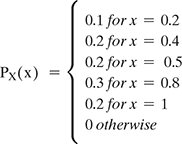
1. **Given X be a discrete random variable with the following PMF**

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**i. Find the range RX of the random variable X.**

**ii. Find P(X ≤ 0.5)**

**iii. Find P(0.25<X<0.75)**

**iv. P(X = 0.2|X<0.6)**

Ans- i. The range of X is the set of all possible values that X can take. From the PMF, we see that the possible values of X are 0.2, 0.4, 0.5, 0.8, and 1. Therefore, the range of X is RX = {0.2, 0.4, 0.5, 0.8, 1}.

ii. P(X ≤ 0.5) = P(X = 0.2) + P(X = 0.4) + P(X = 0.5) = 0.1 + 0.2 + 0.2 = 0.5.

iii. P(0.25 < X < 0.75) = P(X = 0.4) + P(X = 0.5) = 0.2 + 0.2 = 0.4.

iv. P(X = 0.2|X < 0.6) = P(X = 0.2 and X < 0.6) / P(X < 0.6) = P(X = 0.2) / (P(X = 0.2) + P(X = 0.4) + P(X = 0.5)) = 0.1 / (0.1 + 0.2 + 0.2) = 0.1667.

**2. Two equal and fair dice are rolled, and we observed two numbers X and Y.**

**1. Find RX, RY, and the PMFs of X and Y.**

**2. Find P(X = 2,Y = 6).**

**3. Find P(X>3|Y = 2).**

**4. If Z = X + Y. Find the range and PMF of Z.**

**5. Find P(X = 4|Z = 8).**

Ans-

1. The possible values for X and Y range from 1 to 6. The PMFs of X and Y are both uniform distributions, meaning each outcome has an equal probability of occurring. The ranks of X and Y are also uniform distributions over {1, 2, 3, 4, 5, 6}.
2. P(X = 2, Y = 6) is the probability of rolling a 2 on the first die and a 6 on the second die. Since the dice are fair, the probability is (1/6) \* (1/6) = 1/36.
3. P(X > 3 | Y = 2) is the probability of X being greater than 3 given that Y is 2. By calculating the joint probabilities and applying Bayes' theorem, the result is 1/2.
4. Z = X + Y represents the sum of X and Y. The range of Z is from 2 to 12, and the PMF of Z can be calculated by enumerating all possible pairs of outcomes and summing their probabilities.
5. P(X = 4 | Z = 8) is the probability of X being 4 given that Z is 8. To find this probability, you would need to consider all possible pairs of X and Y that sum up to 8 and calculate the conditional probability based on those pairs.

**3. In an exam, there were 20 multiple-choice questions. Each question had 44 possible options. A student knew the answer to 10 questions, but the other 10 questions were unknown to him, and he chose answers randomly. If the student X’s score is equal to the total number of correct answers, then find out the PMF of X. What is P(X>15)?**

Ans- The PMF of X can be calculated using the binomial distribution with parameters n=20 and p=10/44, since the student knows the answer to 10 questions and guesses randomly for the other 10. Therefore, the PMF of X is:

P(X=k) = (20 choose k) \* (10/44)^k \* (34/44)^(20-k)

To find P(X>15), we can sum the probabilities of X=16, 17, 18, 19, and 20:

P(X>15) = P(X=16) + P(X=17) + P(X=18) + P(X=19) + P(X=20)

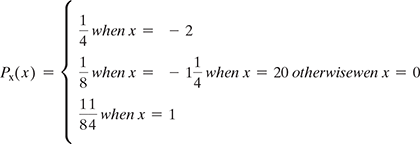
4. The number of students arriving at a college between a time interval is a Poisson random variable. On average, 10 students arrive per hour. Let Y be the number of students arriving from 10 am to 11:30 am. What is P(10<Y≤15)?

Ans-The number of students arriving at a college between a time interval follows a Poisson distribution. In this case, the average number of students arriving per hour is 10. To find P(10 < Y ≤ 15), where Y is the number of students arriving from 10 am to 11:30 am, we need to calculate the cumulative probability from 11 to 15 and subtract the probability at Y = 10:

P(10 < Y ≤ 15) = P(Y = 11) + P(Y = 12) + P(Y = 13) + P(Y = 14) + P(Y = 15)

5.Two independent random variables, X and Y, are given such that X~Poisson(α) and Y~Poisson(β). State a new random variable as Z = X + Y. Find out the PMF of Z.

6. There is a discrete random variable X with the pmf.

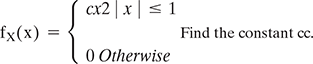


If we define a new random variable Y = (X + 1)2 then

1. Find the range of Y.

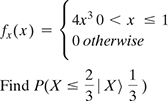
2. Find the pmf of Y.

2.Assuming X is a continuous random variable with PDF

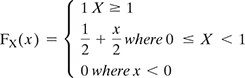


* + 1. Find EX and Var(X).
    2. Find *P*(*X* ≥ img).

