$$F = q(E + v \times B)$$ — COULOMB - LORENTZ FORCE

- TIME INDEPENDENT (STATIC) $E(r)$ AND $B(r)$ DO NOT INTERACT
- TIME DEPENDENT $E(r,t)$ AND $B(r,t)$ → IT'S COMPLICATED

## 2.1.1 ELECTRIC CHARGE

$e = 1.602 \times 10^{-19}$ C — charge (intrinsic property of matter)

particles posses charge in integer multiples of e

however, EM theory develops naturally by defining continuous **charge density**:

$$Q = \int_V d^3r \, \rho(r)$$ — TOTAL Q IN VOLUME V

$$Q = \int_S dS \, \sigma(r_s)$$ — TOTAL Q ON SURFACE S

$$Q = \int_l dl \, \lambda(l)$$ — TOTAL Q ON 1-D LINE $l$

A FREE ELECTRON IS TREATED AS A POINT CHARGE

→ USE DELTA FUNCTION TO REPRESENT CHARGE DENSITY

$$\rho(r) = \sum_{k=1}^{N} q_k \delta(r - r_k)$$

$$\rightarrow Q = \int_V d^3r\, \rho(r) = \int_V d^3r \sum_{k=1}^{N} q_k \delta(r - r_k)$$

## 2.1.2 ELECTRIC CURRENT

CURRENT → CHARGE IN MOTION

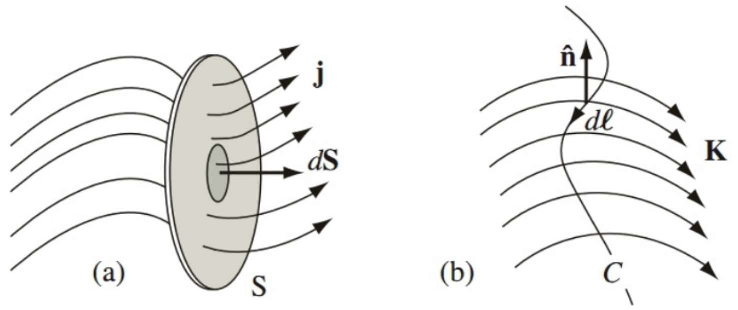


**Figure 2.1:** (a) The current $I$ is the integral over $S$ of the projection of the volume current density $\mathbf{j}$ onto the area element $d\mathbf{S}$; (b) the current $I$ is the integral over $C$ of the projection of $\mathbf{K} \times \hat{\mathbf{n}}$ (the cross product of the surface current density $\mathbf{K}$ and the surface normal $\hat{\mathbf{n}}$) onto the line element $d\boldsymbol{\ell}$.

$j(r,t)$ — current density

$$I = \frac{dQ}{dt} = \int_S d\mathbf{S} \cdot j$$ — total current through S

$$dI = j \cdot d\mathbf{S} = j \cdot \hat{n}\, dS$$

$j(r,t) = \rho v$ — current in a volume

$K = \sigma v$ — current in a suface

$$I = \int_l dl \cdot K \times \hat{n} = \int_l K \cdot (\hat{n} \times dl)$$

## 2.1.3 CONSERVATION OF CHARGE

$$I = \int_V d^3r \, \nabla \cdot j$$ — from divergence theorem

$$I = -\frac{dQ}{dt} = -\frac{d}{dt}\int_V d^3r \, \rho = -\int_V d^3r \frac{\partial \rho}{\partial t}$$

$$\rightarrow \frac{\partial \rho}{\partial t} + \nabla \cdot j = 0$$ (CONTINUITY EQUATION)

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot j$$

## APPLICATION: MOVING POINT CHARGES

$$\rho(r,t) = \sum_{k=1}^{N} q_k \, \delta(r - r_k(t))$$

(CHAIN RULE)

$$\frac{\partial \rho}{\partial t} = \sum_{k=1}^{N} q_k \frac{\partial}{\partial t} \delta(r - r_k(t)) = -\sum_{k=1}^{N} q_k \dot{r}_k(t) \cdot \nabla \delta(r - r_k(t))$$

$$= -\sum_{k=1}^{N} q_k v_k \cdot \nabla \delta(r - r_k(t))$$

$$\nabla \cdot v_k \delta = \cancel{(\nabla \cdot v_k)}^{0} \delta + \nabla \delta \cdot v_k$$

$$= -\nabla \cdot \sum_{k=1}^{N} q_k v_k \delta(r - r_k(t))$$

$$= -\nabla \cdot j(r,t)$$

$$\rightarrow \quad j(r,t) = \sum_{k=1}^{N} q_k v_k \delta(r - r_k)$$