Superconductivity in Niobium

Kirill Klimov
Based partly on Senior Project with Hare Krishna
Dept. of Physics,Stony Brook University
(Dated: May 11, 2020)

Superconductivity is a phenomenon observed when some materials are cooled to some relatively low temperature. The result is a current that flows within the material unimpeded. Various effects can be observed in this state such as the Meissner effect. When separated by a narrow insulator, the Josephson effect can also be observed. We will give an overview of the effects and study them via experiments.

I. INTRODUCTION

Superconductivity was first observed in 1911 by Dr. Heike Onnes when he was studying the effects of ultra cold temperature on current in mercury. At around 4 Kelvin, he noted that resistance was non-existant. Ever since, the scientific community has been on the hunt to understand why that is, and how it can be applied. To this day, many elements have been found to become superconductors at certain temperatures, but not all; while other materials have been shown to exercise superconductive properties at far higher temperatures ($\sim 250K$)[11]. The relation of superconductivity in basic elements to the periodic table is, so far, not obvious, while many theories have been put in place to attempt to understand this phenomenon.

In 1962, Josephson analysed the junction made by two closely spaced superconductors, separated by an insulating barrier (typically an oxide).The Josephson junction has found an important role as amplifiers, SQUID detectors, and oscillators. In this paper, we will explain the theory behind superconductors as well as show measured results of the Josephson junction and compare them to the theoretical values. We will be studying the direct correlation between magnetic field and critical current by setting our Josephson Junction into a pool of liquid helium. We will also be studying the correlation between critical current of the junction and temperature.

II. THEORY

When electrons flow through a material, they are impeded by the resistance $R = \frac{\rho L}{A}$ of said material which is a function of resistivity ρ, length L, and cross-sectional area A. This resisistivity is a variable dependent on temperature of the material, type of material, material purity, etc. As temparature decreases, resistivity also decreases, in most materials. Exceptions include high temperature superconductors whose resistance is not uniformly monatonic. For non-super-conductors, voltage is given as $V = IR$, and power loss as a function of resistance is given by $P = I^2R$. Below some certain temperature T_C, electrons begin to condense into something called Cooper pairs. In this scenario, the electrons turn into Bosons and are able to form very weak bonds with each other due to time-symmetry. This is achieved via quantum entaglement of two electrons. When an observed system consists of an integer multiple of two electrons, the spin of the system is an integer and bosonic properties are achieved. This is why many of the following equations regarding superconductivity include a constant multiple of 2. The transition into Boson states allows for the electrons to inhabit the same energy levels unlike Fermions. However, given the uncertainty principle, temperature cannot be exactly zero. As the temperature decreases, more normal electrons begin to leave the valence shell and condense into Cooper pairs. Cooper pairs are not the only form of electrons still present in this model. Normal electrons are still accounted for. They are however bound under voltages of twice the gap voltage. As the conductor becomes concentrated with Cooper pairs, the gap voltage ($\triangle$) (voltage needed to free an electron in the valence shell into the conductor) increases.

A. Josephson Junction of Normal Metals

The following is a primer on the Josephson Junction in normal metals with an oxide and serves as an example. When constructing the Josephson junction, it is useful to know a diameter such that a relatively large amount of current can pass through. The incident and reflected waves $\Psi_1 = Ae^{ik_1x} + Be^{-ik_1x}$ must pass through a barrier/oxide region given by wave equation $\Psi_2 = Ce^{ik_2x} + De^{-ik_2x}$ and should have some transmitted wave given by $\Psi_3 = Fe^{ik_3x}$ (all of which satisfies a time independent Schroedinger equation for $\Psi * E$). The solution to minimizing the barrier reflection coefficient D leads us to the solution that the coefficient F/A (ratio of probabilities of the initial and transmitted waves traveling in same direction) is maximized when e^{-2k_2w} ($k_2 = \frac{1}{\hbar}\sqrt{2m(U-E)}$) where U is the potential of the barrier and E (energy of an electron) is maximized. Stemming from the uncertainty principle, $w\triangle k_2 \sim 1$[13] assuming all electrons successfully tunnel, we can approximate the width to be

$$w = \hbar\sqrt{\frac{2E_F}{m\psi^2}} \tag{1}$$

where $E_F \approx 1.6 * 10^{-18}J$, mass of the electron is $m = 9.1 * 10^{-31}kg$, and the barrier height is $\psi \approx 3.2 * 10^{-19}J$ is an assumed potential of our given barrier.[13]
Given this, the theoretical length of our oxide barrier should be, $w = 0.6nm$.
The phase shift $\triangle\phi = \phi_1 - \phi_2$ is very important given that it dictates whether or not any given electron can pass through the barrier or not. The current flowing through the oxide can be given by $I_s = I_c \sin(\triangle\phi)$.

