Simulation of Quantum Gates using QUIRK Quantum Simulator

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Abstract: Computer technology has reached another phase with the emerging of Quantum computing. Though in a preliminary stage, the topics on Quantum computing have been entered in to the syllabi of academics. QUIRK Quantum Simulator is one of the simple tools involving hands on learning of the Quantum Gates. The present learning material is aimed at the beginners choosing few single and two qubit gates.

Objective: To learn the operations of certain single and two-qubit gates using the QUIRK quantum simulator.

Apparatus: A computer with internet browser and connection.

Preparing the QUIRK quantum simulator:

(i) Type the URL, *https://algassert.com/quirk* in the internet browser.

(ii) Click, 'Edit Circuit' → Simulation page appears (refer Fig.1) with two tool boxes on top & bottom of the circuit editing area.
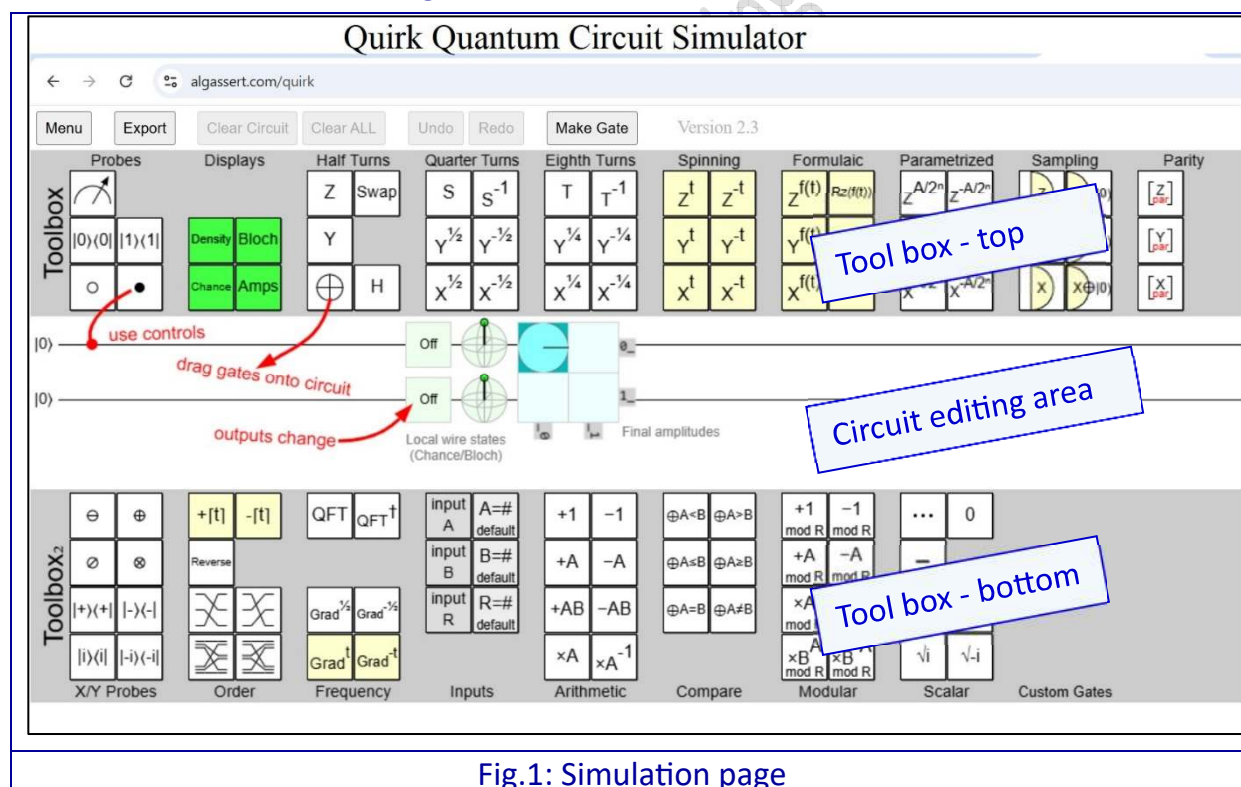

Fig.1: Simulation page

(iii) Observe that in the circuit editing area, there are 2 circuit lines with $|0\rangle$ states as inputs. At the ends of the lines, outputs are indicated in three forms, *viz.*, squares, Bloch spheres and an amplitude display with 4 squares.

