Spring, 2021

COMP5241 Software Engineering and Development

Lecture 10: Research informed Teaching

Models for Decision Making and preference ranking in SDLC

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Outline

- Decision problems in SDLC
- Mathematical Modeling and System Engineering
- Preference Ordering
- Classical Rating Techniques
- Analytical Hierarchy Process
- Cognitive Network Process
- Applications

Prioritization and Selection Problems in System Development Life Cycle

- User Requirement phrase
  - Requirement prioritization and selection
  - Customer order prioritization and selection
- Analysis and Design phrase
  - Engineering design prioritization and selection
  - Technology selection
  - Vendor and outsourcing prioritization selection
  - Component selection
  - Buy or make decision
- Implementation phrase
  - Production plan selection
  - Production Resource selection, e.g. purchasing hardware
  - Maintenance Outsourcing selection

Selection and Prioritization Problem ins SDLC

- Test phrase
  - Test plan selection
- Maintenance
  - Maintenance plan selection
  - Maintenance Outsourcing selection

Mathematical modeling and System Engineering

Mathematical Modeling

"Mathematics is the queen of science"

Carl Friedrich Gauss (1777–1855), German
known as the "prince of mathematicians"

Mathematical Model

- Often when engineers analyze a system to be controlled or optimized, they use a mathematical model.
- In analysis, engineers can build a descriptive model of the system as a hypothesis of how the system could work, or try to estimate how an unforeseeable event could affect the system.
- Similarly, in control of a system, engineers can try out different control approaches in simulations.
- A mathematical model usually describes a system by a set of variables and a set of equations that establish relationships between the variables. The values of the variables can be practically anything; real or integer numbers, boolean values or strings, for example. The variables represent some properties of the system, for example, measured system outputs often in the form of signals, timing data, counters, and event occurrence (yes/no). The actual model is the set of functions that describe the relations between the different variables.

https://en.wikipedia.org/wiki/Mathematical_model

Mathematical Equation

Input/ independent variable

$$f : x \to y$$

function

Output/ dependent variable

or

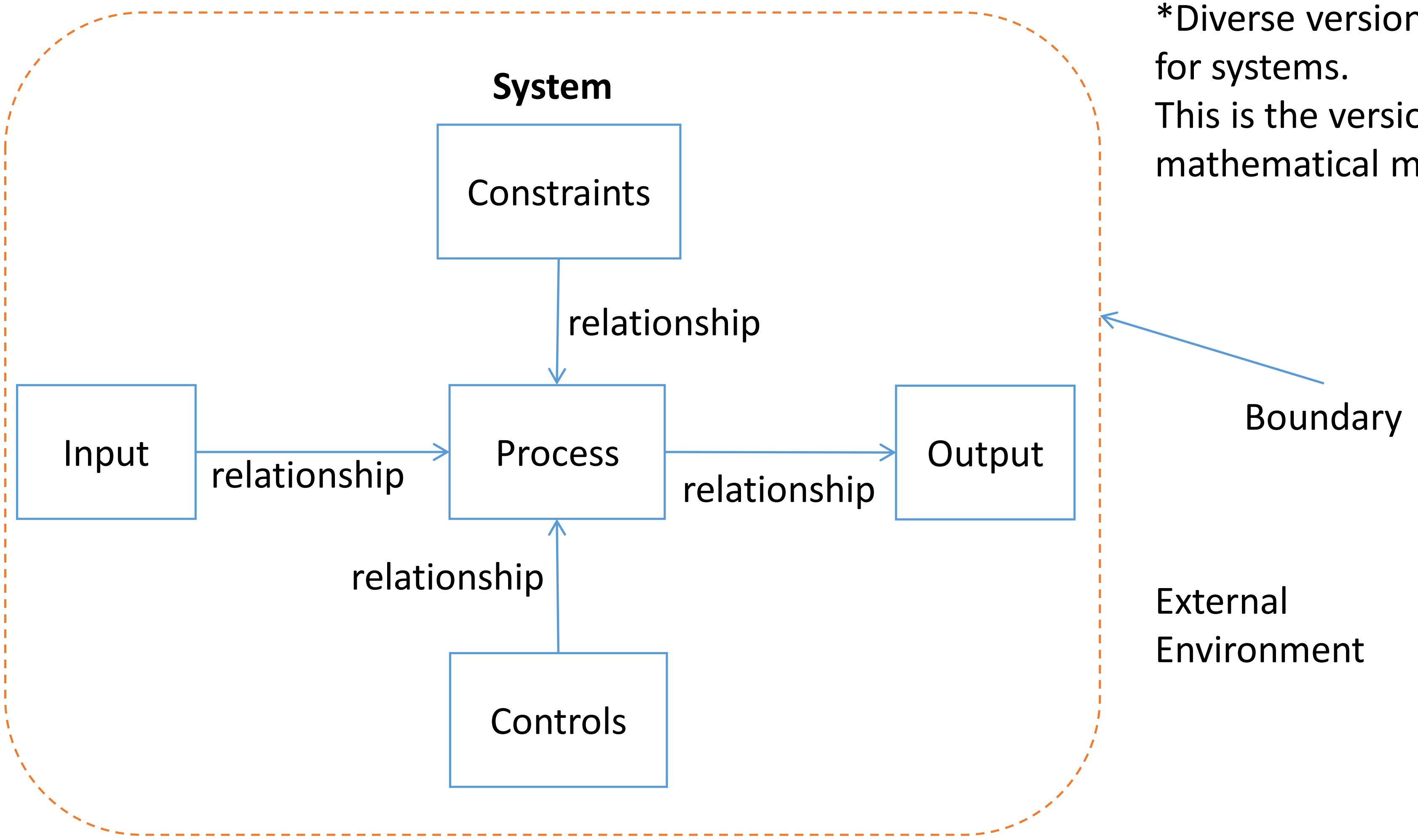
$$y = f(x)$$

Output variable

function name

Input variable

Common characteristics of System

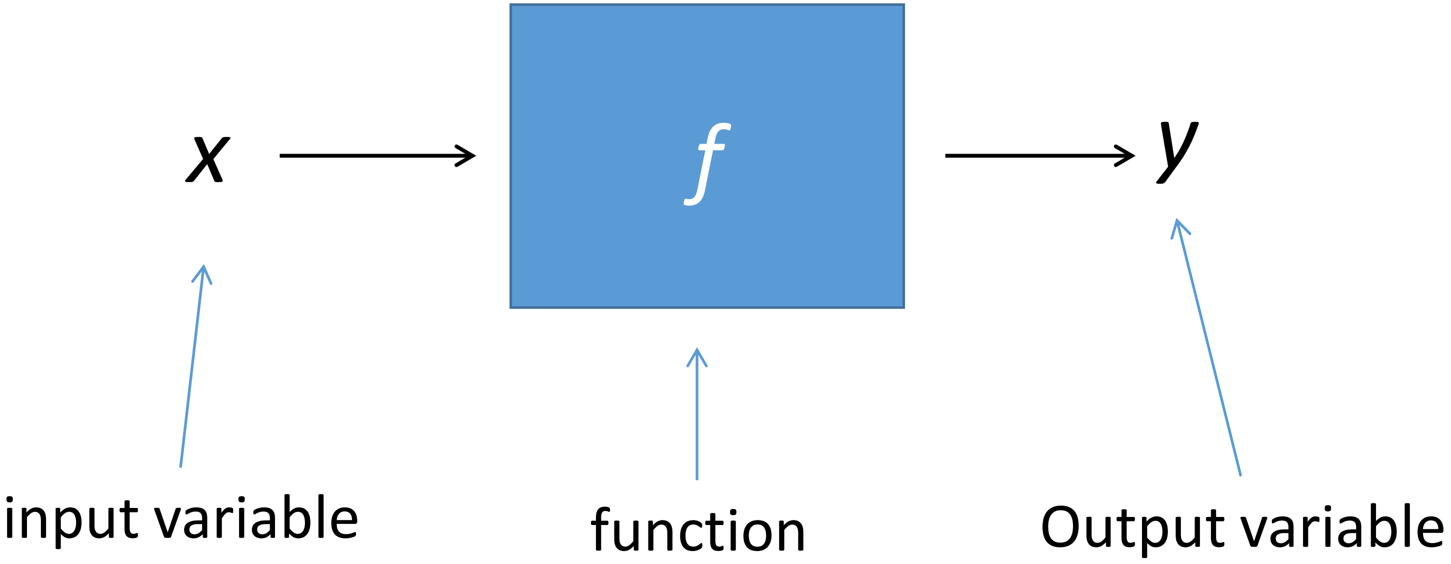


*Diverse versions of definitions for systems.
This is the version I defined for mathematical modeling.

