

# Signal Processing Based on Butterworth Filter: Properties, Design, and Applications

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**Abstract.** This paper presents a comprehensive exploration of the properties, principles, design methodologies and practical applications of Butterworth filters. The Butterworth filters are acknowledged for their flat frequency response in the passband, which is at a maximal level, while introducing minimal distortion to the signal of interest. Therefore, they were commonly applied in various situations. The study centers on a detailed investigation of a specific case involving the analysis of a third-order Butterworth lowpass filter circuit using Sallen-Key circuit topology. The Sallen-Key topology was basically utilized to achieve the Butterworth filter in the second-order circuit design. Moreover, in order to complete the third-order Butterworth filter design, an additional circuit part was also established apart from the second-order Butterworth filter. The simulation of the circuit was done in the software LTspice. This article will provide an overview of the research, discussing the essential characteristics of Butterworth filters and their significance in signal processing. The findings of this study contribute to a deeper understanding of Butterworth filters and their role in various signal processing applications.

**Keywords:** Butterworth filter; Sallen-Key circuit topology; signal processing.

## 1. Introduction

Butterworth filter is a common type of filter. It was first brought up by British engineer Stephen Butterworth in 1930 [1- 3]. Nowadays, it has been widely used in various situations which could use signal processing, including systems of communication, processing sounds and pictures and other fields. The basic quality of Butterworth filter is that it possesses a flat passband response and a steep stopband response, which enables it to effectively filter out undesired noise while preserving information on the useful bandwidth. Another significant characteristic of Butterworth filter is its simplicity comparing to other types of filters. The Butterworth filter has a simple transfer function with only one parameter, making it easy to design and analyze.

The Butterworth filter finds applications in various fields, including medical signal processing, where it is used for filtering electrocardiogram (ECG) and electroencephalogram (EEG) signals. In addition, the Butterworth filter is also used in statistical analysis, where filtering noisy data is essential for accurate results. In recent years, the Butterworth filter has also been applied in image processing, where it is used for smoothing and removing noise from digital images.

To demonstrate the effectiveness of the Butterworth filter, a simulation of a Butterworth low-pass filter (LPF) in the third order will be presented in this article. The simulation will show how the Butterworth LPF can effectively filter out high-frequency noise while preserving the desired signal within the passband. Overall, the Butterworth filter is a versatile and reliable tool for signal processing applications, and its many advantages make it a popular choice for engineers and researchers alike.

This article will introduce the major features of Butterworth filter, followed by a simulation of a third-order Butterworth LPF. Finally, the applications of Butterworth filter in communication systems, medical field, statistics and other domains would also be discussed.

## 2. Features of Butterworth Filter

A Butterworth filter belongs to the category of infinite impulse response (IIR) filters [3]. It has a flattest amplitude response in the passband and a consistent gain. This filter is commonly designed to

achieve attenuation levels of -3 dB and -20 dB [2], making it effective in signal processing applications where frequency response is critical.

Butterworth filter can be implemented as both an analog filter and a digital filter. In the analog domain, Butterworth filter is renowned for its simplicity in design, absence of noticeable performance limitations, which has contributed to its extensive utilization [1]. It should also be noticed that the filter exhibits a monotonic decrease in amplitude as frequency increases, with the amplitude being attenuated at infinite frequency [1]. Due to its qualities as a IIR LPF, the amplitude would not become exactly zero after a finite number of steps in frequency.

The design of Butterworth filter in analog type is notably simpler to achieve compared to other types of filters such as Chebyshev and Elliptic filters [2]. Not only would it maintain the primary shape at higher orders, but also it can be readily converted into digital version [2]. Another advantage of it over other filters is its lower rate of waveform distortion, due to its superior phase linearity [2].

An ideal filter has a frequency response with no loss of gain in insertion in the passband and unlimited attenuation in the stopband to block any signal from going through. However, real-world circuits cannot simultaneously meet these requirements. However, in practical situations, such ideas can only be approximately achieved by real-life designs. Therefore, when designing a circuit, it is necessary to choose an appropriate type of filter that can meet the desired specifications. In order to achieve the concept of Butterworth filter, the Sallen-key circuit topology is a perfect selection to maximally meet the requirements.

### 3. A Practice in the Design of Butterworth LPF Circuit

In this part of the article, a third-order Butterworth LPF circuit has been designed in the software LTspice to further explore the characteristics of Butterworth filter and analyze its performance in filtering the undesired harmonic. In this simulation, the useful bandwidth of the signals is set to be between 0 Hz and 1 kHz. The approach to testing the performance of the filter is by observing how much amplitude of signal has been decreased at 10kHz. The Sallen-Key circuit topology has been adopted to implement the idea of Butterworth filter.

