mlemap

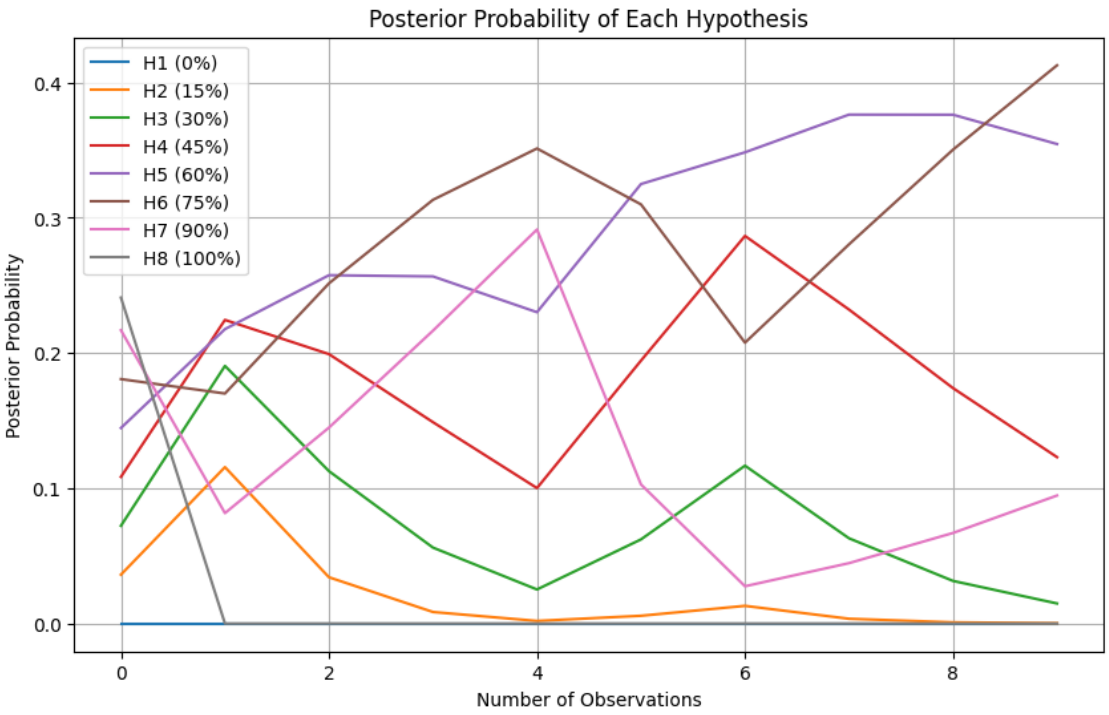
October 26, 2024

Given the coin flip results: [1, 0, 1, 1, 1, 0, 0, 1, 1, 1], perform an MAP and MLE experiment to determine the probability of each hypothesis with respect to the number of coin flips

[1]:

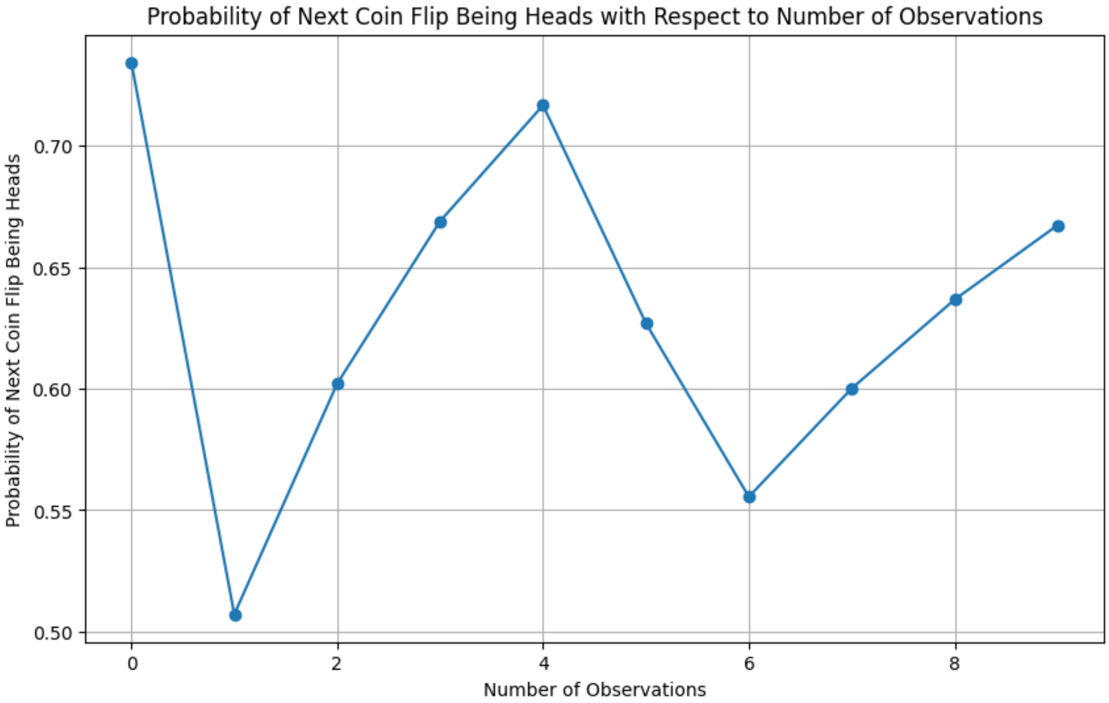
```
import numpy as np
import matplotlib.pyplot as plt

# Define the hypotheses and their probabilities
hypotheses = np.array([0, 0.15, 0.3, 0.45, 0.6, 0.75, 0.9, 1])
coin_flips = np.array([1, 0, 1, 1, 1, 0, 0, 1, 1, 1])

# Calculate the likelihood for the first five coin flips
first_five_flips = coin_flips[:5]
n_heads = np.sum(first_five_flips)
n_tails = len(first_five_flips) - n_heads

likelihoods = hypotheses**n_heads * (1 - hypotheses)**n_tails

# Print the likelihoods for the first five coin flips
for i, h in enumerate(hypotheses):
    print(f"H{i+1}: Likelihood = {likelihoods[i]:.5f}")

# Calculate the posterior probabilities for all coin flips
n_heads_total = np.cumsum(coin_flips)
n_tails_total = np.arange(1, len(coin_flips) + 1) - n_heads_total

posteriors = np.zeros((len(hypotheses), len(coin_flips)))

for i, h in enumerate(hypotheses):
    posteriors[i] = h**n_heads_total * (1 - h)**n_tails_total

# Normalize the posteriors
posteriors /= posteriors.sum(axis=0)

# Plot the posterior probabilities
plt.figure(figsize=(10, 6))
for i, h in enumerate(hypotheses):
    plt.plot(posteriors[i], label=f'H{i+1} ({h*100:.0f}%)')
plt.xlabel('Number of Observations')
plt.ylabel('Posterior Probability')
plt.title('Posterior Probability of Each Hypothesis')
plt.legend()
plt.grid(True)
plt.show()
```

```
H1: Likelihood = 0.00000
H2: Likelihood = 0.00043
H3: Likelihood = 0.00567
H4: Likelihood = 0.02255
H5: Likelihood = 0.05184
H6: Likelihood = 0.07910
H7: Likelihood = 0.06561
H8: Likelihood = 0.00000
```



[2]:

```
# Calculate the probability that the next coin flip is heads
prob_next_heads = np.sum(posteriors * hypotheses[:, np.newaxis], axis=0)

# Plot the probability that the next coin flip is heads
plt.figure(figsize=(10, 6))
plt.plot(prob_next_heads, marker='o')
plt.xlabel('Number of Observations')
plt.ylabel('Probability of Next Coin Flip Being Heads')
plt.title('Probability of Next Coin Flip Being Heads with Respect to Number of␣
 ↪Observations')
plt.grid(True)
plt.show()
```



The most likely hypothesis after all observations are made can be determined by finding the hypothesis with the highest posterior probability at the last observation.

Given the `posteriors` array, we can find the index of the maximum value in the last column and use it to identify the corresponding hypothesis.

[3]:

```
# Find the index of the maximum posterior probability at the last observation
most_likely_hypothesis_index = np.argmax(posteriors[:, -1])

# Get the most likely hypothesis
most_likely_hypothesis = hypotheses[most_likely_hypothesis_index]
print(f"The most likely hypothesis after all observations is:␣
 ↪{most_likely_hypothesis}")
```

```
The most likely hypothesis after all observations is: 0.75
```

[ ]:

```

```