(iv) By clicking $|0\rangle$ states, the inputs may be varied to $|1\rangle$, $|+\rangle$, $|-\rangle$, $|+i\rangle$ and $|-i\rangle$ states. In the present experiment, let us restrict the observation for the inputs $|0\rangle$ and $|1\rangle$ states.

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(v) Note that although 2 circuit lines appear in the editing area, use the upper line for the single qubit gates.

Qubit gates chosen:

Single qubit gates: Pauli X gate (Quantum NOT gate), Pauli Y gate, Pauli Z gate, Hadamard (H) gate, Phase gate and T gate.

Two qubit gates: Controlled X gate (CNOT gate), Controlled Z gate (C Phase gate) and SWAP gate.

Procedure:

(1) Single qubit gates and their outputs:

Pauli X gate or Quantum NOT gate:

(i) Drag the tool button, ⊕ corresponding to the Pauli X gate and drop after the |0⟩ state in the top circuit line.

(ii) Observe that at the right end as shown in Fig.2, the square with 'Off' state turns 'On' and the qubit on top in the Bloch sphere flips down depicting the operation of Pauli X gate (Quantum NOT gate). Further, in the amplitude display represents |1⟩ state.
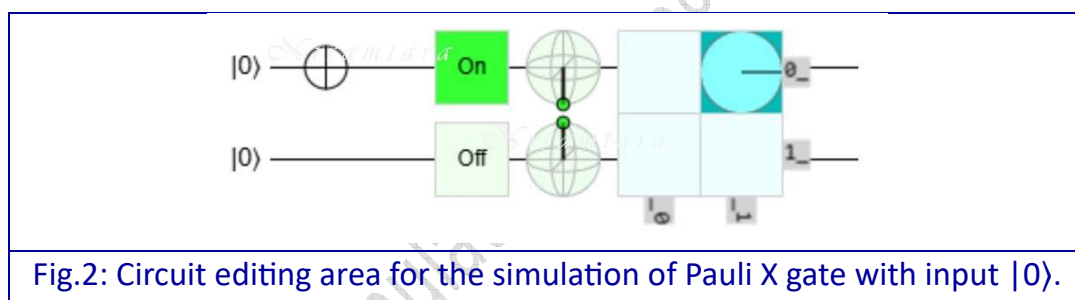

Fig.2: Circuit editing area for the simulation of Pauli X gate with input |0⟩.

(iii) Clicking upon the |0⟩ state turns it into |1⟩ state. Observe that the square with 'On' state turns 'Off' and the qubit at the bottom in the Bloch sphere flips up again depicting the operation of Pauli X gate (Quantum NOT gate). Further, in the amplitude display |0⟩ state is displayed.

(iv) Click the button 'Clear ALL'.

Similarly, the observations for the other chosen single qubit gates *viz*., Pauli Y gate, Pauli Z gate, Hadamard (H) gate, Phase gate and T gate can be made. The observations are present in the Tables 1 and 2.

(2) Two/three qubit gates and their outputs:

CNOT gate with |00⟩:

(i) Drag the tool buttons, ● and ⊕ corresponding to the control and target qubits (here, Pauli X gate), respectively, and drop after the |0⟩ states in the top and bottom circuit lines, respectively.

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(ii) Observe that at the right end as shown in Fig.3, both the upper and lower squares remain 'Off', the qubits in the Bloch spheres remain up and the amplitude display represents |00⟩. Hence, the operation of CNOT gate upon |00⟩ state results in |00⟩ state itself.
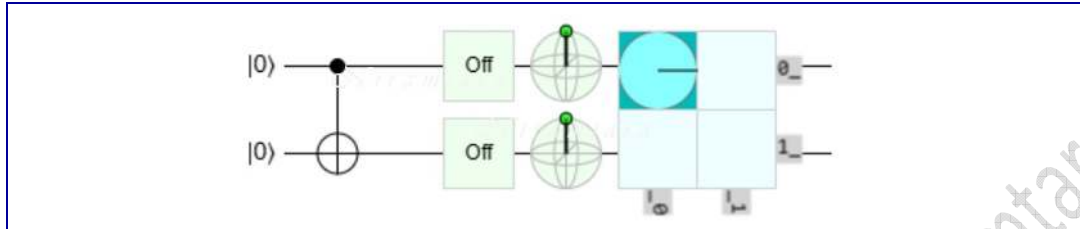

Fig.3: Circuit editing area for the simulation of CNOT gate with input |00⟩.

