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Homework 3

1. $x(n) \rightarrow \boxed{\uparrow 2} \rightarrow$: $x_1(n) = [\cdots 0\ 0\ \underline{1}\ 0\ 2\ 0\ 3\ 0\ 2\ 0\ 1\ 0\ 0\ 0 \cdots]$
$\rightarrow \boxed{H(z)} \rightarrow$: $x_1(n) * h(n) = [\cdots 0\ \underline{1}\ 2\ 2\ 4\ 3\ 6\ 2\ 4\ 1\ 2\ 0\ 0 \cdots]$
$\rightarrow \boxed{\downarrow 3}$: $y(n) = [\cdots 0\ 1\ 4\ 2\ 2\ 0\ 0 \cdots]$

2. $x(n) \rightarrow \boxed{\uparrow 2} \rightarrow$: $x_1(n) = [\cdots 0\ 0\ \underline{1}\ 0\ 2\ 0\ -1\ 0\ 0\ 0\ 1\ 0\ 0\ 0 \cdots]$
$\rightarrow \boxed{H(z^2)} \rightarrow$: $H(z^2) = 1 + z^{-2} \Rightarrow h'(n) = [\underline{1}\ 0\ 1]$
$y(n) = x_1(n) * h'(n) = [\cdots 0\ 0\ \underline{1}\ 0\ 3\ 0\ 1\ 0\ -1\ 0\ 1\ 0\ 1\ 0 \cdots]$

3. $X(z) \rightarrow \boxed{\downarrow M} \rightarrow \frac{1}{M}\sum_{k=0}^{M-1} X(W^k z^{\frac{1}{M}}) \rightarrow \boxed{\uparrow M} \rightarrow \frac{1}{M}\sum_{k=0}^{M-1} X(W^k z) = Y(z)$
when $M=2$, $Y(z) = \frac{1}{2}[X(z) + X(-z)]$.

4. $H_0(z^2) = \frac{1}{2}[H(z) + H(-z)] = \frac{1}{2}\left(\frac{1}{1-cz^{-1}} + \frac{1}{1+cz^{-1}}\right) = \frac{1}{1-c^2z^{-2}}$
$\Rightarrow H_0(z) = \frac{1}{1-c^2z^{-1}}$
$H_1(z^2) = \frac{z}{2}[H(z) - H(-z)] = \frac{c}{1-c^2z^{-2}}$
$\Rightarrow H_1(z) = \frac{c}{1-c^2z^{-1}}$
They each has 1 pole at $z = c^2$.

5. If $h(n)$ is odd length, i.e. Type 1 or 3 FIR filter, its two polyphase components are also linear-phase. Specifically, components of type 1 filter will be type 1 or 2 filters, and components of type 3 filter will be type 3 or 4 filters.
If $h(n)$ is even length, i.e. Type 2 or 4 FIR filter, the compoments would not be linear-phase because they lost symmetry properties.

6. $N=16 \Rightarrow K=4$. $H(z) = \sum_{n=0}^{15} z^{-n} = 1 + z^{-1} + z^{-2} + \cdots + z^{-15}$
$= (1+z^{-1})(1+z^{-2}+z^{-4}+\cdots+z^{-14})$
$= (1+z^{-1})(1+z^{-2})(1+z^{-4}+z^{-8}+z^{-12})$
$= (1+z^{-1})(1+z^{-2})(1+z^{-4})(1+z^{-8})$
$h(n) = \sum_{k=0}^{15} \delta(n-k)$
factor-16 decimator: $\rightarrow \boxed{H(z) = (1+z^{-1})(1+z^{-2})(1+z^{-4})(1+z^{-8})} \rightarrow \boxed{\downarrow 16} \rightarrow$
use noble identity: $(1+z^{-1})(1+z^{-2})(1+z^{-4}) \rightarrow \boxed{\downarrow 8} \rightarrow \boxed{1+z^{-1}} \rightarrow \boxed{\downarrow 2} \rightarrow$
$\vdots$
$\rightarrow \boxed{1+z^{-1}} \rightarrow \boxed{\downarrow 2} \rightarrow \boxed{1+z^{-1}} \rightarrow \boxed{\downarrow 2} \rightarrow \boxed{1+z^{-1}} \rightarrow \boxed{\downarrow 2} \rightarrow \boxed{1+z^{-1}} \rightarrow \boxed{\downarrow 2} \rightarrow$

