Homework 1 - Introduction to Machine Learning for Engineers

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1 Probability

1. Suppose W is a Gaussian random variable with distribution $N(\mu, \sigma^2)$ and U a uniform random variable over the interval $[a, b]$. Assuming that W and U are independent, what is the expected value $\mathbb{E}[Z]$ and variance $\text{Var}[Z]$ of $Z = 3W + 2U$?

- **Expected value:**
$$\mathbb{E}[Z] = \mathbb{E}[3W + 2U] = 3\mathbb{E}[W] + 2\mathbb{E}[U]$$

  Since W is Gaussian with mean μ and U is uniform over $[a, b]$ with mean $\frac{a+b}{2}$:

$$\mathbb{E}[Z] = 3\mu + 2\left(\frac{a+b}{2}\right) = \mathbf{3\mu + (a + b)}$$

- **Variance:** if a random variable X, is scaled by a constant a, then the variance of the scaled random variable is a^2 times the variance of the original random variable. Therefore:

$$\text{Var}[Z] = \text{Var}[3W + 2U] = 3^2\text{Var}[W] + 2^2\text{Var}[U]$$

  Since W is Gaussian with variance σ^2 and U is uniform over $[a, b]$ with variance $\frac{(b-a)^2}{12}$:

$$\text{Var}[Z] = 9\sigma^2 + 4\left(\frac{(b-a)^2}{12}\right) = 9\sigma^2 + \frac{(b-a)^2}{3}$$