Optimising Cross-Asset Carry[1]

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Abstract

The term "carry" has been primarily studied and explored within currency markets where, contrary to the uncovered interest rate parity, borrowing from a low interest rate country and investing in a high interest rate country has historically delivered positive and statistically significant returns. This paper extends the notion of carry to different asset classes by looking at the futures markets of commodities, equity indices and government bonds. We explore the profitability of cross-sectional and time-series variants of the carry strategy within each asset class but most importantly we investigate the benefits of constructing a multi-asset carry strategy after properly accounting for the covariance structure of the entire universe. Multi-asset carry allocations benefit from the low correlation between asset-class specific carry portfolios and do not exhibit significant downside or volatility risk which have been traditionally associated with the FX carry strategy.

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1. Introduction

The term "carry" is generally associated with an FX trading strategy that borrows from a country with a low interest rate and invests in a country with a high interest rate, aiming to capitalise the rate differential as long as the FX rate does not exhibit any adverse move in the meantime which wipes out any gains.[3]

Historically, the short-term FX movements have been unpredictable, hence rendering the FX carry trade a profitable strategy. This profitability constitutes a violation of the no-arbitrage condition of uncovered interest rate parity and the carry premium can only be justified as long as it constitutes compensation for some systematic source of risk; otherwise, it should be related to mispricing. The academic evidence seems conflicting, but the most plausible explanations justify the premium as compensation for bearing currency or equity crash risk. We provide a more detailed review of the suggested explanations later in the paper.

Generalising the above concept and drawing motivation from Koijen, Moskowitz, Pedersen and Vrugt (2017), we define "carry" as the return (or "yield") that an investor enjoys if all market conditions, including the asset's price, remain the same. Put differently, carry measures the value appreciation that accrues to the owner of an asset if there is no expected or unexpected price change:

$$Return = \underbrace{\boldsymbol{Carry} + \boldsymbol{E}(price\ appreciation)}_{\boldsymbol{E}(Return)} + unexpected\ price\ shock \tag{1}$$

Following from this equation, the estimation of carry becomes readily available and therefore requires no model assumptions.[4] Most importantly, it allows us to extend the concept of carry and explore its potential profitability across different asset classes: commodities, government bonds, equity indices and, obviously, FX. Our first objective is therefore to define and measure carry for each asset class and subsequently to document the performance of simple carry portfolios separately for each asset class.

Our second objective is to explore in detail the diversification benefits from constructing a multi-asset carry portfolio and the added value of actively making use of the rich multi-asset covariance structure. Historically, the vast academic work on FX carry has focused on simple cross-sectional (by ranking the currencies based on the Libor rate of the respective country) cash-neutral portfolios. Fewer papers have also looked at the time-series nature of the carry signals (that is, the momentum of carry) and even fewer papers have actively looked at optimising the allocation based on some portfolio optimisation methodology. In all fairness, focusing on a single asset class (FX in this case) can possibly justify the employment of simple portfolio weighting schemes, as the members of a single asset class generally exhibit similar levels of volatility and stable correlations. If there is any breadth in the covariance structure, this is at a multi-asset level. This is exactly what we are after: to construct risk-optimised multi-asset carry portfolios.

The literature on carry is vastly dominated by studies that focus on the FX carry trade. Table 1 provides an overview of the recent academic activity. However, there is very little evidence on carry dynamics across multiple asset classes. The only papers that exist –to our knowledge– are by Ahmercamp and Grant (2013), Baz *et al.* (2015) and Koijen *et al.* (2017). Interestingly, none of these papers focus explicitly on the diversification potential of combining cross-sectional with time-series carry signals, as well as improving the diversification of the multi-asset portfolio by taking into account the covariance structure. This is the literature gap that this paper aims to fill.

The rest of the paper is structured as follows. Section 2 discusses the concept of FX carry and Section 3 describes the extension to the multi-asset space. Section 4 contains details on our dataset and provides diagnostics of the carry metrics across asset classes. Section 5 constitutes the core part of the empirical analysis and includes results for cross-sectional, time-series and optimised variants of carry. Section 6 explores the dependence of carry strategies on crash risk. Section 7 concludes the paper.

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[3] One can argue that the term carry relates to the *"cost of carry"*, a term associated with commodities markets and the theory of *"normal backwardation"*, introduced by Keynes (1930). However, we believe that "carry" has become popular in the investment community when associated with the FX markets.

[4] In a Bayesian framework, one can argue that carry represents the mean of the uninformed prior of expected returns. However, one can even support the idea of carry being the mean of the informative prior of expected returns.

Table 1: Recent Literature on Carry

	Universe	Cross-Sectional	Time-Series	Optimised
Burnside, Eichenbaum & Rebelo (2011)	FX		√	
Olszweski & Zhou (2014)	FX	√		
Barroso & Santa-Clara (2015)	FX	√	√	√
Doskov & Swinkels (2015)	FX	√		
Bekaert & Panayotov (2016)	FX	√		
Ackermann, Pohl & Schmedders (2016)	FX	√		√
Daniel, Hodrick & Lu (2017)	FX	√	√	√
Ahmercamp & Grant (2013)	Multi-asset		√	
Baz, Granger, Harvey, Le Roux & Rattray (2015)	Multi-asset	√		
Koijen, Moskowitz, Pedersen & Vrugt (2017)	Multi-asset	√	√	
This paper	Multi-asset	√	√	√

2. The concept of FX Carry

The concept of carry has been manifested in the foreign exchange markets where historically borrowing at the low interest rate country and investing at the high interest rate country has yielded a statistically strong and positive excess return. Is this positive return justified by asset pricing principles? Does investing in the higher interest rate country come at a higher risk to justify the existence of a premium or is the carry premium an artefact of market inefficiency? To answer these questions, we first take a short detour around the fundamental no-arbitrage concepts of the uncovered and covered interest rate parities.

The **Uncovered Interest Rate Parity (UIRP)** is a no-arbitrage principle that suggests that any interest rate differential between two countries should be completely offset by an adverse movement in the exchange rate. In particular, the low interest rate currency should be expected to appreciate so much as to render an investor indifferent between (a) investing in the domestic Libor market, or (b) investing in an **FX carry trade**, i.e. borrowing in the domestic Libor market, to invest in the foreign –higher-rate– market and converting back any gains at a future date. Put differently, if UIRP holds, then the FX carry trade should not generate any statistically significant positive returns in excess of the domestic Libor market.

Contrary to these theoretical predictions, overwhelming empirical evidence (see for example Hansen and Hodrick, 1980 and Engel, 1996) has shown that the UIRP does not always hold in practice at least for short horizons. In particular, the short-term exchange rate movements appear to be unpredictable (FX rates behave like martingales), which renders the carry trade, on average, profitable. The strategy would only generate negative excess returns if the FX rate exhibits an adverse movement that would wipe out the interest rate differential.

In order to eliminate (or correct) the market inefficiency and reinstate the no-arbitrage principle one can hedge against, or "cover", any unfavourable FX movements with the use of a forward contract that locks the future FX rate at which any gains will be converted back into the local currency. For obvious semantic reasons, this is called the **Covered Interest Rate Parity (CIRP)**. Based on the CIRP, the investor would now be indifferent between (a) investing in the domestic market, or (b) borrowing in the domestic market, to invest in the foreign –higher-rate– market and entering a forward contract to lock the future exchange rate.

If CIRP holds, which is indeed generally the case, then the above strategies should generate the same return, and therefore the forward exchange rate for a contract maturing at time $t+1$, denoted by F_t, is uniquely determined by the spot foreign exchange rate, S_t as follows:

$$CIRP \iff F_t = S_t \cdot \frac{1 + r_t^{\$}}{1 + r_t^{*}} \tag{2}$$

where $r_t^{\$}$ denotes the prevailing at time-t domestic (USD) Libor rate and r_t^{*} denotes the respective foreign rate.

If both UIRP and CIRP hold, then it becomes obvious that the forward price becomes an unbiased predictor[5] of the futures stock price. This is referred to as the *forward rate unbiasedness hypothesis*:

$$\begin{Bmatrix} UIRP \\ CIRP \end{Bmatrix} \Leftrightarrow F_t = \mathbf{E}_t(S_T) \tag{3}$$

However, as already mentioned, the UIRP does not empirically hold, in which case the above hypothesis does not hold either. As a consequence, the prevailing forward price is a biased predictor of the future spot exchange rate. The bias effectively represents the risk premium that the FX carry trade is trying to capitalise (Fama, 1984; Lustig, Roussanov and Verdelhan, 2011).[6]

As to what is driving the risk premium associated with the FX carry trade, there has been a considerable debate in academic literature. The fact that FX carry has historically delivered superior risk-adjusted returns at the expense of a relatively sizeable negative skewness can effectively justify the positive returns as compensation for bearing currency crash risk (Brunnermeier, Nagel and Pedersen, 2008; Rafferty 2012; Jurek, 2014; Farhi and Gabaix, 2016). This risk has been associated to funding liquidity risk (Brunnermeier et al., 2008; Brunnermeier and Pedersen, 2009), FX volatility risk (Bhansali, 2007; Menkhoff, Sarno, Schmeling and Schrimpf, 2012), consumption growth risk (Lustig and Verdelhan, 2007) or even a "peso problem" (Burnside, Eichenbaum, Kleshchelski and Rebelo, 2011)[7]. The cyclicality of the carry trade has also been related to the equity market, hence justifying the premium as compensation for bearing equity downside risk (Dobrynskaya, 2014; Lettau, Maggiori and Weber, 2014). Adding to the debate, Bekaert and Panayotov (2016) and Daniel, Hodrick and Lu (2016) challenge these crash-based explanations.

To summarise, the FX carry trade generally aims to capitalise on the empirical failure of the UIRP by borrowing from countries with lower interest rates and investing in countries with higher interest rates. In other words, a carry strategy for a USD investor would generally overweight countries with large interest rate differential $r_t^* - r_t^\$$ and respectively underweight countries with low (if not negative) interest rate differential.

Following the above, the interest rate differential is of critical importance in deciding the allocation in an FX carry trade. The crucial step that will allow us to extend the notion of carry to other asset classes is to depart from the interest rate differential, which is an FX-specific metric of carry, to an asset-class-free definition. To achieve this, we start from equation (2) and solve for the rate differential:

$$r_t^* - r_t^\$ = \left(1 + r_t^\$\right) \cdot \frac{S_t - F_t}{F_t} \tag{4}$$

Notice that the factor $\left(1 + r_t^\$\right)$ is just a proportionality factor, common across all foreign currencies, which represents the value of \$1 at time $t + 1$. As a consequence, the above equation allows us to generate trading signals for a carry strategy by looking directly at the term structure of futures prices. For instance, a country with a positive interest rate differential would have a futures curve in backwardation, and conversely a country with a negative interest rate differential would have a futures curve in contango. These statements set a clear path in extending the concept of carry to other asset classes outside FX, given that we can simply generate carry signals by looking at the respective futures markets. The next section explains these dynamics.

3. Extending the idea across Asset Classes

Following the detailed discussion of the FX carry dynamics, we can now formally define carry across multiple asset classes by observing directly the behaviour of the respective futures/forward markets:

- Assets that exhibit a term structure of futures in **backwardation** should generate a **positive roll yield** and therefore a positive excess return when market conditions remain unchanged. These assets should therefore be generally **over-weighted** in a carry portfolio.

[5] The expectation is under the physical probability measure, $\mathbb{P}$.

[6] The "risk premium" should not be confused with the so-called "forward premium", which is the difference between the spot exchange rate and the forward rate.

[7] The *"peso problem"* explains the situation under which financial markets might appear inefficient and flawed, but in reality they could just incorporate an unprecedented and very-low probability event that may simply have not yet occurred. Based on Sill (2000), the term is often attributed to Nobel laureate Milton Friedman following his commentary on the Mexican Peso / US Dollar exchange rate movement during the 1970's.

