Homework 2 - Introduction to Probabilistic Graphical Models

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1 Conditional Independence

1. (10 points) State True or False, and briefly justify your answer within 3 lines. The statements are either direct consequences of theorems in Koller and Friedman (2009, Ch. 3), or have a short proof. In the follows, P is a distribution and G is a BN structure.

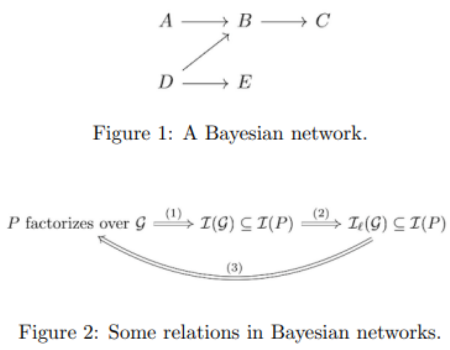

Figure 1: Caption for the image

2. Recall the definitions of local and global independences of G and independences of P.

$$I_l(G) = \{(X \perp \text{NonDescendants}_G(X) \mid \text{Parents}_G(X))\} \quad (1)$$
$$I(G) = \{(X \perp Y \mid Z) : \text{d-separated}_G(X, Y \mid Z)\} \quad (2)$$
$$I(P) = \{(X \perp Y \mid Z) : P(X, Y \mid Z) = P(X \mid Z)P(Y \mid Z)\} \quad (3)$$

(a) In Figure 2, relation 1 is true.
(b) In Figure 2, relation 2 is true.
(c) In Figure 2, relation 3 is true.
(d) If G is an I-map for P, then P may have extra conditional independencies than G.
(e) Two BN structures G_1 and G_2 are I-equivalent if they have the same skeleton and the same set of v-structures.
(f) If G_1 is an I-map of distribution P, and G_1 has fewer edges than G_2, then G_2 is not a minimal I-map of P.
(g) The P-map of a distribution, if it exists, is unique.

2 Exact Inference (Junction Tree a.k.a Clique Tree)

1. Consider the following Bayesian network G:

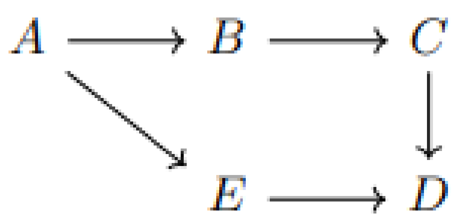

Figure 2: Caption for the image

2. We are going to construct a junction tree T from G. Please sketch the generated objects in each step.

(a) (4 points) Moralize G to construct an undirected graph H.
(b) (7 points) Triangulate H to construct a chordal graph H*. (Although there are many ways to triangulate a graph, for the ease of grading, please try adding fewest additional edges possible.)
(c) (7 points) Construct a cluster graph U where each node is a maximal clique Ci from H* and each edge is the sepset $S_{i,j} = C_i \cap C_j$ between adjacent cliques C_i and C_j.
(d) (7 points) The junction tree T is the maximum spanning tree of U. (The cluster graph is small enough to calculate maximum spanning tree in one's head.)