Motivation

- Problems with Signaling games: the set of PBE is usually large.
  - In addition, some equilibria are insensible ("crazy").
- Hence, how can we restrict the set of equilibria to those prescribing sensible behavior?
- Solutions to refine the set of PBE:
  - Intuitive criterion [Cho and Kreps, 1987] ($\leftarrow$ easy), and
  - "Universal Divinity" criterion [Banks and Sobel, 1987] (also referred as the D_1-criterion)($\leftarrow$not for this course. For references, see links on the course website).

Signaling games

- One player is privately informed.
  - For example, he knows information about market demand, his production costs, etc.
- He uses his actions (e.g., his production decisions, investment in capacity, etc.) to communicate/conceal this information to other uninformed player.

Time Structure

In particular, let us precisely describe the time structure of the game:

1. Nature reveals to player i some piece of private information, $\theta_i \in \Theta$.
   - For instance, $\Theta = \{\theta_L, \theta_H\}$.
2. Then, player i, who privately observes θ_i, chooses an action (or message m) which is observed by other player j.
3. Player j observes message m, but does not know player i's type. He knows the prior probability distribution that nature selects a given type θ_i from Θ, $\mu(\theta_i) \in [0, 1]$.
   - For example, the prior probability for $\Theta = \{\theta_L, \theta_H\}$ can be $\mu(\theta_L) = p$ and $\mu(\theta_H) = 1 - p$.

Time Structure

Continues:

4. After observing player i's message, player j updates his beliefs about player i's type.
    1. Let $\mu(\theta_i|m)$ denote player j's beliefs about player i's type being exactly $\theta = \theta_i$ after observing message m.
    2. For instance, the probability that player i is a Friendly type given that he offered me a gift is $\mu(F|Gift)$
5. Given these beliefs, player j selects an optimal action, a, as a best response to player i's message, m.

Outline of the Intuitive Criterion

Consider a particular PBE, e.g., pooling PBE with its corresponding equilibrium payoffs $u_i^*(\theta)$.

Application of the Intuitive Criterion in two steps:

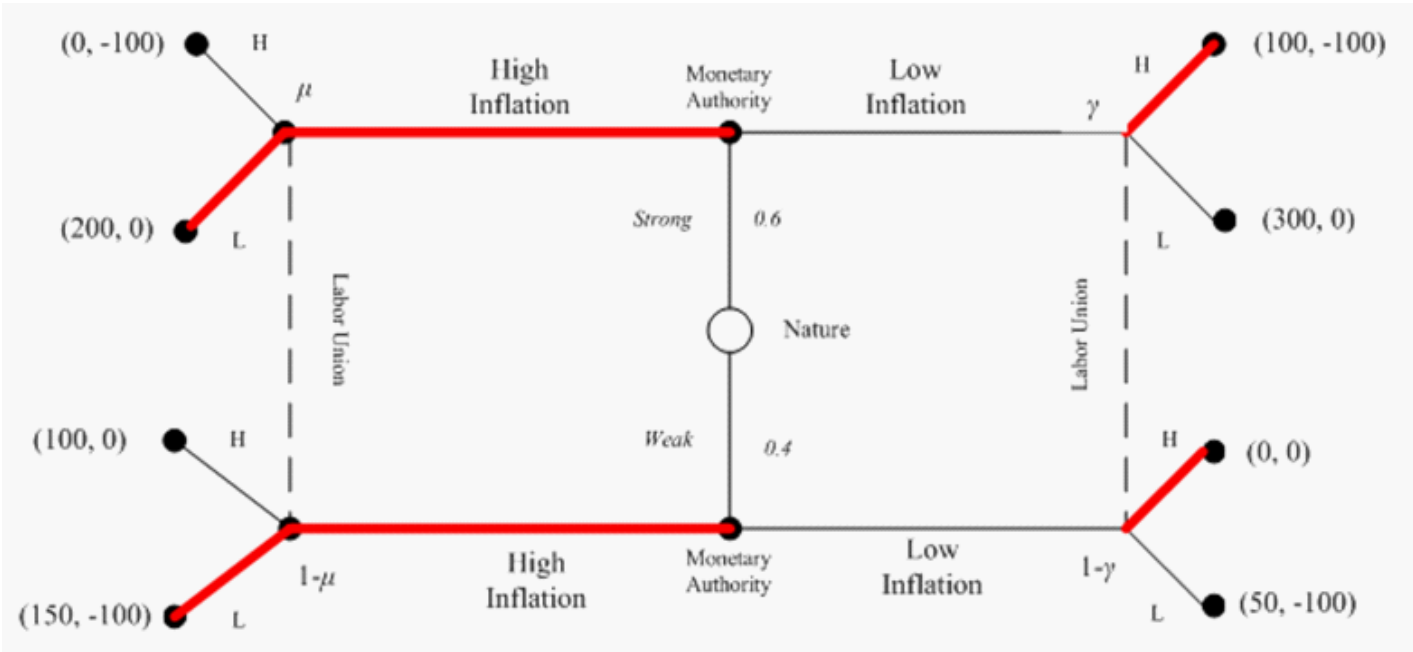
1. **First Step:** Which type of senders could benefit by deviating from their equilibrium message?
2. **Second Step:** If deviations can only come from the senders identified in the First Step, is the lowest payoff from deviating higher than their equilibrium payoff?
    1. If the answer is **yes**, then the equilibrium **violates** the Intuitive Criterion.
    2. If the answer is **no**, then the equilibrium **survives** the Intuitive Criterion.