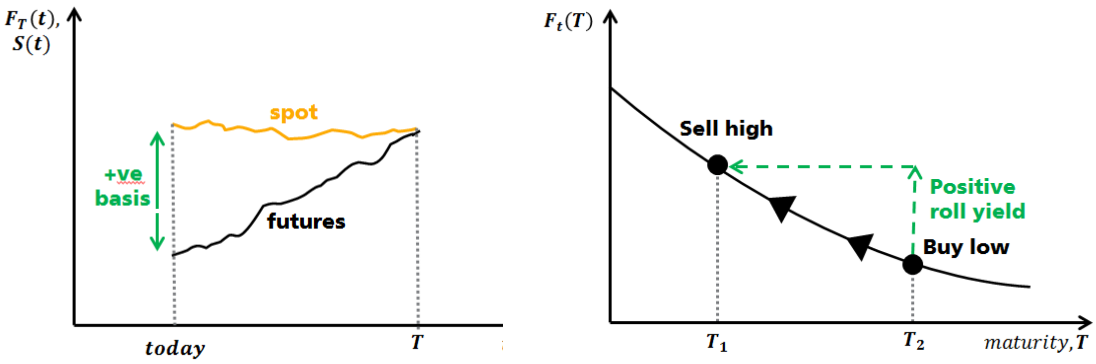
- Assets that exhibit a term structure of futures in **contango** should generate a **negative roll yield** and therefore a negative excess return when market conditions remain unchanged. These assets should therefore be generally **under-weighted** in a carry portfolio.

Panels A and B of Figure 1 explain illustratively the above dynamics. Notice that when the market conditions remain unchanged, the term structure does not move at all and therefore the entirety of asset return is the roll yield.

Figure 1: Carry dynamics when the spot price does not change

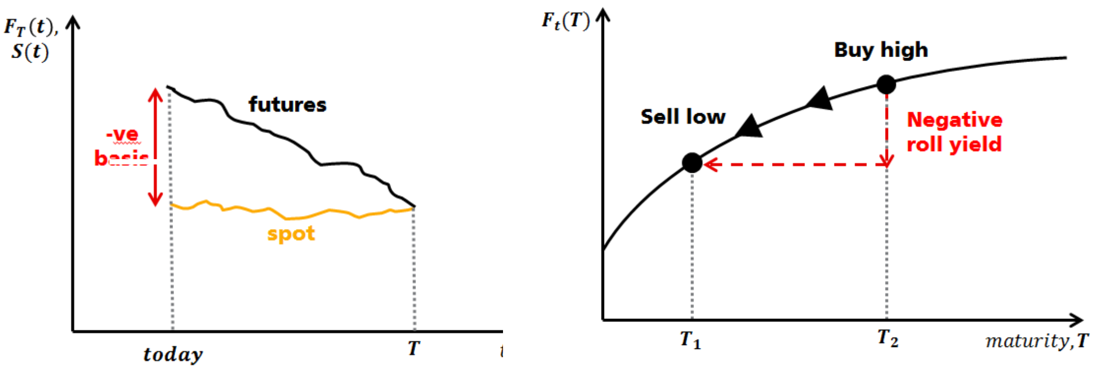
Panel A: Backwardation mechanics

<u>Life of a contract</u> | <u>Term Structure</u>



Panel B: Contango mechanics

<u>Life of a contract</u> | <u>Term Structure</u>



Notes: The figure presents the carry dynamics for an asset in backwardation (Panel A) or in contango (Panel B) as long as the conditions do not change.

The generic carry strategy of over-weighting assets with the strongest backwardation and under-weighting (or even going short; we will return to this point later on) assets with the strongest contango would therefore extract whatever "yield" each asset of any asset class is willing to pay the investor for holding it. In order to comprehend exactly the nature of this yield for each asset class, we start from equation (4) that is FX-specific and define carry, C_t, as the right-hand side of this equation (ignoring the proportionality factor that is common across assets), in line with Koijen *et al.* (2017):

$$C_t = \frac{S_t - F_t}{F_t} \tag{5}$$

Evidently, carry is just the slope of the futures curve. Using the arbitrage-free definition of the futures price per asset class and substituting it in the above equation we can deduce what carry represents across the different asset classes.

Table 2 contains the arbitrage-free future price per asset class, as well as the value of carry based on equation (5) after substituting out F_t. The last column of the Figure explains in detail the type of yield that we can extract from each asset class:

- For FX markets, the investor receives (or pays, if negative) the interest rate of the foreign country in excess of the financing cost, which is obviously the domestic (USD) interest rate.

- For equity index markets, the investor receives the expected future dividend yield in excess of the financing cost.
- For commodity markets, the investor receives the convenience yield in excess of any storage and financing costs.
- For government bonds the investor receives two components of return: (a) the yield to maturity in excess of the financing cost (this is the so-called "term premium"; Fama and French 1993) and (b) the "roll-down" of the bond across the yield curve as this approaches maturity.

Table 2: The notion of Carry across asset classes

Asset Class	Futures Price (assume $T - t = 1\ year$)	Carry	Carry Interpretation
FX	$F_t = S_t \cdot \frac{1 + r_t}{1 + r_t^*}$ r_t^*: foreign currency risk-free rate	$C_t \propto r_t^* - r_t$	Capitalise the foreign risk-free rate <u>in excess of the financing cost</u>
Equity Indices	$F_t = S_t \cdot (1 + r_t - q_t)$ q_t: dividend yield	$C_t \propto q_t - r_t$	Capitalise the expected dividend yield <u>in excess of the financing cost</u>
Commodities	$F_t = S_t \cdot (1 + r_t + c_t - y_t)$ c_t: storage costs y_t: convenience yield	$C_t \propto (y_t - c_t) - r_t$	Capitalise the convenience yield net of the storage costs, <u>in excess of the financing cost</u>
Government Bonds	$F_t = \frac{1 + r_t}{(1 + y_t^{10Y})^{10}}$ $y_t^{9Y},\ y_t^{10Y}$: 9yr, 10yr ZCB yield	$C_t \propto y_t^{10Y} - D_{mod} \cdot (y_t^{9Y} - y_t^{10Y}) - r_t$ D_{mod}: modified duration	Capitalise (a) the yield to maturity and (b) the roll-down of the bond across the yield curve, <u>in excess of the financing cost</u>

Notes: Interpretation of carry, following from Koijen *et al.* (2017).

Given these interpretations one might wonder why a carry strategy can generate positive excess returns. What is the underlying economic rationale? Is there any risk that we are compensated for? We next provide some narrative that can guide us.

FX: *Why should higher yielding currencies outperform lower yielding currencies?*

We have already elaborated on the reasons why the FX carry trade has historically exhibited positive returns higher Libor rates are generally associated with rising inflation, funding liquidity concerns, or consumption growth risk, which render the higher-yielding currencies generally more vulnerable, hence justifying the positive FX carry. For relevant academic literature, see the summary earlier in Section 2 of the paper.

Commodities: *What can justify the convenience yield (in excess of any storage costs) that an investor earns from investing in a backwardated commodity?*

Keynes's (1930) theory of "normal backwardation" suggests that commodity producers take short futures positions in order to hedge against price drops and therefore pay a premium to an investor that offers this insurance and takes a long position in the futures contract; this positive premium comes in the form of the carry premium. An alternative explanation of backwardation that is inventory-related, suggests that storing a commodity (given the costs) should compensate the holder with a positive return (hence "convenience" yield) in periods of short supply; the underlying risk here being that these short supply shocks can turn out to be temporary. Contrary to Keynes's point, one can argue that commodity consumers take long futures positions in order to hedge against unexpected future price surges; in this scenario, they should pay a premium to the investor that offers the insurance and takes a short position, in which case contango arises. In any case, roll yield has been documented as an important driver of commodity returns; see Erb and Harvey (2006, 2016), Gorton and Rouwenhorst (2006), Yang (2013), Gorton, Hayashi, Rouwenhorst (2013), Bharwaj, Gorton and Rouwenhorst (2015)

Government Bonds: *What are the risks of investing in a long-term bond, when the yield curve is upward sloping (hence, the futures curve in backwardation)?*

Holding a bond up to maturity should compensate an investor with the yield-to-maturity in excess of the risk-free rate (short-term end of the yield curve); this is effectively the term spread or the slope of the yield curve, which is one of the main drivers of bond returns (Fama and Bliss, 1987, Campbell and Shiller, 1991 and Fama and French, 1993). Yield curves are typically upward sloping and therefore the term spread is positive in order to compensate long-term bond investors for potential illiquidity risk (longer-dated bonds being less liquid than shorter-dated bonds), tightening monetary policy risk and inflation risk (increasing rates in expectation of higher inflation) or some broader macroeconomic risk.

Equity Indices: *Why should an index with higher expected dividend yield outperform an index with lower expected dividend yield?*

If the equity carry strategy turns out to be profitable and if it constitutes compensation for systematic risk, then surely equity indices with higher expected dividend yield must be fundamentally riskier. This discussion resembles equity value investing, where dividend yields have historically been good predictors of stock returns (Fama and French 1988, Campbell and Shiller, 1988 and Ang and Bekaert 2007). Put differently, the profitability of equity carry can be related to the equity value premium (Fama and French, 1993). However, as very nicely illustrated by Koijen et al. (2017); a typical value strategy would use realised (hence backward-looking) dividend yield data; instead, an equity carry strategy focuses on the forward-looking –risk-neutral– expectation of future dividend yields as implied by market dynamics and therefore by the slopes of equity index futures curves.

4. Data and Carry Diagnostics

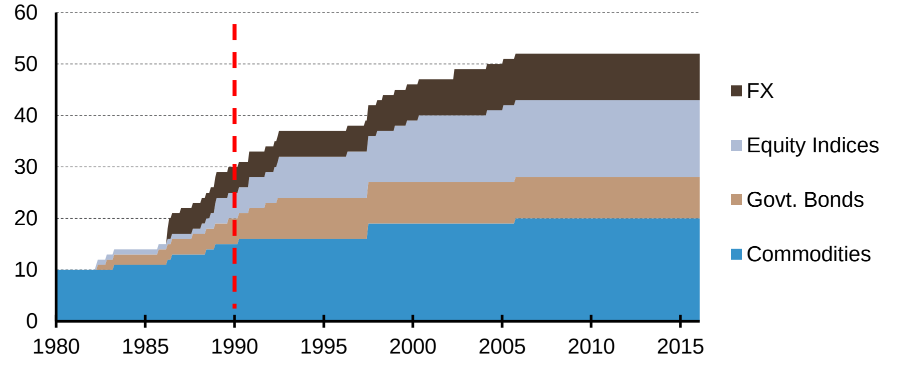
Our empirical focus is on the construction of carry strategies using a broad universe of assets across asset classes. The purpose of this section is to describe our investable universe, to explain how we estimate carry for each asset class and to provide some elementary data diagnostics before we proceed with the core part of our empirical analysis in the next section.

4.1 The Universe of Assets and Asset Classes

For the purpose of our empirical analysis, we collect futures data from Bloomberg for a large cross-section of 52 assets: 20 commodities (constituents of the Bloomberg Commodity index excluding the precious metals, i.e. gold and silver), 8 ten-year government bonds, 9 FX rates (G10 pairs against USD) and 15 equity country indices; see Table 4 for the entire cross-section. We specifically use the roll-adjusted front futures contracts in order to do any back-testing analysis.

Figure 2 presents the number of assets per asset class over time. Our simulations start in January 1990, as this is the first time that we can estimate carry signals for at least five assets per asset class. The sample period ends in January 2016.

Figure 2: Number of Assets per Asset Class



Notes: Data collected from Bloomberg.