B. Josephson Junction Under T_c

When a temperature lower than or equal to T_c is reached, the addition of Cooper pairs creates an energy gap $2\triangle$ in the normal electron energy spectrum. If we were to apply a voltage higher than $2\triangle$ to our junction, normal electrons will be flowing as well as the Cooper pairs. When we achieve current flow in one direction, the Niobium becomes magnetized in that particular direction, so when the current changes direction, a lower critical current is achieved until we reach a Voltage such that it is less than $-2\triangle$. This can be observed in Fig. 4.

Suppose we have two superconductors that are connected by a thin layer of insulating material as shown in Fig. 1. We define Ψ_1 and Ψ_2 as the quantum mechanical wave function of the superconducting state in the left and the right superconductor, respectively. The dynamics of these two wave functions are then governed by the coupled Schrodinger equations Fig. 2, where K represents the coupling across the barrier and μ_1, μ_2 are the lowest energy states on either side. The idea is now to solve the equations in various cases like the DC Josephson effect, AC Josephson effect , Resistively and Capacitively Shunted Junction-model, Magnetic field dependence of the Josephson current, etc.

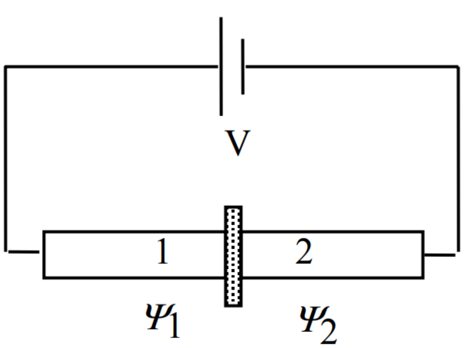

FIG. 1. Two superconductors separated by a thin insulator and wave functions

$$i\hbar\frac{\partial\psi_1}{\partial t} = \mu_1\psi_1 + K\psi_2$$

$$i\hbar\frac{\partial\psi_2}{\partial t} = \mu_2\psi_2 + K\psi_1$$

FIG. 2. Coupled Schrodinger equation

The critical current thus shows a Fraunhofer-like dependence of the magnetic field as seen in Fig. 3.

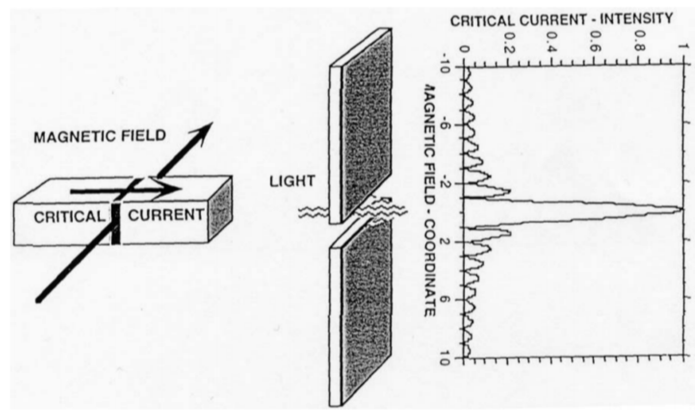

FIG. 3. Analogy of a Josephson junction in a magnetic field and light diffracted in a single slit