In the design below, circuits with amplifiers are adopted in the simulation. Amplifiers play a crucial role in filter circuits by providing gain or amplification to the input signal. They are essential components in the active filter design. Active filters utilize amplifiers along with passive components like resistors and capacitors to create the desired frequency response. The amplifiers provide gain and contribute to filtering the input signal.

#### 3.1. Sallen-Key Circuit Topology

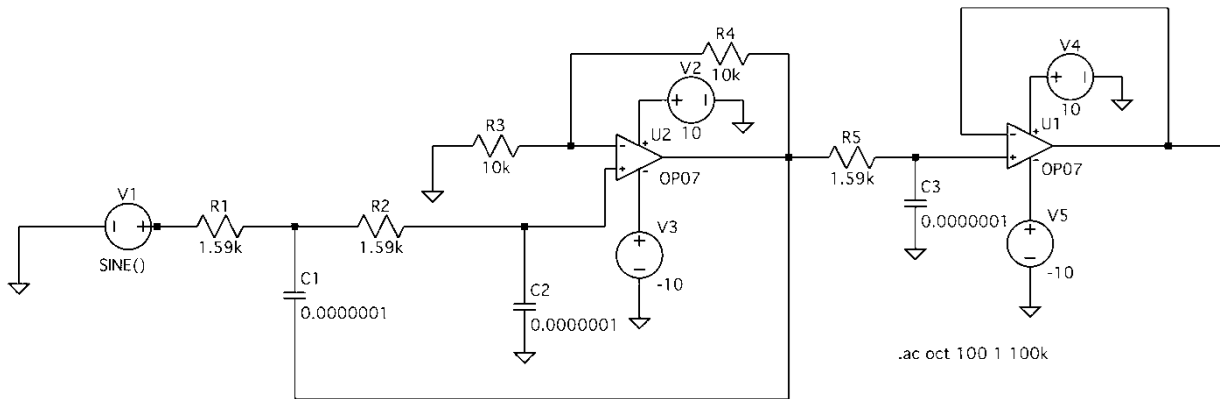
The Sallen-Key topology is a second-order electronic active filter topology that was introduced by Sallen and Key. The idea was published in their manuscript in 1955 [1, 4]. It is a commonly used filter design and commonly used in the implementation of frequency responses with characteristics of low-pass, high-pass, and band-pass filters. [4].

The Sallen-Key filter is based on a feedback circuit using operational amplifiers, offering a simple and flexible structure. It typically employs several capacitors and resistors to achieve the desired filtering characteristics, with the values of these components chosen appropriately to adjust the cutoff frequency and gain of the filter.

Comparing to other implementing methods, the Sallen-Key configuration possesses certain attributes such as a high input impedance, the ability to easily set the gain, which makes it an appropriate selection in this experiment [1].

#### 3.2. Methodology

The circuit of the third-order Butterworth LPF simulated in LTspice is presented in Fig. 1.



**Figure 1.** The circuit design in LTspice

As is shown, a third-order Butterworth LPF design was built with a second-order LPF along with a first-order LPF connecting together. In the second-order LPF, the application of the Sallen-Key filter topology was to achieve the idea of Butterworth filter.

One alternating current (AC) supply was used to generate the small AC parameter with the amplitude of 1V. Two OP07 type of operational amplifiers were adopted, while 4 direct current (DC) bias supply with the voltage of 10 V and -10 V were used to stabilize the operational amplifier in order to make it work regularly. The resistors  $R_1$ ,  $R_2$ ,  $R_5$  and the capacitors  $C_1$ ,  $C_2$ ,  $C_3$  were basic components to build up the Sallen-Key circuit, while  $R_3$  and  $R_4$  were used to adjust the gain of the operational amplifiers.

According to [5], the current transfer function of the third-order filter circuit can be presented as

$$\frac{i_{out}}{i_{in}} = \frac{s^2}{1 + 2s + 2s^2 + s^3} = \frac{s^2}{(s + 1)(s^2 + s + 1)} \quad (1)$$

It suggests that a third-order Butterworth filter circuit is able to be designed by combining a filter in second order with a filter in first order, and the quality factor of the second-filter is 1. The properties of the components are the same as those in the second-order filter, and the value of cutoff frequency in the whole circuit can be decided by the second-order filter.