Table 3: Descriptive Statistics: Average Realised Returns versus Average Carry

		Exc. Returns (ann.)		Carry (ann.)				Exc. Returns (ann.)		Carry (ann.)	
Asset	**Starting Month**	**Geom. Mean**	**Volatility**	**Geom. Mean**	**St. Dev.**	**Asset**	**Starting Month**	**Geom. Mean**	**Volatility**	**Geom. Mean**	**St. Dev.**
Commodities						**Equity Indices**					
Natural Gas	Feb-91	-13.6	57.6	-16.7	8.1	US - S&P500	Apr-83	5.9	15.0	-1.9	0.7
Heating Oil	May-87	7.7	35.9	5.6	6.1	Canada - S&P TSX 60	Oct-01	5.0	13.2	-0.1	0.4
Unl. Gasoline	Aug-06	0.3	35.5	6.6	3.6	Germany - DAX	Jun-92	4.2	21.5	-2.6	0.5
WTI Crude	Jan-84	3.3	35.8	1.9	4.5	UK - FTSE 100	Feb-89	2.4	14.6	-1.5	0.9
Brent Crude	Apr-89	6.7	33.9	3.4	4.0	Korea - Kospi 200	Mar-97	6.4	34.0	-1.9	0.8
Sugar #11	Dec-70	-6.4	41.8	-2.7	4.7	Japan - Nikkei 225	Jul-89	-2.9	21.9	-0.9	0.8
Live Cattle	Dec-70	4.4	17.7	0.7	3.2	Australia - ASX 200	Mar-01	2.6	13.5	-0.3	0.4
Lean Hogs	Mar-87	-5.3	24.6	-6.6	5.4	HK - Hang Seng	Feb-93	6.8	26.5	0.4	0.6
Coffee C	Jun-73	-2.5	39.5	-2.5	4.9	Spain - IBEX 35	May-93	5.6	21.2	0.3	0.9
Cotton #2	Dec-70	-1.1	26.3	-2.1	4.3	Switzerland - SMI	Oct-99	2.1	14.4	1.1	0.5
Soybeans	Jan-71	1.9	28.2	-1.7	2.9	France - CAC 40	Nov-89	2.6	19.2	-0.6	1.1
Corn	Dec-70	-4.1	26.0	-8.8	2.3	Norway - OBX	Apr-06	2.8	21.8	-2.6	0.3
Wheat	Dec-70	-4.1	27.8	-5.2	3.2	Netherlands - AEX 25	Dec-89	5.4	19.8	0.0	0.6
Soybean Oil	Dec-70	2.3	32.7	-0.5	4.9	Italy - MTSE MIB	Feb-05	-2.3	21.4	2.0	0.3
Soybean Meal	Dec-70	6.1	30.7	-0.3	3.7	Sweden - OMX 30	Dec-05	5.4	17.9	1.8	0.4
Kansas Wheat	Dec-70	0.7	26.1	-2.1	2.9						
Copper	Oct-89	2.9	25.7	2.9	2.1	**Government 10-year Bonds**					
Aluminium	May-98	-7.0	19.8	-4.2	0.9	US T-Note	Jun-82	4.9	6.9	3.0	0.5
Nickel	May-98	3.4	36.4	1.5	1.6	Australian GB	Oct-87	0.5	1.2	0.9	0.5
Zinc	May-98	-2.4	27.1	-3.6	0.7	Canadian GB	Oct-89	4.0	6.0	2.1	0.5
						German Bund	Dec-90	4.4	5.2	2.0	0.5
FX						Japanese GB	Nov-85	3.4	5.1	1.9	0.4
EUR	Jan-99	-0.8	10.4	-0.4	0.5	UK Gilt	Dec-82	2.8	7.5	0.4	0.7
JPY	Jan-89	-2.2	11.0	-3.0	1.0	Swiss GB	Jul-92	4.0	4.6	2.1	0.3
GBP	Jan-89	0.7	9.4	4.9	1.9	New Zealand GB	Nov-91	0.3	1.0	0.6	0.5
AUD	Feb-87	2.9	11.5	1.8	0.5						
CAD	Jan-89	0.0	7.8	0.9	0.7						
CHF	Jan-89	0.1	11.3	-2.3	1.4						
NZD	Jun-97	2.4	13.4	1.3	0.2						
SEK	Jun-02	1.2	11.9	0.1	0.3						
NOK	Jun-02	0.5	12.0	0.5	0.2						

Notes: Sample period ends in January 2016.

4.2 Measuring Carry across Asset Classes

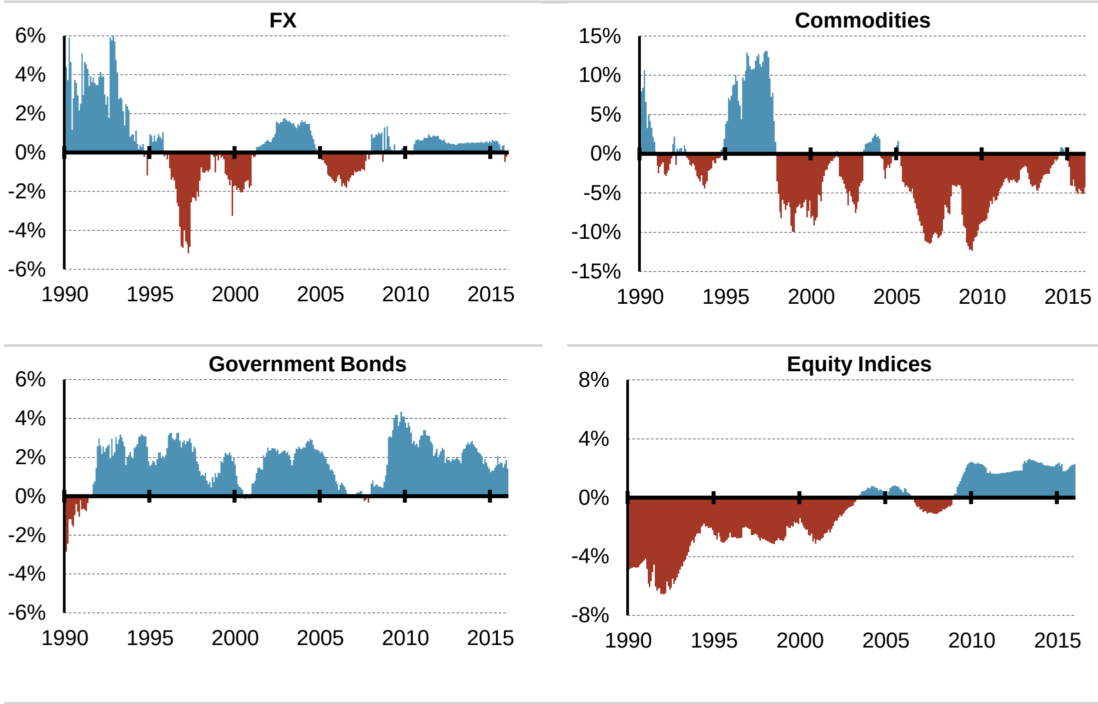
Table 2 in the previous section explains the nature of carry for each asset class in terms of expected yield. However, in order to actually construct carry portfolios we need to estimate the level of carry for each asset of each asset class at the end of each rebalancing period (monthly for the purpose of our analysis) using data readily available at the time.

The level of carry is estimated as the slope of the futures/forward curve for each asset in the spirit of equation (5). In order to measure the slope of the curve one can use either spot and front futures data, or front and back futures data. Due to the various idiosyncrasies of each asset class, as well as due to data availability, we use an asset-class specific metric for the slope of the future curve. Further technical details on these calculations are given in the Appendix. Briefly, for FX markets we use spot and one-month forward data, for equity indices and commodities we use data for the first two (i.e. front and back) futures contracts and seasonally adjust the carry metrics, and for government bonds we use zero coupon bond data in order to calculate spot and synthetic one-month future prices of a 10-year bond.

Table 3 presents annualised average excess returns and volatility for each asset as well as the level of annualised average carry and its respective standard deviation. The Table conveniently reports as well the starting date for each asset (this is the first month that we can generate a carry signal for each asset).

Figure 3 presents the median carry for each asset class at the end of each month over the entire sample period in order to identify asset class shifts between contango and backwardation over time.

Figure 3: Carry in the time-series of assets (1990M01 – 2016M01)



Notes: The figure presents the median carry across all assets of each asset class at the end of each month; January 1990 to January 2016.

Here are some emerging patterns that are worth mentioning:

FX: being the asset class most familiar to us on the carry landscape, typical patterns emerge. JPY and CHF have generally been the currencies with negative carry (so in contango), whereas AUD and NZD have been the currencies with positive carry. On a time-series basis, the median carry effectively tracks the overall strength of USD; positive when Libor US rates are generally lower than the rest of the G10 currencies and negative otherwise.

Commodities: historically, the storage of commodities has provided (positive) convenience yield at times of supply squeezes. However, after the introduction of the Commodity Futures Modernization Act (CFMA) 2000 the universe of commodities has started behaving more like a universe of financial assets, hence turning into contango during the most recent decade.

Government Bonds: being invested in long-term government bonds has historically generated positive excess returns. Upward sloping yield curves correspond to downward sloping bond futures curves and therefore all bonds in our universe (and the universe as a whole) have been mostly in backwardation during our sample period. For the bond markets to turn to contango, the yield curves should invert, which is generally the case prior and during economic recessions, as also witnessed in the time-series plot; notice the zero or negative median carry during the early 1990's recession, after the dot-com bubble in 2001 and during the global financial crisis of 2007-09.

Equity Indices: while historically in contango, as the dividend yield used to be lower than the Libor rate, equity indices have turned into backwardation following the recent financial crisis, as rates have fallen to extremely low levels (if not zero) and therefore an equity investor benefits from the dividend yield if the conditions remain unchanged. This transition between contango and backwardation is obvious from the time-series plot in Figure 3.

5. Constructing Carry Portfolios

With all the pre-work in place, we can now proceed with constructing carry portfolios within each asset class, but most importantly across all asset classes in a multi-asset framework.

In constructing carry portfolios, we explore three different weighting schemes[8]:

1. **Cross-sectional ("XS") Carry**: the **relative strength of the carry** of each asset compared to all other assets in the same asset class is used in order to construct a balanced long-short portfolio in terms of notional exposure.
2. **Times-series ("TS") / Absolute Carry**: the **sign of the carry** of each asset is used to determine the type of position (long or short) in order to construct a portfolio with explicit directional tilts; net long when the majority of assets are in backwardation, and net short when in contango.
3. **Optimised ("OPT") Carry:** both the **relative strength** and the **sign of the carry** are used in order to determine the type (long or short) as well as the gross exposure for each asset. Most importantly, the optimised carry portfolio additionally accounts for the **covariance structure** between assets and asset classes in a way that risk allocation is optimised.

As already noted, the XS form of carry is definitely the most familiar to the investment community due to FX carry. Instead, the TS form has been less of a focus in academic works, whereas the optimised form has never been addressed in a multi-asset framework. Our objective is to discuss all three forms, but eventually focus on the optimised portfolio.

5.1 Cross-sectional Carry

Let N_t denote the number of available contracts at time t. Using the latest carry estimate for each asset, C_t^i, with $i = 1, \dots, N_t$, we rank all assets within the same asset class in increasing order (i.e. high ranks are associated with high-carry assets). We then assign linear weights to the assets, which are proportional to the demeaned ranks[9] (the average rank of N_t assets equals $\frac{N_t+1}{2}$):

$$\widehat{w}_t^{XS,i} = rank(C_t^i) - \frac{N_t + 1}{2} \tag{6}$$

Evidently, assets with higher (either positive or negative) level of carry relative to the rest of the universe, will bear larger gross weights; this is one way that the relative strength of the signals translates into the XS weighting scheme. Obviously, the XS weights are symmetrical around zero, and in order to normalise them so to make no use of leverage (gross exposure of 100%), we simply rescale them by $\sum_{j=1}^{N_t} \left|\widehat{w}_t^{XS,j}\right|$, so the final weights are:

$$w_t^{XS,i} = \frac{\widehat{w}_t^{XS,i}}{\sum_{j=1}^{N_t} \left|\widehat{w}_t^{XS,j}\right|} \tag{7}$$

The weight symmetry results in zero net exposure, $\sum_{j=1}^{N_t} w_t^{XS,j} = 0$. Koijen *et al.* (2017) use the same weighting scheme, with the only difference being that their level of gross exposure is fixed at 200% (sum of positive weights to 100% and sum of negative weights to -100%).

Table 4 presents various performance statistics for XS carry portfolios for each asset class. Broadly speaking, all asset classes generate positive excess returns with Sharpe ratios ranging from 0.19 for commodities to 0.85 for government bonds. The statistical significance of average returns is strongest for bonds (at 1% confidence), as the bonds with steeper yield curves have outperformed the bonds with less steep, if not inverted, yield curves. Equity XS carry has also generated significant excess returns (at 5% confidence), albeit with more volatility. Quite surprisingly, FX and commodities have been the asset classes where their XS carry returns, even though positive over the sample period, do not exhibit strong statistical significance (FX carry is just off the 10% threshold of statistical significance).