Example 1 - Discrete Messages

- Let us consider the following sequential game with incomplete information:
  - A monetary authority (such as the Federal Reserve Bank) privately observes its real degree of commitment with maintaining low inflation levels.
  - After knowing its type (either Strong or Weak), the monetary authority decides whether to announce that the expectation for inflation is High or Low.
  - A labor union, observing the message sent by the monetary authority, responds by asking for high or low salary raises (denoted as H or L, respectively)

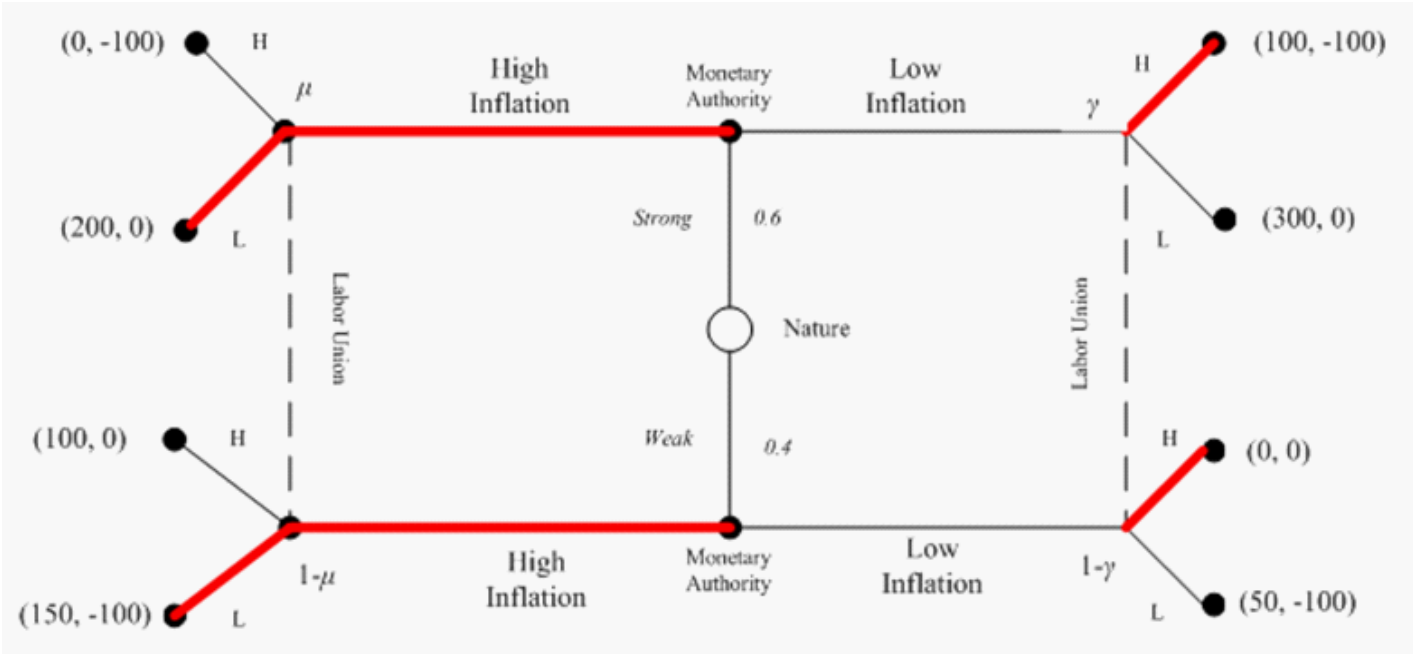
Example 1 - Discrete Messages

- The only two strategy profiles that can be supported as a PBE of this signaling game are:
  - A polling PBE with both types choosing (High, High); and
  - A separating PBE with (Low, High).
- Let us check if (High, High) survives the Intuitive Criterion.



