconfliction of agents' paths

Question

How to obtain a balanced solution?

Problem statement – Remarks on MSPP-PD (IV)

Answer

Multi-objective optimization

- 2 objectives:
  - Minimize total distance traveled by agents
  - Minimize degree respective agents' path conflict
- Linear combination

Mathematical formulation – Definitions for MSPP-PD (I)

Sets

$$
\begin{aligned}
K &: \text{Agents to be routed, indexed by } k\text{, where } \mathcal{K} = |K| \\
N &: \text{Nodes in the network, indexed by } i \text{ or } j\text{, where } \mathcal{N} = |N| \\
&\qquad S \subseteq N : \text{Source nodes. } s^k \text{ source of agent } k \\
&\qquad T \subseteq N : \text{Terminus nodes. } t^k \text{ terminus of agent } k \\
A &: \text{Directed arcs in the network, indexed by } (i,j).\ \mathcal{A} = |A| \\
G(N,A) &: \text{The directed network}
\end{aligned}
$$

Mathematical formulation – Definitions for MSPP-PD (II)

Parameters

d_{ij} : Non-negative length of arc (i, j)

Decision variables

$$x_{ij}^{k} : \begin{cases} 1 & \text{if agent } k \text{ traverses arc } (i, j) \\ 0 & \text{otherwise} \end{cases}$$

p : Penalty for conflicts between agent paths. Computed via $g(\mathbf{x})$, different for each penalty metric

Mathematical formulation – MSPP-PD Formulation (I)

Formulation

$$
\begin{aligned}
&\min_{\mathbf{x},p} \big(f(\mathbf{x}),\ p \big) \\
&\text{s.t.} \quad f(\mathbf{x}) = \sum_{(i,j)\in A} \sum_{k\in K} d_{ij}\, x_{ij}^k \\
&p = g(\mathbf{x}) \\
&\sum_{i:(i,j)\in A} x_{ij}^k - \sum_{j:(i,j)\in A} x_{ji}^k = \begin{cases} 1 & \text{if } i = s^k \\ -1 & \text{if } i = t^k \\ 0 & \text{otherwise} \end{cases} && \forall\, k \in K \\
&x_{ij}^k \in \{0, 1\} && \forall\, (i,j) \in A, k \in K
\end{aligned}
$$

Mathematical formulation – MSPP-PD Formulation (II)

Parameters

w_d weight of distance-related objective function

w_p weight of penalty-related objective function

Formulation (*linear combination*)

$$\min_{\mathbf{x},p}\big(f(\mathbf{x}),\ p\big) = \min_{\mathbf{x},p}\big(w_d\, f(\mathbf{x}) + w_p\, p\big)$$

$$\vdots$$

Mathematical formulation – MSPP-PD Formulation (III)

Examples (MSPP-PD(ABP) variant)

MSPP-PD formulation augmented with:

$$g_{total}^{arc-b}(\mathbf{x}) = \sum_{(i,j)\in A} \psi_{ij}$$

$$\frac{1}{\mathcal{K}}\left[\left(\sum_{k\in K} x_{ij}^{k}\right) - 1\right] \leq \psi_{ij} \qquad \forall\,(i,j)\in A$$

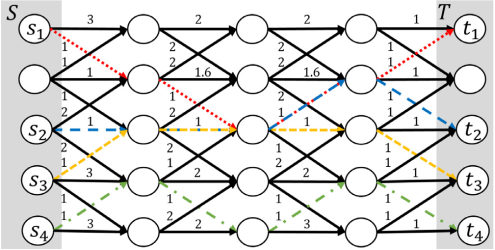
$$\psi_{ij} \in \{0,1\} \qquad \forall\,(i,j)\in A$$

Implementation results – Tools

- Programming language
  - Python 3.9
- Solver
  - Gurobi 10.0

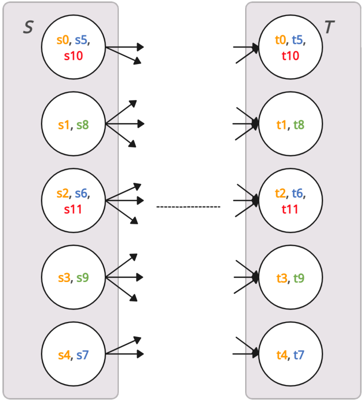
Implementation results – Network topology

- Provided by authors
- $m \times n$ grid network
- Directed acyclic graphs (DAGs)
- Weighted arcs (w_{ij})