[8] One can draw parallels between cross-sectional and time-series forms of carry and the equivalent forms for momentum: cross-sectional (Jegadeesh and Titman, 1993, 2001), and time-series (Moskowitz, Ooi and Pedersen, 2012).
[9] For further details on this weighting scheme, please also see Asness, Moskowitz and Pedersen (2013) and Baltas, Jessop and Jones (2014).

Table 4: Performance Statistics for XS Carry Strategies across Asset Classes

	FX	Commodities	Govt. Bonds	Equity Indices
Average Excess Return (%)	1.36	1.83	1.63***	3.09**
Annualised Volatility (%)	4.39	9.59	1.93	6.43
Skewness	-0.77	-0.19	-0.04	0.80
Kurtosis	4.87	3.48	4.18	7.39
Maximum Drawdown (%)	15.80	26.28	5.15	18.23
Sharpe Ratio (annualised)	0.31	0.19	0.85	0.48
Sortino Ratio (annualised)	0.43	0.28	1.42	0.83
Calmar Ratio	0.08	0.05	0.31	0.16
Monthly Turnover (%)	19.44	18.48	19.02	15.87
Correlation Matrix				
- FX	1.00			
- Commodities	0.01	1.00		
- Government Bonds	0.06	-0.10	1.00	
- Equity Indices	0.12	0.00	0.20	1.00

Notes: The table presents performance statistics for cross-sectional (XS) demeaned-rank weighted carry strategies within each asset class (FX, commodities, government bonds and equity indices) as well as the correlations between each other. The statistical significance of the average return is indicated with *, ** and *** for 1%, 5% and 10% confidence levels, using White (1980) standard errors. The Sortino ratio is defined as the annualised excess return divided by downside volatility and the Calmar ratio is defined as the annualised geometric return divided by the maximum drawdown. The sample period is January 1990 to January 2016.

Focusing on higher moments, FX carry exhibits a large negative skewness (-0.77), which is consistent with the literature, followed by commodities and bonds that also have a small, though possibly insignificant negative skewness. However, in line with Koijen *et al.* (2017), the equity XS carry strategy exhibits positive skewness, which can cast doubt on a crash/downside risk explanation of multi-asset XS carry. We return to this point with a detailed analysis at a later stage, in Section 6.

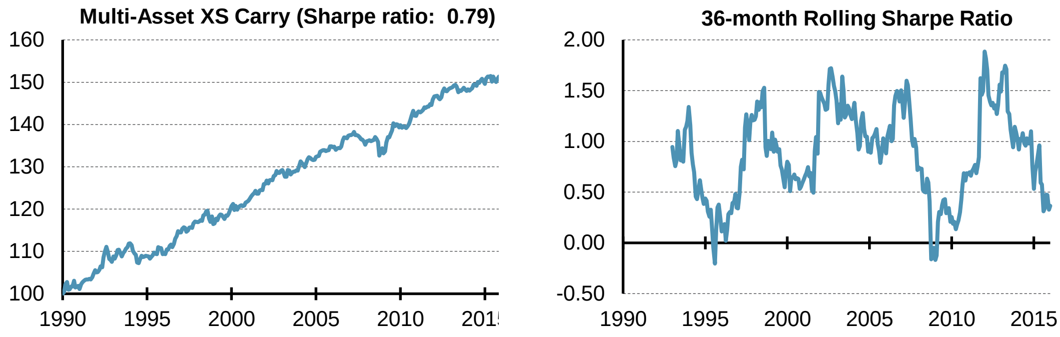
Next, we explore the diversification benefits from pulling together all XS carry portfolios, in order to construct a multi-asset XS portfolio. As reported in Table 4, there is little correlation between these portfolios, which, on the one hand, means that building a unified framework that explains the XS carry patterns across all asset classes might be a challenging task but, on the other hand, it means that blending the different asset classes in a multi-asset portfolio can deliver superior returns.

The fact that the XS portfolios have rather different volatilities – this is due to the nature of the respective asset classes – justifies the use of a risk-based scheme for the construction of the multi-asset portfolio. To keep things simple, we suggest combining the XS portfolios on an inverse-volatility basis. The multi-asset portfolio is rebalanced on a monthly basis using volatilities estimated using a 100-day rolling window of data. The cumulative returns of the portfolio as well as its 36-month rolling Sharpe ratio are presented in Figure 4. Performance statistics are reported in Table 5, including a levered version of the strategy that targets a volatility of 7%.

The multi-asset XS carry strategy delivers statistically strong (at 1% confidence) average excess returns, generating a Sharpe ratio of 0.79, albeit at a negative skewness of -0.44 (in line with Koijen *at al.*, 2017) and with excess kurtosis. The portfolio volatility is 2.04% annualised, which is the result of the inverse-volatility scheme that we have employed, as well as of the low correlation between the XS carry portfolios across the asset classes. For this reason, the volatility-targeted version of the strategy, which generates similar performance statistics, requires an average leverage of 4x, so to achieve the required 7% level of volatility.

In unreported results (available upon request) we find that all XS strategies (across asset classes and at the multi-asset level) exhibit very low betas against various passive broad market indices (MSCI World Index, Bloomberg Commodity Index, JPMorgan Aggregate Bond Index and Trade-weighted USD). This market neutrality is largely driven by the cash-neutral nature of the XS strategies. Importantly enough, the betas of the multi-asset portfolio across all these market indices are largely shrunk towards zero. This finding highlights the diversification benefits of the multi-asset XS carry portfolio.

Figure 4: Multi-Asset Cross-Sectional Carry (unlevered)



Notes: The figure presents cumulative monthly returns for the cross-sectional (XS) multi-asset carry portfolio as well as its 36-month rolling Sharpe ratio. The multi-asset portfolio is constructed as the inverse-volatility weighted portfolio of four each asset XS demeaned-rank weighted portfolios across FX, commodities, government bonds, and equity indices. The portfolio is rebalanced monthly and the volatilities are estimated using a 100-day window. The sample period is January 1990 to January 2016.

Table 5: Performance Statistics for Multi-Asset XS Carry Strategies

	Unlevered	Levered – 7% Target
Average Excess Return (%)	1.60***	5.88***
Annualised Volatility (%)	2.04	7.30
Skewness	-0.44	-0.37
Kurtosis	4.99	4.86
Maximum Drawdown (%)	4.19	18.61
Sharpe Ratio (annualised)	0.79	0.81
Sortino Ratio (annualised)	1.25	1.31
Calmar Ratio	0.38	0.31
Monthly Turnover (%)	21.83	23.92
Average Leverage	1x	4.0x
25th – 75th percentiles	1x to 1x	3.2x to 4.5x

Notes: The table presents performance statistics for the cross-sectional (XS) multi-asset carry portfolio both on unlevered basis as well as on levered basis, assuming 7% target volatility. The multi-asset portfolio is constructed as the inverse-volatility weighted portfolio of four each asset XS demeaned-rank weighted portfolios across FX, commodities, government bonds, and equity indices. The portfolio is rebalanced monthly, the volatility target for the levered strategy is also applied on a monthly basis and all volatilities are estimated using a 100-day window. The statistical significance of the average return is indicated with *, ** and *** for 1%, 5% and 10% confidence levels, using White (1980) standard errors. The Sortino ratio is defined as the annualised excess return divided by downside volatility and the Calmar ratio as the annualised geometric return divided by the maximum drawdown. In the estimation of turnover, the absolute change in portfolio weights is normalised by the total gross exposure of the strategy in the beginning of the period. The sample period is January 1990 to January 2016.

5.2 Time-series Carry

Instead of focusing on the relative strength of the carry signals, one can construct a time-series carry (TS) strategy, by simply focusing on the sign of the carry of each asset. The TS form of the strategy would then assume a long position on assets with a positive carry (that is, assets in backwardation) and a short position on assets with a negative carry (that is, assets in contango), and assign equal gross weights across all assets (one can think of it as the *"momentum of carry"*):

$$w_t^{TS,i} = \frac{sign(C_t^i)}{N_t} \tag{8}$$

Contrary to the XS form, the TS variant (a) is not taking into account the relative strength of the carry metrics, but only their sign and (b) is not a cash-neutral strategy, $\sum_{j=1}^{N_t} w_t^{TS,j} \neq 0$ (notice, however, that we maintain the 100% gross exposure as in the XS variant for comparability). Instead, the TS has explicit directional tilts and is net long (short) when the majority of assets are in backwardation (contango). The net exposure of the strategy for each asset class roughly tracks the median carry of each asset class that was presented Figure 3.

Before proceeding with the presentation of our TS results, we should note than an alternative scheme to the equal gross weights $(1/N_t)$ would be to use inverse-volatility gross weights. Given that we first construct portfolios for each asset class separately (before aggregating the asset classes to a multi-asset portfolio), we can argue that the volatilities of the assets in the same asset class are in practice rather close and therefore the two weighting schemes (equal weights and inverse-volatility weights) will be numerically similar (see also Table 3).[10] In unreported results we have conducted this analysis and obtained very similar results to those reported below.

Table 6 presents various performance statistics for TS carry portfolios for each asset class. Similar to the XS analysis, all asset classes generate positive excess returns with Sharpe ratios ranging from 0.10 for commodities to 0.88 for government bonds; this constitutes evidence that the level and sign of carry experience some degree of positive serial dependence. The statistical significance of average returns is again strongest for bonds (at 1% confidence). Interestingly enough, given the popularity of the XS nature of the FX carry, its TS form appears to generate even larger and statistically stronger returns (at 1% confidence). Equity indices and commodities, though generating positive returns over the sample period, fail to exhibit any statistical significance.

Table 6: Performance Statistics for TS Carry Strategies across Asset Classes

	FX	**Commodities**	**Govt. Bonds**	**Equity Indices**
Average Excess Return (%)	3.02***	0.98	2.62***	2.74
Annualised Volatility (%)	5.25	9.87	2.99	12.00
Skewness	-0.58	0.10	-0.01	0.70
Kurtosis	4.45	4.40	3.23	5.26
Maximum Drawdown (%)	14.50	40.21	7.91	61.94
Sharpe Ratio (annualised)	0.57	0.10	0.88	0.23
Sortino Ratio (annualised)	0.85	0.15	1.51	0.37
Calmar Ratio	0.20	0.01	0.33	0.03
Monthly Turnover (%)	8.99	12.22	6.27	8.72
Rank Correlation with XS	0.51	0.56	0.68	0.30
Correlation Matrix				
- FX	1.00			
- Commodities	-0.16	1.00		
- Government Bonds	-0.00	0.11	1.00	
- Equity Indices	0.09	-0.03	0.01	1.00

Notes: The figure presents performance statistics for time-series (TS) equal-gross weighted carry strategies within each asset class (FX, commodities, government bonds and equity indices). The statistical significance of the average return is indicated with *, ** and *** for 1%, 5% and 10% confidence levels, using White (1980) standard errors. The Sortino ratio is defined as the annualised excess return divided by downside volatility and the Calmar ratio as the annualised geometric return divided by the maximum drawdown. The table also reports the return rank correlation of the strategies with the respective XS strategies for the same asset class. The sample period is January 1990 to January 2016.

As far as the skewness of the strategies is concerned, it is only the FX carry that still exhibits large negative skewness (-0.58). All other asset classes generate either close to symmetrical return distribution or positive skewness, with the equity TS strategy exhibiting again (as in the XS case) the strongest positive skewness (0.70). As in the XS case, this evidence can cast doubt on a crash/downside risk explanation of multi-asset TS carry. We will return to this point at a later stage in Section 6.

Table 6 also reports the rank correlation of monthly returns between the TS carry strategies and the respective XS carry strategies for each asset class. Broadly speaking, the two forms of strategies share common features and the correlations range from 0.30 for equity indices to 0.68 for government bonds. Quite expectedly, when the carry signals are symmetrically distributed around zero, a TS strategy would very much resemble an XS strategy. Baz *et al.* (2015) and Goyal and Jegadeesh (2015) offer an interesting discussion on the XS versus TS dynamics.