(iii) Clicking upon the |0⟩ state in the lower line turns it into |1⟩ state. In the lower line, observe that the square with 'Off' state turns 'On' and the qubit in the Bloch sphere flips down. Further, the amplitude display represents |01⟩. Hence, the operation of CNOT gate upon |01⟩ state results in |01⟩ state itself.

Similarly, the observations for the other chosen two qubit gates *viz.*, Controlled X gate (CNOT gate), Controlled Z gate (C Phase gate) and SWAP gate can be made. The circuit editing areas corresponding to Controlled Z gate (C Phase gate) and SWAP gate are shown in Fig.4. The observations are present in the Tables 3, 4 and 5.
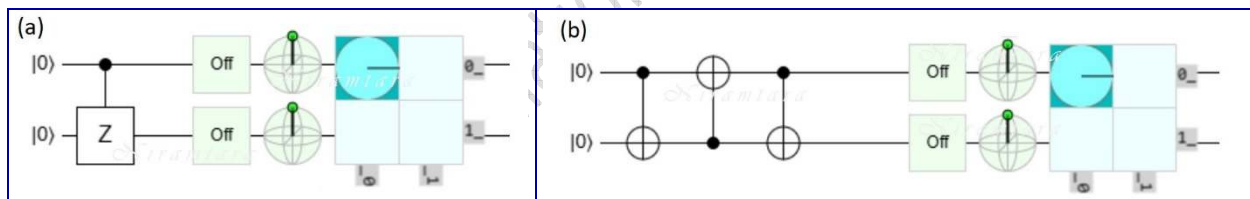

Fig.4: Circuit editing areas for the simulation of (a) Controlled Z gate and (b) SWAP gate with inputs |00⟩.

Result: The operations of few single and two-qubit gates are studied, and their output states are predicted using QUIRK quantum simulator.

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Observations: By above procedure, the observations made for 6 single qubit gates and 3 two qubit gates are tabulated below.

Table 1: Output observations with single qubit gates viz., Pauli X gate, Pauli Y gate, Pauli Z gate upon $|0\rangle$ and $|1\rangle$ states.
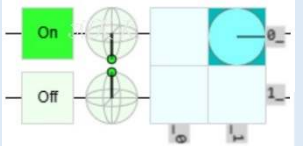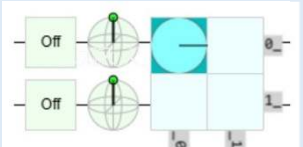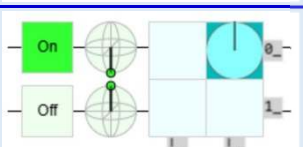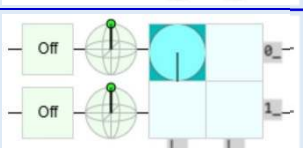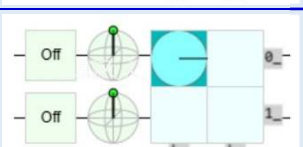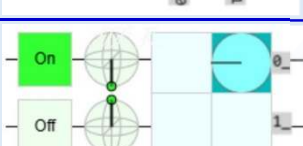
Gates applied	I/P state	O/P observation	Observation explained (in the upper circuit line)	O/P state
Pauli X	$|0\rangle$		The square turns 'On', the qubit in the Bloch sphere flips down and the amplitude display represents $|1\rangle$. Hence, the operation of Pauli X gate upon $|0\rangle$.	$|1\rangle$
Pauli X	$|1\rangle$		The square remains 'Off', the qubit in the Bloch sphere remains up and the amplitude display represents $|0\rangle$. Hence, the operation of Pauli X gate upon $|1\rangle$.	$|0\rangle$
Pauli Y	$|0\rangle$		The square turns 'On', the qubit in the Bloch sphere flips down and the amplitude display represents $i|1\rangle$. Hence, the operation of Pauli Y gate upon $|0\rangle$.	$i|1\rangle$
Pauli Y	$|1\rangle$		The square remains 'Off', the qubit in the Bloch sphere remains up and the amplitude display represents $-i|0\rangle$. Hence, the operation of Pauli Y gate upon $|1\rangle$.	$-i|0\rangle$
Pauli Z	$|0\rangle$		The square remains 'Off', the qubit in the Bloch sphere remains up and the amplitude display represents $|0\rangle$. Hence, the operation of Pauli Z gate upon $|0\rangle$.	$|0\rangle$
Pauli Z	$|1\rangle$		The square turns 'On', the qubit in the Bloch sphere flips down and the amplitude display represents $-|1\rangle$. Hence, the operation of Pauli Z gate upon $|1\rangle$.	$-|1\rangle$

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Table 2: Output observations with single qubit gates viz., Hadamard (H) gate, Phase gate and T gate upon $|0\rangle$ and $|1\rangle$ states.

