Efimov Physics – The Three-Body Problem

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The Peculiar Efimov Effect

- A quantum effect
- Resonant 2-body forces can give rise to a series of bound energy levels in 3-particle systems
- When the two-body s-wave scattering length $a \to \pm\infty$ the # of bound states is infinite
- # of 3-body bound states is *reduced* as the two-body interaction is made more attractive
- Emerge irrespective of the nature of the 2-body forces and can *in principle* be observed in all quantum mechanical systems.

Scattering Length

- The 2-body s-wave scattering length describes the strength of the interparticle interaction. Definition:

$$a = \lim_{k \to 0} -\frac{\tan \delta_0(k)}{k} \tag{1}$$

- Negative scattering lengths correspond to an attractive effective interaction
- Positive scattering lengths correspond to a repulsive effective interaction

Solving The 3-body Problem: Step 1, Jacobi Coordinates

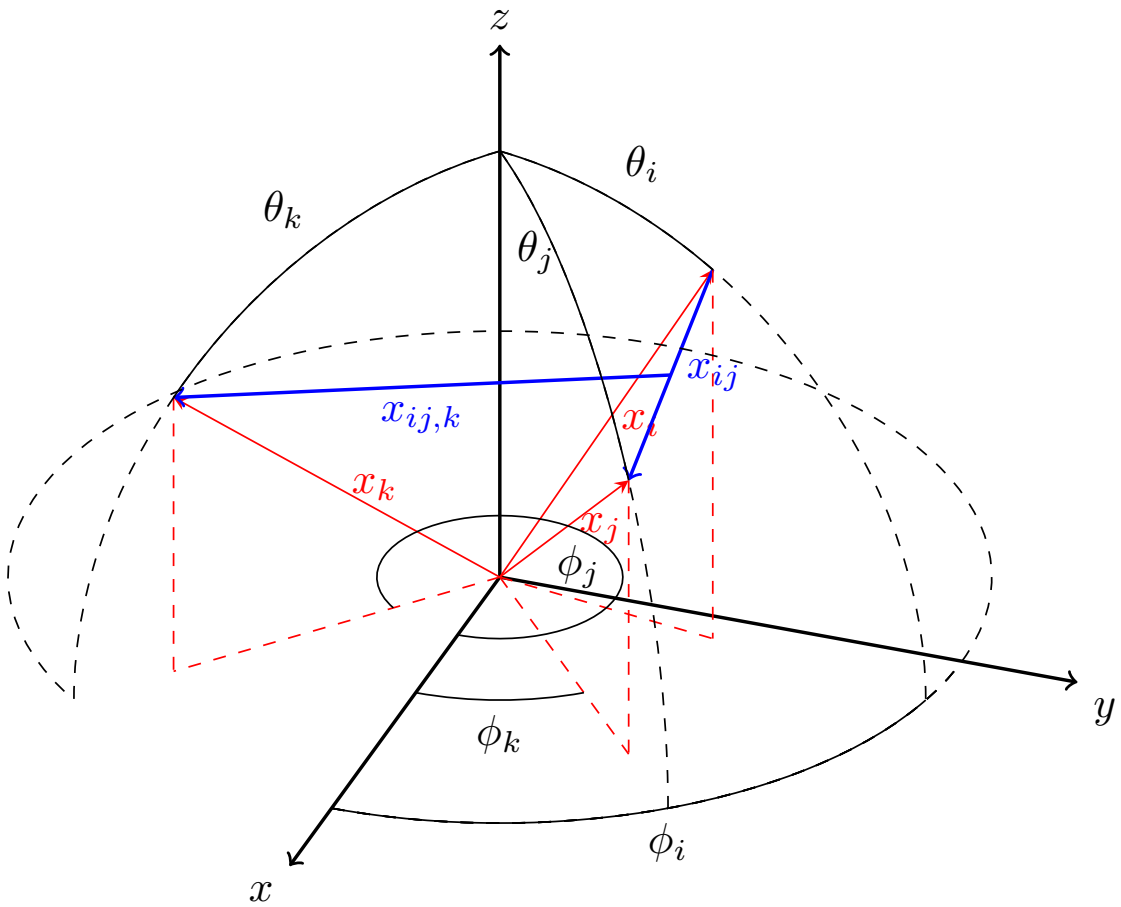
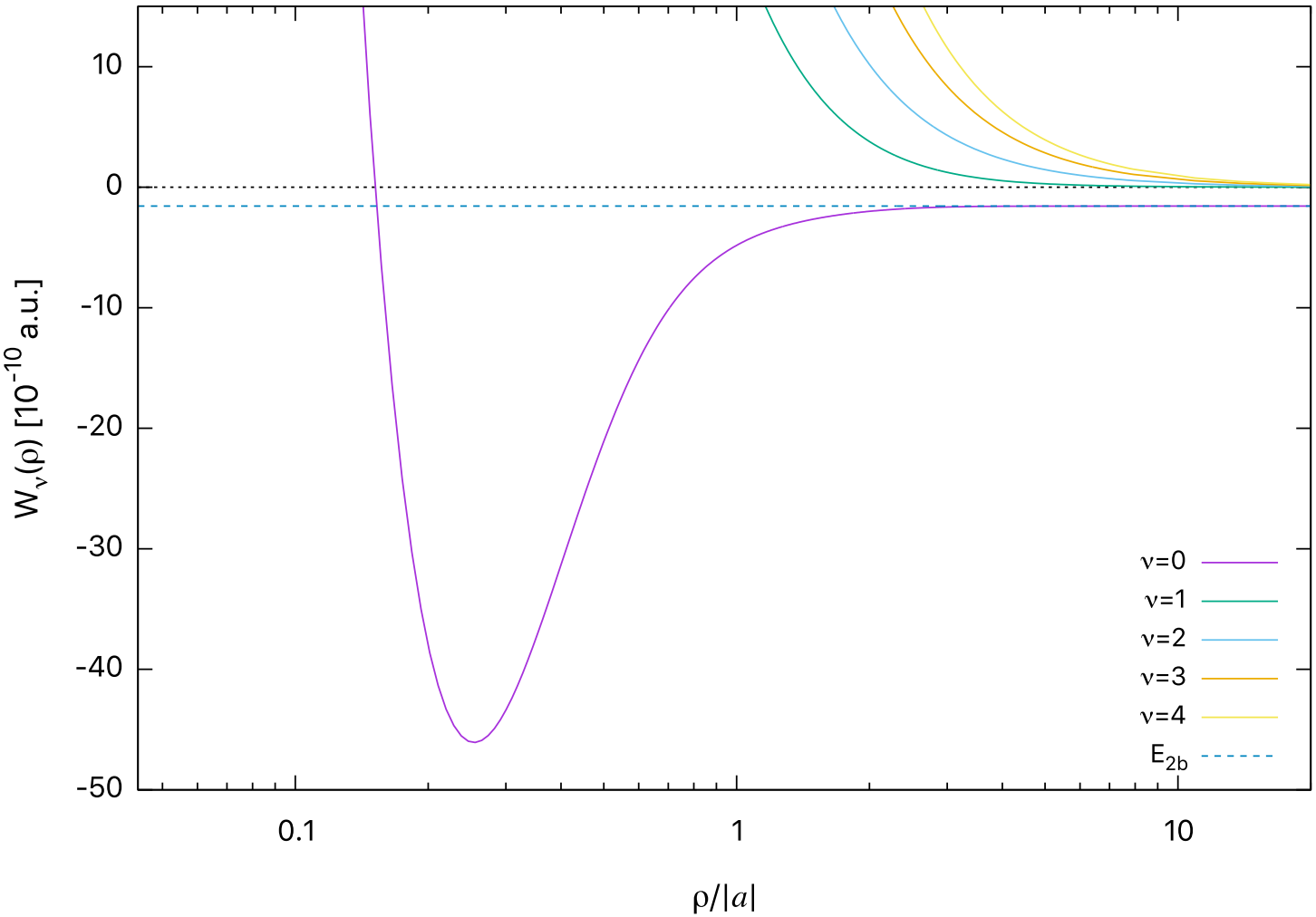


Figure: Spatial positions of three particles.