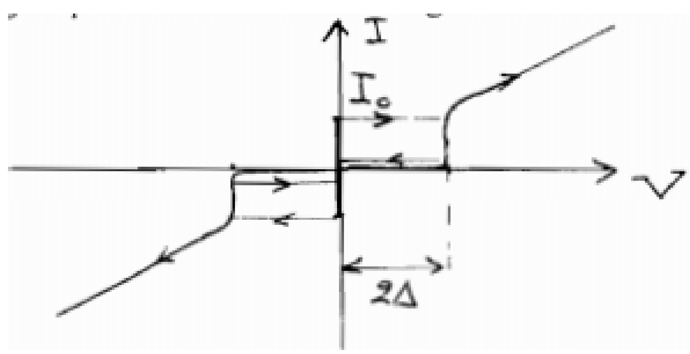

FIG. 4. Current vs. Voltage at $T < T_c$

The energy gap and critical current dependence on temperature are written below. The energy gap at zero temperature is given as

$$\Delta(0) = 1.76k_BT_c \tag{2}$$

Our measured T_c was around 7K, as we could not differentiate critical current for higher temperatures although the actual temperature should have been 9K. We then introduce δ which is given as the energy gap at some temperature divided by the energy gap at zero temperature.

$$\delta = \frac{\Delta(T)}{\Delta(0)} \tag{3}$$

δ is given by an integral equation in 'Principles of Superconductive Devices and Circuits, T. Van Duzer and C.W. Turner, Elsevier(1981)'. The solution that we will use is found in 'Thomas P. Sheahens, Phys. Rev. B, 149, 368 (1966)' and is of the form

$$\delta^2 = \cos(\frac{(\pi)(t^2)}{2}) \tag{4}$$

where

$$t = \frac{T}{T_c} \tag{5}$$

We can then calculate an approximation for an energy gap at a given temperature with the following equation:

$$\Delta(T) = \sqrt{\cos(\frac{\pi(T^2)\Delta(0)}{2(T_c)^2})}\Delta(0) \tag{6}$$

The critical current is related to the energy gap in the form of

$$I_c = \frac{\pi\Delta(T)\tanh(\frac{\Delta(T)}{2k_BT})}{2eR_n} \tag{7}$$

This equation describes the current of the Cooper pairs and not normal electrons. For voltages less than 2δ, the quasiparticle (normal electron) current is not able to pass through the junction. R_n is the resistance for voltages higher than the gap voltage.

C. Meissner Effect

In regards to the Meissner effect, we try to determine the depth of penetration (λ) for the magnetic field. The penetration depth is dependent on the concentration of Cooper pairs. Since this concentration is dependent on temperature, so is the penetration depth. A magnetic field acts on the critical current when applied on the plane of the junction and will change it in the form of a Fraunhofer diffraction pattern. The flux through the junction is given by:

$$\Phi = \omega(t_{ef})(B) \tag{8}$$

where ω is given to be the junction width and $t_(ef)$ is the effective barrier thickness of the junction given by

$$t_{ef} = 2\lambda + d \tag{9}$$

where d is oxide thickness given to be about 5nm. As one can clearly see, due to the dependence of I_c on magnetic field in the form of a Fraunhofer diffraction pattern, the dependence is relative to λ which we can therefore measure.

D. Fraunhoffer Diffraction Pattern

Given the quantized unit of magnetic flux to be $\Phi_o = \frac{h}{2e}$ and the maximum current at some arbitrary temperature less than T_c, there is an interesting effect as shown in the graph below. Due to the fact that superconductors try to expel all magnetic fields, a current forms on the surface of the Josephson junction perpendicular to the magnetic field so as to counteract the magnetic flux. This results in a deviation of the current as shown as magnetic flux changes from 0. When some constant voltage is applied to the junction, it emits radiation at a frequency of $\frac{eV}{h\pi} = 483.5\frac{MHz}{\mu V}$(13)

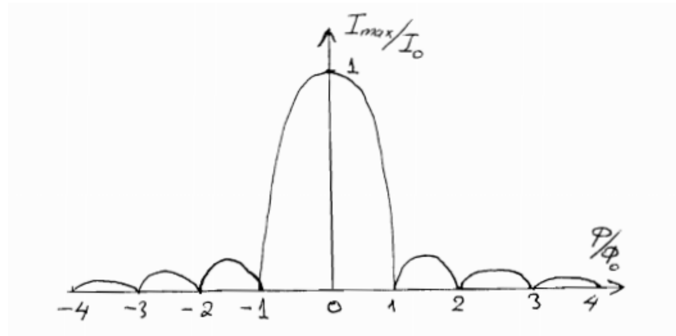

FIG. 5.

