Institutt for fysikk, NTNU

Exam FY8305 Functional integral methods in condensed matter physics
14:00 December 9- 14:00 December 16, 2019

An interacting spinless fermion system is defined on a two-dimensional square lattice. The Hamilton-operator for this system is given by

$$H = \sum_{\langle i,j \rangle} \left[-t \, c_i^\dagger \, c_j + V \, n_i \, n_j \right] - \mu N.$$

Here, $\sum_{\langle i,j, \rangle}$ means summation over nearest neighbor lattice points i, j, $t > 0$ is a matrix element controlling hopping of fermions from one lattice point to a nearest neighbor lattice point, $V > 0$ is an electrostatic repulsive nearest neighbor density-density interaction, and μ is the chemical potential of the system. Furthermore, $n_i = c_i^\dagger \, c_i$, and $N = \sum_i n_i$. We have that $\{c_i^\dagger, c_i\}$ are fermion-operators with standard anti-commutation relations.

a) Determine μ such that the system is half-filled, and illustrate what you would expect the ground state to look like on the lattice at half-filling when $V \gg t$.

b) Write down the partition function of the system as a functional integral over Grassmann-variables.

c) Introduce the bosonic field $\phi_\delta(\mathbf{r}_i) = \langle c_i^\dagger c_j \rangle$, where $\mathbf{r}_j = \mathbf{r}_i + \delta$. Perform a Hubbard-Stratonovic-decoupling of the interaction in H, and write down the partition function for the system as a functional integral over Grassman-variables and c-number variables.

d) Integrate out the Grassmann-variables such that the partition function is given as a functional integral over c-number variables with an effective action. Give the expression for this action in terms of a part quadratic in ϕ and a fermion-determinant. The fermion-determinant is to be given as the trace of the natural logarithm of a matrix, and you should give an explicit expression for the matrix.

e) Write down a mean-field version of the effective action, denoting it S_0. Mimimize S_0 and find an equation determining the mean-field order parameter. Approximate the density of states for fermions by its value on the Fermi surface, and solve the equation for the order parameter, with special emphasis on the V-dependence. Determine whether or not one can detect any ordering in the mean-field order parameter by computing to finite order in perturbation theory in V.