As required, the cutoff frequency was set as 1 kHz. In the second-order filter, it was shown in [1] that the cutoff frequency and the quality factor can be calculated with the formulas,

$$f = \frac{1}{2\pi\sqrt{C_1 C_2 R_1 R_2}} \quad (2)$$

$$Q = \frac{\sqrt{C_1 C_2 R_1 R_2}}{(C_1(R_1 + R_2) + (1 - K)C_1 R_1)} \quad (3)$$

K represents the gain. By analysing the amplifier circuits, the gain can be calculated as

$$K = 1 + \frac{R_4}{R_3} \quad (4)$$

To simplify the problem, the resistance of  $R_1$ ,  $R_2$  and the capacitance of  $C_1$ ,  $C_2$  were both set equivalent, and the capacitance was set as 1  $\mu$ F. In this case, the expressions of the cutoff frequency and the quality factor can be rewritten as

$$f = \frac{1}{2\pi RC} \quad (5)$$

$$Q = \frac{1}{3 - K} \quad (6)$$

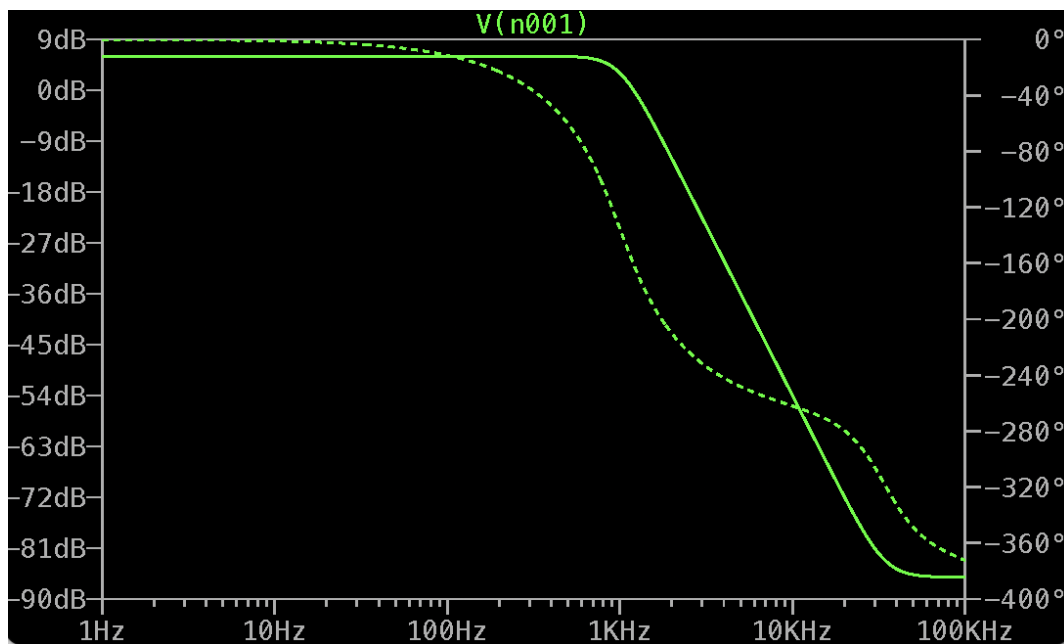
The symbols R and C represent the mutual value of the resistance of the resistors and the capacitance of the capacitors. Therefore, the resistances of R<sub>1</sub>, R<sub>2</sub> and R<sub>5</sub> were calculated approximately to be 1.59 kΩ. Moreover, the quality factor of the third-order Butterworth LPF circuit was 1, thus the gain K was calculated as 2. Combining it with the formula (4), it indicated that the resistances of R<sub>3</sub> and R<sub>4</sub> were equivalent. According to this, the resistances of R<sub>3</sub> and R<sub>4</sub> were set as 10kΩ.

The order '.ac oct 100 1 100k' shown above was written to show the AC frequency analysis of this circuit. The frequency response curve and the phase response curve were plotted due to this order. The setting 'oct' means it is on an octave band. The following numbers mean sequentially the interval number of points, the starting frequency of testing and the ending frequency.

After connecting and setting up value for all the components, all the nodes on the external side of the wires (except the node for measurement) were connected to the ground. Finally, the LTspice directive was edited and the simulation was run.

### 3.3. Results and Analysis

Here is the frequency response curve of the simulation shown in Fig. 2.



**Figure 2.** The frequency response curve of the simulation

As expected, the filter exhibits a passband ranging from 0 Hz to 1 kHz, allowing the useful frequencies of the signal to pass through with minimal attenuation. At 10 kHz, the filter provides an attenuation of more than 40 dB, effectively suppressing the undesired harmonic signal. Further more, the amplitude curve appears to be smooth. Between the useful bandwidth, the curve is flat, while out of the useful bandwidth, the curve decreases steeply, which satisfies the features of Butterworth filter mentioned before. However, the phase curve of the filter appears to be slightly steep, which is against the great phase linearity property mentioned in the feature part. In suspicion, it might be due to the flat magnitude response quality of Butterworth filter. The Butterworth filter is designed to have minimal amplitude variations within the passband, resulting in a sharply changing phase response.

