The multiple shortest path problem with path deconfliction

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Overview

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Paper

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Discrete Optimization

The multiple shortest path problem with path deconfliction

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Problem statement – Common scenario

- Problem:
  - Routing multiple agents over a network from sources to terminus nodes
- Solution:
  - Identify agents' respective shortest paths
- Multiple Shortest Path Problem (**MSPP**)

Problem statement – Less common scenario (I)

- Problem:
  - Routing multiple agents over a network from sources to terminus nodes in a contested environment
- Solution (?):
  - Identify agents' respective shortest paths

Question

Is the previous solution still valid?

Problem statement – Less common scenario (II)

Answer

NO

- Respective agents' path *conflict*
- Adversary can observe agents by observing a limited number of location
- Agents' paths deconfliction *is* important

Problem statement – Less common scenario (III)

- Problem:
  - Routing multiple agents over a network from sources to terminus nodes in a contested environment
- Balanced solution:
  - Agents' respective shortest paths
  - Spatial deconfliction of agents' paths
- Multiple Shortest Path Problem with Path Deconfliction (**MSPP-PD**)

Problem statement – Remarks on MSPP-PD (I)

- Balanced solution:
  - Agents' respective shortest paths
  - Spatial deconfliction of agents' paths

Questions

1. When a path conflict arise?
2. How to quantitatively measure it?

Problem statement – Remarks on MSPP-PD (II)

Answers

1. Arise when more than one agent traverse an arc and/**or** node
2. Alternative penalty metrics:
   - Binary
   - Linear
   - Quadratic

1 MSPP-PD problem $\longleftrightarrow$ 6 variants

Problem statement – Remarks on MSPP-PD (III)

- Balanced solution:
  - Agents' respective shortest paths
  - Spatial deconfliction of agents' paths

Question

How to obtain a balanced solution?

Problem statement – Remarks on MSPP-PD (IV)

Answer

Multi-objective optimization

- 2 objectives:
  - Minimize total distance traveled by agents
  - Minimize degree respective agents' path conflict
- Linear combination

Mathematical formulation – Definitions for MSPP-PD (I)

Sets

$$
\begin{aligned}
K &: \text{Agents to be routed, indexed by } k\text{, where } \mathcal{K} = |K| \\
N &: \text{Nodes in the network, indexed by } i \text{ or } j\text{, where } \mathcal{N} = |N| \\
&\qquad S \subseteq N : \text{Source nodes. } s^k \text{ source of agent } k \\
&\qquad T \subseteq N : \text{Terminus nodes. } t^k \text{ terminus of agent } k \\
A &: \text{Directed arcs in the network, indexed by } (i,j)\text{. } A = |A| \\
G(N,A) &: \text{The directed network}
\end{aligned}
$$

Mathematical formulation – Definitions for MSPP-PD (II)

Parameters

d_{ij} : Non-negative length of arc (i, j)

Decision variables

$$x_{ij}^{k} : \begin{cases} 1 & \text{if agent } k \text{ traverse arc } (i, j) \\ 0 & \text{otherwise} \end{cases}$$

p : Penalty for conflicts between agent paths. Computed via $g(\mathbf{x})$, different for each penalty metric

Mathematical formulation – MSPP-PD Formulation (I)

Formulation

$$
\begin{aligned}
&\min_{\mathbf{x},p} \big(f(\mathbf{x}),\ p\big) \\
\text{s.t.}\quad & f(\mathbf{x}) = \sum_{(i,j)\in A} \sum_{k\in K} d_{ij}\, x_{ij}^k \\
& p = g(\mathbf{x}) \\
& \sum_{i:(i,j)\in A} x_{ij}^k - \sum_{j:(i,j)\in A} x_{ji}^k = \begin{cases} 1 & \text{if } i = s^k \\ -1 & \text{if } i = t^k \\ 0 & \text{otherwise} \end{cases} && \forall\, k \in K \\
& x_{ij}^k \in \{0, 1\} && \forall\, (i,j) \in A, k \in K
\end{aligned}
$$

Mathematical formulation – MSPP-PD Formulation (II)

Parameters

w_d weight of distance-related objective function

w_p weight of penalty-related objective function

Formulation (*linear combination*)

$$\min_{\mathbf{x},p} \big(f(\mathbf{x}),\ p \big) = \min_{\mathbf{x},p} \big(w_d\, f(\mathbf{x}) + w_p\, p \big)$$

$$\vdots$$

Mathematical formulation – MSPP-PD Formulation (III)

Examples (MSPP-PD(ABP) variant)

MSPP formulation augmented with:

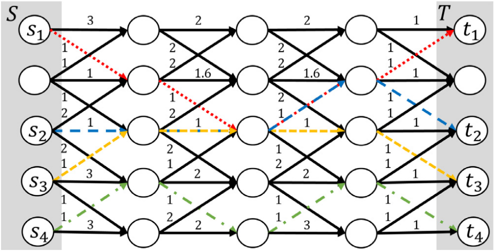
$$
\begin{aligned}
& g_{total}^{arc-b}(\mathbf{x}) = \sum_{(i,j)\in A} \psi_{ij} & \\
& \frac{1}{K}\left[\left(\sum_{k\in K} x_{ij}^{k}\right) - 1\right] \leq \psi_{ij} & \forall\,(i,j)\in A \\
& \psi_{ij} \in \{0,1\} & \forall\,(i,j)\in A
\end{aligned}
$$

Implementation results – Tools

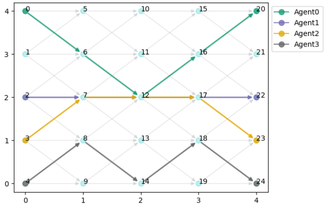
- Programming language
  - Python 3.9
- Solver
  - Gurobi 10.0

Implementation results – Network topology

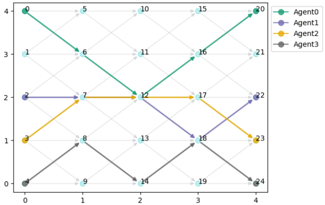
- 2D space tessellation
- $m \times n$ grid network
- Weighted arcs (w_{ij})
- Directed acyclic graphs (DAGs)
- Fine/coarse tessellation matters



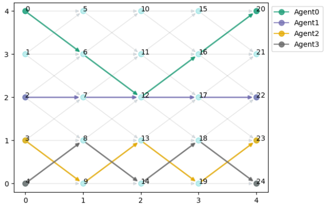
Implementation results – Comparison of optimal solutions



(a) *MSPP*, $f(\mathbf{x}^*) = 16$



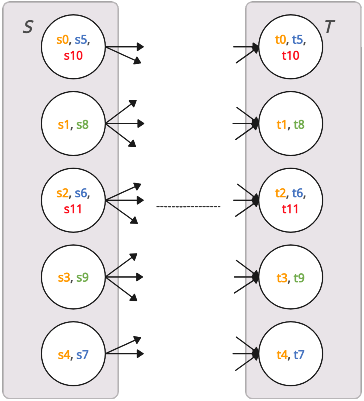
(b) *MSPP-PD(ABP)*, $(w_d, w_p) = (0.5, 0.5)$, $(f(\mathbf{x}^*), p^*) = (16, 1)$



(c) *MSPP-PD(NQP)*, $(w_d, w_p) = (0.5, 0.5)$, $(f(\mathbf{x}^*), p^*) = (17, 1)$

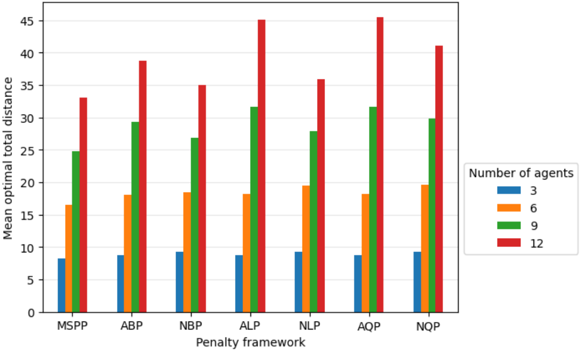
Implementation results – Agents' sources and termini

- Ordinally sorted from top-to-bottom among the respective sets S and T
- If $\mathcal{K} > m$ added alternatively to the even and odd rows



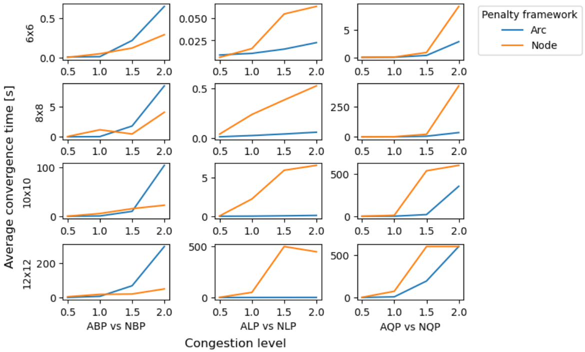
Implementation results – Effects of network congestion on optimal solutions

- 150 instances, $(m, n) = (6, 6)$, $w_{ij} = U[0, 2]$ and $(w_d, w_p) = (1, 1)$ for MSPP-PD
- congestion level $= \dfrac{\text{number of agents}}{\text{number of rows}}$
- Dynamic relationship
- Certain variants inadequately incentivize path deconfliction