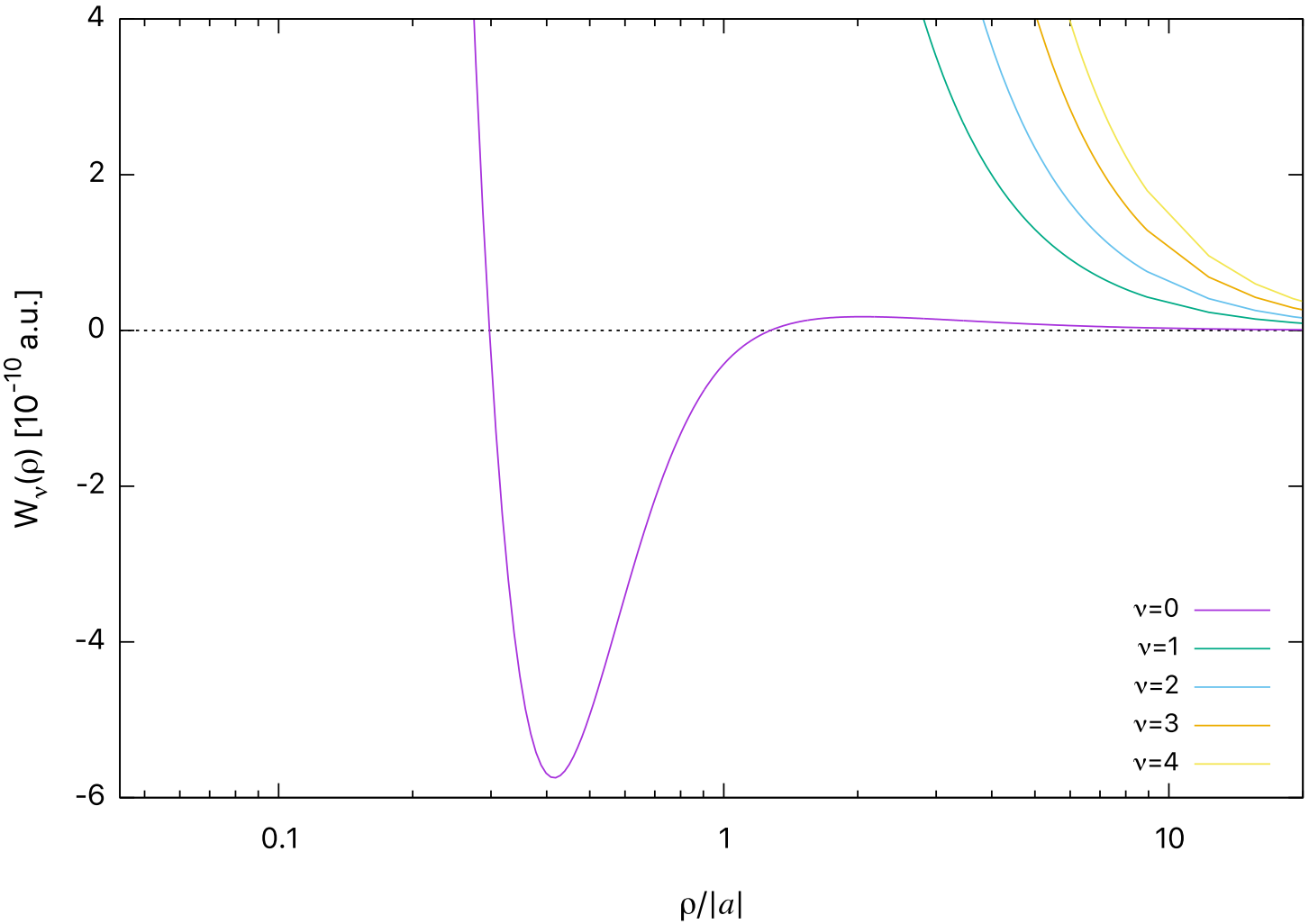
The Model Potential

$$v(r) = d \cosh^{-2}(r/r_0), \tag{2}$$

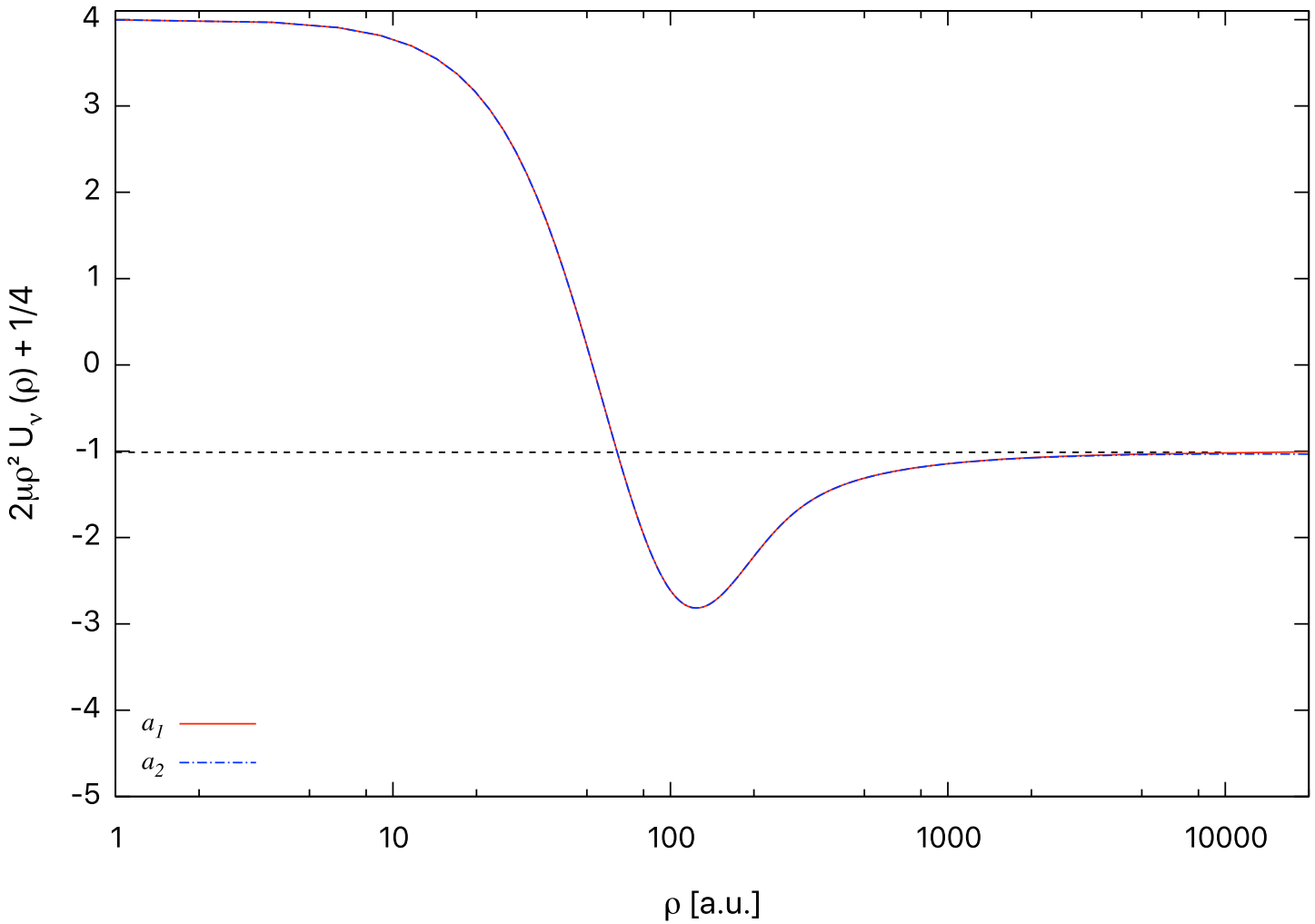
$a > 0$