Gates applied	I/P state	O/P observation	Observation explained (in the upper circuit line)	O/P state
Hadamard (H)	$|0\rangle$		The square turns '50% On', the qubit in the Bloch sphere is in '+X' and the amplitude display represents 'superposition', $\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$. Hence, the operation of H gate upon $|0\rangle$.	$\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$
Hadamard (H)	$|1\rangle$		The square turns '50% On', the qubit in the Bloch sphere is in '−X' and the amplitude display represents 'superposition', $\frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$. Hence, the operation of H gate upon $|1\rangle$.	$\frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$
Phase gate or S gate	$|0\rangle$		The square remains 'Off', the qubit in the Bloch sphere remains up and the amplitude display represents $|0\rangle$. Hence, the operation of Phase (S) gate upon $|0\rangle$.	$|0\rangle$
Phase gate or S gate	$|1\rangle$		The square turns 'On', the qubit in the Bloch sphere flips down and the amplitude display represents $i|1\rangle$. Hence, the operation of Phase (S) gate upon $|1\rangle$.	$i|1\rangle$
T gate or $\frac{\pi}{8}$ gate	$|0\rangle$		The square remains 'Off', the qubit in the Bloch sphere remains up and the amplitude display represents $|0\rangle$. Hence, the operation of T gate upon $|0\rangle$.	$|0\rangle$
T gate or $\frac{\pi}{8}$ gate	$|1\rangle$		The square turns 'On', the qubit in the Bloch sphere flips down and the amplitude display represents $e^{i\pi/4}|1\rangle$. Hence, the operation of T gate upon $|1\rangle$.	$e^{i\pi/4}|1\rangle$

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Table 3: Output observations with CNOT two qubit gate upon various combination of $|0\rangle$ and $|1\rangle$ states.
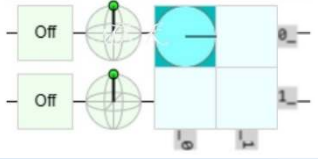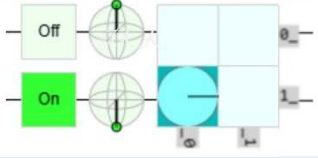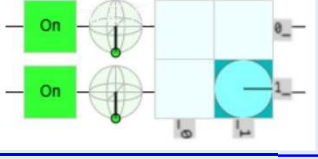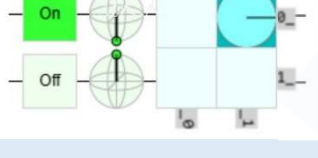
Gate applied	I/P state	O/P observation	Observation explained (observation in the upper & lower circuit lines)	O/P state
Controlled NOT gate -or- CNOT gate -or- Controlled X gate	$|00\rangle$		Both the squares remain 'Off', the qubits in the Bloch spheres remain up and the amplitude display represents $|00\rangle$. Hence, the operation of CNOT gate upon $|00\rangle$.	$|00\rangle$
	$|01\rangle$		In the upper line, the square remains 'Off' and the qubit remains up in the Bloch sphere. In the lower line, the square turns 'On' and the qubit flips down in the Bloch sphere. The amplitude display represents $|01\rangle$. Hence, the operation of CNOT gate upon $|01\rangle$.	$|01\rangle$
	$|10\rangle$		Both the squares turn 'On', the qubits in the Bloch spheres flip down and the amplitude display represents $|11\rangle$. Hence, the operation of CNOT gate upon $|10\rangle$.	$|11\rangle$
	$|11\rangle$		In the upper line, the square turns 'On' and the qubit flips down in the Bloch sphere. In the lower line, the square remains 'Off' and the qubit remains up in the Bloch sphere. The amplitude display represents $|10\rangle$. Hence, the operation of CNOT gate upon $|11\rangle$.	$|10\rangle$