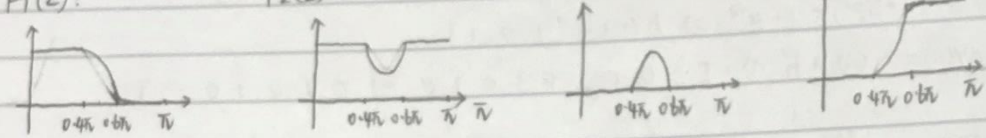
7. a) $F_1(z) = H(z)H(z^2)$

$F_2(z) = H(z)G(z^2)$

$F_3(z) = G(z)H(z^2)$

$F_4(z) = G(z)G(z^2)$

b) $F_1(z)$: $F_2(z)$: $F_3(z)$: $F_4(z)$:



c) $F_1(z)$: lowpass. $F_4(z)$: highpass.

$F_2(z)$: bandstop $F_3(z)$: bandpass.

8. a) Not equivalant

Let $x(n) = [1\ 2\ 3\ 4]$

$x(n) \rightarrow \boxed{A} \rightarrow [1\ 0\ 3\ 0]$

$x(n) \rightarrow \boxed{B} \rightarrow [1\ 2\ 3\ 4]$

b) Not equivalant

Let $x(n) = [1\ 2\ 3\ 4\ 5]$

$x(n) \rightarrow \boxed{A} \rightarrow [1\ 0\ 0\ 3\ 0\ 0\ 5\ 0\ 0]$

$x(n) \rightarrow \boxed{B} \rightarrow [1\ 0\ 0\ 3\ 0\ 0\ 5\ 0]$

c) Not equivalant

Let $x(n) = [1\ 2\ 3\ 4]$

$x(n) \rightarrow \boxed{A} \rightarrow [1\ 0\ 0\ 0\ 3\ 0\ 0\ 0]$

$x(n) \rightarrow \boxed{B} \rightarrow [1\ 0\ 2\ 0\ 3\ 0\ 4\ 0]$

9. a) $\rightarrow \boxed{\downarrow 2} \rightarrow \boxed{\downarrow 3} \rightarrow \Leftrightarrow \rightarrow \boxed{\downarrow 6} \rightarrow$

b) $\rightarrow \boxed{\uparrow 2} \rightarrow \boxed{\uparrow 2} \rightarrow \Leftrightarrow \rightarrow \boxed{\uparrow 4} \rightarrow$

c) $\rightarrow \boxed{\uparrow 5} \rightarrow \boxed{\downarrow 10} \rightarrow \boxed{\uparrow 2} \rightarrow \Leftrightarrow \rightarrow \boxed{\uparrow 5} \rightarrow \boxed{\uparrow 4} \rightarrow \boxed{\downarrow 10} \rightarrow$

$\Leftrightarrow \rightarrow \boxed{\uparrow 20} \rightarrow \boxed{\downarrow 10} \rightarrow \Leftrightarrow \rightarrow \boxed{\uparrow 2} \rightarrow$

10. $x(n) \rightarrow \downarrow 2 \rightarrow H(z) \rightarrow \downarrow 5 \rightarrow G(z) \rightarrow \uparrow 3 \rightarrow y(n)$

$x(n) \rightarrow H(z^2) \rightarrow \downarrow 2 \rightarrow \downarrow 5 \rightarrow \uparrow 3 \rightarrow G(z^{\frac{1}{2}}) \rightarrow y(n)$

$x(n) \rightarrow H(z^2) \rightarrow \downarrow 10 \rightarrow \uparrow 3 \rightarrow G(z^{\frac{1}{2}}) \rightarrow y(n)$

$x(n) \rightarrow H(z^2) \rightarrow \uparrow 30 \rightarrow \downarrow 10 \rightarrow G(z^{\frac{1}{2}}) \rightarrow y(n)$