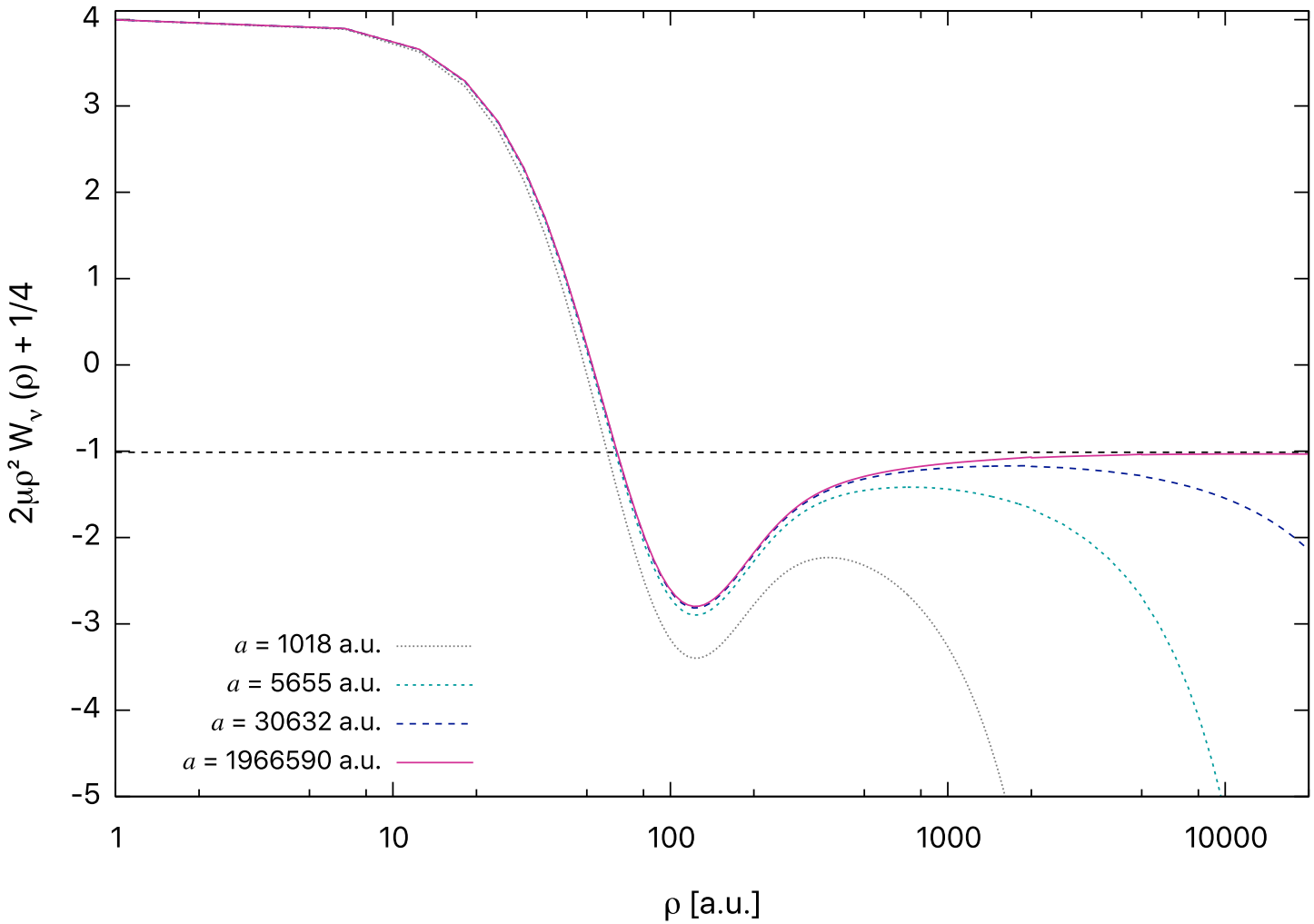
$a < 0$



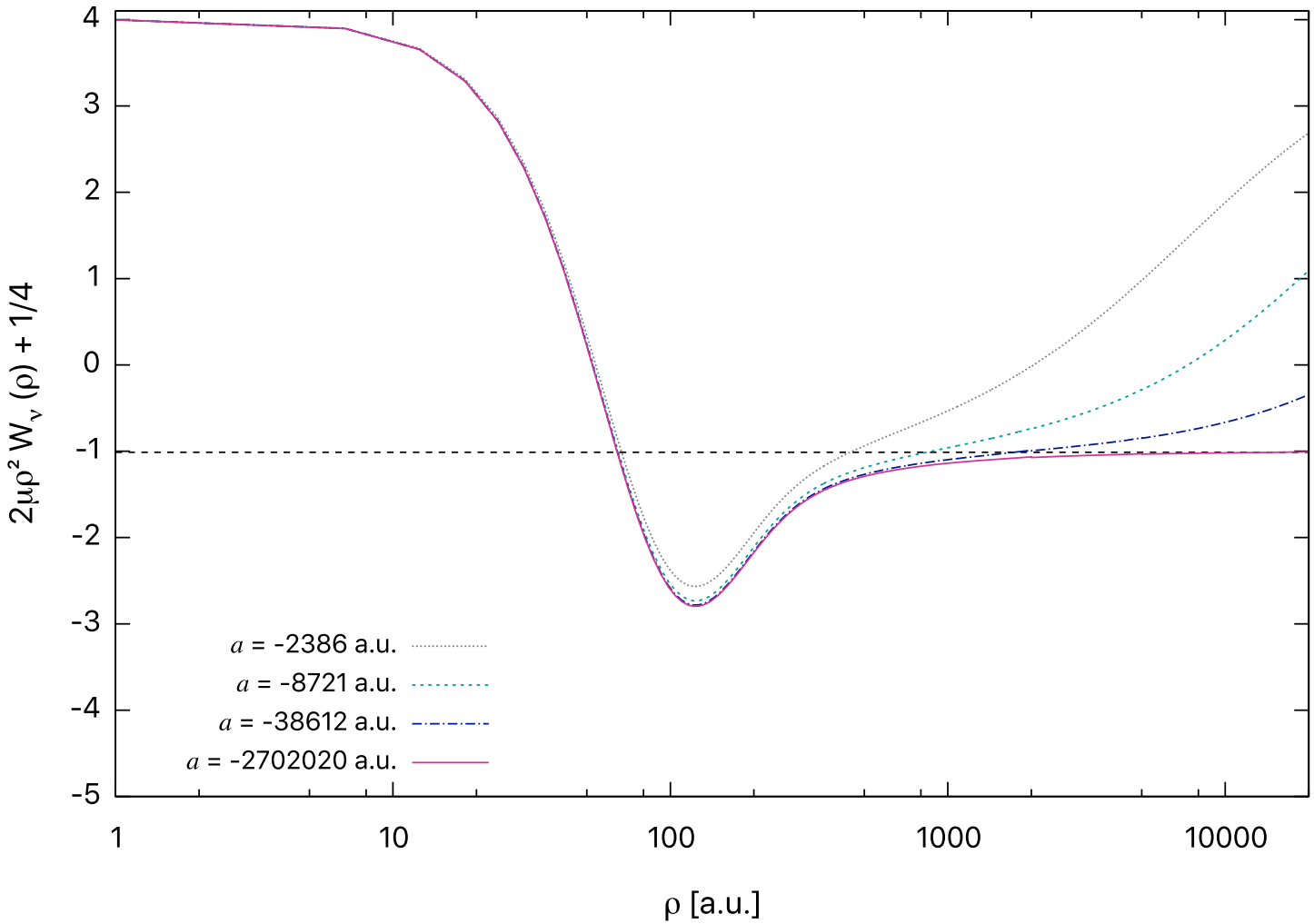
$a \to \pm\infty$, $-s_0^2 (\simeq -1.0125$ for $J = 0^+$ states)



$a > 0$


4
3
2
1
0
-1
-2
-3
-4
-5
1
10
100
1000
10000
2μρ² W_ν (ρ) + 1/4
ρ [a.u.]
a = 1018 a.u.
a = 5655 a.u.
a = 30632 a.u.
a = 1966590 a.u.

$a > 0$


4
3
2
1
0
-1
-2
-3
-4
-5
1
10
100
1000
10000
$2\mu\rho^2 W_\nu(\rho) + 1/4$
ρ [a.u.]
a = -2386 a.u.
a = -8721 a.u.
a = -38612 a.u.
a = -2702020 a.u.