We next combine the four TS carry portfolios into a multi-asset portfolio using inverse-volatility weights (monthly rebalancing using 100-day volatility estimates). Figure 5 presents the cumulative returns and the rolling Sharpe ratio of the multi-asset TS portfolio; Table 7 reports several performance

[10] One can contrast this to a time-series strategy that is constructed using all assets from all asset classes, like the default time-series momentum strategy of Moskowitz *et al.* (2012). In such an environment the use of inverse-volatility gross weights is obviously mandatory. For further information see also Baltas (2015).

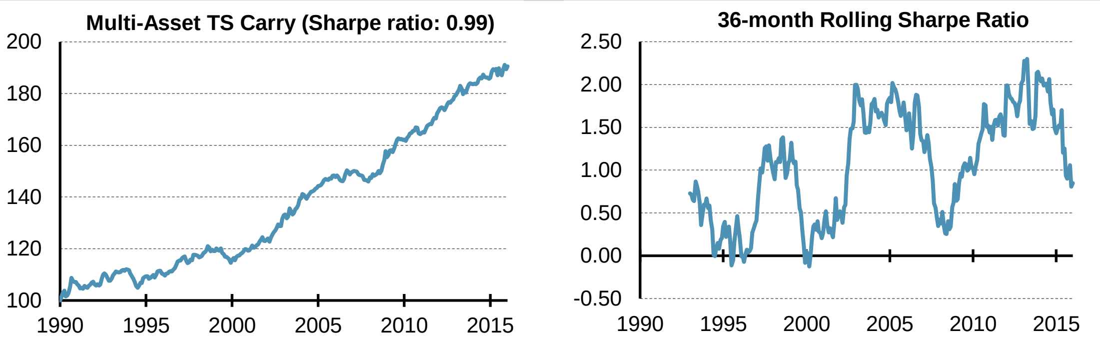
statistics both for the unlevered strategy as well as for a levered version of the strategy that targets a volatility of 7%.

The multi-asset TS carry strategy delivers a superior return profile compared to all the asset class TS portfolios. The average excess return is statistically strong (at 1% confidence) and the Sharpe ratio is 0.99 for our sample period. Most importantly, the multi-asset TS carry portfolio is positively skewed (as opposed to its XS variant), and the kurtosis is just slightly higher than 3. This tail behaviour could be due to the diversification coming from the different asset classes as well as due to the occasional active (net long or net short) tilts of each asset class TS portfolio.

Due to the occasional directional tilts, the multi-asset TS carry strategy exhibits an annualised volatility of 2.60%, which is, as expected, higher than that of its XS variant that maintains a more balanced profile (2.04% volatility from Table 5). As a result, the TS carry strategy requires an average leverage of 3.3x to achieve the required 7% level of volatility.

In unreported results (available upon request) we look at the betas of all TS strategies against various passive broad market indices (MSCI World Index, Bloomberg Commodity Index, JPMorgan Aggregate Bond Index and Trade-weighted USD). Contrary to the XS strategies that are generally market-neutral, the TS strategies exhibit relatively larger betas, even though not large in value. However, the multi-asset TS portfolio exhibits rather small betas against the various markets. The diversification benefits of the multi-asset class framework are again clearly exposed.

Figure 5: Multi-Asset Time-Series Carry (unlevered)



Notes: The figure presents cumulative monthly returns for the time-series (TS) multi-asset carry portfolio as well as its 36-month rolling Sharpe ratio. The multi-asset portfolio is constructed as the inverse-volatility weighted portfolio of four each asset TS equal-gross weighted portfolios across FX, commodities, government bonds, and equity indices. The portfolio is rebalanced monthly and the volatilities are estimated using a 100-day window. The sample period is January 1990 to January 2016.

Table 7: Performance Statistics for Multi-Asset TS Carry Strategies

	Unlevered	Levered – 7% Target
Average Excess Return (%)	2.57***	8.14***
Annualised Volatility (%)	2.60	7.89
Skewness	0.18	-0.06
Kurtosis	3.78	2.92
Maximum Drawdown (%)	6.40	18.36
Sharpe Ratio (annualised)	0.99	1.03
Sortino Ratio (annualised)	1.79	1.85
Calmar Ratio	0.40	0.44
Monthly Turnover (%)	12.16	14.85
Average Leverage	1x	3.3x
25^{th} – 75^{th} percentiles	1x to 1x	2.8x to 3.9x

Notes: The figure presents performance statistics for the time-series (TS) multi-asset carry portfolio both on unlevered basis as well as on levered basis, assuming 7% target volatility. The multi-asset portfolio is constructed as the inverse-volatility weighted portfolio of four each asset TS equal-gross weighted portfolios across FX, commodities, government bonds, and equity indices. The portfolio is rebalanced monthly, the volatility target for the levered strategy is also applied on a monthly basis and all volatilities are estimated using a 100-day window. The statistical significance of the average return is indicated with *, ** and *** for 1%, 5% and 10% confidence levels, using White (1980) standard errors. The Sortino ratio is defined as the annualised excess return divided by downside volatility and the Calmar ratio is defined as the annualised geometric return divided by the maximum drawdown. In the estimation of the turnover, the absolute change in portfolio weights is normalised by the total gross exposure of the strategy. The sample period is January 1990 to January 2016.

5.3 The relationship between XS and TS carry strategies

The analysis so far has shown that carry strategies, both in their XS and TS form, generate positive excess returns across all asset classes. The correlations between asset class portfolios are small, hence benefiting their multi-asset combination. Table 8 summarises the overall rank correlation structure between XS and TS carry strategies across asset classes and also at the multi-asset class level.

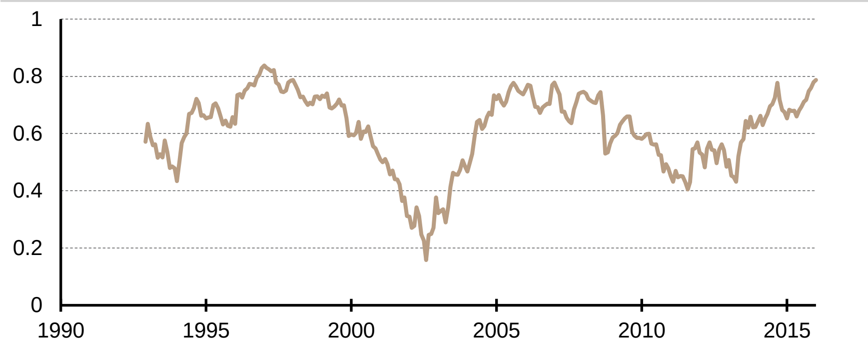
Table 8: Return Rank Correlation between XS and TS Carry Strategies

	Cross-Sectional Carry				
Time-Series Carry	**Commodities**	**Govt. Bonds**	**FX**	**Equity Indices**	**Multi-Asset**
Commodities	0.56	0.06	-0.10	0.10	0.31
Govt. Bonds	-0.12	0.68	-0.02	0.11	0.30
FX	0.06	-0.02	0.51	0.07	0.30
Equity Indices	0.04	0.02	0.02	0.30	0.21
Multi-Asset	0.27	0.38	0.22	0.28	0.61

Notes: The table presents the rank correlation between XS and TS carry strategies. The estimation uses monthly returns series and the sample period is from January 1990 to January 2016.

Broadly speaking, the correlation estimates are not extreme, which leads us to our ultimate objective in this paper. It is one thing to benefit from diversification potential across asset classes, but it is a different thing to – additionally – benefit from the diversification potential of balanced (i.e. XS) and active (i.e. TS) strategies.[11] Put differently, we next explore whether there is any return and/or diversification benefit from combining cross-sectional and time-series carry signals across all asset classes. Figure 6 presents a 36-month rolling rank correlation between our multi-asset XS and TS carry strategies and appears to answer the question in an affirmative sense. The following section focuses on the dynamics of combining XS and TS signals, while additionally optimising the overall portfolio risk by actively making use of the rich cross-asset covariance structure.

Figure 6: 36m Rolling Rank Correlation between XS and TS Carry Strategies



Notes: The figure presents the 36-month rolling rank correlation between XS and TS multi-asset carry strategies. The sample period is from January 1990 to January 2016.

5.4. Optimised Carry

In constructing a carry portfolio, the academic literature seems to have completely ignored the fact that its constituents have a proper covariance structure, which could be theoretically used to optimise portfolio risk and return. The only three exceptions are the very recent papers by Barroso and Santa-Clara (2015), Ackermann *et al.* (2016) and Daniel *et al.* (2017), which highlight the benefits of portfolio optimisation but only for FX carry portfolios. However, if there is any benefit from building optimised portfolios one would expect this benefit to be magnified for a multi-asset universe, as it is characterised by a much richer and more dynamic covariance structure at the intra-asset-class and most importantly at the inter-asset-class level. Following the above discussion, we consider this paper to be the first paper to ever explore optimised multi-asset carry portfolios.

[11] To a certain extent, one can draw parallels with an investor that holds an equity long-only portfolio and seeks performance improvement from incorporating market-neutral long-short alternative beta strategies.

In constructing a multi-asset portfolio, we generally have two broad options[12]:

- *Two-step approach*: first construct a portfolio for each asset class and then combine these portfolios to generate the multi-asset portfolio. This has been the process that we have followed so far for the XS and TS strategies.
- *One-step approach*: construct a portfolio across all assets in one step using all assets from all asset classes. It is of critical importance to notice that this approach must certify that the assets from each asset class are somehow "equally" treated in risk-return terms. A counterexample will help understanding this: think of a XS portfolio that ranks all the assets from different classes based on their carry metric. This procedure is flawed, unless the carry metric of each asset class is accordingly adjusted (e.g. scaled by asset volatility) to allow for cross-sectional comparison.

In order to construct our optimised multi-asset portfolio, we expect a greater diversification benefit from using the one-step approach which assumes the full 52x52 covariance structure of all assets (even though we do acknowledge that this comes with more estimation error and potentially more turnover), than using the two-step approach, in which case the covariance structure is shrunk to the 4x4 covariance structure of the asset class portfolios.[13] For this reason, we proceed with the one-step approach; next, we discuss the portfolio optimisation methodology.

Jessop *et al.* (2013) show how a risk-parity[14] allocation can be extended by introducing (a) expected returns and (b) long and short positions. The findings of this report have been applied to a multi-asset trend-following portfolio by Baltas (2015). Using the same principles, we next present the steps to the **long-short risk-budgeting weighting scheme** that we use for the multi-asset optimised carry. This can be also considered as an extension to the Bruder and Roncalli (2012) risk-budgeting framework.

In the absence of any model of expected returns (we will relax this shortly), we suggest applying the risk-parity principles and allocate equal amount of risk across the four asset classes; so each asset class should contribute 25% of the overall portfolio volatility. This 25% risk contribution of each asset class is equally allocated across all the constituents of the asset class. To achieve this in one step, we simply extend risk-parity to risk-budgeting with each asset i contributing $25\%/N_t^i$ amount of risk, where N_t^i denotes the number of assets that belong in the same asset class as asset i at time t:

$$N_t^i = \sum_{j=1}^{N_t} \mathbb{I}\{Class_j = Class_i\}, \quad \forall i \tag{9}$$

where $\mathbb{I}\{Class_j = Class_i\}$ denotes an indicator function that equals one if assets i and j belong to the same asset class and zero otherwise. To give an example, at the end of our sample period we have 52 assets: 20 commodities, 8 government bonds, 9 FX rates and 15 equity country indices. In this case N_t^i is equal to 20 if asset i denotes a commodity, 8 if asset i denotes a bond and so on so forth.

Based on the above, our risk-budgeting (RB) framework can be described by:

$$w_t^{RB,i} \cdot MCR_t^i = \frac{25\%}{N_t^i} \cdot \sigma_P, \quad \forall i \tag{10}$$

where σ_P denotes the overall portfolio volatility and MCR is that marginal contribution to risk for each asset and is defined as the marginal change in portfolio volatility for a marginal change in the asset weight. To ease notation, we can drop the term $(25\% \cdot \sigma_P)$, as it's a common factor across all assets and our focus is really on the relative (and not absolute) distribution of risk. We can therefore re-write equation (10) using a proportionality symbol:

$$w_t^{RB,i} \cdot MCR_t^i \propto \frac{1}{N_t^i}, \quad \forall i \tag{11}$$

We can contrast this against the standard risk-parity (RP) allocation, where the weighted MCR is equal across all assets (instead of being proportional to the number of assets in the same asset class):

$$w_t^{RP,i} \cdot MCR_t^i = constant, \quad \forall i \tag{12}$$

[12] For an interesting discussion on a relevant topic see Fitzgibbons, Friedman, Pomorski and Serban (2016).

