Master Theorem

Test methods

Yet another test method

$$T(n) = aT\left(\frac{n}{b}\right) + \Theta(n^c log_b{}^p n)$$

$a \geq 1, b > 1, c \geq 0,$ and $p\ is\ some\ real\ number$

$Case\ 1: if\ \dfrac{a}{b^c} > 1, then\ T(n) = \Theta\left(n^{\log_b a}\right)$

$Case\ 2: if\ \dfrac{a}{b^c} = 1, and$

i. $if\ p < -1, then\ T(n) = \Theta\left(n^{\log_b a}\right)$

ii. $if\ p = -1, then\ T(n) = \Theta\left(n^{\log_b a} \times \log_b \log_b n\right)$

iii. $if\ p > -1, then\ T(n) = \Theta\left(n^{\log_b a} \times log_b{}^{p+1}\ n\right)$

Yet another test method

$$T(n) = aT\left(\frac{n}{b}\right) + \Theta(n^c {log_b}^p n)$$

$a \geq 1, b > 1, c \geq 0,$ and $p\ is\ some\ real\ number$

$Case\ 3: if\ \frac{a}{b^c} < 1, and$

i. $if\ p < 0, then\ T(n) = O(n^c)$

ii. $if\ p \geq 0, then\ T(n) = \Theta(n^c \times {log_b}^p\ n)$

Solve the following recurrence equation using Master Theorem (if possible)

- $T(n) = 3T(^{n}/_{2}) + n^2$
- $T(n) = 4T\left(\frac{n}{2}\right) + n^2$
- $T(n) = 16T\left(\frac{n}{4}\right) + n$
- $T(n) = 2T\left(\frac{n}{2}\right) + n \lg n$
- $T(n) = 0.5T\left(\frac{n}{2}\right) + \frac{1}{n}$
- $T(n) = 64T\left(\frac{n}{8}\right) - n^2 \lg n$

$$T(n) = 3T\left(\frac{n}{2}\right) + n^2$$

$$a = 3, b = 2, c = 2, p = 0$$

$$\frac{a}{b^c} = \frac{3}{2^2} < 1, and\ p = 0, then$$

$$T(n) = \Theta(n^c log_b{}^p n) = \Theta\left(n^2 log_2{}^0 n\right) = \Theta(n^2)$$

$$T(n) = 4T\left(\frac{n}{2}\right) + n^2$$

$$a = 4, b = 2, c = 2, p = 0$$

$$\frac{a}{b^c} = \frac{4}{2^2} = 1, and\ p > -1, then$$

$$T(n) = \Theta\left(n^{\log_b a} log_b{}^{p+1} n\right)$$

$$= \Theta\left(n^{\log_2 4} log_2{}^{1} n\right) = \Theta(n^2 \log_2 n)$$

$$T(n) = 16T\left(\frac{n}{4}\right) + n$$

$$a = 16, b = 4, c = 1, p = 0$$

$$\frac{a}{b^c} = \frac{16}{4^1} > 1, then$$

$$T(n) = \Theta\left(n^{\log_b a}\right)$$

$$= \Theta\left(n^{\log_2 4}\right) = \Theta(n^2)$$

$$T(n) = 2T\left(\frac{n}{2}\right) + n\log_2 n$$

$a = 2, b = 2, c = 1, p = 1$

$\frac{a}{b^c} = \frac{2}{2^1} = 1, and\ p > -1\ then$

$$T(n) = \Theta\left(n^{\log_b a} {log_b}^{p+1} n\right)$$
$$= \Theta\left(n^{\log_2 2} {log_2}^{2} n\right) = \Theta\left(n {log_2}^{2} n\right)$$

$$T(n) = 0.5T\left(\frac{n}{2}\right) + \frac{1}{n}$$

- $a = 0.5, b = 2, c = -1, f(n) = \frac{1}{n}$
- a is less than 1, hence master theorem cannot be applied to determine the running time complexity of this recurrence relation. Expansion and substitution will be used instead.

$$T(n) = 64T\left(\frac{n}{8}\right) - n^2 \log_2 n$$

$a = 64, b = 8, c = 2, p = 1$

$\frac{a}{b^c} = \frac{64}{8^2} = 1, and\ p > -1\ then$

$$T(n) = \Theta\left(n^{\log_b a} {log_b}^{p+1} n\right)$$

$$= \Theta\left(n^{\log_8 64} {log_8}^{2} n\right) = \Theta\left(n^2 {log_8}^{2} n\right)$$