Title goes here

Some information about a cool theorem or anything i find interesting, related equations (here: Dirac-equation)

$$(i\gamma_\mu \partial^\mu - m)\psi = 0$$

Some more information here

Ehrenfests teorem

Hvorfor kollapser ikke bølgefunksjonen til hverdagslige ting (som f.eks. en stol)? Korrespondanseprinsippet forteller oss at i grensen av store antall partikler og høye energier, må kvantemekanikk være ekvivalent med klassisk fysikk. Paul Ehrenfest fant en måte å relatere tidsutviklingen i forventningsverdien av kvantemekaniske operatorer til forventningsverdien av kraften som virker på systemet. Fra Newtons andre lov er dette forventet, men Ehrenfests teorem gir en matematisk trygghet. En generalisering av teoremet kan skrives på formen

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle O\rangle = -i\hbar\,\langle[O,\mathcal{H}]\rangle + \left\langle \frac{\partial O}{\partial t} \right\rangle$$

hvor O er en operator som korresponderer til en observerbar størrelse og $\mathcal{H}$ er Hamiltonian til systemet.

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The Hubbard Model

The Hubbard model is often called the "Ising model" of strongly correlated quantum systems, and an exact solution to this lattice model is only known in one dimension except in special limits. It is given by the Hamiltonian

$$\mathcal{H} = -\sum_{i,j,\sigma} t_{ij} c^{\dagger}_{i\sigma} c_{j\sigma} + \sum_{i,\sigma} U_i n_{i,\sigma} n_{i,-\sigma}.$$

where the first term is a tight-binding approximation of the system near its Fermi-energy, and the second term describes the on-site energy cost of having doubly occupied lattice sites.

Hubbard-Stratonovich-decoupling

The HS-decoupling is a useful identity if you are troubled by too many interacting fermions in your system and would prefer a theory written in terms of (hopefully) noninteracting bosons. The transformation is based on the identity

$$\mathrm{e}^{-\frac{a}{2}\psi^2} = \frac{1}{\sqrt{2\pi a}} \int\limits_{-\infty}^{\infty} \mathrm{d}\varphi \, \mathrm{e}^{-\left(\frac{\varphi^2}{2a} + i\varphi\psi\right)},$$

where ψ represent fermions through Graßman variables, and φ represent bosonic field(s). This transformation is used in all kinds of quantum field theories. A discrete version can be used to transform the *quantum mechanical* Hubbard model in d dimensions into a *classical* Ising-like model in $d+1$ dimensions.

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Superconductivity - BCS theory (1)

The Bardeen-Cooper-Schrieffer theory of superconductivity is the most celebrated microscopic theory describing the macroscopic phenomena of superconductivity. In this theory, an effective attractive potential between electrons in vicinity of the Fermi surface leads to the condensation of "Cooper pairs", reducing the free energy of the system. After some crude simplifications, a model Hamiltonian reads

$$\mathcal{H} = \sum_{k,\sigma} \varepsilon_k c^\dagger_{k\sigma} c_{k\sigma} + \sum_{k,k'} V_{kk'} c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow} c_{k'\downarrow} c_{k'\uparrow},$$

where $V_{kk'}$ is attractive in a thin shell close to the Fermi surface. This attractive potential can for instance be mediated by interacting with quantized lattice vibrations – phonons.

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Superconductivity - BCS theory (2)

Since this Hamiltonian is quartic in fermion-operators, solving it exactly is very difficult. However, a mean field treatment is possible by letting $c_{-k\downarrow}c_{k\uparrow} = \langle c_{-k\downarrow}c_{k\uparrow}\rangle +$ fluctuations, and the system might be solved. The self-consistent BCS gap-equation states

$$\Delta_k = -\sum_{k'} V_{kk'}\Delta_{k'}\chi_{k'}$$

$$\chi_k = \frac{1}{\sqrt{\varepsilon_k^2 + \Delta_k^2}}\tanh\left(\frac{\beta}{2}\sqrt{\varepsilon_k^2 + \Delta_k^2}\right).$$

This is a "gap"-equation precisely because the condensation of Cooper pairs opens a gap Δ_k in the electronic excitation spectrum. A goal of modern physics is to find systems whose superconducting gap is as large as possible.

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