$x(n) \rightarrow H(z^2) \rightarrow \uparrow 3 \rightarrow G(z^{\frac{1}{2}}) \rightarrow y(n)$

$x(n) \rightarrow \uparrow 3 \rightarrow H(z^6) \rightarrow G(z^{\frac{1}{2}}) \rightarrow y(n)$

$\therefore M=3,\ N=1$

$LTI = H(z^6)\,G(z^{\frac{1}{2}})$

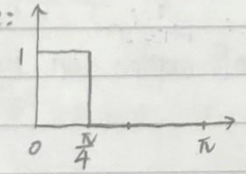
11. $f_0 = 12$ (samples/s)

$f_n = 9$ (samples/s) $= \frac{3}{4} f_0$

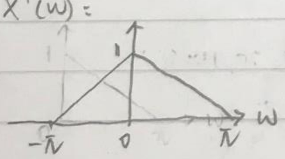
$\rightarrow \boxed{\uparrow 3} \rightarrow \boxed{H(z)} \rightarrow \boxed{\downarrow 4} \rightarrow$

cut-off frequency $w_0 = \min\left(\frac{\pi}{3}, \frac{\pi}{4}\right) = \frac{\pi}{4}$

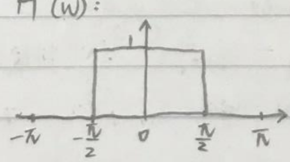
frequency response:



12. $X^f(w)$:



$H^f(w)$:



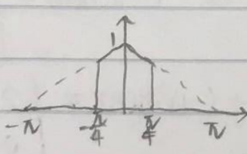
$x(n) \rightarrow \boxed{H(z)} \rightarrow \boxed{\downarrow 2} \rightarrow \boxed{H(z)} \rightarrow \boxed{\downarrow 2} \xrightarrow{s(n)} \boxed{\uparrow 4} \rightarrow y(n)$

$\Updownarrow$

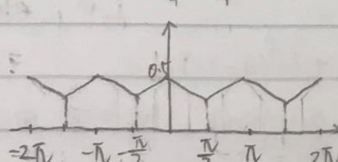
$x(n) \rightarrow \boxed{H(z)\,H(z^2)} \rightarrow \boxed{\downarrow 4} \xrightarrow{s(n)} \boxed{\uparrow 4} \rightarrow y(n)$

$X^f(w)\,H^f(w)\,H^f(2w)$

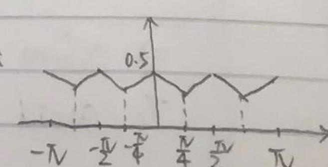
$= \left(1 - \frac{|w|}{\pi}\right) \cdot \begin{cases} 1, & |w| < \frac{\pi}{4} \\ 0, & \text{else} \end{cases}$



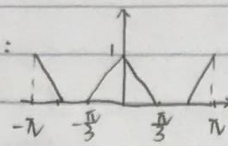
$S^f(w)$:



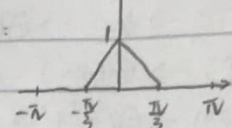
$\Rightarrow Y^f(w)$:



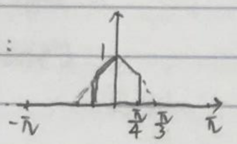
13. a) $\rightarrow \boxed{\uparrow 3} \rightarrow$:



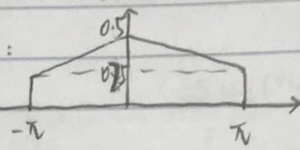
$\rightarrow \boxed{H(z)} \rightarrow$:



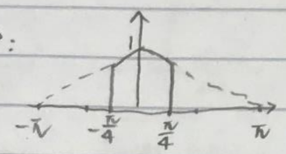
$\rightarrow \boxed{G(z)} \rightarrow$:



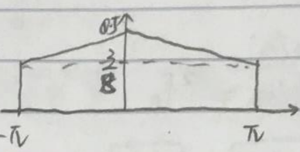
$\rightarrow \boxed{\downarrow 4} \rightarrow$:



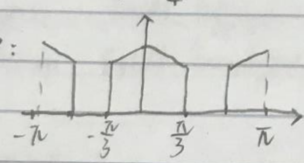
b) $\rightarrow \boxed{G(z)} \rightarrow$:



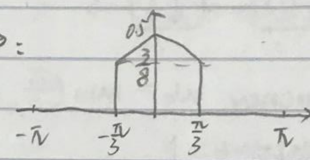
$\rightarrow \boxed{\downarrow 4} \rightarrow$:



$\rightarrow \boxed{\uparrow 3} \rightarrow$:



$\rightarrow \boxed{H(z)} \rightarrow$:



c) A is better, for B lost too much information on high frequency.