Example 1 - Discrete Messages

- Notice that the pooling PBE prescribes a somewhat insensible behavior from the Strong monetary authority:
  - It announces a High inflation target for next year.
- Let us check if this behavior survives the Intuitive Criterion.



First Step

- **First Step:** Which types of monetary authority have incentives to deviate towards Low inflation?
  - *Low inflation* is an off-the-equilibrium message.
- Let us first apply condition (1) to the **Strong** type,

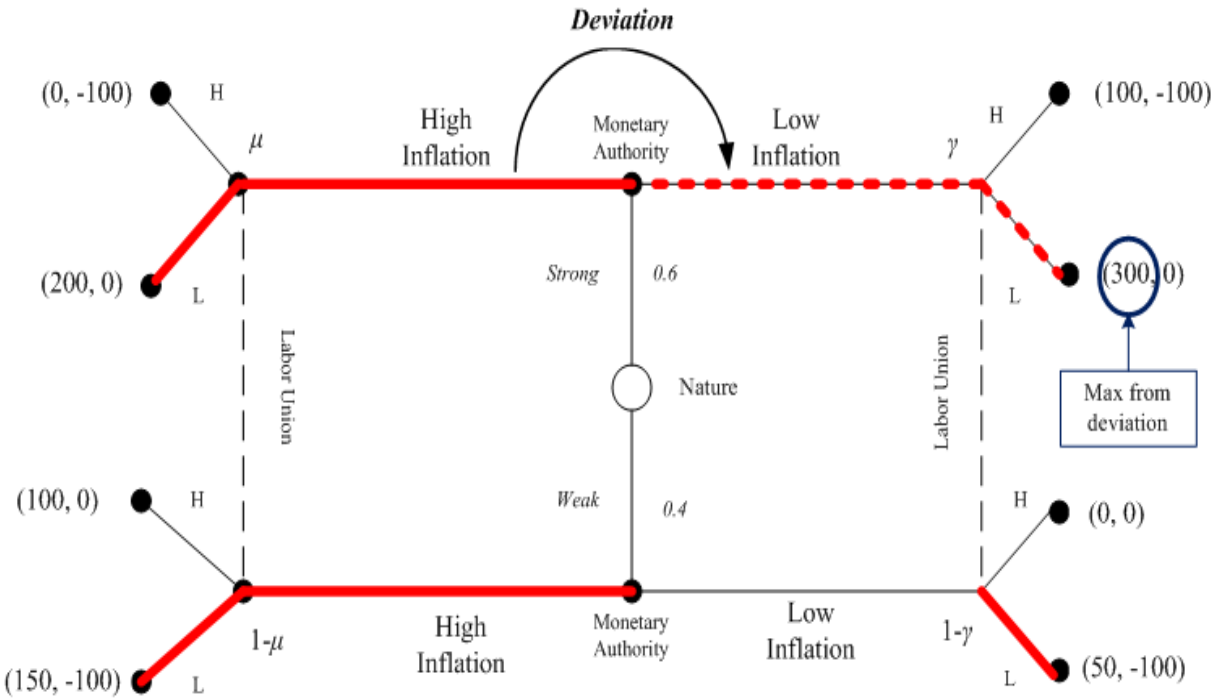
$$\underbrace{u^*_{Mon}\left(High|Strong\right)}_{\text{Equil. Payoff}} < \underbrace{\max_{a_{Labor}} u_{Mon}\left(Low|Strong\right)}_{\text{Highest payoff from deviating to Low}}$$

$$200 < 300$$

- Hence, the Strong type of monetary authority **has incentives** to deviate towards Low inflation.