[13] On a technical note, if each asset class follows a CAPM-like single factor structure, then the two approaches would result in very similar allocations.

[14] To avoid any confusion in the terminology, when we use the term "risk-parity" we effectively refer to the equal risk contribution (ERC) scheme and not to the basic inverse-volatility scheme, which we typically call "volatility-parity" in our publications.

The next step in defining the multi-asset portfolio optimisation for the carry portfolio is to introduce expected returns to the framework. In the history of portfolio optimisation, starting from Markowitz's (1952) modern portfolio theory, one of the most challenging parts has been the determination of expected returns. The absence of a well-agreed and accurate forecasting model[15], as well as the sensitivity/instability of mean-variance optimisation to estimation errors[16] have been the primary reasons as to why risk-based investing (starting from minimum-variance portfolios) became progressively more popular with the passage of time.

Following the above, one could justifiably question why we should even bother introducing expected returns in our multi-asset carry portfolio. But here lies probably one very simple, yet fundamental, idea. As already discussed in the introduction of the paper and in equation (1), carry constitutes the expected returns of an asset if conditions remain the same. Put differently, carry, which is readily available at any point in time and measured directly from the slope of the future/forward curve, is indeed a measure of expected returns that would even realise if the price of the asset does not move. We can therefore introduce the level of carry in our optimisation framework and aim to optimise our allocation so to tilt the portfolio towards assets with higher carry potential at each point in time.

Following the above, we suggest extending the risk-budgeting framework in order to increase the risk allocation (and therefore the gross weights) for assets that have higher level of carry, either positive or negative. To maintain transparency we decide to take long positions for assets with positive carry and short positions for assets with negative carry as was the case in the time-series form of the strategy.

Regarding the absolute level of carry of each asset and given that our universe contains assets from different asset classes, it becomes likely that we would end up with a vector of expected returns that are not directly comparable to each other; given that we run an optimisation across all assets, we must make sure that there is fair comparability in the expected return dimension. To achieve that we scale the level of carry for each asset with the level of volatility (measured over a rolling window of 100 days). Along these lines, we express the level of carry in risk units; hence, the ratio between carry and recent volatility expresses the ex-ante Sharpe ratio of each asset at each rebalancing point:

$$Ex - ante\ Sharpe\ Ratio(i) = \frac{C_t^i}{\sigma_t^i}, \quad \forall i \tag{13}$$

This measure of risk-adjusted carry constitutes a fairer metric to enter the optimisation framework.

Putting all the pieces together:

- At the single asset level, assets with positive carry bear a long position and assets with negative carry bear a short position. In this way we succeed in **incorporating the time-series definition of carry in our optimised framework**.
- Assets with higher level of risk-adjusted carry (either positive or negative) bear higher risk contribution (and therefore gross weight). Evidently, the relative strength of the carry signals does matter for the overall portfolio and therefore in this way we succeed in **incorporating the cross-sectional definition of carry in our optimised framework**.
- At the asset class level, we assume equal contribution to risk from each asset class, as long as all assets have the same ex-ante Sharpe ratio. When this is not the case, the **risk allocation becomes proportional to the collective risk-adjusted carry of each asset class**, after taking into account all the **cross-correlation dependencies** between all assets from all asset classes.

Overall, combining the proportionality statement (11) with equation (13), we end up with the final expression for the weighted marginal contribution to risk in our final long-short risk-budgeting framework, which we plan to apply to construct the optimised (OPT) multi-asset carry portfolio:

$$w_t^{OPT,i} \cdot MCR_t^i \propto \frac{|C_t^i|}{N_t^i \cdot \sigma_t^i}, \quad \forall i \tag{14}$$

To ease notation, let us denote the right-hand side of the above relationship by s_t^i that stands for the "score" of each asset at each point in time that controls the overall risk contribution of this asset:

$$s_t^i \equiv \frac{|C_t^i|}{N_t^i \cdot \sigma_t^i}, \quad \forall i \tag{15}$$

[15] Forecasting returns is probably the hardest empirical task in financial economics.
[16] See for example Merton (1980) and Chopra and Ziemba (1993).

Solving for the optimised portfolio that satisfies (14) requires a proper risk-based optimisation. This is achieved by restating our current problem in the following optimisation format (see Baltas, 2015):

$$\boldsymbol{w}_t^{OPT} = \text{argmax} \sum_{i=1}^{N_t} |s_t^i| \cdot log(|w_t^i|) \\ \text{subject to } \sqrt{\boldsymbol{w}_t' \cdot \boldsymbol{\Sigma}_t \cdot \boldsymbol{w}_t} \leq \sigma_{TGT} \tag{16}$$

where Σ_t denotes the covariance matrix of the universe and $\boldsymbol{w}_t$ denotes the vector of (net) weights. To put this in words, we solve for a long-short portfolio that maximises the log-weighted carry, given a certain portfolio volatility target σ_{TGT}. It is easy to show that the Lagrangian of this optimisation coincides with the proportionality expression (14), so the two expressions are indeed equivalent (Baltas, 2015). The optimisation problem does not require any further constraints, but we must discuss a few final technical points:

- The weights of the optimisation would not add up to 100% in gross terms due to the volatility constraint. One solution to this is to normalise the weights post-optimisation, acknowledging of course that the portfolio volatility will obviously change. This is the approach that we follow in our results in the following pages, so that we always maintain an unlevered portfolio. When a volatility-targeted portfolio is required for the purposes of our analysis, we apply this after first normalising the gross weights so that we can properly track the amount of required leverage.

- The optimisation (16) preserves by construction – due to the use of the logarithmic function – the sign of the weights and no additional constraints are required. As a result, the signs of the initial weights are of critical importance as they will be preserved during the optimisation procedure.[17] For this reason the initial vector of weights for the optimisation (16) should contain positive values for assets with positive scores (assets in backwardation) and negative values for assets with negative scores (assets in contango). To make things easier for the optimisation, we decide to start the search for the optimal allocation from the risk-budgeting solution if all pairwise correlations were equal, which is the inverse-volatility score-adjusted solution (so if all pairwise correlations are indeed equal then we are already at the optimum):

$$w_t^{OPT,i,initial} = \frac{s_t^i / \sigma_t^i}{\sum_{j=1}^{N_t} |s_t^j| / \sigma_t^j}, \quad \forall i \tag{17}$$

This concludes our long description of our suggestion for a multi-asset carry portfolio that actively makes use of the relative strength of carry metrics, of the sign of the carry metrics as well as of the covariance structure of the universe.

As already discussed the OPT portfolio follows the TS portfolio in terms of which assets to go long and which assets to go short. However, the gross weights are purely determined by the optimisation and, broadly speaking, we expect that assets with higher level of carry (over volatility) in absolute value should bear higher gross weights, so an XS type of ranking is also indirectly incorporated in the process. Finally, it becomes obvious that the OPT portfolio is not cash-neutral (as the XS portfolio is) but it will have proper directional tilts. However, contrary to the TS portfolio that is net long (short) if the majority of the assets are in backwardation (contango), the OPT portfolio might still end up being net long even if most assets are in contango, if the few assets that are in backwardation have much larger carry (over volatility) signals and also diversify much more risk away for the overall portfolio in a way that their gross allocation exceeds that of the assets in contango. This is generally the idea of residing to an optimisation so as to optimally allocate risk across the various assets of all asset classes.

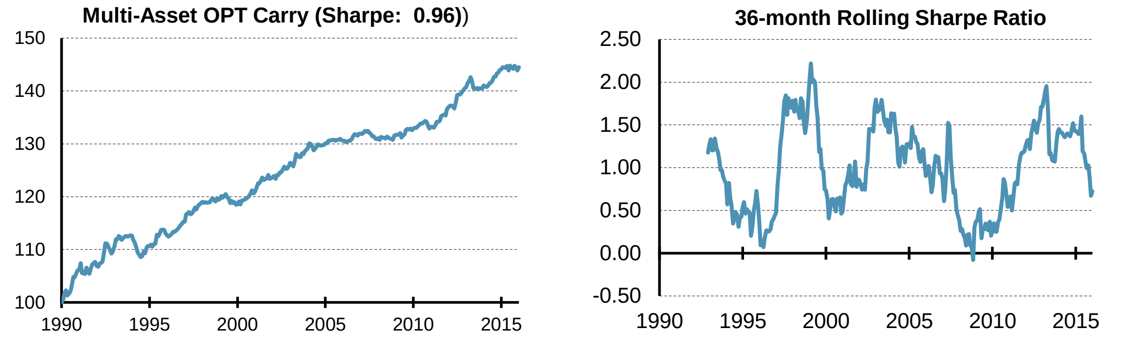
Given that our optimised framework should benefit mostly from an application across all assets from all asset classes, we only present of our results directly at the multi-asset level.[18] Figure 7 presents the cumulative returns and the rolling Sharpe ratio of the unlevered multi-asset OPT carry portfolio (so gross exposure is set at 100%), which is rebalanced on monthly basis using 100-day covariance matrix estimates between all 52 assets of our universe. Additionally, Panel A of Table 9 reports performance

[17] The objective function of the optimisation (16) pushes the positive weights away from zero towards the positive territory, with the relative effects being more aggressive for assets with larger (positive) scores and equivalently pushes the negative weights away from zero towards the negative territory, with the effects being more aggressive for assets with larger (in absolute value, yet negative) scores.

[18] Optimised portfolios for each asset class are available upon request.

statistics both for the unlevered strategy as well as for a levered version of the strategy that targets a volatility of 7%.

Figure 7: Multi-Asset Optimised Carry (unlevered)



Notes: The figure presents cumulative monthly returns for the optimised (OPT) multi-asset carry portfolio as well as its 36-month rolling Sharpe ratio. The multi-asset portfolio is constructed using a long-short risk-budgeting framework for 52 assets across FX, commodities, government bonds, and equity indices. The portfolio is rebalanced monthly and the covariance structure is estimated using a 100-day window. The sample period is January 1990 to January 2016.

Table 9: Performance Statistics for Multi-Asset Carry Strategies

	Panel A: Optimised Carry Portfolio		Panel B: Alternative XS & TS Combinations	
	Unlevered	**Levered – 7% Target**	**Equal Weights – 7% Target**	**Inv. Volatility – 7% Target**
Average Excess Return (%)	1.42***	9.16***	7.58***	7.89***
Annualised Volatility (%)	1.48	8.77	7.64	7.81
Skewness	0.26	0.30	-0.15	-0.13
Kurtosis	5.32	4.42	3.50	3.36
Maximum Drawdown (%)	3.66	18.86	15.50	17.53
Sharpe Ratio (annualised)	0.96	1.04	0.99	1.01
Sortino Ratio (annualised)	1.76	1.96	1.70	1.76
Calmar Ratio	0.39	0.49	0.49	0.45
Monthly Turnover (%)	27.31	29.95	20.85	20.67
Average Leverage	1x	7.1x	2.8x	3.0x
25th – 75th percentiles	1x to 1x	5.1x to 9.1x	2.3x to 3.2x	2.5x to 3.5x

Notes: Panel A presents performance statistics for the optimised (OPT) multi-asset carry portfolio both on unlevered basis as well as on levered basis, assuming 7% target volatility. The multi-asset portfolio is constructed using a long-short risk-budgeting framework for 52 assets across FX, commodities, government bonds, and equity indices. For comparison purposes, Panel B presents equally-weighted and inverse-volatility weighted portfolios of multi-asset cross-sectional (XS) and time-series (TS) portfolios, both levered at 7% target volatility. The portfolios are rebalanced monthly, the volatility target for the levered strategies is also applied on a monthly basis and all volatilities and covariance structure are estimated using a 100-day window. The statistical significance of the average return is indicated with *, ** and *** for 1%, 5% and 10% confidence levels, using White (1980) standard errors. The Sortino ratio is defined as the annualised excess return divided by downside volatility and the Calmar ratio is defined as the annualised geometric return divided by the maximum drawdown. In the estimation of turnover, the absolute change in portfolio weights is normalised by the total gross exposure of the strategy in the beginning of the period. The sample period is January 1990 to January 2016.