14. $x(n) \rightarrow \boxed{\uparrow 2} \rightarrow v(n) = \begin{cases} x(n) & n \text{ is even} \\ 0 & n \text{ is odd} \end{cases}$

half-band $h(n) = \begin{cases} 1 & n=0 \\ 0 & n\neq 0, n \text{ is even} \\ h(n) & n \text{ is odd.} \end{cases}$

$v(n) * h(n) = \begin{cases} x(0) & n=0 \\ h(n) & n \text{ is odd} \\ x(n) & n \text{ is even}, n\neq 0 \end{cases}$

$\Rightarrow [\downarrow 2]\ y(n) = x(n)$

15. $H^f(\pi - w) \leftrightarrow e^{j\pi n} h(n) = (-1)^n h(n) =$

$h(n) = \begin{cases} 1, & n=0 \\ 0, & n \text{ even} \\ h_1(n), & n\neq 0, \text{odd} \end{cases} \Rightarrow (-1)^n h(n) = \begin{cases} 1, & n=0 \\ 0, & n \text{ even} \\ -h_1(n), & n\neq 0, \text{odd} \end{cases}$

$H^f(w) + H^f(\pi - w) \leftrightarrow h(n) + (-1)^n h(n) = \begin{cases} 2, & n=0 \\ 0, & \text{else} \end{cases} = 2\delta(n)$

$\Rightarrow 2\delta(n) \leftrightarrow 2 = H^f(w) + H^f(\pi - w)$

To avoid aliasing and gapping, $w_0 = \frac{\pi}{2}$ such that $H^f(w_0) = 1$.

16. a) $H(z) = \frac{1}{2}(1+z^{-1})$

b) $H(z) = \frac{1}{4}(1+2z^{-1}+z^{-2}) = \frac{1}{4}(1+z^{-1})^2$

c) $H(z) = \frac{1}{8}(1+3z^{-1}+3z^{-2}+z^{-3}) = \frac{1}{8}(1+z^{-1})^3$

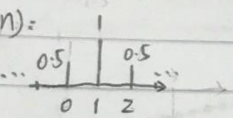
As shown in the output signals (attached), they preserved poles of the original signal x. It is because all of the filters are forms of $Q(z)(1+z^{-1})^{d+1}$.

17. $d=1$: $H(z) = Q(z)(1+z^{-1})^2 = (1+2z^{-1}+z^{-2})Q(z) \leftrightarrow h(n) = q(n)+2q(n-1)+q(n-2)$

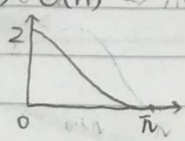
q(n) should be length of 3 $\Rightarrow$ $\left.\begin{array}{l} q(0)+2q(1)+q(2) = 2q(0)+2q(1) = 1 \\ q(1) = 0 \end{array}\right\} \Rightarrow q = 0.5\delta(n-1)$

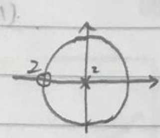
$\Rightarrow h(n) = [0.5, 1, 0.5] \Rightarrow h(2n+1) = \delta(n)$

$h(n)$:



$|H^f(w)|$:

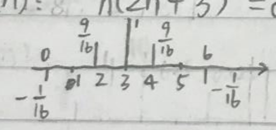




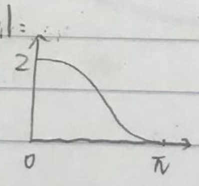
$d=3$: $H(z) = Q(z)(1+z^{-1})^4 \leftrightarrow h(n) = q(n)+4q(n-1)+6q(n-2)+4q(n-3)+1q(n-4)$