Overall, the results of our study suggest that Butterworth LPF is considerably effective in the field of attenuating the undesired harmonics, whereas the phase response preservation might not be ideal. However, there are plenty of limitations that still exists in this experiment. One of the most noticeable flaws is that the third-order Butterworth LPF is analyzed without any comparisons. To make improvements, second-order LPF or LPF with higher orders can also be simulated and analysed as

comparisons to the filter explored here. Apart from that, other analog filter techniques such as Chebyshev or Elliptic filters can also be achieved, which in that case, the different qualities in the passband and stopband of the curve of frequency response, along with the phase response curve can be explored. By achieving the comparative experiment, the performance of the Butterworth filter can be further explored with more details. Although the limitations do exist, this experiment still has exhibited the powerful ability of signal denoising that the Butterworth filter possesses.

#### 4. Applications of Butterworth Filter

The Butterworth filter has a wide range of applications in different fields which can be attributed to its simple design and effective performance in signal denoising (The wide usage of it in engineering is apparently obvious, and its specific application would not be mentioned here except in communication systems).

Butterworth filters are used in modulation for signal processing purposes commonly. Modulation is the process of transforming an information signal into a form suitable for transmission or storage, and it includes both analog modulation and digital modulation. In analog modulation, Butterworth filters can be employed for processing the modulating signals. In phase modulation (PM) or frequency modulation (FM), Butterworth filters can filter out unwanted frequency components or limit the signal within the desired frequency range. In digital modulation, the role that Butterworth filter plays is often in the demodulation process. For instance, in amplitude shift keying (ASK), which is a common type of digital modulation skills, the Butterworth filter can significantly remove the carrier signal from the original baseband signal, which enables the recovery of the original digital signal. In summary, Butterworth filter can play a as the foundation part of the basic principle of different types of modulation skills, which provides endless approaches of more effective modulation process.

Butterworth filter also has the potential of being used in other domains. One of the hugest field it has been widely used is in medical fields. To start with, Butterworth filter plays a critical role in single photon emission computed tomography (SPECT) imaging [6], as it is able to remove various types of noise and artifacts caused by radioactive decay and physical factors [6], which increases accuracy of the images. Another technology that Butterworth filter influences is ECG [7]. As a matter of fact, the digital filter algorithm was initially brought up decades ago to achieve the detection of the noise in ECG [7]. The detection is necessary because that the ECG signals are often not huge in frequency and amplitude, leading to them taking high risk of being corrupted [7]. Therefore, the filter is incredibly necessary in this case. As an effective LPF, Butterworth filter is a wise selection to attenuate the noises in ECG.

Study in [8] suggests that Butterworth filter can also be used to make estimations for the long term trend and business cycles. In a traditional way, the Hodrick and Prescott (HP) filter is often utilized to make such estimations for economic uses. However, various problems occur with the model of HP filter. For instance, the incorrect datas are often produced and enormous amount of noise can't be removed effectively [8]. Compared with HP filter, Butterworth filter has a significantly better performance dealing with the high-frequency mutation, which result in less incorrect datas inducing [8]. Besides that, Butterworth filter can efficiently remove noise from the model, making it more accurate [8].

The usage of Butterworth filter is far more than in the limited domains mentioned above. In many domains which do not have many relations with the electronic engineering, LPF with more simple designs has often been adopted in practical situations, such as oceanographic data processing [9], or Structural health monitoring [10]. It should be realized that with the use of Butterworth filter, the processes can become significantly more effective and precise.

One thing should be noticed that despite the fact that the previous simulation of Butterworth filter was of the analog filter, digital filters are actually more preferred in modern signal processing applications [7]. It was due to the better flexibility and the more accurate replication of the digital filters [7].

## 5. Conclusion

In this paper, we have explored the properties and principles of Butterworth filters, as well as their design and applications. The study focused on a specific example of a third-order Butterworth filter circuit design.

Through our analysis, we have gained a deeper understanding of the characteristics and behavior of Butterworth filters. It highlighted the importance of setting appropriate value of the electronic components. To make a thorough conclusion in the Butterworth filter circuit simulation, the undesired harmonic in high frequency can be effectively attenuated, whereas the phase linearity has not been exhibited properly due to low magnitude response quality of Butterworth filter.

Butterworth filters contain several advantages in practical applications. Their flat frequency response in the passband, along with good stopband attenuation, makes them suitable for many signal processing tasks. They are commonly used in various fields, including engineering related subjects like communication and audio processing. Moreover, it also has various applications in medical field, statistics, etc. In all, digital Butterworth filter would be a superior choice in practical uses.

In conclusion, the study of Butterworth filters has provided valuable insights into their properties, design considerations, and applications. This knowledge can be applied in diverse engineering disciplines, contributing to the advancement of signal processing techniques and systems.

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