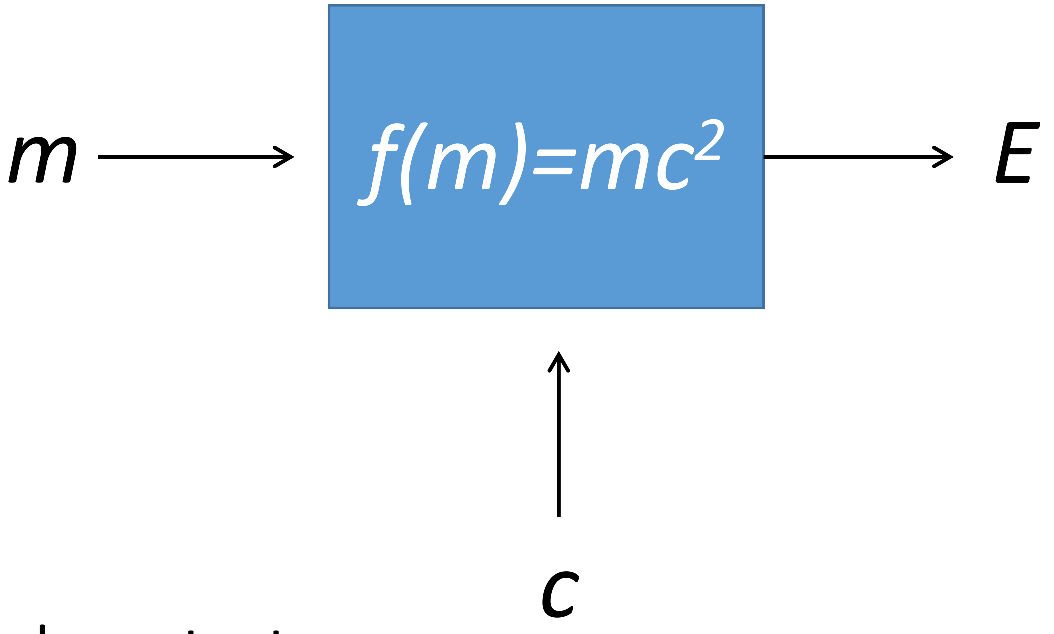
External Environment

Modeling simple mathematical system

$$f : x \to y$$

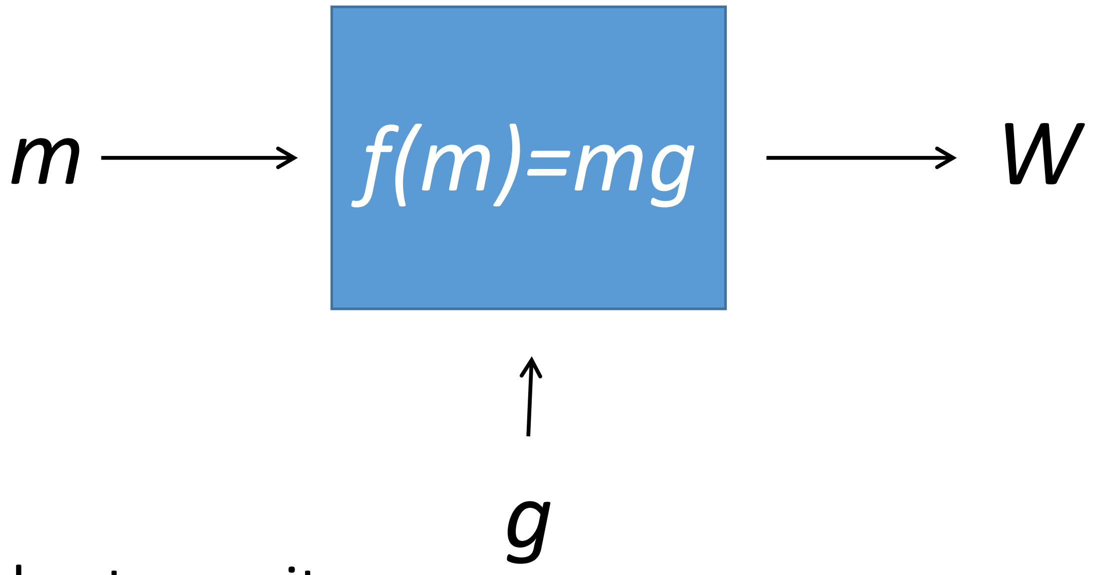


Example: Mass–energy equivalence



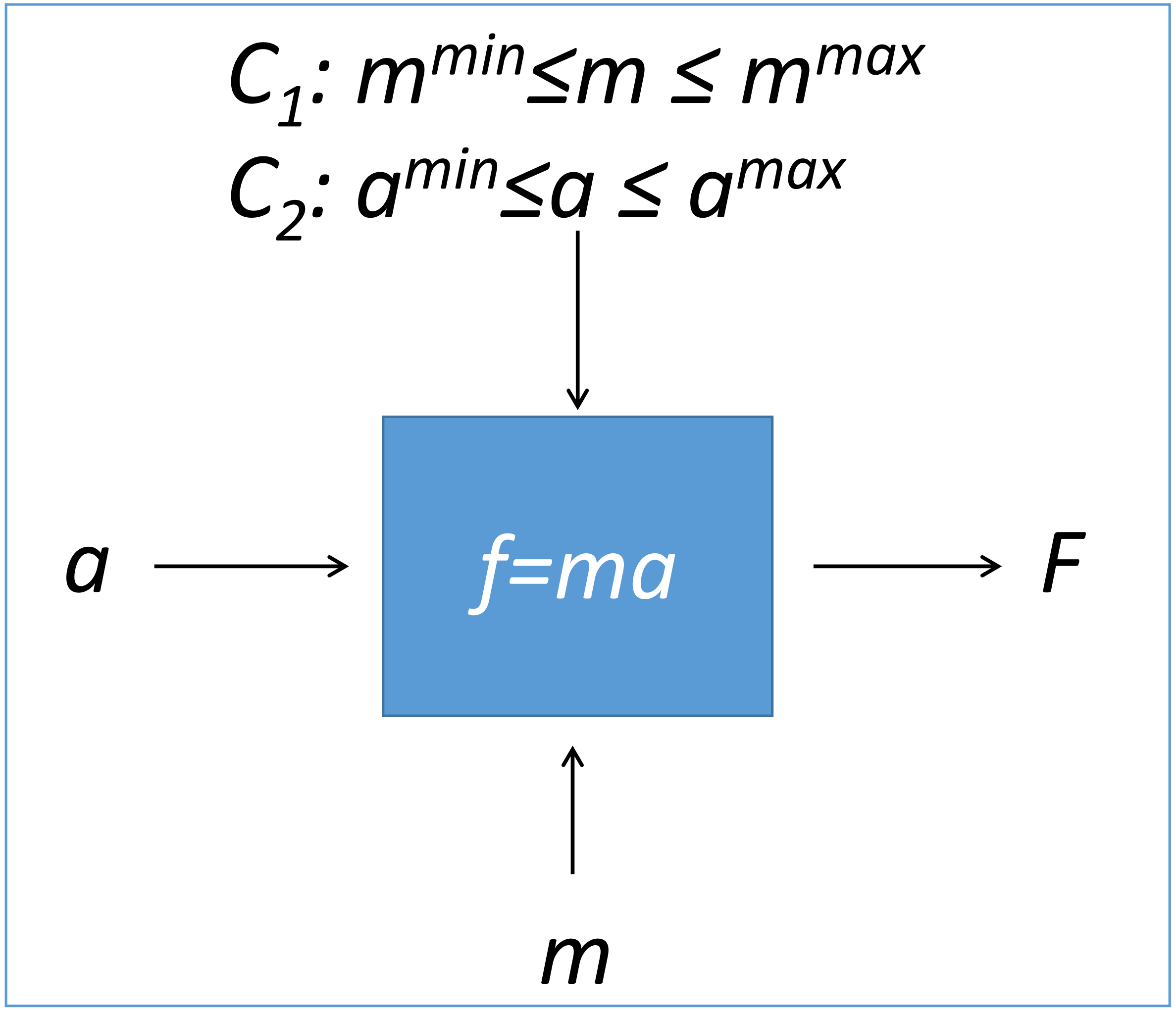
E: energy
c: light travel speed constant
m: mass
f: function