First Step

- Graphically, we can represent the incentives of the Strong monetary authority to deviate towards Low inflation as follows:



First Step

- Let us now check if the **Weak** type also has incentives to deviate towards Low:

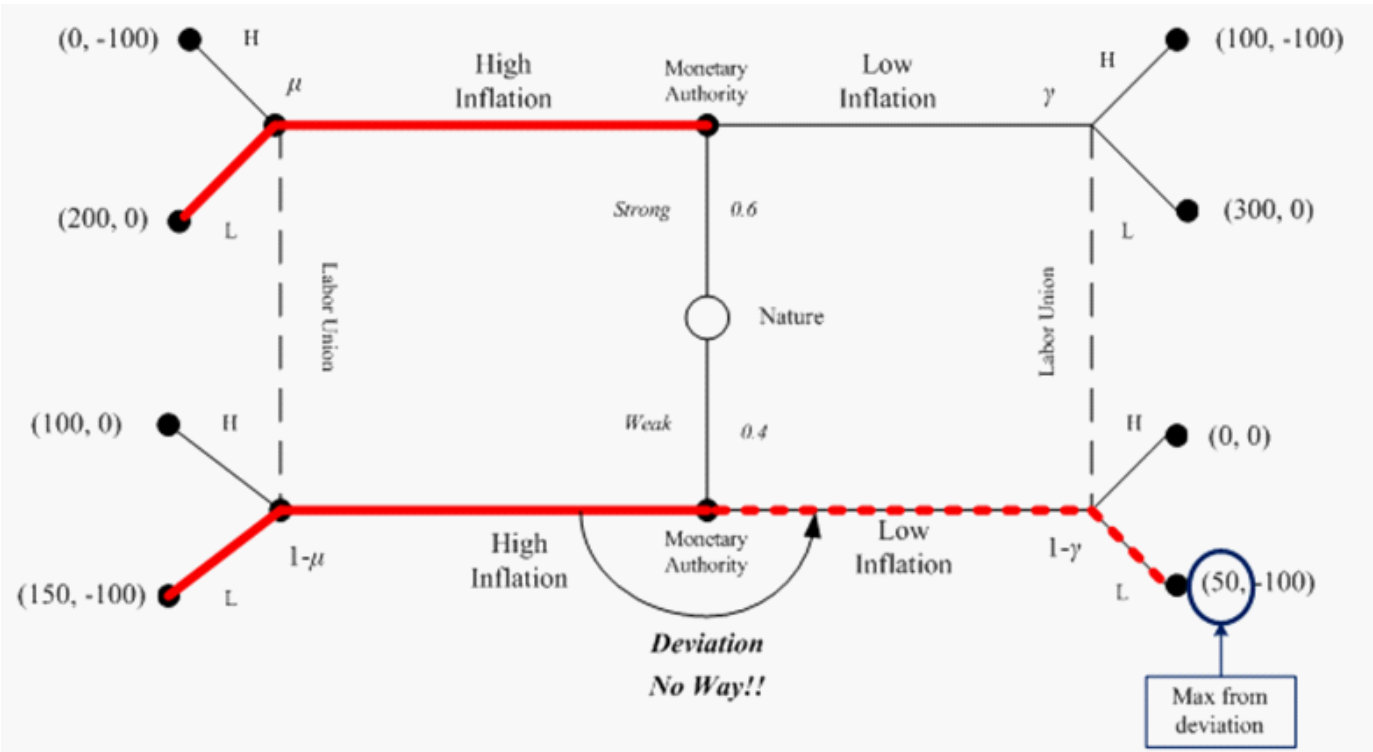
$$\underbrace{u^*_{Mon}\left(High|Weak\right)}_{\text{Equil. Payoff}} < \underbrace{\max_{a_{Labor}} u_{Mon}\left(Low|Weak\right)}_{\text{Highest payoff from deviating to Low}}$$

$$150 > 50$$

- Thus, the Weak type of monetary authority **does not have incentives** to deviate towards Low inflation.