III. EXPERIMENTAL SET UP AND PROCEDURE

It is important to mention the setup we are using. Our results strictly depend on the sensitivity of our instruments. The main components of the apparatus are

1.The cryostat probe which is put into liquid helium. The probe has a vacuum can holding the junctions as well as a thermometer and heater which regulates temperature.

2. A pumping system to act on the vacuum can in order to create thermal isolation from the helium bath.

3. An amplifier rack with circuit boxes. A box labeled 'probe' is connected to a cable providing electrical leads to the probe

4. The Ersatz Probe which is used to test the measurement circuit without using the real probe

5. A DC voltage source located on the amplifier rack

6. For temperature measurements, a "General Resistance" decade resistor and 2 Keithley multimeters.

7. A Wavetek sweep generator for the I-V curve source and a computer for I-V measurement.

8. An HP digital oscilloscope.

9. Two HP 34401A multimeters.

10. One "General Radio" decade resistor and one Keithley 195A multimeter for magnetitc field vs current measurements.

The schematics of the Ersatz probe is shown in Fig. 6.

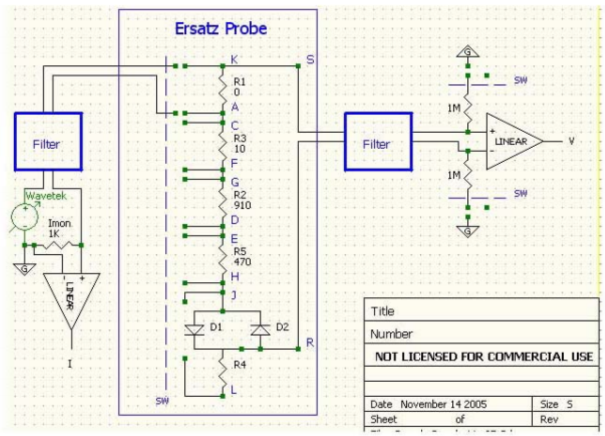

FIG. 6. Schematics of the pseudo sample circuits in the ersatz probe box

A. Temperature regulation and measurements

Temperature is measured by a standard Lake Shore germanium resistor thermometer that has a well measured relationship between resistance and temperature. Calibration tables for the thermometer are given in the equipment manuals. The thermometer resistance R_T is measured using Keithley digital multimeters (DMMs) using the current and voltage across the thermometer, in a four wire resistance measurement. The variable voltage source provides a current I_T which is measured by using a DMM to measure the voltage drop across a known resistance provided by a decade resistor. The circuit diagram is shown in Fig. 7.

B. Current-voltage curve measurement

In our setup, we have various probes which all have various resistors or diodes. We determined the I-V curve of each, in order to determine resistance values. The probes are attached to Ammeters and Voltmeters and

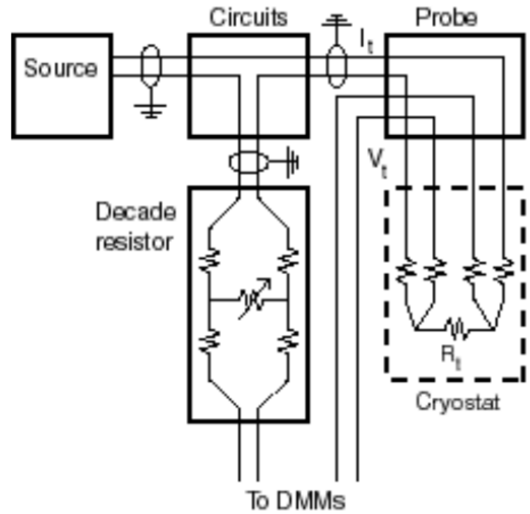

FIG. 7. : Schematic of the temperature measurement circuit

are ultimately connected to Lab-View which is a computer program that reads current and plots it for varying voltages. From the slope of the I-V curve, we determined the resistance values of the resistors as well as the diodes.

