

Homework 2 - Introduction to Probabilistic Graphical Models

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1 Conditional Independence

- (10 points) State True or False, and briefly justify your answer within 3 lines. The statements are either direct consequences of theorems in Koller and Friedman (2009, Ch. 3), or have a short proof. In the follows, P is a distribution and G is a BN structure.

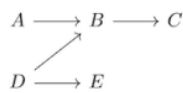


Figure 1: A Bayesian network.

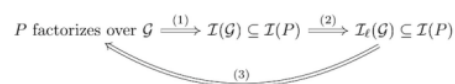


Figure 2: Some relations in Bayesian networks.

Figure 1: Caption for the image

- Recall the definitions of local and global independences of G and independences of P .

$$I_l(G) = \{(X \perp \text{NonDescendants}_G(X) \mid \text{Parents}_G(X))\} \quad (1)$$

$$I(G) = \{(X \perp Y \mid Z) : \text{d-separated}_G(X, Y \mid Z)\} \quad (2)$$

$$I(P) = \{(X \perp Y \mid Z) : P(X, Y \mid Z) = P(X \mid Z)P(Y \mid Z)\} \quad (3)$$

- In Figure 1, relation 1 is true.
- In Figure 1, relation 2 is true.
- In Figure 1, relation 3 is true.
- If G is an I-map for P , then P may have extra conditional independencies than G .
- Two BN structures G_1 and G_2 are I-equivalent if they have the same skeleton and the same set of v-structures.
- If G_1 is an I-map of distribution P , and G_1 has fewer edges than G_2 , then G_2 is not a minimal I-map of P .
- The P-map of a distribution, if it exists, is unique.