Example: *W=f(m)=mg,*



m: mass
g: acceleration constant due to gravity
W: weight
f: function

Example F=ma



Testing environment

m: mass
a: acceleration
F: force
f: function

m and *a* can be exchanged, depending on objectives

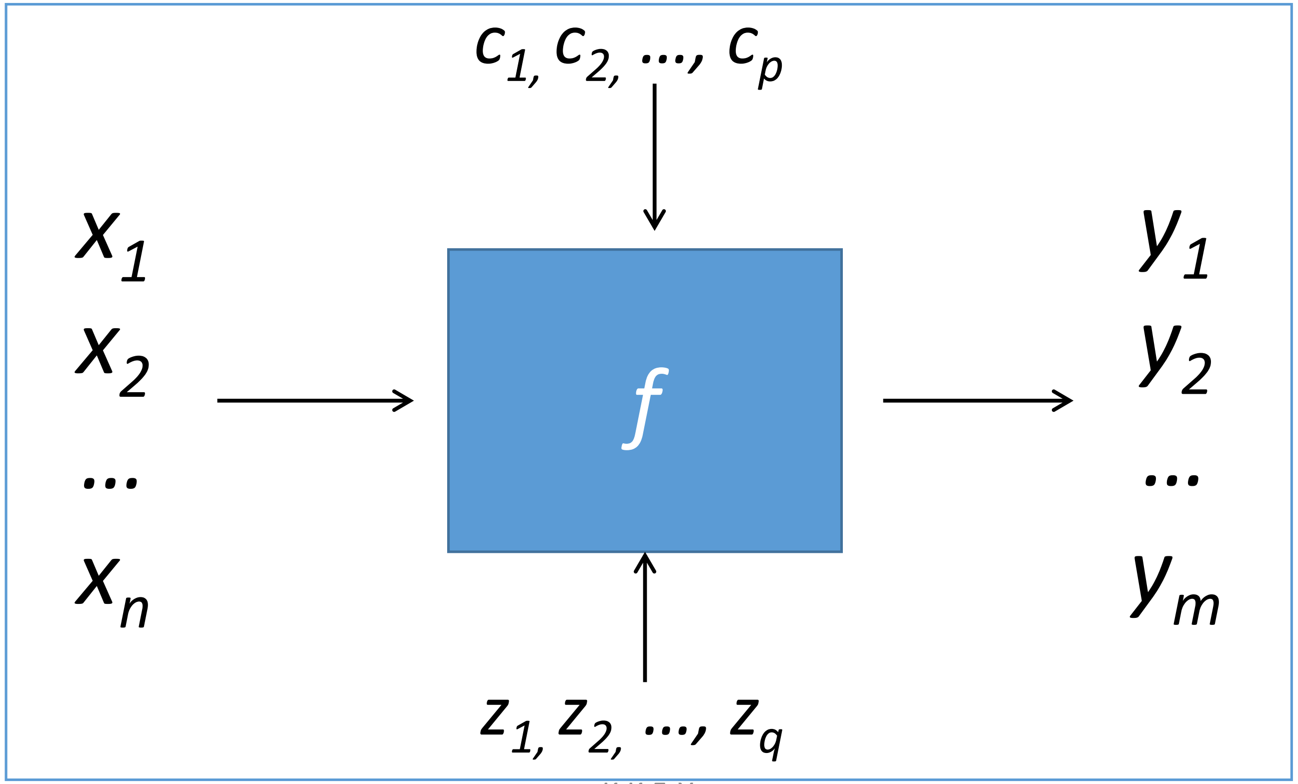
Multi-input and multi-output system

$$f : X \rightarrow Y$$

$$f : \{x_1, x_2, \ldots x_n\} \rightarrow \{y_1, y_2, \ldots y_m\}$$

where $X=\{x_{1,} x_{2,} \ldots, x_n\}$
$Y=\{y_{1,}\ y_{2,} \ldots, y_n\}$

Subject to $C=\{c_{1,}\ c_{2,} \ldots, c_n\}$



Decision Model

Criteria

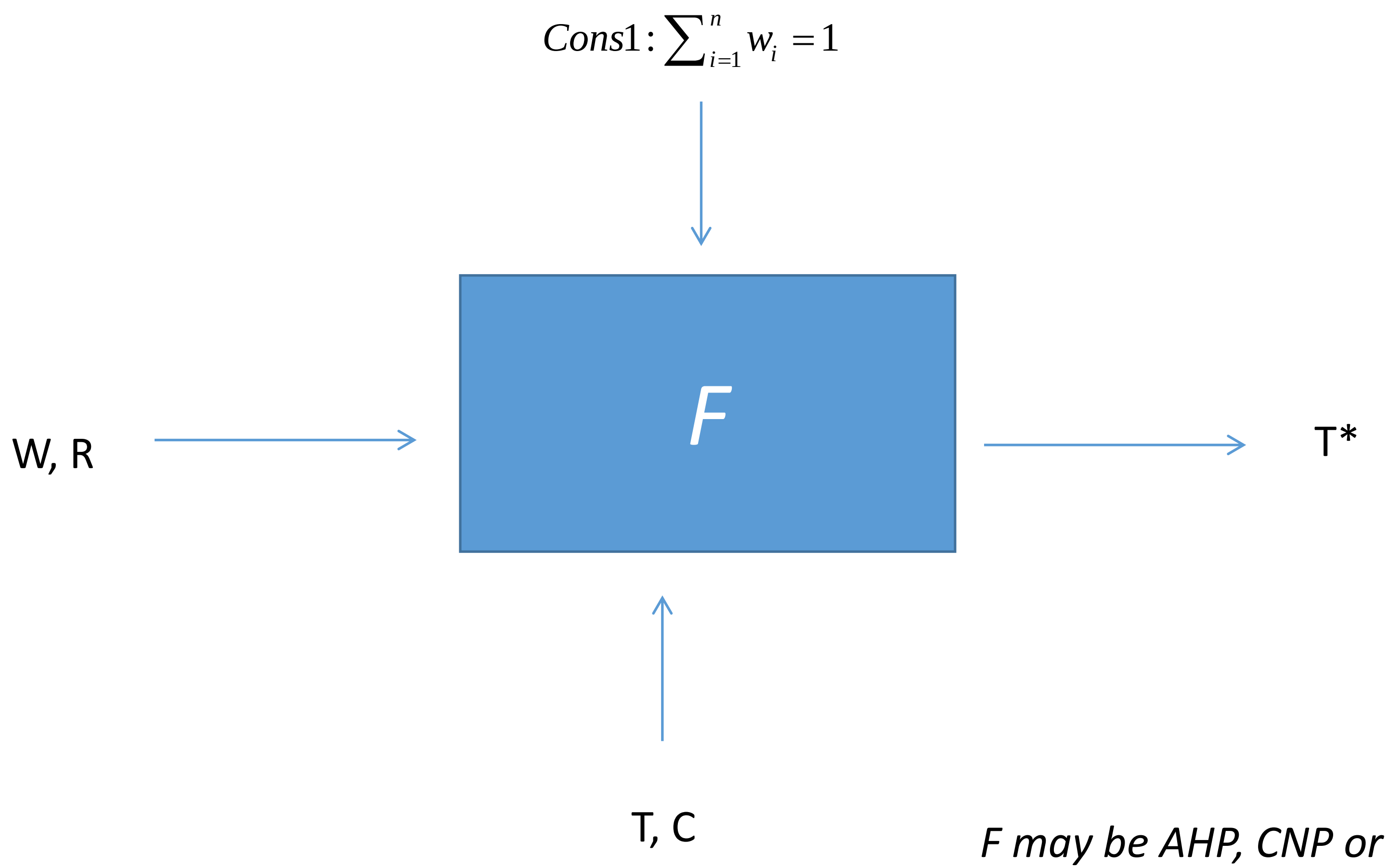
Weights

$$A=\begin{array}{cc} & \begin{matrix} c_1 & \cdots & c_j & \cdots & c_n \\ (w_1 & \cdots & w_j & \cdots & w_n) \end{matrix} \\ \begin{matrix} T_1 \\ \vdots \\ T_i \\ \vdots \\ T_m \end{matrix} & \begin{pmatrix} & & \\ & r_{ij} & \\ & & \end{pmatrix} \end{array}$$

Utility scores

Alternatives

Use system diagram to model decision matrix and its desired output such as the best alternative

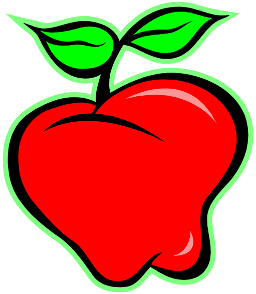


F may be AHP, CNP or others

Preference Ordering & Classical Rating Techniques

A simple reference order problem

- Rank the following fruit according to your preference.

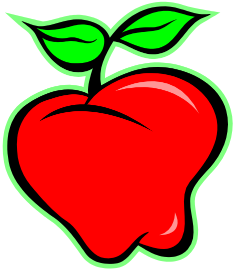


What methods could we use to rank these items to reflect rater's preference?

1.Assign an order score to each item

Give a number from 1 to 3 for the below items to show your preference.

The least value of number means highest preference.



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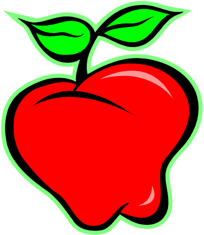
Example

Give a number from 1 to 3 for the below items to show your preference. The least value of number means highest preference.

Assume the results are as below.

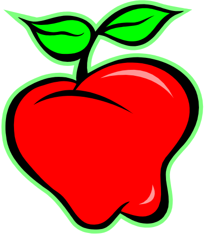


(2)

(1)

(3)

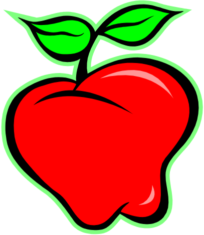
So, the preference is as below

 $\succcurlyeq$  $\succcurlyeq$ 

What are the problems for this method?

Think about this ranking problem?

Give a number from 1 to 8 for the below items to show your preference.
The least value of number means highest preference.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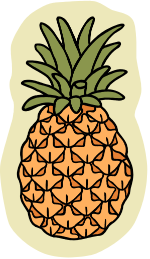
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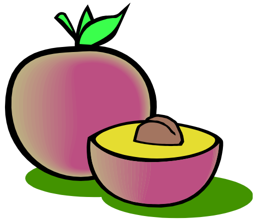
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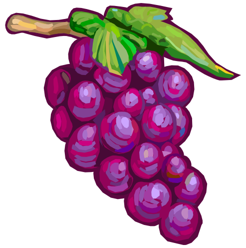
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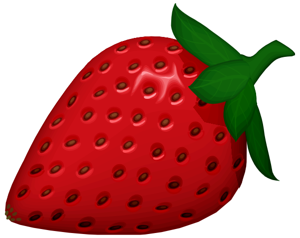
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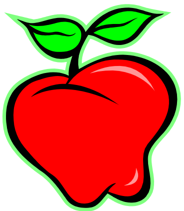
Limitations

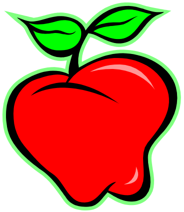
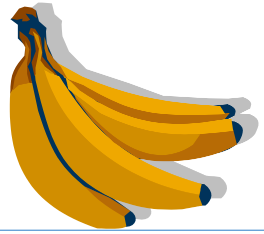
- Difficult to rank if the number of items is not small
- We do not know how strong one item over the other items.

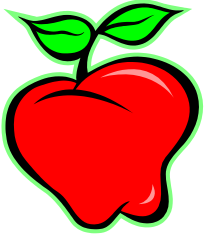
2. Choose a score in Likert Scale

Please circle the number to best express your preference about the fruit.

	Strongly Dislike				Strongly Like
	1	2	3	4	5
	1	2	3	4	5
	1	2	3	4	5

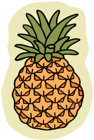
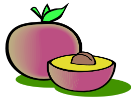
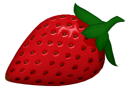
Suppose…

	Strongly Dislike		normal		Strongly Like
	1	2	3	4	5
	1	2	3	4	5
	1	2	3	4	5

 $\succcurlyeq$  $\succcurlyeq$ 

It seems that we can observe how much an item is over the other items.

How about this?

	Strongly Dislike		normal		Strongly Like
	1	2	3	4	5
	1	2	3	4	5
	1	2	3	4	5
	1	2	3	4	5
	1	2	3	4	5
	1	2	3	4	5
	1	2	3	4	5
	1	2	3	4	5

- Still not easy to evaluate the preference
- We have a number of items with the same preferences.

Limitations

- Weight determination is not reliable;
- Direct rating is too subjective as no reference object is used for comparison.
- Usually raters may randomly finish their ratings. It is very difficult to check the rating consistency.

Introduction of AHP

Analytical Hierarchy Process (AHP)

3. The Analytical Hierarchy Process Approach

- Found by L.A Saaty, 1980
- Used to prioritize alternatives by pairwise comparisons
- Very controversial!

Applications of AHP

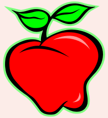
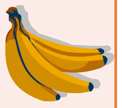
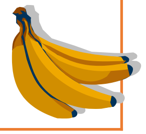
- Rock sciences [1], organization capital measurement [3], quality management [6], work system [12], supplier selection [13,44], technology management [14], product design [17], medical waste management [18], transportation management [21], job evaluation [22], energy technology [23], brand decision [24], mining method[25], information system[26,32], capital budgeting problem[29], portfolio selection[31], hospital site selection [33].

(Yuen, 2014, IEEE Fuzzy)

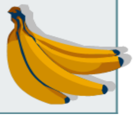
3. The Analytical Hierarchy Process Approach

Compare the relative preference for each pair of items as below,

and circle the score accordingly

Item	Extremely	Very, very Strongly	Very Strongly	Strong Plus	Strongly	Moderately Plus	Moderately	Weakly	Equally	Weakly	Moderately	Moderately	plus Strongly	Strong Plus	Very Strongly	Very very Strongly	Extremely	Item
	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	
	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	
	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	

How to calculate the results by AHP

w_1		9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9		w_2
w_1		9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9		w_3
w_2		9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9		w_3

 : w_1

 : w_2

 : w_3

$$\begin{cases} \dfrac{w_1}{w_2} = 2 \\ \dfrac{w_1}{w_3} = 4 \\ \dfrac{w_2}{w_3} = 2 \\ w_1 + w_2 + w_3 = 1 \end{cases} \Rightarrow \begin{cases} w_1 - 2w_2 = 0 \\ w_1 - 4w_3 = 0 \\ w_2 - 2w_3 = 0 \\ w_1 + w_2 + w_3 - 1 = 0 \end{cases} \Rightarrow \begin{cases} w_1 = \dfrac{4}{7} \\ w_2 = \dfrac{2}{7} \\ w_3 = \dfrac{1}{7} \end{cases}$$

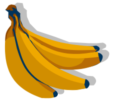
$$W = \left(\frac{4}{7}, \frac{2}{7}, \frac{1}{7} \right)$$

However, what if.....

