2.2.1 ELECTROSTATICS

$$\mathbf{F} = q\,\mathbf{E}(\mathbf{r})$$

$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\int d^3r'\,\rho(\mathbf{r}')\,\frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|^3}$$

ELECTRIC FIELD
(VECTOR FIELD)

SUPERPOSITION PRINCIPLE:

E produced by ρ is the vector sum of the E of it's constituent pieces.

USING THE FOLLOWING IDENTITIES

$$\nabla \frac{1}{|\mathbf{r}-\mathbf{r}'|} = -\frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|^3} \qquad \nabla^2 \frac{1}{|\mathbf{r}-\mathbf{r}'|} = -4\pi\delta(\mathbf{r}-\mathbf{r}')$$

$$\nabla \cdot \mathbf{E} = \nabla \cdot \left(\frac{1}{4\pi\epsilon_0}\int d^3r'\,\rho(\mathbf{r}')\,\frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|^3} \right)$$

$$= \nabla \left(\frac{-1}{4\pi\epsilon_0}\int d^3r'\,\rho(\mathbf{r}')\,\nabla\frac{1}{|\mathbf{r}-\mathbf{r}'|} \right)$$

$$= -\frac{1}{4\pi\epsilon_0}\int d^3r'\,\rho(\mathbf{r}')\,\nabla^2\frac{1}{|\mathbf{r}-\mathbf{r}'|}$$

$$= \frac{1}{\epsilon_0}\int d^3r'\,\rho(\mathbf{r}')\,\delta(\mathbf{r}-\mathbf{r}')$$

$$\boxed{\nabla \cdot E = \rho / \epsilon_0}$$ — GAUSS' LAW

$$\boxed{\nabla \times E = 0}$$ (ONLY VALID FOR ELECTROSTATICS)

CURL OF A GRADIENT = 0

$$\nabla \times \nabla \frac{1}{|r - r'|} = 0$$

2.2.3 MAGNETOSTATICS

$$F = -\frac{\mu_0}{4\pi} \oint I\, dl \cdot \sum_{k=1}^{N} \oint I_k\, dl_k \frac{r - r_k}{|r - r_k|^3}$$

$$F = \oint I\, dl \times B(r)$$

$$B(r) = \frac{\mu_0}{4\pi} \sum_{k=1}^{N} \oint I_k\, dl_k \times \frac{r - r_k}{|r - r_k|^3}$$

$$I \oint dl \rightarrow \int d^3r\, j$$

linear circuits → volume distribution (section 9.3.1)

↓

$$B(r) = \frac{\mu_0}{4\pi} \int d^3r' \frac{j(r') \times (r - r')}{|r - r'|^3}$$

STEADY CURRENT

$$\boxed{\nabla \cdot B = 0} \qquad \boxed{\nabla \times B = \mu_0 J}$$

CONDITION $\nabla \cdot j = 0$

SUPERPOSITION PRINCIPLE:

B PRODUCED BY A STEADY CURRENT DISTRIBUTION j IS THE VECTOR SUM OF B OF IT'S CONSTITUENT PIECES.

2.2.4 FARADAY'S LAW

$$-\frac{d}{dt}\int_S dS \cdot B = IR$$

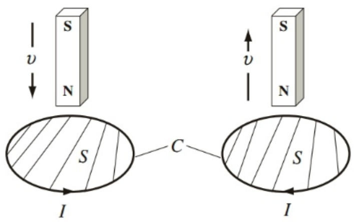

Figure 2.3: A typical experiment which reveals Faraday's law. Current flows in opposite directions in the filamentary wire C when the permanent magnet moves upward or downward. The area S is bounded by the wire C.

change in B over time $\Rightarrow I(t)$

$$IR = \oint_C dl \cdot E$$

$$-\int_S dS \cdot \frac{dB}{dt} = \underbrace{\oint_C dl \cdot E = \int_S dS \cdot \nabla \times E}_{\text{STOKES' THEOREM}}$$

$$\boxed{\nabla \times E = \frac{-\partial B}{\partial t}}$$