Homework 2 - Applied Stochastic Processes

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1. Discrete Random Variables and Real Life Applications (16 Points)

   (a) A factory produces electronic components, with each component either passing a quality check or being rejected. Let X represent the number of components that pass out of 5 tested components in a day, where the probability of each component passing the quality check is p = 0.8.

      i. (a) (2 points): Define the probability mass function (PMF) for X as a binomial distribution. Show that the total probability is 1 by summing the PMF over all possible values of X. PMF for X as a binomial distribution is given by:

$$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$$

      where $n = 5$ and $p = 0.8$. The total probability is 1:

$$\sum_{k=0}^{5} P(X = k) = \sum_{k=0}^{5} \binom{5}{k} 0.8^k (1 - 0.8)^{5-k} = 1$$

$$\sum_{k=0}^{5} P(X = k) = 1$$

$$\binom{5}{0}0.8^0(1-0.8)^5+\binom{5}{1}0.8^1(1-0.8)^4+\binom{5}{2}0.8^2(1-0.8)^3+\binom{5}{3}0.8^3(1-0.8)^2+\binom{5}{4}0.8^4(1-0.8)^1+\binom{5}{5}0.8^5(1-0.8)^0 = 1$$

      ii. (3 points): Suppose the factory wants to predict the likelihood of a specific number of components passing the quality check. Find the expected value and variance of X, and explain how the factory can use this information to estimate daily production quality. The expected value of X is given by:

$$E(x) = np = \text{number of trials} \times \text{probability of success}$$

$$E(X) = np = 5 \times 0.8 = 4$$

      The variance of X is given by:

$$Var(X) = np(1-p) = \text{number of trials} \times \text{probability of success} \times \text{probability of failure}$$

$$Var(X) = np(1-p) = 5 \times 0.8 \times 0.2 = 0.8$$