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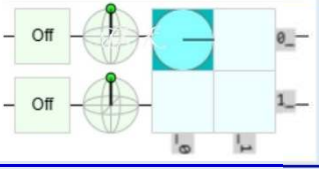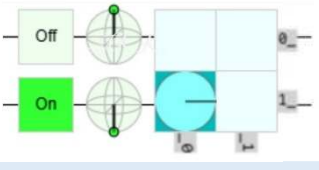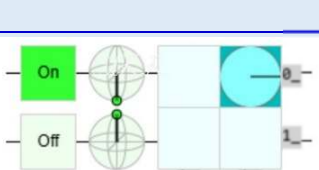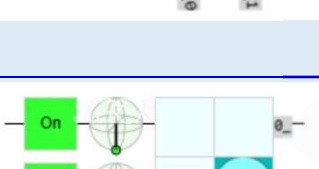Table 4: Output observations with Controlled Z gate (C Phase gate) upon various combination of |0⟩ and |1⟩ states.
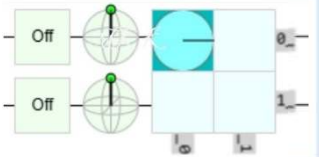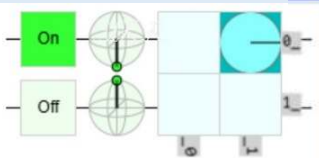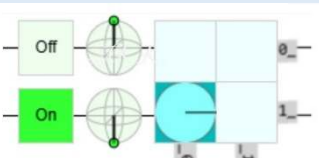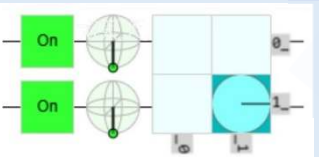
Gate applied	I/P state	O/P observation	Observation explained (in the upper & lower circuit lines)	O/P state
Controlled Z gate -or- C Phase gate	|00⟩		Both the squares remain 'Off', the qubits in the Bloch spheres remain up and the amplitude display represents |00⟩. Hence, the operation of Controlled Z gate upon |00⟩.	|00⟩
	|01⟩		In the upper line, the square remains 'Off' and the qubit remains up in the Bloch sphere. In the lower line, the square turns 'On' and the qubit flips down in the Bloch sphere. The amplitude display represents |01⟩. Hence, the operation of Controlled Z gate upon |01⟩.	|01⟩
	|10⟩		In the upper line, the square turns 'On' and the qubit flips down in the Bloch sphere. In the lower line, the square remains 'Off' and the qubit remains up in the Bloch sphere. The amplitude display represents |10⟩. Hence, the operation of Controlled Z gate upon |10⟩.	|10⟩
	|11⟩		Both the squares turn 'On', the qubits in the Bloch spheres flip down and the amplitude display represents −|11⟩. Hence, the operation of Controlled Z gate upon |11⟩.	−|11⟩

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Table 5: Output observations with SWAP gate upon various combination of |0⟩ and |1⟩ states.

Gate applied	I/P state	O/P observation	Observation explained (in the upper & lower circuit lines)	O/P state
SWAP gate	|00⟩		Both the squares remain 'Off', the qubits in the Bloch spheres remain up and the amplitude display represents |00⟩. Hence, the operation of SWAP gate upon |00⟩.	|00⟩
	|01⟩		In the upper line, the square turns 'On' and the qubit flips down in the Bloch sphere. In the lower line, the square remains 'Off' and the qubit remains up in the Bloch sphere. The amplitude display represents |10⟩. Hence, the operation of SWAP gate upon |10⟩.	|10⟩
	|10⟩		In the upper line, the square remains 'Off' and the qubit remains up in the Bloch sphere. In the lower line, the square turns 'On' and the qubit flips down in the Bloch sphere. The amplitude display represents |01⟩. Hence, the operation of SWAP gate upon |01⟩.	|01⟩
	|11⟩		Both the squares turn 'On', the qubits in the Bloch spheres flip down and the amplitude display represents |11⟩. Hence, the operation of SWAP gate upon |10⟩.	|11⟩

Attention: The learners are advised to read critically the above slides and experiment in Quirk portal. For any clarifications, the author may be contacted via <*uchitavire@gmail.com*>. Happy Learning.

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