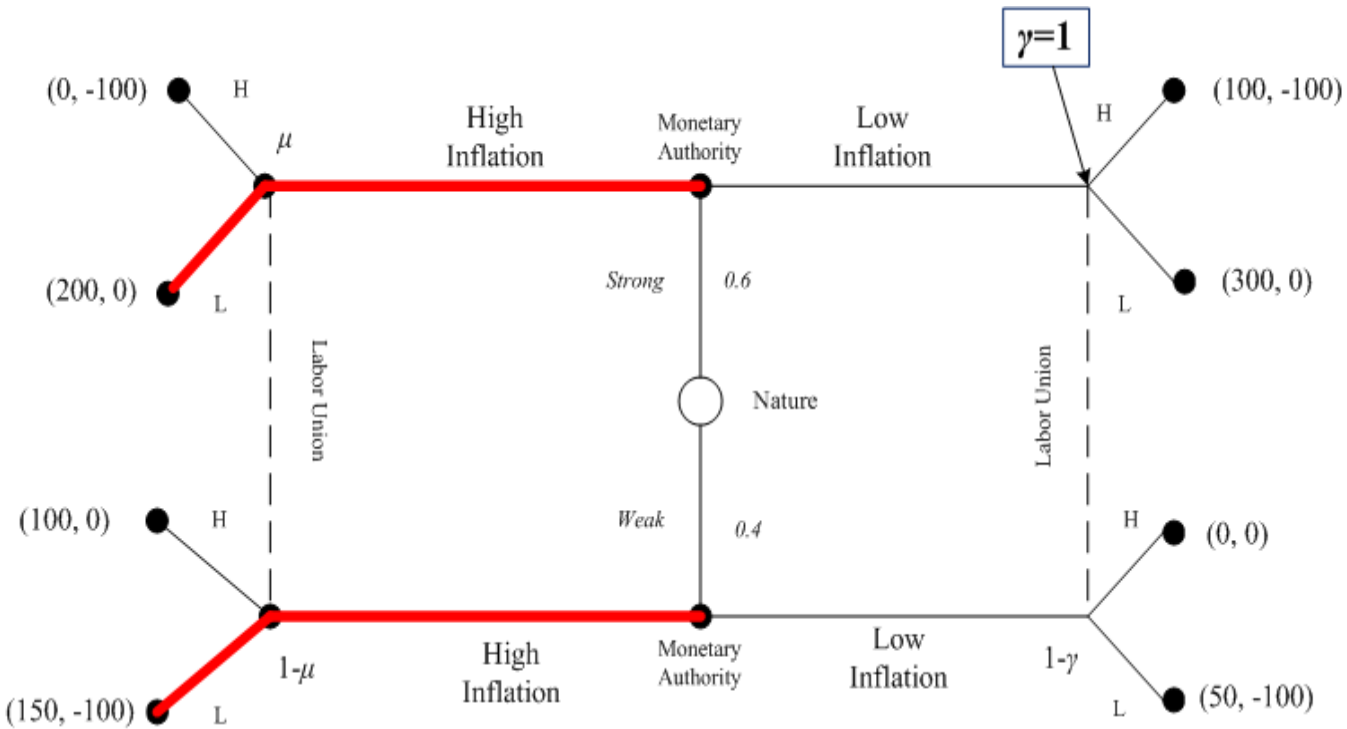
First Step

- Graphically, we can represent the lack of incentives of the Weak monetary authority to deviate towards Low inflation as follows:



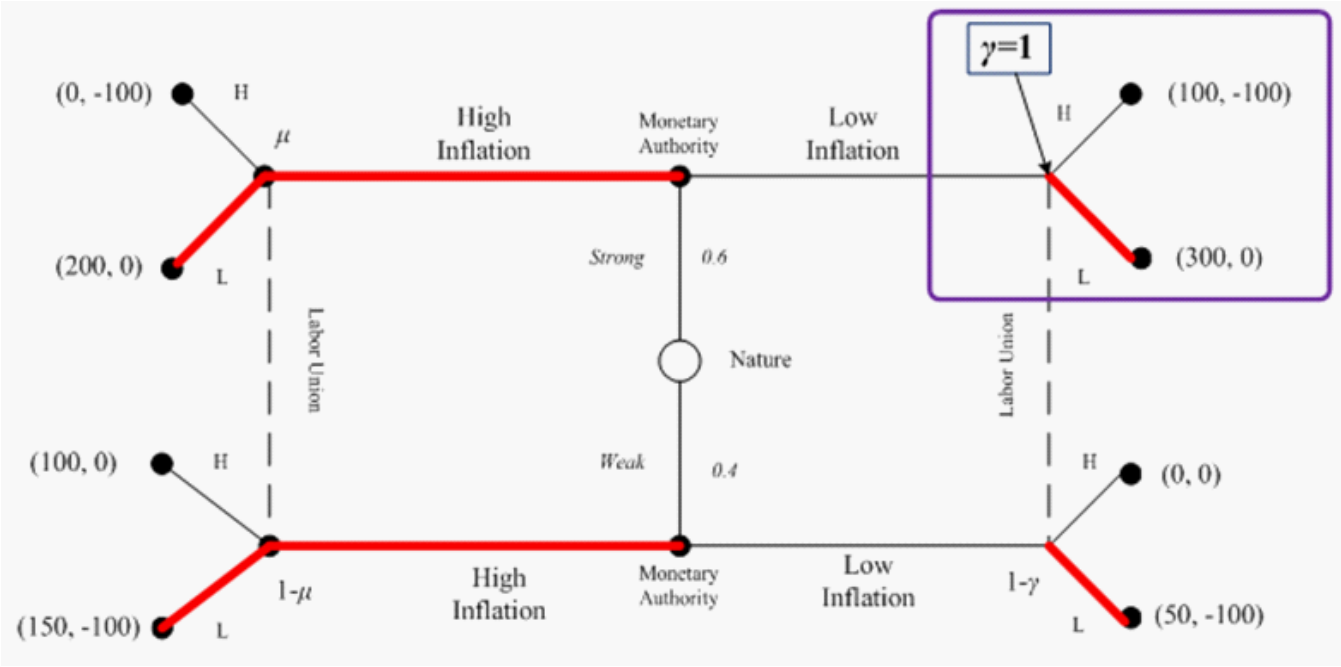
First Step

- Hence, the only type of Monetary authority with incentives to deviate is the Strong type, $\Theta^{**}(Low) = \{Strong\}$.
- Thus, the labor union beliefs after observing *Low inflation* are restricted to $\gamma = 1$. (Not arbitrary, $\gamma \in [0, 1]$, anymore)



First Step

- This implies that the labor union chooses *Low wage demands* after observing *Low inflation*. (0 is larger than -100, in the upper right-hand node).

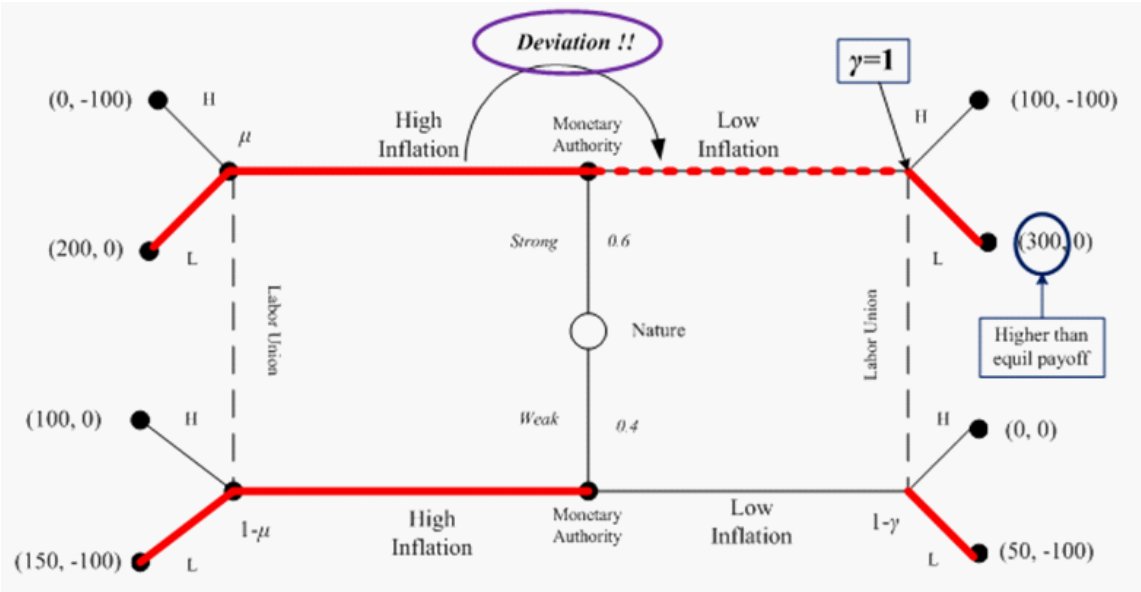


Second Step

- Study if there is a type of monetary authority and a message it could send such that condition (2) is satisfied:

$$\min_{a \in A^*(\Theta^{**}(m),m)} u_i\left(m, a, \theta\right) > u_i^*\left(\theta\right).$$

which is indeed satisfied since $300 > 200$ for the Strong monetary authority.



As a result...

- The pooling PBE of (High, High) *violates* the Intuitive Criterion:
  - there exists a type of sender (Strong monetary authority) and
  - a message (Low)
  - which gives to this sender a higher utility level than in equilibrium, regardless of the response of the follower (labor union).