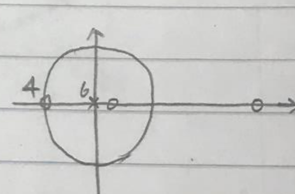
q(n) should be length of 3: $\left.\begin{array}{l} 4q(0)+6q(1)+4q(2) = 1 \\ 4q(0)+q(1) = 0 \\ q(0) = q(2) \end{array}\right\} \Rightarrow q = [-\frac{1}{16}, \frac{1}{4}, -\frac{1}{16}]$

$\Rightarrow h(n)$: $h(2n+3) = \delta(n)$

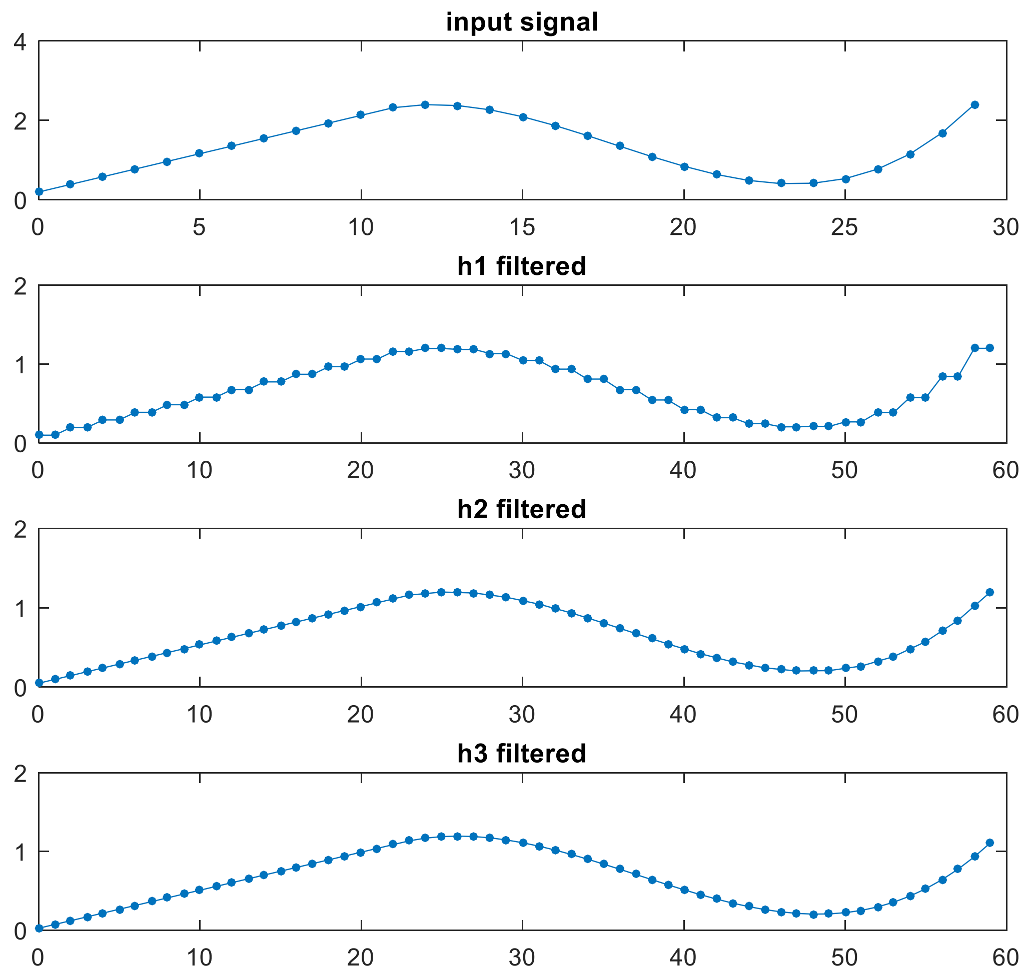


$|H^f(w)|$:





16. interpolated signals



17. d=5 case: h(2n+5)=imp(n)

```
h =

    0.0117   -0.0000   -0.0977   -0.0000    0.5859    1.0000    0.5859    0.0000   -0.0977   -0.0000    0.0117
```