Garch and std-NTS Fit

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1 Introduction

In the field of financial forecasting, there has been major upheaval. Many of the most popular models were based on assumptions that returns follow the normal distribution. However, the observance of semi-frequent financial meltdowns leads one to consider new ways of forecasting volatility for asset returns. We will consider using General AutoRegressive Conditional Heteroskedastic (GARCH) processes, coupled with a Standard Normal Tempered Stable distribution in predicting future residuals of Microsoft stock.

2 GARCH

2.1 Describing the Model

It is sufficient to say that the daily volatility of asset prices follows a distinct pattern. When the volatility of a stock is high on one day, it is likely for it to be high the next day as well. When volatility is low, it is usually low for the next day. This is referred to as volatility clustering and is what leads us to consider the GARCH model, given that it uses the volatility and residual of the previous p, q days respectively, we can create a model that reflects the pattern observed in financial markets.
The GARCH(p,q) model is given as:

$$r_t = \mu_t + \epsilon_t \tag{1}$$

$$\epsilon_t = \sigma_t z_t \tag{2}$$

$$\sigma_t^2 = \omega + \alpha_1 \epsilon_{t-1} + ... + \alpha_p \epsilon_{t-p} + \beta_1 \sigma_{t-1}^2 + ... + \beta_q \sigma_{t-q}^2 \tag{3}$$

where r_t is the log return (at time t)
μ is the mean value of the returns
σ_t is the conditional volatility
ϵ_t is the residual
e_t is standardized residual
ω, α, β are all coefficients of the GARCH model.

2.2 Fitting the GARCH Model

We will use the returns of Microsoft stock (MSFT) for dates between January, 1st 2014, and February 1st, 2020. Applying the Partial Autocorrelation function squared, we get

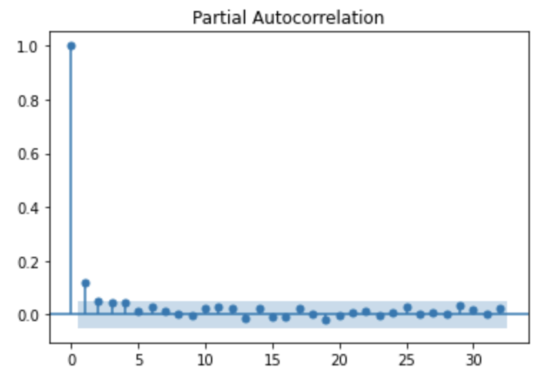

Figure 1: PACF of MSFT mean-corrected returns

We see that only a time-delta of 1 is the best for explaining mean corrected returns for our asset. Moreover, GARCH(1,1) models are generally the best given that the residual and volatility at $t-1$ are also dependant on $t-2, t-3, \ldots$. We determine the GARCH coefficients using Maximum Likelihood Estimation (MLE) using a Student's T distribution with an unspecified number of degrees of freedom ν. Our likelihood function is

$$L(\nu, \omega, \alpha_1, \beta_1 | \epsilon_1, \ldots \epsilon_t) \tag{4}$$

By MLE, we determine that for our example, $(\nu, \omega, \alpha_1, \beta_1) = (5, .1296, .1396, .8090)$.

2.3 Fitting the stdNTS to the Residuals

Having calculated the residuals using our GARCH model, we can now determine the NTS parameters. We will use the Standard NTS distribution given by the characteristic function

$$\phi_X(u) = (-\beta ui - \frac{2\theta^{1-\frac{\alpha}{2}}}{\alpha}((\theta - i\beta u + (1-\beta^2(\frac{2-\alpha}{2\theta}))\frac{u^2}{2})^{\frac{\alpha}{2}} - \theta^{\frac{\alpha}{2}})) \tag{5}$$

where $\alpha \in (0,2), |\beta| < \sqrt{\frac{2\theta}{2-\alpha}}, \theta > 0$

To obtain the coefficients of our Standard-NTS model, we assume that our returns follow the Levy process where $X_{\delta t}$ represents the mean corrected return of any day relative to the previous. Because we have a time delta of one day, we can optimize our coefficients relative to $stdNTS(\alpha, \theta, \beta)$ using MLE. In our example, we get the coefficients $(1.9990, 6.89574976, 3.61038786)$. Plugging into our characteristic function, and integrating $\frac{1}{2\pi} \int_{-100}^{100} e^{-iux}\phi(u)\,du$ for various $x \in \Re$. To compare our new NTS model to actual observed residuals, we use KDE (Kernel Density Estimate) using the normal distribution to obtain a probability density function (Fig. 2).

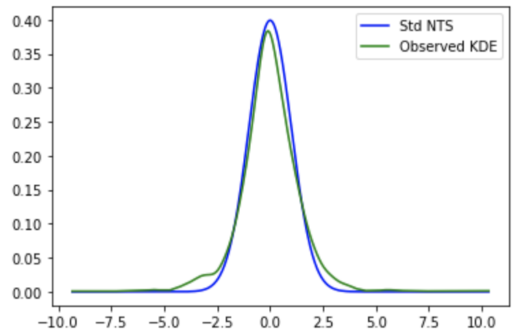

Figure 2: KDE estimate vs Std-NTS distribution

We perform goodness of fit tests by determining the CDF of both distributions (Fig:3)

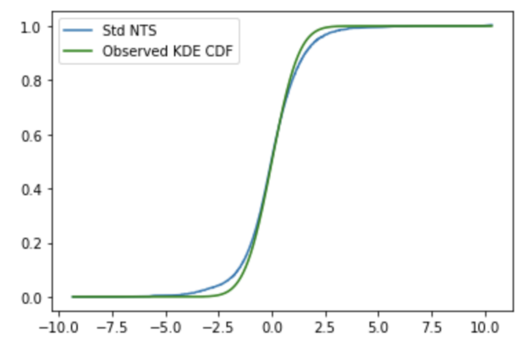

Figure 3: KDE CDF vs Std-NTS CDF estimate

Now, we perform the K-S (Kolmogorov-Smirnov) by finding the D statistic to be 0.04410. We can further examine the QQ plot by determining the quantiles for each CDF. Leaving out the 0 and 100 percentiles, we create a QQ plot using $10th, 20th, ...$ percentiles (Fig:4).

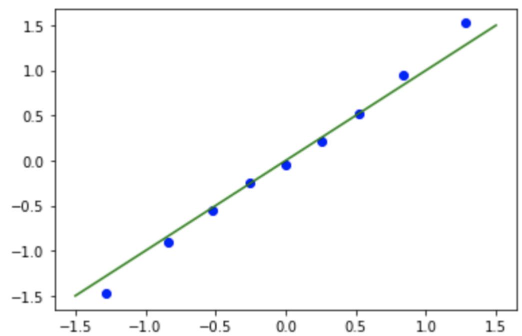

Figure 4: KDE vs Std-NTS QQ-plot vs 45°
line

2.4 Conclusion

It is evident that the STD-NTS distribution is successful in describing the mean corrected returns for MSFT stock. The model is very flexible and can explain the particularly fat-tailed, left skew properties seen within asset returns.