	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	
	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	
	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	

 : w_1

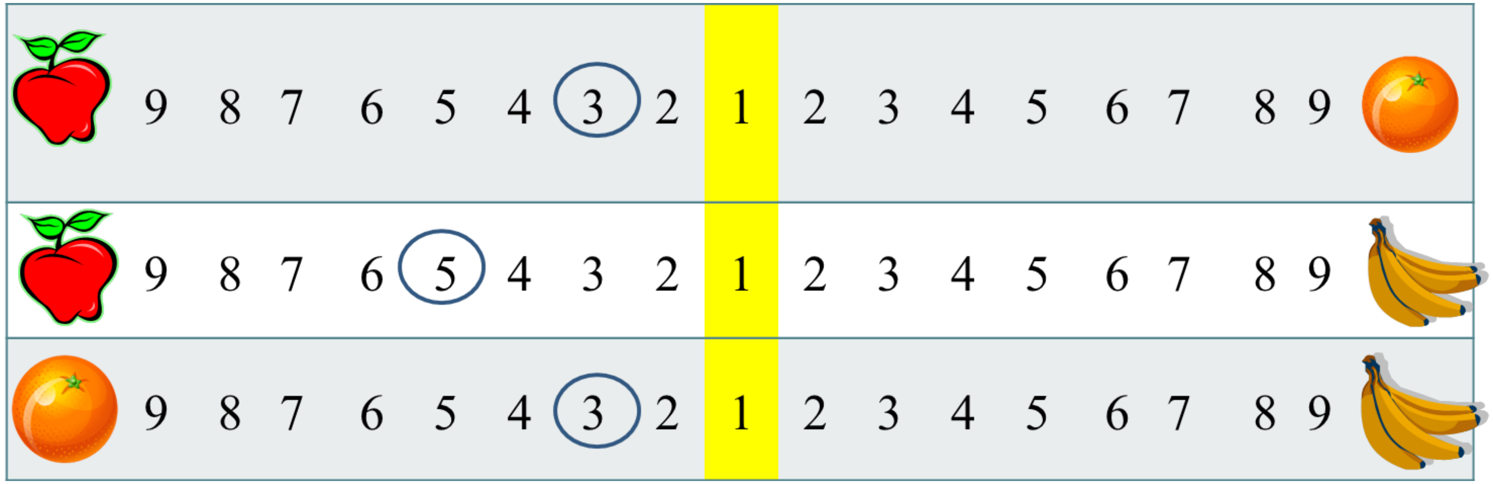
 : w_2

 : w_3

$$\begin{cases} \dfrac{w_1}{w_2} = 3 \\ \dfrac{w_1}{w_3} = 5 \\ \dfrac{w_2}{w_3} = 3 \\ w_1 + w_2 + w_3 = 1 \end{cases} \Rightarrow \begin{cases} w_1 - 3w_2 = 0 \\ w_1 - 5w_3 = 0 \\ w_2 - 3w_3 = 0 \\ w_1 + w_2 + w_3 = 1 \end{cases} \Rightarrow$$

No solution

Consistency and Inconsistency



Ratings should be **numerically consistent**

– That means that

If *A* is 3 times of *B*

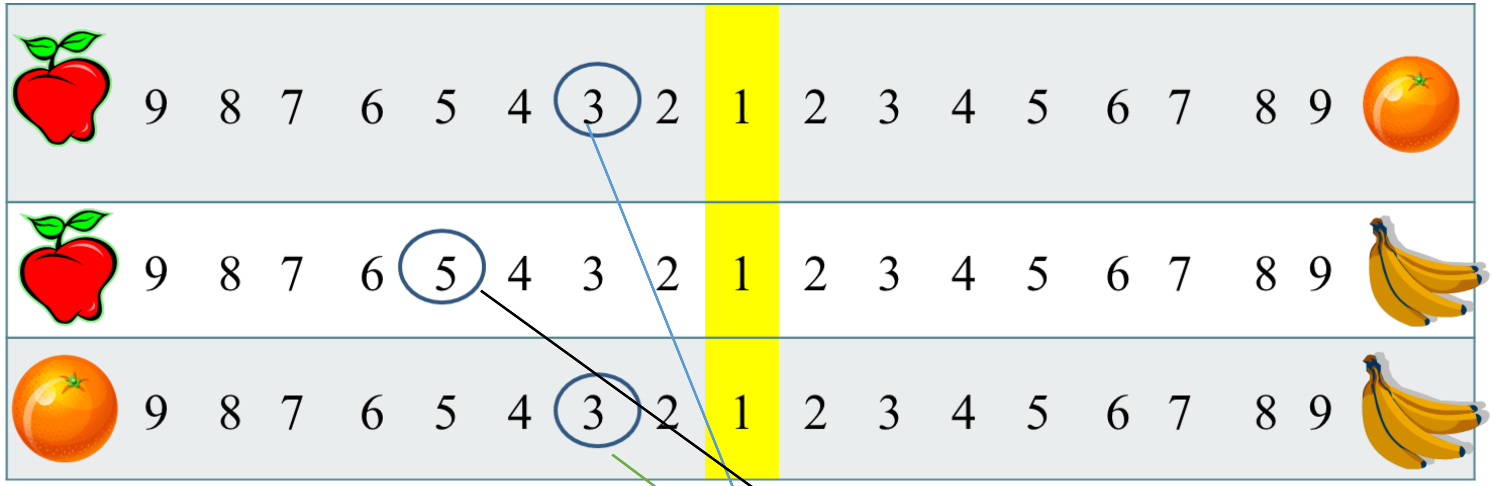
and *B* is 3 times of *C*

then *A* must be 9 times of *C*

A=3B
B=3C
A=9C

AHP still accepts some errors induced by rating

Weights for Inconsistent Matrices



	1	**3**	**5**
	1/3	1	3
	1/5	1/3	1

1. There are several methods to compute Inconsistent matrices.
2. Typically, Eigen system method, the original one proposed by Saaty.

Eigenvalue/Eigenvector Method

– Defined as follows:

$$\mathbf{Ax} = \lambda\mathbf{x}$$

A: square matrix *A*

λ: Eigenvalue of A

x is the nonzero eigenvector associated with λ

For example, A is

1	**3**	**5**
1/3	1	3
1/5	1/3	1

Usually we need to use computer program to compute λ and **x** .

Eigen system method for Inconsistent Matrix

Eigensystem[A] = **Ax = λx**

List of Eigen values | List of Eigen vectors

$$\left(\begin{array}{c} 3.03851 \\ -00192555+0341534i \\ -00192555-0341534i \end{array}\right), \left(\begin{array}{ccc} 6.0822 & 2.46621 & 1. \\ -30411+526734i & -123311-21358i & 1. \\ -30411-526734i & -123311+21358i & 1. \end{array}\right)$$

3.039 is the principle Eigen value; {6.082, 2.466, 1} is its corresponding Eigen vector.

$$\text{Consistency Index} = C.I. = \frac{\lambda^{\max} - n}{n-1} = \frac{3.039 - 3}{3-1} = 0.0192$$

n	1	2	3	4	5	6	7	8
Random Index (R.I)	0	0	0.52	0.89	1.11	1.25	1.35	1.4

$$\text{Consistency Ratio} = \frac{C.I.}{R.I.} = \frac{0.0192}{0.52} = 0.037$$

If CR < 0.1, the survey is acceptable.
Otherwise, the survey needs to be revised again.

Eigen system method for Inconsistent Matrix

$$\mathbf{Ax = \lambda x}$$

Eigensystem[A] =

List of Eigen values, List of Eigen vectors

$$\begin{pmatrix} 3.03851 \\ -00192555+0341534i \\ -00192555-0341534i \end{pmatrix}, \begin{pmatrix} 6.0822 & 2.46621 & 1. \\ -30411+526734i & -123311-21358i & 1. \\ -30411-526734i & -123311+21358i & 1. \end{pmatrix}$$

- 3.039 is the Principle Eigen value;
- {6.082, 2.466, 1} is its corresponding Eigen vector.

Normalization of the corresponding Eigen vector , priority vector is

$$W=\left\{\frac{6.082}{9.548},\frac{2.466}{9.548},\frac{1}{9.548}\right\}=\{0.637, 0.258, 0.105\}$$

where $6.082+2.466+1=9.548$

The result is similar to geometric mean

More methods to solve this problem

Geometric Mean

$$w_i' = \prod_{j=1}^{n} a_{ij}^{1/n} \quad i = 1, 2, \ldots, n$$

$$w_i = \frac{w_i'}{\sum_{i=1}^{n} w_i'} \quad i = 1, 2, \ldots, n$$

Procedure:

1. Compute geometric mean of each row
2. Normalize each column

- Or vise versa

Limitation: it cannot use consistency measure directly (e.g. no principle of Eigen value)

Yuen K.K.F. (2010), "Analytic Hierarchy Prioritization Process in the AHP Applications Development: A Prioritization Operator Selection Approach". ***Applied Soft Computing***, 10, pp.975-989.

Geometric means

	F	Q	P
F	1	3	5
Q	1/3	1	3
P	1/5	1/3	1

$\xrightarrow{\text{Geometric means}}$

W'
2.466
1.000
0.405

$\xrightarrow{\text{Normalized}}$

W
0.637
0.258
0.105

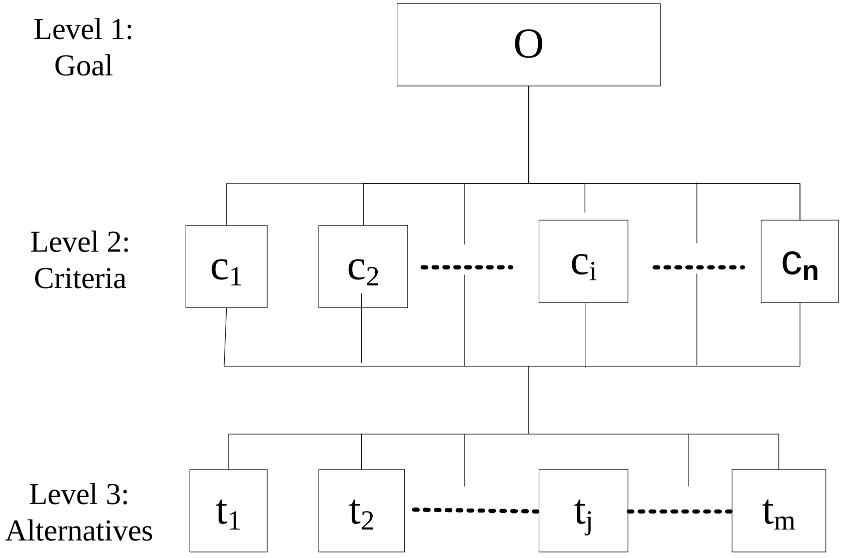
$$w'_1 = \sqrt[3]{1 \cdot 3 \cdot 5} = 2.466$$

$$w'_2 = \sqrt[3]{\frac{1}{3} \cdot 1 \cdot 3} = 1.000$$

$$w'_3 = \sqrt[3]{\frac{1}{5} \cdot \frac{1}{3} \cdot 3} = 0.405$$

Analytic Hierarchy Process

Step 1) Define Hierarchy Model



Step 2)
Evaluate in pairwise matrix by question survey forms

$$A = \{a_{ij}\} \qquad a_{ij} = {w_i}/{w_j} \qquad 0 < a_{ij} = a_{ji}^{-1}$$

$$\tilde{A} = \begin{bmatrix} \frac{w_1}{w_1} & \frac{w_1}{w_2} & \cdots & \frac{w_1}{w_n} \\ \frac{w_2}{w_1} & \frac{w_2}{w_2} & \cdots & \frac{w_2}{w_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{w_n}{w_1} & \frac{w_n}{w_2} & \cdots & \frac{w_n}{w_n} \end{bmatrix} \cong \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} == A$$