We first set our probe into the liquid helium and set the position so that the temperature would stay fairly constant throughout the process of measuring. We then used the solenoids in order to create a magnetic field and observe the Meissner effect which is the expulsion of magnetic field in a superconductor. We initially began with a very small magnetic field and increased it in intervals until the critical current had disappeared.

Once this was completed, we used the Ersatz probe as well as an actual probe to place into the liquid helium. During the first part of the experiment, we slowly lifted our probe by one-two centimeters for every measurement. Once we did, we changed our variable resistor so that our two ammeters would read the same values. The resistance at which this was achieved would correspond to a set temperature of the rod. This is achieved by a source providing a current-I_T, which is then measured with a DMM which measures the voltage drop across a resistance (given by our variable resistor). Once our resistance value is known, we use the "Lake Shore Cryotronics, Inc." plot to determine the relative temperature at which the I-V plot was determined.
The current is read by measuring the voltage drop across a $1KOhm$ resistor in the "CIRCUITS" box. See 8. Current and voltage signals are amplified as well as isolated from ground by differential amplifiers. These amplifier outputs can be put on the oscilloscope operating in the XY mode and monitored by two HP 34401A DMMs.

The junctions are located on a silicon chip in thermal

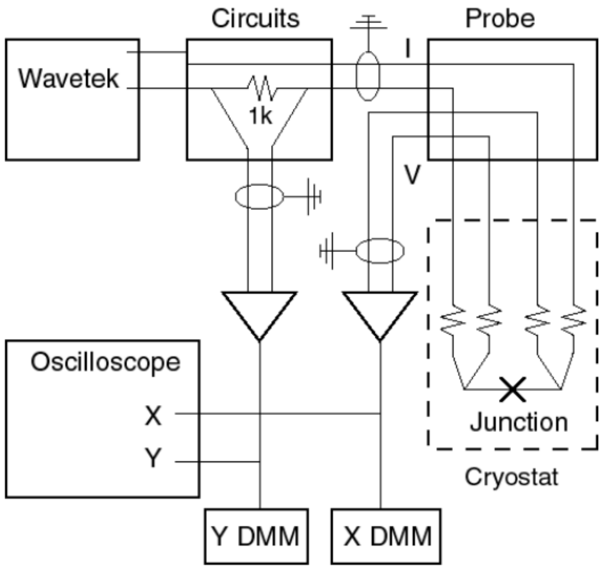

FIG. 8. Schematic of the setup for the measurement of the junction I-V curve

contact with the temperature controlled platform. Five junctions with a range of sizes are connected. The wiring diagram for the junctions in the probe are not informative and are not included here. The actual picture of the probe vacuum can, which contains the junction, is shown in Fig. 9.

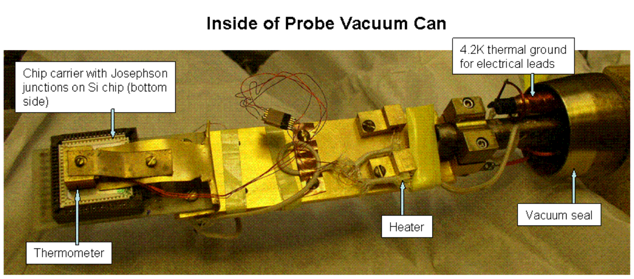

FIG. 9. Inside of a probe vacuum can

IV. ANALYSIS

A. Critical current dependence of applied magnetic field

We begin with introducing the data obtained in the experiment. Fig. 10 shows behaviour of Critical current (I_c) measured in Amp against applied magnetic field (Gauss). In the experiment, we systematically changed the applied current which has been converted into magnetic field by the conversion: $1mAmp = 0.25Gauss$. We applied the errors associated in measurement of

B	Ic	Ic
gauss	Amp	Amp
0	2.3833E-5	1E-9
0.00418	2.2455E-5	1E-9
0.00938	1.3144E-5	1E-9
0.01405	1.176E-5	1E-9
0.01963	9.074E-6	1E-9
0.02428	4.349E-6	1E-9
0.02865	0	1E-9

FIG. 10. Table of critical current against magnetic field

current, but they are relatively insignificant. The behaviour should look like the Fig. 3 diffraction pattern obtained in single slit experiment of light. We have plotted Critical current against the applied magnetic field in Fig.11.

