長庚大學期中、期末考試答案用紙

1-8 學年度 第 二 學期 中 考 資 工 系 姓名 [redacted] 學號 [redacted]

一、

1. $\frac{1}{10}$ 紅 $\frac{9}{10}$ 不紅

$\sum_{x=0}^{10} (x; 10, \frac{1}{10})$

$= C_x^{10} (\frac{1}{10})^x \cdot (\frac{9}{10})^{10-x}$

$f_X(x)$:
- $x=0$ $b(0, 10, \frac{1}{10}) \doteqdot 0.3487$
- $x=1$ $b(1, 10, \frac{1}{10}) \doteqdot 0.3874$
- $x=2$ $b(2, 10, \frac{1}{10}) \doteqdot 0.1937$
- $x=3$ $b(3, 10, \frac{1}{10}) \doteqdot 0.0574$
- $x=4$ $b(4, 10, \frac{1}{10}) \doteqdot 0.0112$
- $x=5$ $b(5, 10, \frac{1}{10}) \doteqdot 0.0015$
- $x=6$ $b(6, 10, \frac{1}{10}) \doteqdot 0.0001$
- $x=7$ $b(7, 10, \frac{1}{10}) \doteqdot 0$
- $x=8$ $b(8, 10, \frac{1}{10}) \doteqdot 0$
- $x=9$ $b(9, 10, \frac{1}{10}) \doteqdot 0$
- $x=10$ $b(10, 10, \frac{1}{10}) \doteqdot 0$
- $0, \quad x \in R \setminus S$

2. x 的期望值 $n \cdot p = 10 \cdot \frac{1}{10} = 1$

3. x 的標準差 $\sigma^2 = np(1-p) = 10 \cdot \frac{1}{10} \cdot \frac{9}{10} = \frac{9}{10}$

$\sigma = \frac{\sqrt{9}}{\sqrt{10}} = 0.9487$

4. 取 10 個不放回

$$f(x) \begin{cases} \sum_{x=0}^{10} \dfrac{C_x^{10} \cdot C_{10-x}^{90}}{C_{10}^{100}} & \text{有 } x \text{ 個紅球} \\ 0, & x \in R \setminus S \end{cases}$$

5. $E[Y] + std[Y] = E[x] + std[x] = 1.9487$

負二項分布

6. $b^*(x; k, p) = \binom{x-1}{k-1} p^k q^{x-k}$

$b^*(z; 5, 0.1) = \binom{z-1}{4} 0.1^5 \, 0.9^{z-5}$

$$f_Z(z) \begin{cases} \sum_{z=5}^{95} \binom{z-1}{4} 0.1^5 \cdot 0.9^{z-5} \\ 0, \quad z \in R \setminus S \end{cases}$$

二、

1. $\lambda = 1 \quad t = 100$

$\mu = \lambda t = 1 \cdot 100 = 100$

$p(x; \lambda t) \; \frac{e^{-\lambda t} (\lambda t)^x}{x!}, \quad x = 0, 1, 2, \cdots$

$f_W(w) = p(w; \lambda t) = \frac{e^{-100} \cdot 100^w}{w!}, \quad w = 0, 1, 2 \cdots$

2. $E(w) + std[w] = \lambda t + \sqrt{\lambda t} = 100 + 10 = 110$

3. $p(|W - 100| \le 2 \cdot 10) = p(98 \le w \le 102) \; \sum_{w=0}^{102} P(w; 100) - \sum_{w=0}^{98} P(w, 100)$

$= 0.6047 - 0.4074 = 0.1973$

4. $P(W>120) = 1 - P(W \le 120) = 1 - \sum_{W=0}^{120} P(120, 100) = 1 - 0.6047 = 0.3953$

5. 拒絕他 $\lambda \cdot t = 100 \to$ 期望期

當"$W>120$"時常發生時，"$W<80$"並無時常發生，"$W>120$"時常發生時的期望值也應 >120，與100矛盾

三、

$p = 0.05$, $n = 100$

1. 超出10個 $\to >9$

$P(x>9) = \binom{100}{x}(0.05)^x \cdot (0.95)^{100-x}$ $x = 10, 11, 12, \ldots$ $P(x>9)$

$= 1 - \binom{100}{x}(0.05)^x \cdot (0.95)^{100-x}$ $x = 0, 1, 2, 3 \cdots 9$ 全部 1 − $x = 0 \sim 9$ 的

$= 1 - 0.9718$

$= 0.0282$ #

2. 接受，雖然機率很小(0.0282)，但還是有發生的可能

四、 $b(x; n, p)$ $np = \mu$ $p = \frac{\mu}{n}$

$= \binom{n}{x} p^x (1-p)^{n-x}$ $\binom{n}{x} = \frac{n!}{x!(n-x)!}$

$= \binom{n}{x} \left(\frac{\mu}{n}\right)^x \left(1 - \left(\frac{\mu}{n}\right)\right)^{n-x}$

$= \left(\frac{n!}{x!(n-x)!}\right) \left(\frac{\mu}{n}\right)^x \left(1 - \left(\frac{\mu}{n}\right)\right)^{n-x}$

$= \frac{\mu^x}{x!} \frac{n!}{(n-x)!} \cdot \frac{1}{n^x} \left(1 - \left(\frac{\mu}{n}\right)\right)^{n-x}$

$= \frac{\mu^x}{x!} \left(1 - \frac{1}{n}\right)\left(1 - \frac{2}{n}\right) \cdots \left(1 - \frac{x+1}{n}\right)\left(1 - \frac{\mu}{n}\right)^{n-x}$

$= \frac{\mu^x}{x!} \left(1 - \frac{1}{n}\right)\left(1 - \frac{2}{n}\right) \cdots \left(1 - \frac{x+1}{n}\right)\left(1 - \frac{\mu}{n}\right)^{n}\left(1 - \frac{\mu}{n}\right)^{-x}$

$\Rightarrow \lim_{n \to \infty} \binom{n}{x} p^x (1-p)^{n-x}$

$= \frac{\mu^x}{x!} \cdot \lim_{n \to \infty} \overset{\to 1}{\left(1 - \frac{1}{n}\right)} \overset{\to 1}{\left(1 - \frac{2}{n}\right)} \cdots \overset{\to}{\left(1 - \frac{x+1}{n}\right)} \cdot \overset{= e^{-\mu}}{\lim_{n \to \infty} \left(1 - \frac{\mu}{n}\right)^n} \cdot \overset{\to 1}{\lim_{n \to \infty} \left(1 - \frac{\mu}{n}\right)^{-x}}$

$= \frac{\mu^x}{x!} e^{-\mu}$