$$CR = \frac{CI}{RI}$$

3) Prioritize with the matrices

$$PO : A \to W$$

$$W = \{w_1, \ldots, w_n\} \quad \sum_{i=1}^{n} w_i = 1$$

4) Aggregation

$$Agg : \{W\} \to V$$

$$V = \{v_1, \ldots, v_n\}$$
$$\{W\} = \{W_1, \ldots, W_m\}$$

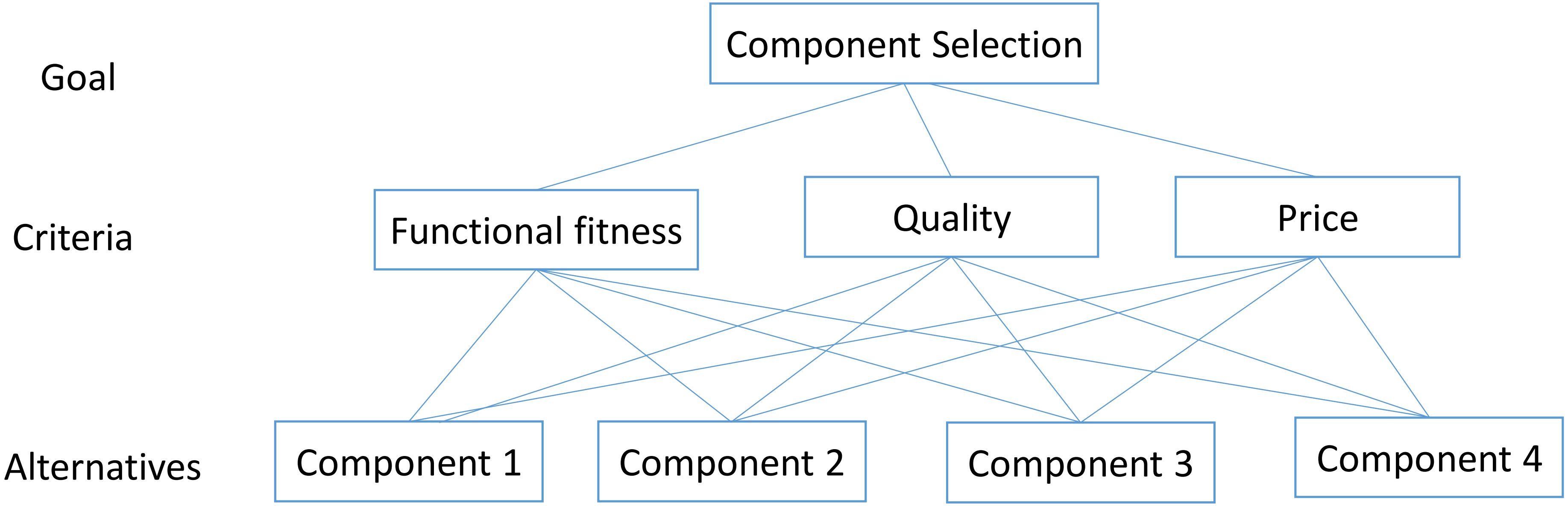
5) Select best Alternative

$$S : V \to v^*$$

W: local priority vector; V: global priority vector

Component Selection

Step 1) Define Hierarchy Model



1 -9 Scale

Intensity of Importance	Definition
1	Equal Importance
3	Moderate Importance
5	Strong Importance
7	Very Strong Importance
9	Extreme Importance
2, 4, 6, 8	For compromises between the above
Reciprocals of above	In comparing elements i and j - if i is 3 compared to j - then j is 1/3 compared to i
Rationals	Force consistency Measured values available

Step 2 &3 Evaluation and Prioritization

Importance of three criteria with respect to Component Selection																		
Criteria	Extremely (9)	Very, very Strongly (8)	Very Strongly (7)	Strong Plus (6)	Strongly (5)	Moderately Plus (4)	Moderately (3)	Weakly (2)	Equally (1)	Weakly (2)	Moderately (3)	Moderately plus (4)	Strongly (5)	Strong Plus (6)	Very Strongly (7)	Very very Strongly (8)	Extremely (9)	Criteria
Functional fitness F							X											Quality Q
Functional fitness F					X													Price attraction P
Quality Q							X											Price attraction P

$$
A0 = \begin{array}{c|ccc} & F & Q & P \\ \hline F & 1 & 3 & 5 \\ Q & 1/3 & 1 & 3 \\ P & 1/5 & 1/3 & 1 \end{array}
$$

$$
\text{Consistency Ratio} = \frac{C.I.}{R.I.} = \frac{0.0192}{0.52} = 0.037
$$

$$
W = \left\{ \frac{6.082}{9.548}, \frac{2.466}{9.548}, \frac{1}{9.548} \right\} = \{0.637, 0.258, 0.105\}
$$

where $6.082 + 2.466 + 1 = 9.548$

Eigensystem[A0] =

List of Eigen values

List of Eigen vectors

$$\begin{pmatrix} 3.03851 \\ -00192555+0341534i \\ -00192555-0341534i \end{pmatrix}, \begin{pmatrix} 6.0822 & 2.46621 & 1. \\ -30411+526734i & -123311-21358i & 1. \\ -30411-526734i & -123311+21358i & 1. \end{pmatrix}$$

3.039 is the principle Eigen value; {6.082, 2.466, 1} is its corresponding Eigen vector.

$$\text{Consistency Index} = C.I. = \frac{\lambda^{\max} - n}{n-1} = \frac{3.039 - 3}{3-1} = 0.0192 \qquad \text{Consistency Ratio} = \frac{C.I.}{R.I.} = \frac{0.0192}{0.52} = 0.037$$

n	1	2	3	4	5	6	7	8
Random Index (R.I)	0	0	0.52	0.89	1.11	1.25	1.35	1.4

Since the consistency ratio is acceptable, by normalization of the corresponding Eigen vector , priority vector is

$$W=\left\{\frac{6.082}{9.548}, \frac{2.466}{9.548}, \frac{1}{9.548}\right\} = \{0.637, 0.258, 0.105\}$$ where $6.082+2.466+1=9.548$

Comparing alternatives with respect to Functional Fitness

Criteria	Extremely (9)	Very, very Strongly (8)	Very Strongly (7)	Strong Plus (6)	Strongly (5)	Moderately Plus (4)	Moderately (3)	Weakly (2)	Equally (1)	Weakly (2)	Moderately (3)	Moderately plus (4)	Strongly (5)	Strong Plus (6)	Very Strongly (7)	Very very Strongly (8)	Extremely (9)	Criteria
Com1												X						Com2
Com1						X												Com3
Com1														X				Com4
Com2						X												Com3
Com2												X						Com4
Com3					X													Com4

F	Com1	Com2	Com3	Com4
Com1	1	1/4	4	1/6
Com2	4	1	4	1/4
Com3	1/4	1/4	1	1/5
Com4	6	4	5	1

$$CR = 0.162$$

$$W = \{0.116, 0.247, 0.06, 0.577\}$$

Comparing alternatives with respect to Quality

Criteria	Extremely (9)	Very, very Strongly (8)	Very Strongly (7)	Strong Plus (6)	Strongly (5)	Moderately Plus (4)	Moderately (3)	Weakly (2)	Equally (1)	Weakly (2)	Moderately (3)	Moderately plus (4)	Strongly (5)	Strong Plus (6)	Very Strongly (7)	Very very Strongly (8)	Extremely (9)	Criteria
Com1								X										Com2
Com1					X													Com3
Com1									X									Com4
Com2							X											Com3
Com2								X										Com4
Com3												X						Com4

Q	Com1	Com2	Com3	Com4
Com1	1	2	5	1
Com2	½	1	3	2
Com3	1/5	1/3	1	1/4
Com4	1	1/2	4	1

$$CR = 0.072$$

$$W = \{0.379, 0.290, 0.074, 0.257\}$$

Price(quantitative information)

	$ K	Normalized		Price Attraction
Com1	34	0.301	Reverse the order	0.212
Com2	27	0.239		0.248
Com3	24	0.212		0.301
Com4	28	0.248		0.239
	113	1.0		

Step 4-5: Aggregation and Selection

Selecting
A component
1.0

Functional Fitness
0.637

- Com1 0.116
- Com2 0.247
- Com3 0.06
- Com4 0.577

Quality
0.258

- Com1 0.379
- Com2 0.290
- Com3 0.074
- Com4 0.257

Price Attraction
0.105

- Com1 0.212
- Com2 0.248
- Com3 0.301
- Com4 0.239

	F	Q	P
W	0.637	0.258	0.105
Com1	0.116	0.379	0.212
Com2	0.247	0.290	0.248
Com3	0.006	0.074	0.301
Com4	0.577	0.257	0.239

→

0.194
0.258
0.055
0.459

Pros and Cons of AHP

Pros

- The consistency measure can *filter a number of "random" survey*. So the only valid survey is chosen.

Cons

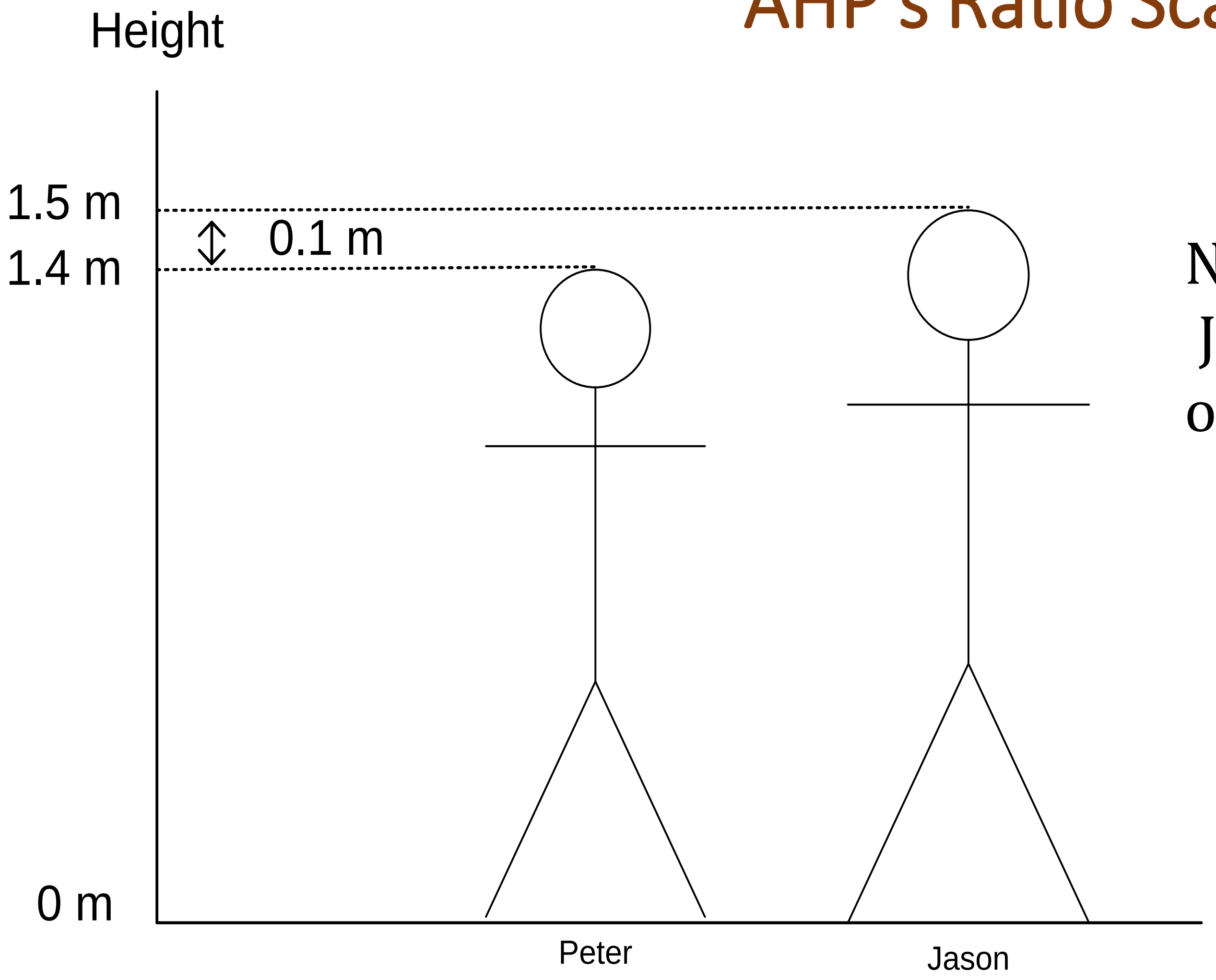
- It is very difficult for human to observe **ratio** relationship between two objects.
- Ratio relationship is not appropriate to present the difference of two objects.
- Since the ratio scale is not appropriate to represent the users' perception, it is very easy to produce the inconsistent survey.
- Computational method for the matrix is too complicated.