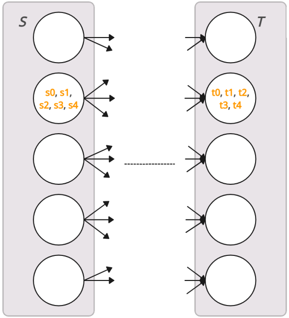


Implementation results – MSPP-PD variants practability

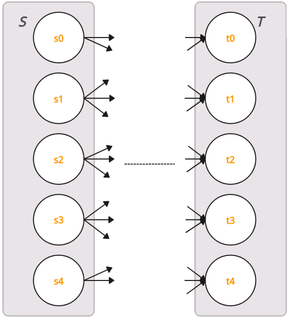
- 5 instances, $m = n$,
  $m = 6, 8, 10, 12$ and
  Congestion level $= 0.5, 1, 1.5, 2$
- Maximum run time of 10 min (600 s)
- Both network size and congestion level increase convergence time
- MSPP-PD(ALP) fastest model



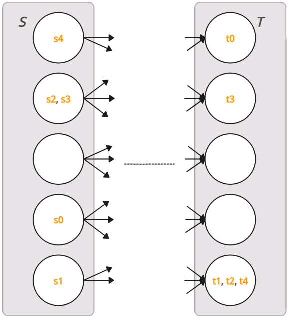
Extra – Impact of agents' symmetry (I)



(d) *High symmetry*



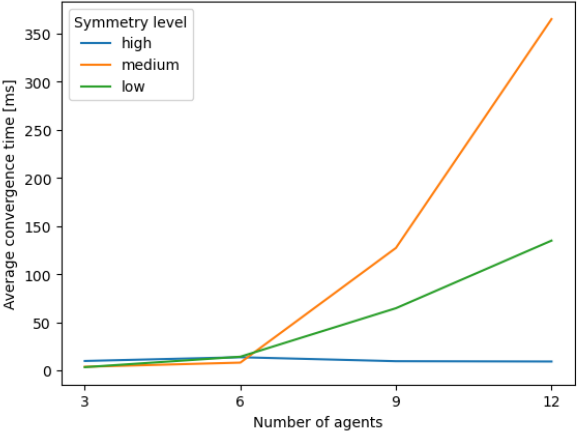
(e) *Medium symmetry*



(f) *Low symmetry*

Extra – Impact of agents' symmetry (II)

- 150 instances, $(m, n) = (6, 6)$, $w_{ij} = U[0, 2]$ and $(w_d, w_p) = (1, 1)$ for MSPP-PD(ABP)
- Sophisticated patterns cause convergence time increase faster



Extra – Real case application (I)

Problem statement
A company has a facility where humans and robots work together.
8 Robots have to move on predefined tracks marked on the floor, this tracks form a 10×10 grid-like system. Each grid's link has two direction of travel and if two robots follow opposite directions on the same link there is no risk of head-on collision. On the other hand if more robots follow the same direction on the same link, collisions may occur.
Humans worker perform most of their tasks at the center of the factory, whereas each robot has to reach a different grid's crossroad in order to perform its assignment. Robots' starting points depend on where they were left the day before.
The company wants to minimize at the same time the road traveled by robots to reach their end-points and the risk of both robot-robot and human-robot collision.

Extra – Real case application (II)

Definitions

K : Robots that has to be routed in the facility, $\mathcal{K} = 8$

N : Tracks' crossroads, $\mathcal{N} = 100$

S, T : Robots' starting and ending points. Selected randomly

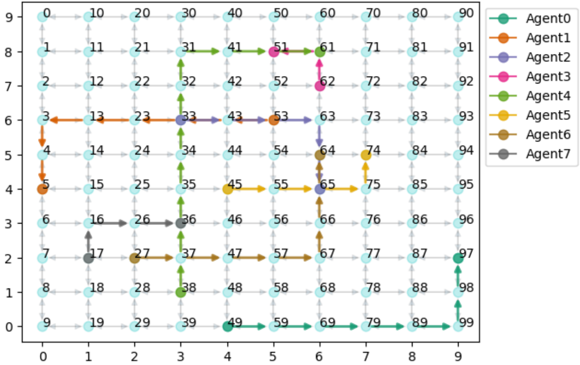
A : Tracks with their direction

d_{ij} : Level of human worker presence on a track's link. Values follow a Gaussian shape

$$x_{ij} : \begin{cases} 1 & \text{if robot } k \text{ traverse truck connecting crossroad } i \text{ to crossroad } j \\ 0 & \text{otherwise} \end{cases}$$

Extra – Real case application (III)

- MSPP-PD(ABP)
- $(w_d, w_p) = (1, 1)$
- $\left(f(\mathbf{x}^*), p^*\right) = (62.25, 0)$



Conclusions

1. If in doubt: MSPP-PD(ALP)
2. Choice of the parameters is crucial

References

Michael S. Hughes, Brian J. Lunday, Jeffrey D. Weir, Kenneth M. Hopkinson (2021)
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European Journal of Operational Research, vol. 292, no. 3, pp. 818–829,

Kevin Marzio (2023)
Source code
https://github.com/kkevin98/Multiple_shortest_path_problem_with_path_deconfliction

Thanks for the attention