Possible speech

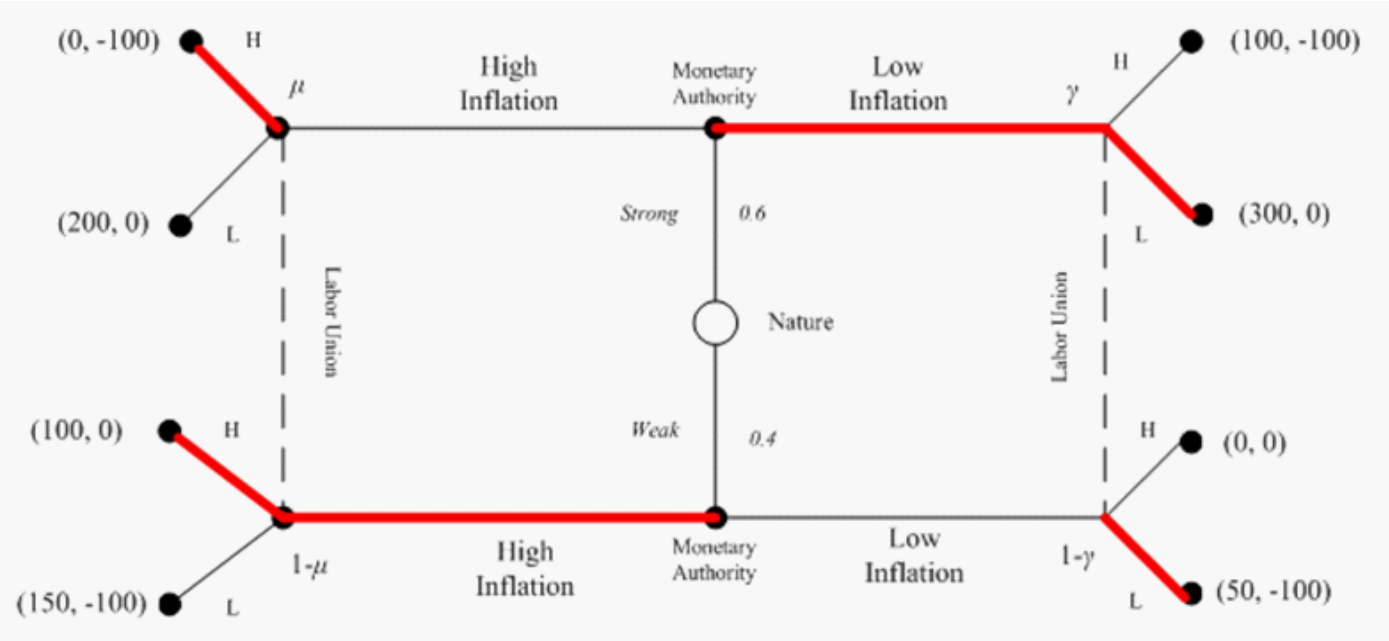
Possible speech from the sender with incentives to deviate (Strong monetary authority):

"It is clear that my type is in $\Theta^{**}(m) = \{Strong\}$. If my type was Weak I would have no chance of improving my payoff over what I can obtain at the equilibrium (condition (1)) by selecting Low inflation. We can therefore agree that my type is Strong. Hence, update your believes as you wish, but restricting my type to be in $\Theta^{**}(m) = \{Strong\}$.

Given these beliefs, your best response to my message improves my payoff over what I would obtain with my equilibrium strategy (condition (2)). For this reason, I am sending you such off-the-equilibrium message of Low inflation."

Only separating PBE survives

Therefore, there is only one equilibrium in this game that survives the Intuitive Criterion: the separating PBE with (Low, High)



One second...

Why can't we apply the Intuitive Criterion to the above separating PBE, to test if this equilibrium survives the Intuitive Criterion?

- In this separating PBE there is no off-the-equilibrium message, since all messages are used by either type of sender.
- Recall that in the Intuitive Criterion we start by checking if a sender has incentives to deviate towards an off-the-equilibrium message, then we restrict the responder's off-the-equilibrium beliefs, etc.

Insight

- This implies that, in signaling games with two types of senders and two available actions (messages) for the senders, the separating PBEs always survive the Intuitive Criterion.
- What if two types of senders can choose among three possible messages?
  - Type t_1 sends message m_1,
  - Type t_2 sends message m_2, and
  - Nobody sends message m_3!! Then, m_3 is an off-the-equilibrium message.
    - In this case, we can check if this separating PBE survives/violates the Intuitive Criterion.