Cognitive Network Process

Free download at

https://theses.lib.polyu.edu.hk/handle/200/4716

Example of Fundamental Problems on AHP's Ratio Scale



Natural Language:

Jason is slightly taller than Peter by our observation

Jason is slightly (or weakly) taller than Peter by our observation

By AHP's Ratio Scale

$$a_{ij} = \frac{w_i}{w_j} = \frac{Jason}{Peter} = 2$$

Jason is 2 times as tall as Peter

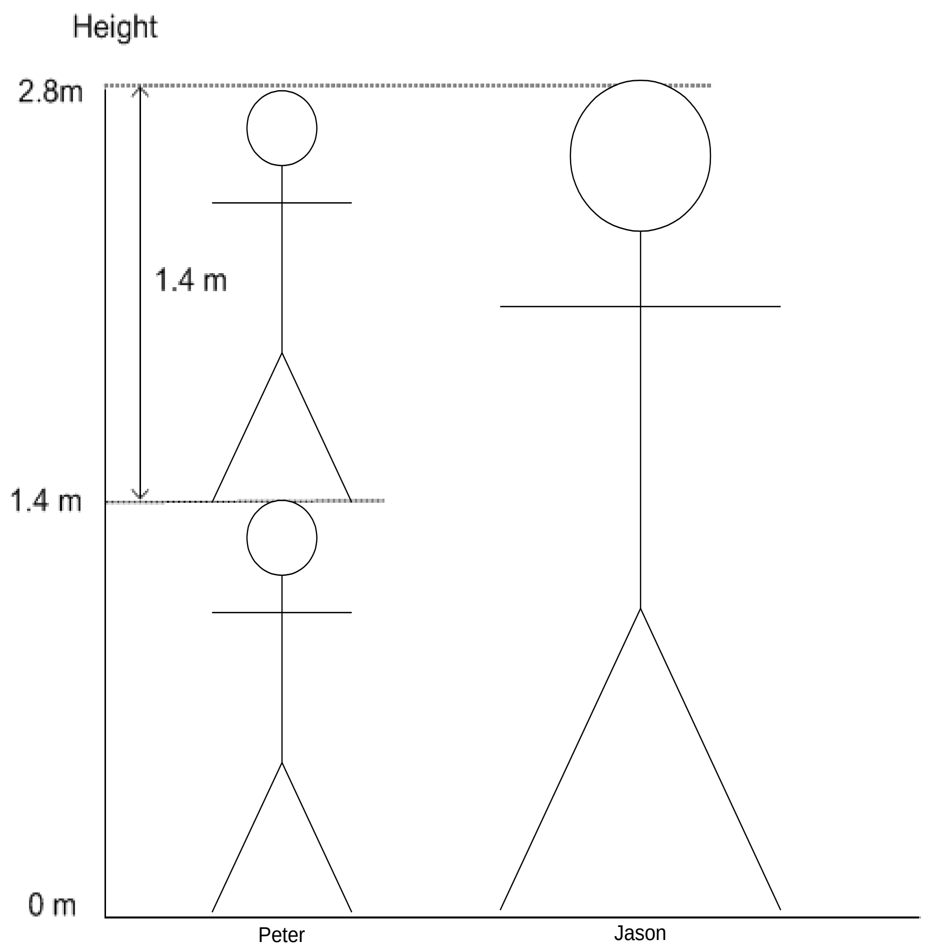
If Peter is 1.4 m, and Jason is 1.7 m

By Our perception

Jason is much taller than Peter.

By AHP's Ratio Scale

Jason is 5 times or 7 times as much as Peter



Ridiculous!!

Motivation for CNP

- Paired Interval scales are more appropriate in above cases.

Jason is slightly taller than
Peter by our observation

$\updownarrow$ match

Jason is 0.1m taller than Peter

$$a_{ij} = w_i / w_j \longrightarrow b_{ij} = v_i - v_j$$ **(Yuen, 2009)**

AHP CNP

More example

Assumption: Peter is 60 kg and Jason is 61 kg.

↓

Statement of natural language:
Jason is slightly heavier than Peter, by our observation

↓

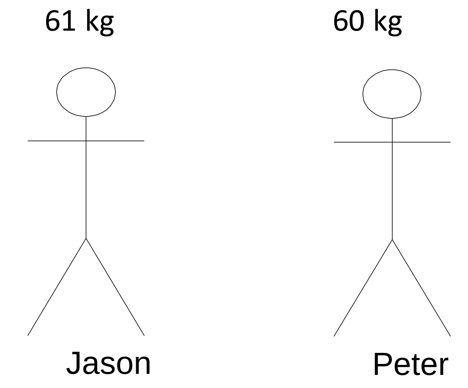
If we only know Peter's weight, what is the expectation value of Jason, according to the above statement?

P-CNP Perception

Cognitive Paired Comparison:
Jason is 1 kg heavier than Peter

Semantic form of paired interval scale:

$$b_{ij} = v_i - v_j$$

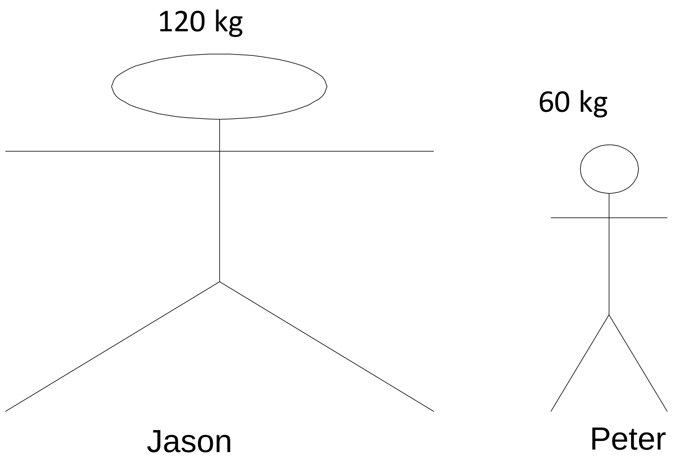


AHP Perception

Analytic Paired Comparison:
Jason is 2 times as heavy as Peter

Semantic form of paired ratio scale:

$$a_{ij} = w_i / w_j$$



Motivation for Cognitive Network Process

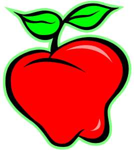
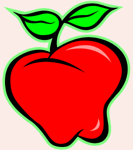
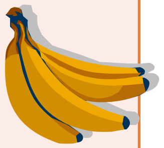
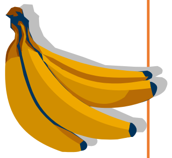
- The word "cognitive" means correction of cognition for *paired difference* in AHP.
- The different axioms of paired comparisons from CNP lead to different propositions, theorems, algorithms of the computation for the perception.
- KKF Yuen firstly proposed Cognitive Network Process, which is the ideal alternative of AHP, in his thesis.
- The method has been being recognized.

Core CNP References

1. K.K.F. Yuen, Cognitive network process with fuzzy soft computing technique for collective decision aiding, The Hong Kong Polytechnic University, **Ph.D. thesis**, 2009.
2. K.K.F. Yuen, The primitive cognitive network process: comparisons with the analytic hierarchy process, **International Journal of Information Technology & Decision Making**, 10(4), 2011, pp.659-680.
3. K.K.F. Yuen, The pairwise opposite matrix and its cognitive prioritization operators: the ideal alternatives of the pairwise reciprocal matrix and analytic prioritization operators, **Journal of the Operational Research Society** 63, 2012, pp.322-338.
4. K.K.F. Yuen, Combining Compound Linguistic Ordinal Scale and Cognitive Pairwise Comparison in the Modified Fuzzy TOPSIS Method for Group Decision Making, **Fuzzy Optimization and Decision Making**, 2014, 13 (1), *pp.105-130*.
5. K.K.F. Yuen, The primitive cognitive network process in medical decision making: Comparisons with the Analytic Hierarchy Process, **Applied Soft Computing**, 14, 2014, pp.109–119.
6. K.K.F. Yuen, Fuzzy Cognitive Network Process: Comparisons with Fuzzy Analytic Hierarchy Process in New Product Development Strategy, **IEEE Transactions on Fuzzy Systems**, 22(3), 2014, pp.597-610.
7. K.K.F. Yuen, A Hybrid Fuzzy Quality Function Deployment Framework using Cognitive Network Process and Aggregative Grading Clustering: An Application to Cloud Software Application Development, **Neurocomputing**, 142, 2014, pp.95-106.
8. Yuen, K.K.F. (2021), Decision Models for Information Systems Planning Using Primitive Cognitive Network Process: Comparisons with Analytic Hierarchy Process, Operational Research, https://doi.org/10.1007/s12351-021-00628-3.

3. The Cognitive Network Process Approach

Compare the relative preference for each pair of items as below, and circle the score accordingly

Item	Extremely	Very, very Strongly	Very Strongly	Strong Plus	Strongly	Moderately Plus	Moderately	Weakly	Equally	Weakly	Moderately	Moderately plus	Strongly	Strong Plus	Very Strongly	Very very Strongly	Extremely	Item
	8	7	6	5	4	3	2	1	0	1	2	3	4	5	6	7	8	
	8	7	6	5	4	3	2	1	0	1	2	3	4	5	6	7	8	
	8	7	6	5	4	3	2	1	0	1	2	3	4	5	6	7	8	

What are V1,V2, and V3?