The multi-asset OPT carry strategy, in its unlevered form, delivers positive average excess returns that are statistically strong (at 1% confidence) and a Sharpe ratio of 0.96. Similar to the TS carry strategy, it is positively skewed, but it also comes along with higher kurtosis of 5.32; along these lines, it appears to exhibit superior positive tail behaviour compared to the XS or TS variants of the strategy.

Most importantly, the fact that the OPT portfolio is optimised from a risk-diversification perspective, given the risk-budgeting framework that is employed, results in rather low portfolio volatility, equal to 1.48% (compared to 2.04% for XS and 2.60% for TS). Consequently, the OPT carry strategy requires more leverage (on average 7.1x) to achieve the required 7% level of volatility in its levered form.

Before presenting further results on the multi-asset OPT portfolio, it is fair to compare it against simpler portfolios of the multi-asset XS and TS portfolios that we presented in the previous sections. For this reason, Panel B of Table 9 reports performance statistics for a static 50%-50% allocation between these portfolios as well as a dynamic inverse-volatility allocation between the same portfolios. Both combinations target a 7% level of volatility so that they can be compared with the levered OPT portfolio. We find that the OPT portfolio generates higher average returns and risk-adjusted returns than the simple portfolios of XS and TS strategies; the differences might not be very large, but we should appreciate that the OPT portfolio in fact consists of both XS and TS patterns so the similarities are more than expected. Probably the most important point in this comparison exercise relates to downside risk and skewness. The basic XS-TS portfolios exhibit negative skewness (-0.15 and -0.13), whereas the OPT portfolio exhibits positive and sizeable skewness (0.30). This leads to very pronounced differences in downside-risk-adjusted performance, as captured by the Sortino ratio: 1.96 for the OPT portfolio, as opposed to 1.70 and 1.76 for the two XS-TS portfolios.

Table 10 presents full-sample results from regressing the monthly returns of the OPT portfolio on the returns of the multi-asset XS and TS portfolios. All portfolios are in their levered form, targeting 7% volatility. As expected, the OPT portfolio is strongly exposed to both XS and TS portfolios either on a univariate basis or a multivariate basis.[19] However, in all cases the OPT portfolio manages to generate positive and statistically significant alpha, above and beyond what is already attained from the exposure to the underlying strategies. This constitutes evidence that the optimised framework adds value on top of the documented cross-sectional and time-series patterns.

Table 10: Exposures of the Levered (7% Vol.) Multi-Asset Optimised Carry

Ann. alpha (%)	Cross-Sectional (7%)	Time-Series (7%)	adjusted R^2
5.08***	0.69***		33.3%
2.73**		0.79***	50.4%
2.33*	0.29***	0.63***	53.8%

Notes: The table presents the results of univariate and multivariate regressions of the optimised (OPT) multi-asset carry portfolio on the respective cross-sectional (XS) and time-series (TS) carry portfolios. All strategies are levered at 7% target volatility. The portfolios are rebalanced monthly, the volatility target is also applied on a monthly basis and all volatilities and covariance structure for the OPT portfolio are estimated using a 100-day window. The regressions are conducted using monthly returns between January 1990 and January 2016. The statistical significance of the average return is indicated with *, ** and *** for 1%, 5% and 10% confidence levels, using Newey and West (1987) standard errors with 6 lags.

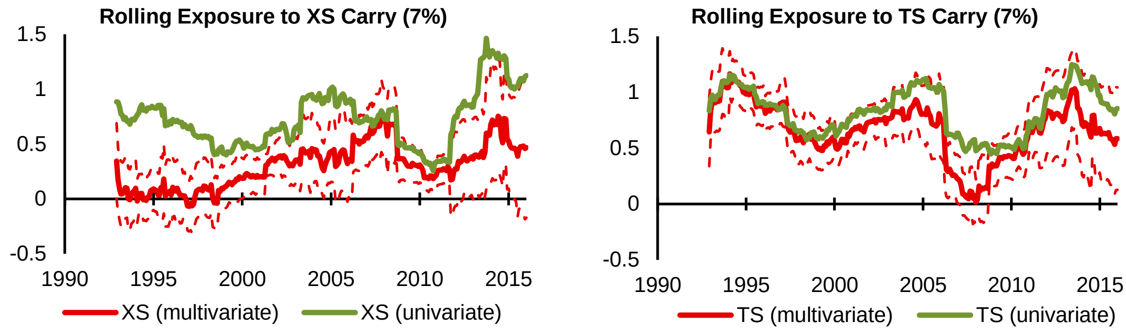
As for the betas against the XS and TS strategies, it is the one against the TS variant of carry that is stronger, both in magnitude and statistical significance. This is to a large extent to be expected, as the OPT strategy has been designed so that it is aligned with the TS strategy on the type of positions; additionally both the OPT and TS strategies exhibit directional tilts, whereas the XS strategy is net-zero. However, even if this is the case, the OPT strategy is also strongly exposed to the XS strategy even after controlling for its exposure to the TS strategy, as the multivariate regression results show in the last row of Table 10. The multivariate alpha is obviously smaller than the univariate alphas and its statistical significance falls, but still remains strong.

Corroborating these full-sample results, Figure 8 presents the 36-month rolling univariate betas and multivariate betas (including a 10% confidence interval band) of the OPT carry strategy on XS and TS multi-asset carry strategies. It becomes obvious that controlling for TS, the beta to the XS strategy falls; however, it maintains its statistical significance for the most part of the post-2000 era. As for TS, the exposure of OPT to it remains almost unaffected by the inclusion of XS in the multivariate regression. The statistical significance is strong except for a few years in the late 2000's, when the exposure to XS picks up.

As last piece of performance diagnostics of the multi-asset OPT carry strategy, Figure 9 presents the 36-month rolling univariate betas against the standard benchmark indices of the different asset class markets (MSCI World Index, Bloomberg Commodity Index, JPMorgan Aggregate Bond Index and Trade-weighted USD). Broadly speaking, the OPT carry strategy exhibits occasional directional tilts, which generally fall from its large dependence on the TS nature of signals. One important observation is that the strategy did not just benefit from the long-standing bond rally, as for about one third of time it exhibits a negative beta to the JPMorgan Aggregate Bond Index.

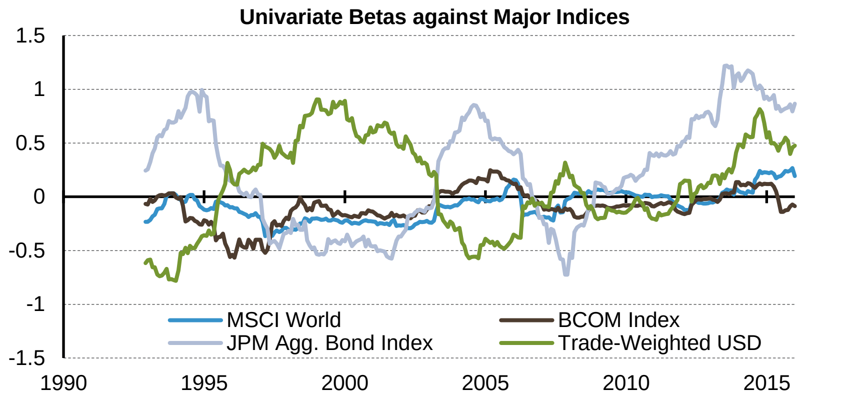
[19] On a technical note, the fact that we use 100-day volatility estimates in the volatility-targeting overlay can give rise to serial correlation in the variables of the regressions. In order to control for this serial dependence we use Newey and West (1987) standard errors for the calculation of t-statistics, using 6 lags, as 100 business days are roughly 5-6 months.

Figure 8: 36-month Rolling Betas of the Levered (7% Vol.) Multi-Asset Optimised Carry



Notes: The figure presents 36-month rolling betas of the optimised (OPT) multi-asset carry portfolio on the respective cross-sectional (XS) and time-series (TS) carry portfolios, using both univariate regressions as well as a multivariate regression; for the multivariate case the charts additionally include a 10% rolling confidence interval in dashed line based on Newey and West (1987) standard errors with 6 lags. All strategies are levered at 7% target volatility. The portfolios are rebalanced monthly, the volatility target is also applied on a monthly basis and all volatilities and covariance structure for the OPT portfolio are estimated using a 100-day window. The regressions are conducted using monthly returns between January 1990 and January 2016.

Figure 9: 36-month Rolling Betas of the Levered Multi-Asset OPT Carry



Notes: The figure presents 36-month rolling univariate betas of the unlevered optimised (OPT) multi-asset carry portfolio on four broad indices: MSCI World Index, Bloomberg Commodity (BCOM) Index, JPMorgan Aggregate Bond Index and Trade-Weighted USD. The regressions are conducted using monthly excess returns between January 1990 and January 2016.

6. Is it Crash Risk?

The paper so far has documented strong multi-asset carry dynamics that can be captured in various forms (XS, TS and OPT portfolios). The important question when it comes to a strategy with positive and statistically strong excess returns is whether the returns are positive because they compensate an investor for bearing some type of systematic risk (so that it is justified as "risk premium") or they simply constitute an artefact of mispricing and behavioural biases.

As already explained early in this paper, FX carry has been linked with FX crash risk as well as equity downside risk in the academic literature; hence, it is generally thought of as a cyclical short-volatility strategy. Without this being a formal test (yet), our FX carry strategies, both in XS and TS form, have been characterised by large negative skewness, which can possibly validate the academic claims.

Our task here is harder and broader. We have documented carry patterns across different asset classes and most importantly at the multi-asset level, most of which have either lower negative skewness, if not positive. Can crash risk at the asset class level (or even broad equity crash risk) justify the positive returns?

We next present regression results between levered carry strategies across asset classes as well as at the multi-asset level against a number of factors F:

$$r_{Carry,t} = const. + \boldsymbol{\beta}' \cdot \boldsymbol{F}_t + \epsilon_t \tag{18}$$

When the factors are tradable portfolios, the constant of the regression can also be interpreted as "alpha". Our analysis is split between two sets of factors.

6.1 Downside risk analysis

First, we conduct a downside risk analysis, where carry portfolios of each asset class are regressed against asset-class-specific downside risk variables as well as against equity downside risk variables. The downside risk variables used for this analysis are:

- Market squared: the squared market return variable helps to uncover any dependence on large scale market moves, either positive or negative.
- Henrikson and Merton (1981) downside risk variable:

  $$r_{MKT,t,down} = -r_{MKT,t} \cdot \mathbb{I}\{r_{MKT,t} < 0\} \tag{19}$$

  The downside risk variable captures only the months with negative returns.
- Lettau, Maggiori and Weber (2014) tail risk variable:

  $$r_{MKT,t,tail} = -r_{MKT,t} \cdot \mathbb{I}\{r_{MKT,t} < -\sigma_{MKT}\} \tag{20}$$

  The tail risk variable captures only the months with negative returns that are more extreme than one standard deviation (σ_{MKT}). We use the full-sample standard deviation estimates, acknowledging the presence of forward-looking bias. The focus is only to understand the negative tail dynamics and its effect on carry profitability, so this is not directly impacting our analysis.

  We use the minus sign in equations (19) and (20) so that a negative exposure to all three downside variables can be interpreted as loading on a some form of downside risk, whereas a positive exposure to these variables can be interpreted as a hedge against downside risk.

For the asset-class specific downside risk analysis, the "market" is proxied by the MSCI World Index for equities, the Bloomberg Commodity Index for commodities, the JPMorgan Aggregate Bond Index for government bonds and the Trade-Weighted USD index for FX.[20] For the equity downside risk analysis, the "market" is always the MSCI World Index.