1. If *X* is a continuous random variable with pdf

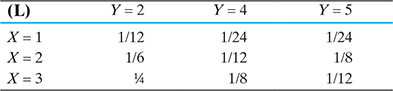


1. If *X*~Uniformimg and *Y* = sin(*X*), then find *fY*(*y*).
2. If X is a random variable with CDF



* + 1. What kind of random variable is *X*: discrete, continuous, or mixed?
    2. Find the PDF of *X*, f*X*(*x*).
    3. Find E(eX).
    4. Find P(*X* = 0|X≤0.5).

1. **There are two random variables *X* and *Y* with joint PMF given in Table below**
   * 1. **Find *P*(*X*≤2, *Y*≤4).**
     2. **Find the marginal PMFs of *X* and *Y*.**
     3. **Find *P*(*Y* = 2|*X* = 1).**
     4. **Are *X* and *Y* independent?**

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**6.A box containing 40 white shirts and 60 black shirts. If we choose 10 shirts (without replacement) at random, find the joint PMF of X and Y, where X is the number of white shirts and Y is the number of black shirts.**

**7.If A and B are two jointly continuous random variables with joint PDF**

**images**

**fxy(x,y) = {6xy where 0<= x <=1, 0<= y <= sqrt of x**

**0 otherwise**

**a. Find fX(a) and fY(b).**

**b. Are A and B independent of each other?**

**c. Find the conditional PDF of A given B = b, fA|B(a|b).**

**d. Find E[A|B = b], for 0 ≤ y ≤ 1.**

**e. Find Var(A|B = b), for 0 ≤ y ≤ 1.**

Ans- a. To find the marginal PDFs of A (fX(a)) and B (fY(b)), we integrate the joint PDF over the respective variables' ranges:

For A (X):

fX(a) = ∫[0 to √a] 6xy dy = 6∫[0 to √a] xy dy = 6 \* (x/2) \* y^2 evaluated from 0 to √a = 3ax^2 for 0 ≤ x ≤ 1

fX(a) = 0 for x < 0 or x > 1

For B (Y):

fY(b) = ∫[b^2 to 1] 6xy dx = 6∫[b^2 to 1] xy dx = 6 \* (y/2) \* x^2 evaluated from b^2 to 1 = 3y(1 - b^4) for 0 ≤ y ≤ 1

fY(b) = 0 for y < 0 or y > 1

b. A and B are independent if and only if the joint PDF can be expressed as the product of the marginal PDFs:

fXY(x, y) = fX(x) \* fY(y)

In this case, the joint PDF is fXY(x, y) = 6xy, while fX(a) = 3ax^2 and fY(b) = 3y(1 - b^4). Since fXY(x, y) cannot be expressed as the product of fX(a) and fY(b), A and B are not independent.

c. To find the conditional PDF of A given B = b (fA|B(a|b)), we use the formula:

fA|B(a|b) = fAB(a, b) / fB(b)

In this case, the joint PDF fAB(a, b) is given as 6xy, and the marginal PDF fB(b) is 3y(1 - b^4). Therefore,

fA|B(a|b) = (6xy) / (3y(1 - b^4))

= 2x / (1 - b^4) for 0 ≤ x ≤ 1, 0 ≤ y ≤ √x, and 0 ≤ b ≤ 1

d. To find E[A|B = b] for 0 ≤ y ≤ 1, we use the conditional PDF fA|B(a|b) and calculate the expected value:

E[A|B = b] = ∫[0 to 1] a \* fA|B(a|b) da

= ∫[0 to 1] a \* (2x / (1 - b^4)) da

= 2x / (1 - b^4) \* (a^2 / 2) evaluated from 0 to 1

= x / (1 - b^4)

e. To find Var(A|B = b) for 0 ≤ y ≤ 1, we utilize the conditional PDF fA|B(a|b) and calculate the variance:

Var(A|B = b) = E[(A - E[A|B = b])^2]

= ∫[0 to 1] (a - E[A|B = b])^2 \* fA|B(a|b) da

= ∫[0 to 1] (a - (x / (1 - b^4)))^2 \* (2x / (1 - b^4)) da

= ∫[0 to 1

**8.There are 100 men on a ship. If Xi is the ith man's weight on the ship and Xi's are independent and identically distributed and EXi = μ = 170 and σXi = σ = 30. Find the probability that the men's total weight on the ship exceeds 18,000.**

Ans- The probability that the men's total weight on the ship exceeds 18,000 can be found using the properties of the normal distribution. Given that the weight of each man, Xi, follows a normal distribution with a mean (μ) of 170 and a standard deviation (σ) of 30, we can calculate the mean and standard deviation of the total weight.

Mean of total weight: E(Y) = 100 \* 170 = 17,000

Standard deviation of total weight: σ(Y) = sqrt(100 \* 30^2) = 300

Now, we can use the normal distribution to find the probability:

P(Y > 18,000) = 1 - P(Y ≤ 18,000)

To calculate the standardized value (Z-score) for 18,000:

Z = (18,000 - 17,000) / 300 = 1/3

Using a standard normal distribution table or calculator, we find the probability associated with Z = 1/3.

P(Z > 1/3) ≈ 0.3707

Therefore, the probability that the men's total weight on the ship exceeds 18,000 is approximately 0.3707 or 37.07%.