$V1 - V2 = 1$

$V1 - V3 = 3$

$V2 - V3 = 1$

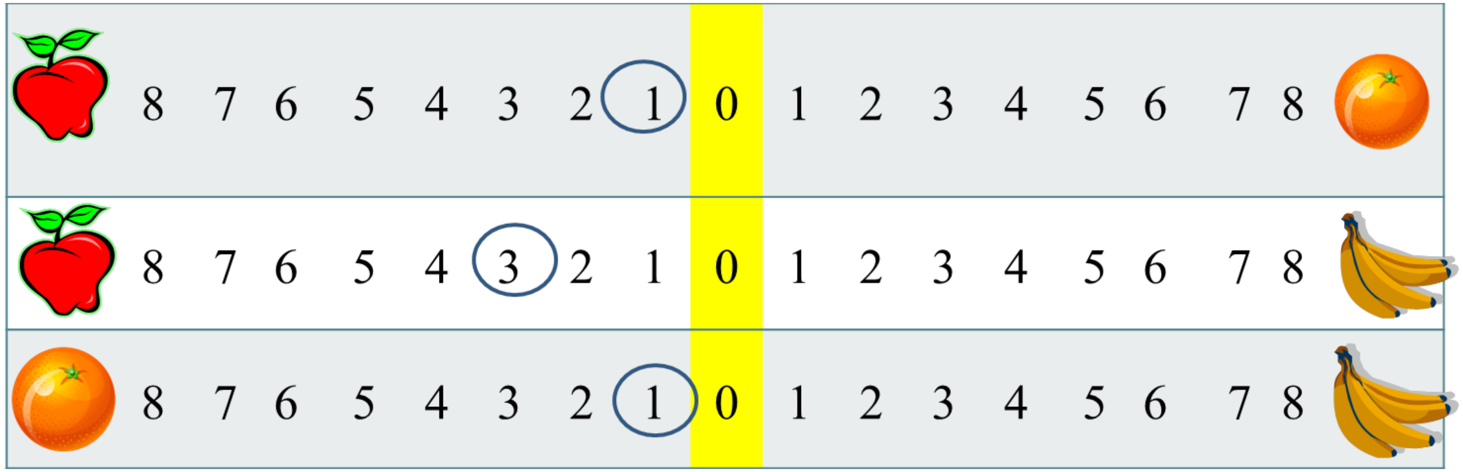
Scale assignment

Scale schema for pairwise interval comparison

i	Verbal scales	CNP Interval Scales
0	Equally	0
1	Weakly	1κ/8
2	Moderately	2κ/8
3	Moderately plus	3κ/8
4	Strongly	4κ/8
5	Strong Plus	5κ/8
6	Very Strongly	6κ/8
7	Very, very strongly	7κ/8
8	Extremely	κ
{-i}	Opposites of Above	(from -κ to 0)

Κ (kappa) is the normal utility and is set to 8 by default.

How to calculate the results by CNP



	0	1	3
	-1	0	1
	-3	-1	0
Col. Sum			

Row Average	RAU	Normalization
1.333	9.333	0.39
0.000	8.000	0.33
-1.333	6.667	0.28
0.000	24.000	1.000

Row Average plus the normal Utility (RAU)

$$v_i = \left(\frac{1}{n}\sum_{j=1}^{n} b_{ij}\right) + \kappa \quad , \qquad \forall i \in \{1,\ldots,n\}$$

Accordance Index

$$AI = \frac{1}{n^2}\sum_{i=1}^{n}\sum_{j=1}^{n} d_{ij}$$

$$d_{ij} = \sqrt{Mean\left(\left(\frac{1}{\kappa}\left(B_i + B_j^T - b_{ij}\right)\right)^2\right)} \qquad ,i,j \in (1,\ldots,n)$$

where $AI \geq 0$, and κ is the normal utility.

If $AI = 0$, then B is perfectly accordant;

If $0 < AI \leq 0.1$, then B is satisfactory.

If $AI > 0.1$, then B is unsatisfactory

$$B=\left[b_{ij}\right]$$

B_1	0	4	-1
B_2	-4	0	-5
B_3	1	5	0

$$B^T =$$

B_1^T	0	-4	1
B_2^T	4	0	5
B_3^T	-1	-5	0

	bij	$B_i+B_j^T$			$B_i+B_j^T-b_{ij}$			$\left(\frac{1}{\kappa}\left(B_i+B_j^T-b_{ij}\right)\right)^2$			$d_{ij}=\sqrt{Mean\left(\left(\frac{1}{\kappa}\left(B_i+B_j^T-b_{ij}\right)\right)^2\right)}$
d11	0	0	0	0	0	0	0	0	0	0	0
d12	4	4	4	4	0	0	0	0	0	0	0
d13	-1	-1	-1	-1	0	0	0	0	0	0	0
d21	-4	-4	-4	-4	0	0	0	0	0	0	0
d22	0	0	0	0	0	0	0	0	0	0	0
d23	-5	-5	-5	-5	0	0	0	0	0	0	0
d31	1	1	1	1	0	0	0	0	0	0	0
d32	5	5	5	5	0	0	0	0	0	0	0
d33	0	0	0	0	0	0	0	0	0	0	0
									AI=Average		0

$$B = \left[b_{ij}\right]$$

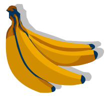
B_1	0	1	1
B_2	-1	0	2
B_3	-1	-2	0

$$B^T =$$

B_1^T	0	-1	-1
B_2^T	1	0	2
B_3^T	1	-2	0

	bij	$B_i + B_j^T$			$B_i + B_j^T - b_{ij}$			$\left(\frac{1}{\kappa}\left(B_i + B_j^T - b_{ij}\right)\right)^2$			$d_{ij} = \sqrt{Mean\left(\left(\frac{1}{\kappa}\left(B_i + B_j^T - b_{ij}\right)\right)^2\right)}$
d11	0	0	0	0	0	0	0	0	0	0	0
d12	1	1	1	-1	0	0	-2	0	0	0.0625	0.1443
d13	1	1	3	1	0	2	0	0	0.0625	0	0.1443
d21	-1	-1	-1	1	0	0	2	0	0	0.0625	0.1443
d22	0	0	0	0	0	0	0	0	0	0	0
d23	2	0	2	2	-2	0	0	0.0625	0	0	0.1443
d31	-1	-1	-3	-1	0	-2	0	0	0.0625	0	0.1443
d32	-2	0	-2	-2	2	0	0	0.0625	0	0	0.1443
d33	0	0	0	0	0	0	0	0	0	0	0
										AI=Average	0.0962

Discussion

			
	0	1	3
	-1	0	1
	-3	-1	0

What is AI for this case?

The Primitive Cognitive Network Process

Step 1
Problem Cognition Process
- Stating Objective
- Listing criteria
- Listing alternative
- Inviting experts & users

Step 2
Cognitive Assessment Process
- Justifying rating scales
- Assessing criteria weights
- Computing Accordance Index
- Evaluating alternatives

Step 3
Cognitive Computation Process
- Computing criteria weights

Steps 4 & 5
Information fusion & Decision Volition Process
- Aggregating granular results
- Producing ranking results

Applications

Discussion

- Use the CNP method to calculate the AHP examples of component selection.

Evaluating software component quality from vendors using the primitive cognitive network process with ISO/IEC 9126

ComComAp
11-13 Jan 2012

Kevin Kam Fung Yuen, PhD

Yuen K.K.F (2012), "Evaluating software component quality from vendors using the primitive cognitive network process with ISO/IEC 9126", IEEE Proceedings of 2012 Computers, Communications and IT Applications Conference, pp.288-293.

Introduction

- Selecting the high quality software components for a final system is essential step in software development process.
- This paper proposes the primitive cognitive network process (P-CNP) approach for evaluating the vendors' software component quality. The criteria of software quality adopt the international software standard ISO/IEC9126.
- The primitive cognitive network process is used to evaluate the software components with respect to these software criteria.
- The proposed model can help developers and testers to evaluate vendors' software components and select the best alternative through user experiences.

Review of SOFTWARE QUALITY MODEL ISO/IEC9126-1: 2001

There are various hierarchical models of software quality attributes such as Factor-Criteria-Metrics Model, McCall's Model, Boehm's Model, FURPS and Dromey's Model, which are reviewed by [7,8].

Functionality	the capability of the software product to provide functions which meet stated or implied needs when the software is in use under specified conditions.
Reliability	the capability of the software product to maintain a specified level of performance when used under specified conditions.
Usability	the capability of the software product to be understood learned, used, and attractive to the user under specified conditions.

ISO/IEC9126-1: 2001

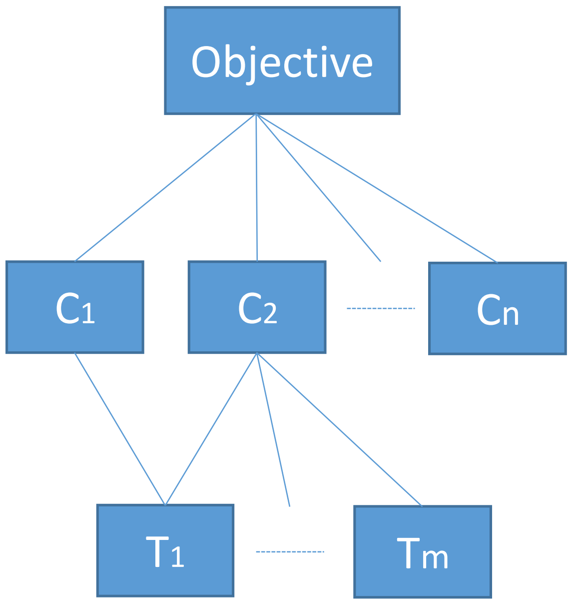
Efficiency	**the capability of the software product to provide appropriate performance, relative to the amount of resources used, under stated conditions**
Maintainability	the capability of the software product to be modified. Modifications may include corrections, improvements or adaptation of the software to changes in environment, and in requirements and functional specifications
Portability	the capability of the software product to be transferred from one environment to another

Primitive Cognitive Network Process

- Due to the difference of the fundamental definition from the AHP, the Primitive Cognitive Network Process is proposed.
- The Cognitive Network Process is the cognitive architecture which comprises five cognitive decision processes:
  1. Problem Cognition Process (PCP)
  2. Cognitive Assessment Process (CAP),
  3. Cognitive Prioritization Process (CPP),
  4. Multiple Information Fusion Processes (MIP)
  5. Decisional Volition Process (DVP).

Step 1: Problem Cognition Process

- Problem Cognition Process is to formulate the decision problem as the measurable Structural Assessment Network (SAN) model, which comprises four units:
- an objective *O*
- a criteria structure (or structural criteria) *C*
- a set of alternatives *T* using a measurement scale schema .



Step 2: Cognitive Assessment Process

$$\tilde{B} = \left[\tilde{b}_{ij}\right] = \begin{bmatrix} 0 & v_1 - v_2 & \cdots & v_1 - v_n \\ v_2 - v_1 & 0 & \cdots & v_2 - v_n \\ \vdots & \vdots & \ddots & \vdots \\ v_n - v_1 & v_n - v_2 & \cdots & 0 \end{bmatrix} \cong \begin{bmatrix} 0 & b_{12} & \cdots & b_{1n} \\ b_{21} & 0 & \cdots & b_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \cdots & 0 \end{bmatrix} = \left[b_{ij}\right] = B$$

B is validated by the Accordant Index (AI) of the below form.

$$AI = \frac{1}{n^2}\sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} d_{ij}},$$

$$d_{ij} = \sqrt{Mean\left(\left(\frac{1}{\kappa}\left(B_i + B_j^T - b_{ij}\right)\right)^2\right)},\quad i, j \in (1, \ldots, n)$$

where $AI \geq 0$, and κ is the normal utility.