Beer-Quiche game (Breakfast in the Far West)

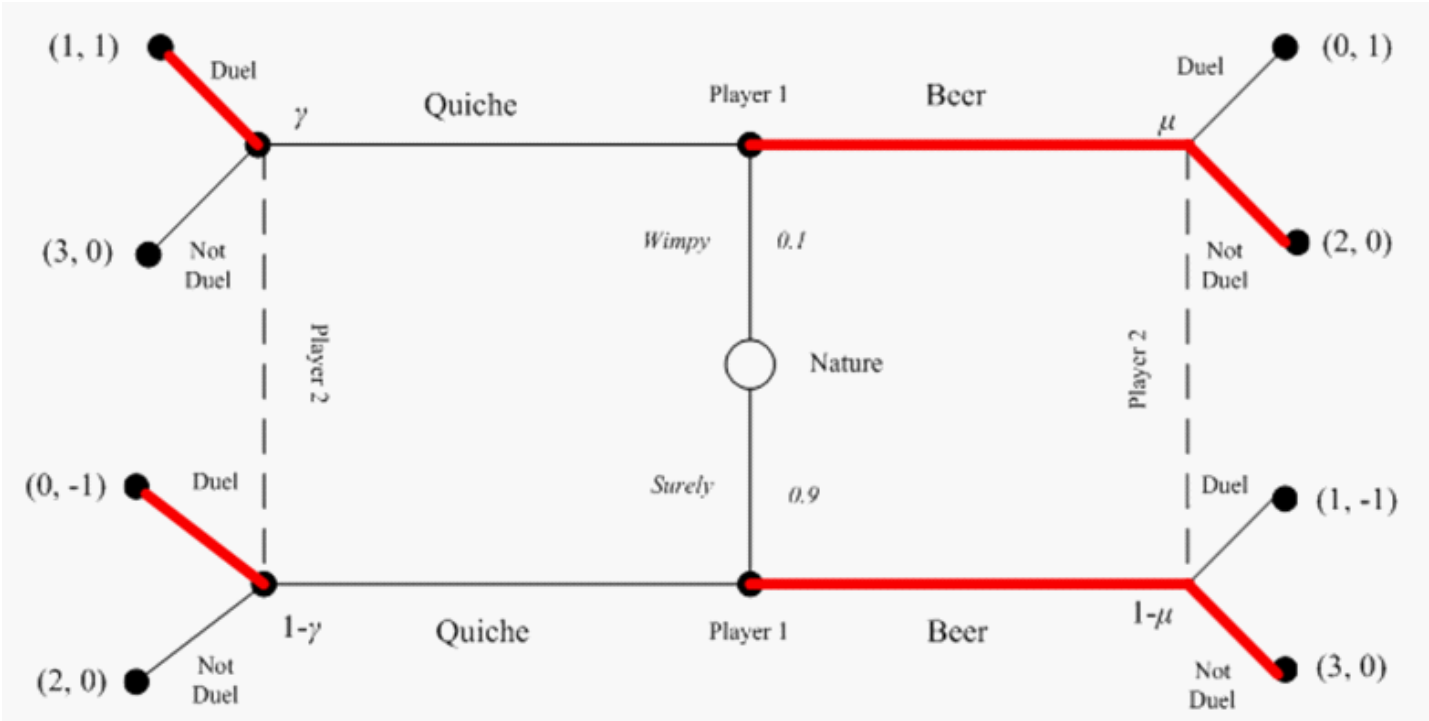
Final exam

1. Player 1 just moved into town, and nobody else but him knows whether he is "Wimpy" or "Surely" (i.e., Weak or Strong).
2. At the moment in the morning the Saloon is a quite place, and he is deciding what to have for breakfast:
    1. Quiche (something that he really enjoys if he is a Wimpy type), or
    2. Beer (something he prefers when he is of the Surely type).
3. Then, player 2 (the typical character looking for trouble in this kind of films) enters into the Saloon and observes the newcomer having breakfast...
    1. but does not know whether he is Surely or Wimpy.

Beer-Quiche game (Breakfast in the Far West)

Part (a) of the exercise

1. Check if the pooling equilibrium in which both types of player 1 have Beer for breakfast survive the Cho and Kreps' (1987) Intuitive Criterion.



Beer-Quiche game (Breakfast in the Far West)

Part (b) of the exercise

1. Check if the pooling equilibrium in which both types of player 1 have Quiche for breakfast survives the Cho and Kreps' (1987) Intuitive Criterion.