The analysis is similar to Fraunhofer diffrac-

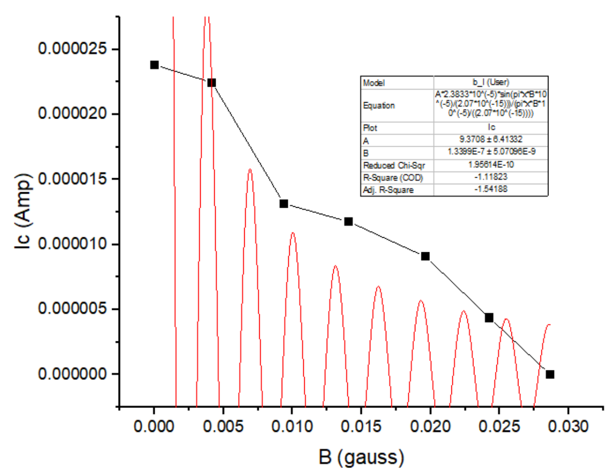

FIG. 11. critical current against applied magnetic field

tion. We have fitted our data sets against $I(\Phi) = I(\Phi_0)|\frac{sin(\pi\Phi/\Phi_0)}{(\pi\Phi/\Phi_0)}|$. Here $\Phi = w * t_{eff} * B$ corresponds to magnetic flux through the junction. $w = 10\mu m$ is the width of the junction. $t_{eff} = 2\lambda + d$, Here λ is the penetration depth and $d = 5nm$ is the thickness of junction. From fitting of our function we got $t_{eff} = 134 \pm 5nm$, which corresponds to $\lambda = 64.5 \pm 5nm$. We should have taken more data points in order to get better fit to the function, but due to lack of time(issues relating to getting liquid helium), we couldn't take more data. Our results confirm that Niobium has penetration depth of around $100nm$. These results were found under the temperature of $4.2K$.

B. Critical current variation against Temperature

We now turn the focus to finding the variation of Critical Current as well as the energy gap versus Temperature. To achieve a varying change in temperature, we pulled the Josephson Junction (submerged in liquid helium) upwards. The idea is that the upper regions of the helium tank have more helium vapor and therefore have a higher temperature. We were, therefore, able to have a incrementally changing temperature with which we could observe the junction.

Fig. 12 shows the variation of Critical current and energy gap against Temperature. The temperature measurements have already been discussed in a previous section. We determined the temperature by methodically observing the resistance. We varied the temperature from $5K$ to $6.8K$ only. Beyond that, we lost the super-current. The energy gap, however, was still observable. We were not able to study the relationship to $9K$ as we had limited time.

R	Temp	IC(Amp)	Delta	VC(V)	Temp(err)	VC(error)	IC(err)
524	6.8	1.329E-6	8.319E-4	0.00235	1E-4	1E-6	1E-9
539	6.6	3E-6	8.425E-4	0.00238	1E-4	1E-6	1E-9
558	6.5	6.83E-6	8.496E-4	0.0024	1E-4	1E-6	1E-9
594	6.3	7.145E-6	8.602E-4	0.00243	1E-4	1E-6	1E-9
652	6.2	9.292E-6	9.027E-4	0.00255	1E-4	1E-6	1E-9
700	6	1.2782E-5	9.31E-4	0.00263	1E-4	1E-6	1E-9
726	5.9	1.4596E-5	9.3458E-4	0.00264	1E-4	1E-6	1E-9
761	5.7	1.6804E-5	9.416E-4	0.00266	1E-4	1E-6	1E-9
796	5.6	1.9354E-5	9.522E-4	0.00269	1E-4	1E-6	1E-9
831	5.5	1.9279E-5	9.593E-4	0.00271	1E-4	1E-6	1E-9
847	5.45	2.0056E-5	9.664E-4	0.00273	1E-4	1E-6	1E-9
895	5.25	2.1119E-5	9.77E-4	0.00276	1E-4	1E-6	1E-9
945	5.1	2.3478E-5	9.806E-4	0.00277	1E-4	1E-6	1E-9
962	5.05	2.4637E-5	9.841E-4	0.00278	1E-4	1E-6	1E-9