If $AI = 0$, then B is perfectly accordant;

If $0 < AI \leq 0.1$, then B is satisfactory.

If $AI > 0.1$, then B is unsatisfactory

Table 1: Scale schema for pairwise opposite comparison

i	Verbal scales ℵ	CNP (Interval Scales)
0	Equally	0
1	Weakly	$\kappa/8$
2	Moderately	$\kappa/4$
3	Moderately plus	$3\kappa/8$
4	Strongly	$\kappa/2$
5	Strong Plus	$5\kappa/8$
6	Very Strongly	$3\kappa/4$
7	Very, very strongly	$7\kappa/8$
8	Extremely	κ
{-i}	Opposites of Above	(from $-\kappa$ to 0)

Step 3: Cognitive prioritization Process

Primitive Least Squares Optimization

$$\mathrm{PLS}\left(B^{+},\kappa\right)=$$

$$\text{Min} \quad \bar{\Delta}=\sum_{i=1}^{n}\sum_{j=i+1}^{n}\left(b_{ij}-v_i+v_j\right)^2$$

$$\text{s.t.} \quad \sum_{i=1}^{n} v_i = n\kappa,$$

where $n=\left|\{v_i\}\right|$, and κ is the normal utility.

The closed form solution is
the Row Average plus the normal Utility (RAU)

$$v_i=\left(\frac{1}{n}\sum_{j=1}^{n} b_{ij}\right)+\kappa \quad , \quad \forall i\in\{1,\ldots,n\}$$

Least Penalty Squares (LPS) or
Discrete Least Squares (DLS)

$$\text{Min} \quad \hat{\Delta}=\sum_{i=1}^{n}\sum_{j=i+1}^{n}\beta_{ij}\cdot\left(b_{ij}-v_i+v_j\right)^2$$

$$, \quad \beta_{ij}=\begin{cases}\beta_1, & \mathrm{v}_i>v_j \ \& \ \mathrm{b}_{ij}>0 \\ & \text{or } \mathrm{v}_i<v_j \ \& \ \mathrm{b}_{ij}<0 \\ \beta_2, & \mathrm{v}_i=v_j \ \& \ \mathrm{b}_{ij}\neq 0 \\ & \text{or } v_i\neq v_j \ \& \ \mathrm{b}_{ij}=0 \\ \beta_3, & \textit{otherwise}\end{cases}, \quad 1=\beta_1\le\beta_2\le\beta_3$$

$$\text{s.t.} \quad \begin{matrix}\sum_{i=1}^{n} v_i = n\kappa, \\ \mathrm{v}_i\ge 0, i=1,2,\ldots,n\end{matrix},$$

, where $n=\left|\{v_i\}\right|$, and κ is the normal utility,

$\{\beta_1,\beta_2,\beta_3\}$ is a set of penalty indices.

To normalize the $\{v_i\}$, then $W=\left\{w_i : w_i=\dfrac{v_i}{n\kappa}, \forall i\in\{1,\ldots,n\}\right\}$, $\sum_{i\in\{1,\ldots,n\}} v_i = n\kappa$.

Step 4: Multiple Information Fusion Processes

Multiple Information Fusion Process is the process to aggregate a list of result sets of priority pairs $\{Y\}$ to represent a vector of the objective priorities T with respective to the Structural Access Network by an aggregation operator, i.e. $AO:\{Y\}\to T$. The levels of aggregation is subject to the levels of criteria. Weighted arithmetic mean is chosen as default aggregation operator:

$$wam(V',V)=\sum_{i=1}^{n} v'_i v_i$$

Step 5: Decisional Volition Process

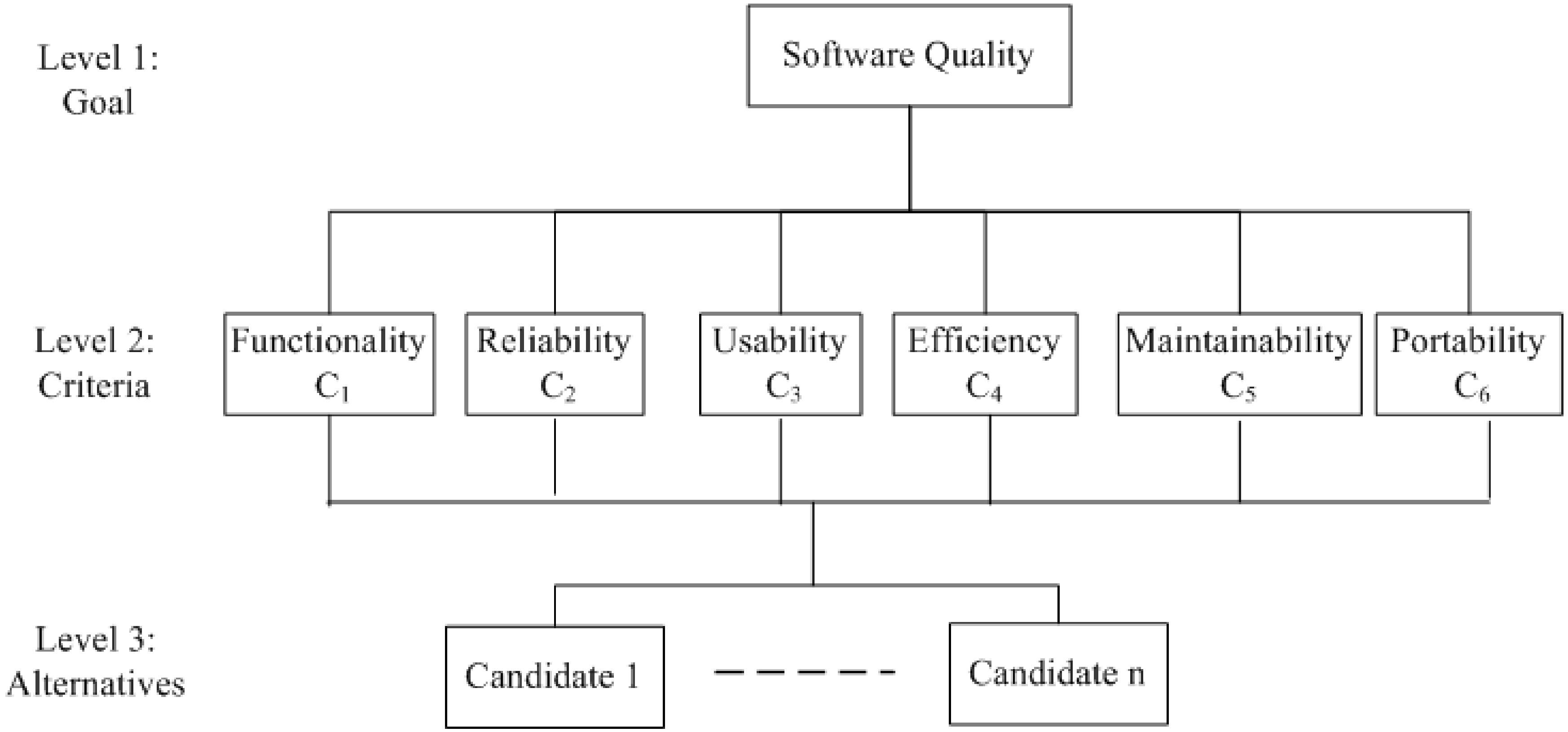
The Decisional Volition Process is the process to decide the final decision $t^* \in T = [T_1, \dots, T_m]$ with respect to the aggregation results by the volition function $VL : T \to t^*$. The best alternative is determined by the highest score $T^* = Max(T)$, and its position γ is returned by the argument of the maximum function $\arg\max$.

$$t^* = VL(T) = T_\gamma, \text{ where } \gamma = \underset{i \in \{1,2,\dots,m\}}{\arg\max}\left(\{T_1, T_2, \dots, T_i, \dots, T_m\}\right)$$

5. Numerical example

- A software company received a customer order from a hospital to develop a tailor-made medical information system. In the design phrase, in order to reduce the software development time, the company decided to find a component to construct the interactive graph display, rather than self-development.
- The primitive cognitive network process is used to access software quality.
- The rating scale is shown in table 1 with k=8.

Structural Assessment Network



Cognitive pairwise matrices

Table 2: Cognitive pairwise matrix for the importance of six criteria and their local weights

Criteria	C_1	C_2	C_3	C_4	C_5	C_6	w
C_1	0	-1	-2	-2	3	2	0.167
C_2	1	0	-1	-1	4	5	0.194
C_3	2	1	0	0	5	6	0.215
C_4	2	1	0	0	6	7	0.222
C_5	-3	-4	-5	-6	0	1	0.108
C_6	-2	-5	-6	-7	-1	0	0.094
AI=0.085							

Row Average		Normal Utility		RAU		Normalization
0.000		8		8.000		0.167
1.333		8		9.333		0.194
2.333	+	8	=	10.333	-->	0.215
2.667		8		10.667		0.222
-2.833		8		5.167		0.108
-3.500		8		4.500		0.094
			sum=nk=	48	sum=	1

Table 3: Cognitive pairwise matrix of the three candidates with respect to functionality C_1 and their local weights

Candidates	T_1	T_2	T_3	w
T_1	0	0	0	0.333
T_2	0	0	0	0.333
T_3	0	0	0	0.333
AI=0				

Table 4: Cognitive pairwise matrix of the three candidates with respect to reliability C_2 and their local weights

Candidates	T_1	T_2	T_3	w
T_1	0	1	2	0.365
T_2	1	0	1	0.351
T_3	-2	-1	0	0.284
AI=0.080				

Table 5: Cognitive pairwise matrix of the three candidates with respect to usability C_3 and their local weights

Candidates	T_1	T_2	T_3	w
T_1	0	4	-1	0.375
T_2	-4	0	-5	0.208
T_3	1	5	0	0.417
AI=0				

Table 6: Cognitive pairwise matrix of the three candidates with respect to efficiency C_4 and their local weights

Candidates	T_1	T_2	T_3	w
T_1	0	5	-1	0.389
T_2	-5	0	-6	0.181
T_3	1	6	0	0.431
AI=0				

Table 7: Cognitive pairwise matrix of the three candidates with respect to maintainability C_5 and their local weights

Candidates	A_1	A_2	A_3	w
T_1	0	7	7	0.535
T_2	-7	0	1	0.254
T_3	-7	-2	0	0.211
AI=0.088				

Table 8: Cognitive pairwise matrix of the three candidates with respect to portability C_6 and their local weights

Candidates	A_1	A_2	A_3	w
T_1	0	6	-1	0.403
T_2	-6	0	-5	0.181
T_3	1	5	0	0.417
AI=0.096				

Final Decision

Table 9: Synthesis of local fuzzy weights of three candidates and their Global Weight

	Criterion 1	Criterion 2	Criterion 3	Criterion 4	Criterion 5	Criterion6	Global weights
	0.167	0.194	0.215	0.222	0.108	0.094	
T_1	0.333	0.365	0.375	0.389	0.535	0.403	0.389*
T_2	0.333	0.351	0.208	0.181	0.254	0.181	0.253
T_3	0.333	0.284	0.417	0.431	0.211	0.417	0.358

Q&A
Thank you