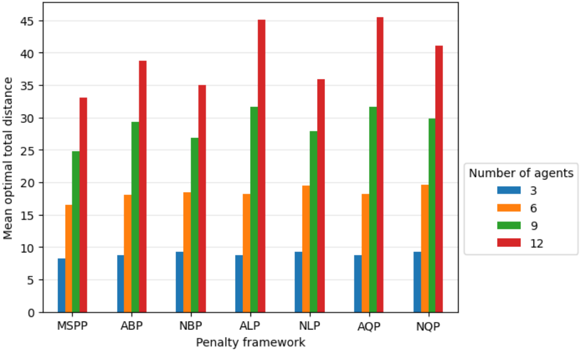
Implementation results – Agents’ sources and termini

- Ordinally sorted from top-to-bottom among the respective sets S and T
- If $\mathcal{K} > m$ added alternatively to the even and odd rows



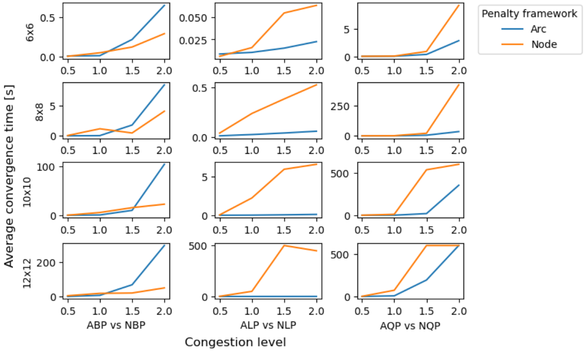
Implementation results – Effects of network congestion on optimal solutions

- congestion level $= \frac{\text{number of agents}}{\text{number of rows}}$
- 150 instances, $(m, n) = (6, 6)$, $w_{ij} = U[0, 2]$ and $(w_d, w_p) = (1, 1)$ for MSPP-PD
- Dynamic relationship
- Certain variants inadequately incentivize path deconfliction

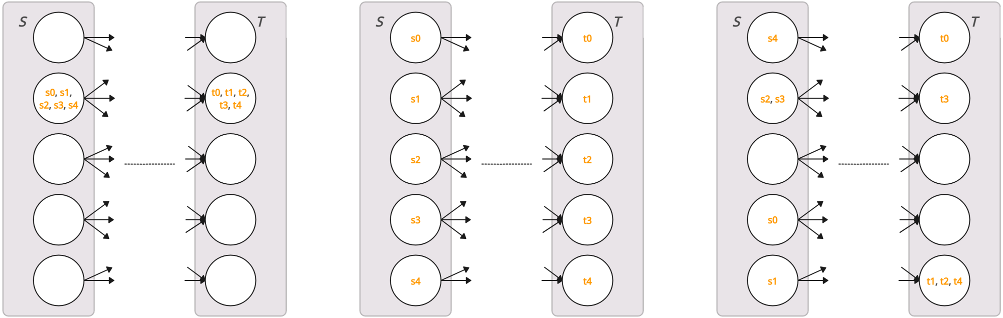


Implementation results – MSPP-PD variants practability

- 5 instances, $m = n$, $m = 6, 8, 10, 12$, $w_{ij} = U[0, 2]$ and Congestion level $=$ $0.5, 1, 1.5, 2$. $(w_d, w_p) = (1, 1)$
- Maximum run time of 10 min (600 s)
- Both network size and congestion level increase convergence time
- MSPP-PD(ALP) fastest model



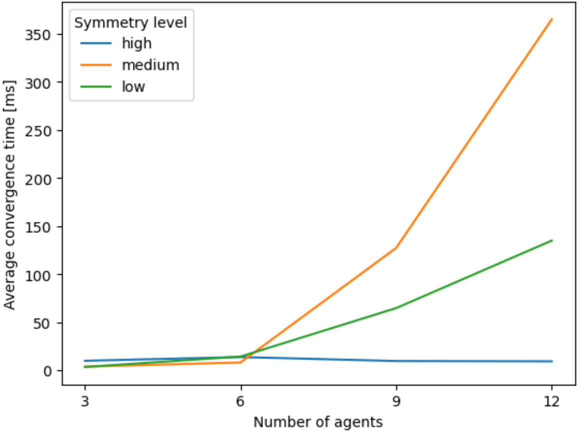
Implementation results – Impact of agents' symmetry (I)



(a) *High symmetry* (b) *Medium symmetry* (c) *Low symmetry*

Implementation results – Impact of agents' symmetry (II)

- 150 instances, $(m, n) = (6, 6)$, $w_{ij} = U[0, 2]$ and $(w_d, w_p) = (1, 1)$ for MSPP-PD(ABP)
- Sophisticated patterns cause convergence time increase faster



Conclusions

1. If in doubt: MSPP-PD(ALP)
2. Choice of the parameters is crucial

References

Michael S. Hughes, Brian J. Lunday, Jeffrey D. Weir, Kenneth M. Hopkinson (2021)
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Kevin Marzio (2023)
Source code
https://github.com/kkevin98/Multiple_shortest_path_problem_with_path_deconfliction

Thanks for the attention