FIG. 12. Table of critical current and energy gap against temperature

Fig. 14 shows the variation of energy gap $\Delta(T)$ against Temp. In this plot We fitted the function $\delta^2 = Cos(\pi t^2/2)$. $\delta = \frac{\Delta(T)}{\Delta(0)}, t = T/T_c$ and $\Delta(0) = 1.76K_B t_c$. Energy gap should follow this behaviour. Near T_c it will follow: $\delta = 1.74(1-t)^{1/2}$. It is very easy to understand this from phase transition point of view. Near criticality, systems have universal behaviour and are dependent on critical exponents (1/2 in our case). From the plot we can see the theoretical function matches quite nicely with our data sets.

The next step is to understand the behaviour of Critical current versus temperature. To do this, we need

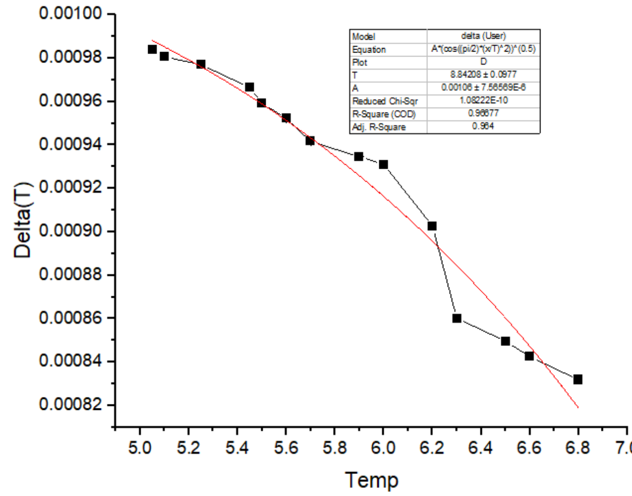

FIG. 13. energy gap against Temp

to understand quantum tunneling better. Super-current vanishes at $6.8K$ in our experiment but a gap was still observed. The equation governing this behaviour is

$$\Phi = \omega(t_{ef})(B) \tag{10}$$

We have fitted our data against this function.

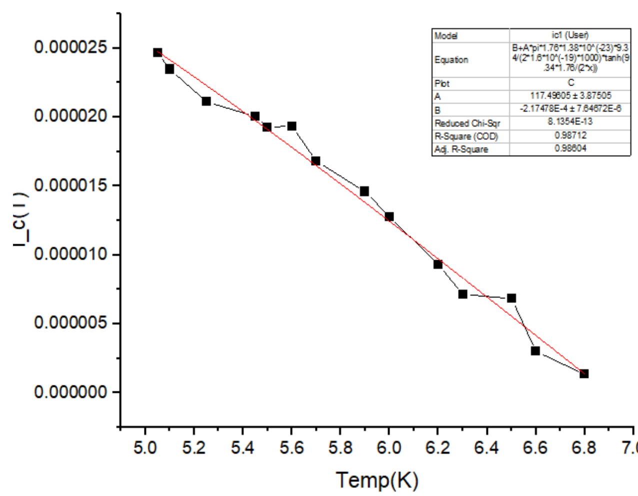

FIG. 14. Critical current against Temperature

V. RESULTS AND DISCUSSIONS

We have studied the Temperature dependence of $\Delta(T)$ and $I_c(T)$. Our data sets match with theoretical predictions(see reduced chi value in plot). We have also studied the variation of Critical current against magnetic field. From this data set, we calculated the value of penetration depth $\lambda = 64.5 \pm 5$ nm. It is close to the actual value of Niobium penetration depth of around $100nm$. We have also studied how temperature effects the I-V curve. This is quite transparent in our plot. By increasing the temperature, we got a small $\Delta(T)$. In the plot on Lab-view, we saw the transition zone on the voltage axis shrink. We didn't go beyond 6.8 K, but the expectation is above critical temperature. We should have taken more data in the study of Critical current dependence on the magnetic field, to get the theoretical the Fraunhofer diffraction pattern. We have also studied the I-V curve of various probes in the "Ersatz probe". It looks like the curve of a normal conductor. The experiment was therefore relatively successful and has confirmed existing models of Superconductivity.

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