Starting from the asset-class specific downside risk analysis (see Table 11, Panels A1, B1, C1 and D), the evidence shows that FX carry, both in XS and TS, is heavily exposed to currency downside risk, which is generally in line with existing academic evidence. However, the only other sizeable exposure to downside risk appears for the XS (but not the TS) carry strategy is within commodities, which suffers when the underlying commodity market experiences large losses.

Contrary to the above the equity TS carry strategy appears to offer a hedge against equity downside risk. Finally, as already documented at several points in this paper, the government bond carry strategies (both XS and TS) are the only ones to load positively on their underlying market and therefore benefiting to a certain extent from the bond rally over the recent decades.

Regarding the equity downside risk analysis (see Table 11, Panels A2, B2, C2 and D), apart from the equity TS carry strategy, which offers a hedge against equity downside risk, as already highlighted above, no other XS or TS carry strategy is exposed, either positively or negatively, to equity downside risk.

Interestingly, all betas of XS and TS carry strategies against the MSCI World Index are low in magnitude, even though statistically strong in most cases. In line with existing academic evidence, the FX carry strategy (XS and TS) exhibits a cyclical behaviour as deduced by the positive and statistically significant equity market betas. Conversely, the bond carry strategy (XS and TS) is characterised by strong negative market betas, hence offering a hedge against equity market crashes.

The documented dynamics seem to imply that carry strategies across different asset classes constitute a diversified universe and therefore, when combined to a multi-asset portfolio, most of the asset-class specific dynamics are expected to be suppressed. We investigate this after we first look at the equity volatility risk analysis.

[20] The US 3m Libor rate is used to construct excess monthly returns for these indices.

Table 11: Downside Risk analysis for XS and TS portfolios of each asset class

Cross-Sectional Carry Strategies						Time-Series Carry Strategies					
const.	***MKT***	MKT^2	MKT_{down}	MKT_{tail}	$adj.R^2$	***const.***	***MKT***	MKT^2	MKT_{down}	MKT_{tail}	$adj.R^2$
Panel A1: Commodities (market: BCOM Index)											
0.10	0.00				0.00%	0.12	-0.22***				18.64%
0.27**	-0.02	-0.95***			1.92%	0.13	-0.22***	-0.04			18.10%
0.41**	-0.11*		-0.20**		1.01%	0.18	-0.24***		-0.04		18.16%
0.26**	-0.07			-0.16**	1.16%	0.16	-0.24***			-0.04	18.21%
Panel A2: Commodities (market: MSCI World Index)											
0.07	0.06**				1.25%	0.11	-0.10***				4.24%
0.11	0.05*	-0.21			0.71%	0.08	-0.10***	0.19			3.70%
0.05	0.06		0.01		0.61%	0.11	-0.10*		0.00		3.62%
0.12	0.03			-0.05	0.79%	0.11	-0.10**			0.00	3.62%
Panel B1: Gov't Bonds (market: JPM Aggregate Bond Index)											
0.46***	0.38***				9.57%	0.48***	0.57***				15.34%
0.53***	0.40***	-2.76			9.33%	0.52***	0.59***	-1.55			14.87%
0.66***	0.24*		-0.32		9.60%	0.63***	0.48***		-0.22		15.00%
0.55***	0.31***			-0.24	9.57%	0.51***	0.55***			-0.07	14.83%
Panel B2: Gov't Bonds (market: MSCI World Index)											
0.58***	-0.09***				3.69%	0.68***	-0.12***				4.51%
0.60***	-0.09***	-0.14			3.12%	0.74***	-0.13***	-0.30			4.05%
0.60***	-0.09*		-0.01		3.07%	0.71***	-0.13**		-0.02		3.91%
0.58***	-0.09**			0.00	3.07%	0.64***	-0.10**			0.04	3.98%
Panel C1: FX (market: Trade-Weighted USD)											
0.16	-0.09				0.55%	0.31**	-0.15				1.37%
0.50***	-0.11	-11.87***			6.95%	0.55***	-0.16**	-8.37**			4.01%
0.75***	-0.58***		-0.89***		4.90%	0.84***	-0.59***		-0.81***		4.57%
0.34***	-0.30**			-0.47**	2.85%	0.47***	-0.34***			-0.42**	2.95%
Panel C2: FX (market: MSCI World Index)											
0.16	0.12***				6.30%	0.32***	0.12***				5.49%
0.28**	0.11***	-0.63			6.65%	0.26**	0.12***	0.35			5.14%
0.31	0.07		-0.09		6.04%	0.22	0.15**		0.06		5.02%
0.17	0.12***			-0.01	5.70%	0.21	0.17***			0.11	5.78%
Panel D: Equity Indices (market: MSCI World Index)											
0.18*	-0.05**				1.45%	0.21*	-0.18***				12.07%
0.22*	-0.05**	-0.17			0.91%	-0.01	-0.16***	1.14**			14.04%
0.15	-0.04		0.02		0.84%	-0.26*	-0.03		0.28***		14.23%
0.11	-0.02			0.06	1.25%	-0.06	-0.07			0.25***	15.64%

Notes: The figure presents the results from regressing XS and TS carry portfolios of each asset class to the respective market (BCOM Index for commodities, JPMorgan Aggregate Bond Index for government bonds, Trade-Weighted USD for FX and MSCI World Index for equity indices) as well as a number of associated downside risk variables. All strategies are levered at 7% target volatility. Any statistical significance is indicated with *, ** and *** for 1%, 5% and 10% confidence levels, using Newey and West (1987) standard errors with 6 lags. The constant of each regression is multiplied by 100. The regressions are conducted using monthly returns between January 1990 and January 2016.

6.2 Equity volatility risk analysis

Next, we conduct an equity volatility risk analysis, where carry portfolios of each asset class are regressed against monthly changes in VIX; see Table 12. Broadly in line with the findings so far, the FX carry (both XS and TS) is very strongly exposed to changes of VIX, hence justifying the academic evidence in this space. Additionally, the XS form of FX carry, which is the most heavily studied form of carry, maintains a strong and positive market beta alongside a negative exposure on the changes in VIX. The only other specification with negative exposure on the changes in VIX is the commodity XS carry strategy, which however turns insignificant when we control for the overall equity market.

Contrary to the above, government bonds appear to provide a hedge against changes in VIX, in line with our earlier findings above. It is worth commenting that all TS carry strategies, except for FX, seem to constitute a hedge against increases in equity market volatility.

Table 12: Volatility Risk across XS and TS Carry portfolios for each asset class

Cross-Sectional Carry Strategies				Time-Series Carry Strategies			
$const.$	$MSCI$	ΔVIX	$adj.R^2$	$const.$	$MSCI$	ΔVIX	$adj.R^2$
Commodities							
0.08		-0.06**	1.37%	0.10		0.08**	2.35%
0.08	0.03	-0.04	0.91%	0.12	-0.10***	0.01	3.86%
Government Bonds							
0.57***		0.07**	2.12%	0.64***		0.07**	1.72%
0.58***	-0.08**	0.01	3.22%	0.66***	-0.12***	0.00	3.60%
FX							
0.18		-0.12***	6.11%	0.34***		-0.10***	4.17%
0.16	0.08**	-0.07*	7.01%	0.32***	0.09**	-0.04	5.43%
Equity Indices							
0.17*		0.01	0.03%	0.16		0.09***	2.81%
0.19*	-0.08***	-0.05	1.53%	0.20*	-0.22***	-0.06	11.74%

Notes: The figure presents the results from regressing XS and TS carry portfolios of each asset class to the MSCI World Index and to monthly changes in VIX (ΔVIX). All strategies are levered at 7% target volatility. Any statistical significance is indicated with *, ** and *** for 1%, 5% and 10% confidence levels, using Newey and West (1987) standard errors with 6 lags. The constant of each regression and the exposure to ΔVIX are multiplied by 100. The regressions are conducted using monthly returns between January 1990 and January 2016.

All in all, asset-class-specific or equity downside risks do not seem to justify the positive returns of carry strategies with the only exception being within FX. Put differently, crash risk does not appear to provide a unifying framework for explaining the carry patterns. We finally look at the multi-asset level.

6.3 Multi-asset carry analysis

Having established econometrically the dependencies of carry strategies of each asset class with the respective market as well as with the overall global equity market, we conclude the empirical analysis of this paper with regression results for the multi-asset XS, TS and OPT carry portfolios in Table 13.

The multi-asset XS carry portfolio is effectively equity market neutral and only exhibits a negative exposure in changes in VIX, which might justify the premium as compensation for volatility risk (in line with Koijen *et al.*, 2017). Instead, the multi-asset TS carry portfolio exhibits negative equity market betas, which generally remain significant unless we control for equity downside risk, in which case the portfolio appears to constitute a hedge against this risk. So, if anything, the TS carry appears to be a good diversifier for an equities portfolio.

Given the above dependences, the multi-asset OPT carry portfolio appears to benefit from the optimised risk allocation and completely eliminates any exposure to downside or volatility risk. It only exhibits a small in magnitude –at times significant– beta against the market. This finding is extremely interesting and highlights the benefits of forming multi-asset portfolios, as well as employing an optimisation framework to optimise the risk allocation.

Table 13: Downside and Volatility Risk for Multi-Asset Carry Strategies

$const.$	$MSCI$	$MSCI^2$	$MSCI_{down}$	$MSCI_{tail}$	ΔVIX	$adj.R^2$
XS Multi-Asset						
0.49***	0.02					0.23%
0.63***	0.01	-0.76				0.94%
0.59**	-0.01		-0.06			-0.26%
0.50***	0.02			-0.01		-0.39%
0.49***					-0.06	1.46%
0.50***	-0.03				-0.08**	1.01%
TS Multi-Asset						
0.70***	-0.13***					6.13%
0.57***	-0.11***	0.68*				6.45%
0.43**	-0.04		0.16*			6.46%
0.49***	-0.04			0.20***		8.22%
0.66***					0.05	1.03%
0.69***	-0.16***				-0.05	5.78%
OPT Multi-Asset						
0.78***	-0.08**					1.80%
0.81***	-0.08**	-0.15				1.20%
0.80***	-0.09		-0.01			1.17%
0.74***	-0.06			0.04		1.25%
0.77***					0.03	0.33%
0.79***	-0.10**				-0.03	1.39%

Notes: The figure presents the results from regressing multi-asset XS, TS and OPT carry portfolios to the MSCI World Index and a number of associated downside risk variables as well as changes in VIX. All strategies are levered at 7% target volatility. The constant of each regression and the exposure to ΔVIX are multiplied by 100. The regressions use monthly returns between January 1990 and January 2016.

7. Concluding Remarks

The carry trade is probably the most well-known trading strategy in the foreign exchange market, but there is little evidence and analysis on a multi-asset scale. This paper contributes to the literature by first extending the concept of carry across asset classes and second, by highlighting the benefits of diversification that an investor can enjoy when invested in a systematic multi-asset carry strategy.

Carry signals are found to have predictive power for future asset returns, not just within FX, but also across commodities, equity indices and government bonds, hence offering an additional source of return for an investor who looks for yield, especially during sideways markets. Most importantly, carry strategies outside FX do not appear to have significant exposure to downside risk –either with respect to their respective market or to the broad equity market– hence justifying their inclusion to a multi-asset allocation framework.

The greater benefit from extending carry to a multi-asset concept is, in fact, the diversification potential from combining carry strategies from different asset classes. The fact that not all carry strategies fail at the same time renders the multi-asset carry portfolio robust to equity downside risk and volatility spikes. We have provided an original risk-based optimisation framework that optimises the allocation of risk across asset classes, while tilting the portfolio towards asset classes that bear higher collective carry at any point in time. This way, we have managed to optimally combine the relative strength (cross-sectional) with the absolute (time-series) nature of carry signals while also accounting for the multi-asset covariance structure. Based on this analysis the optimised multi-asset carry portfolio has an attractive risk-return profile, with positive skewness and a small and negative exposure to the broad equity market, without being exposed to any downside risk.

Appendix - Estimating the Carry metric for each asset class

In order to estimate the level of carry of each asset from each asset class at the end of each month, we can theoretically estimate the slope of the futures curve, as already explained in the main body of the paper. However, various idiosyncrasies of the different asset classes that we focus on make the estimation of carry a tedious process that makes use of various data sources and required careful pre-processing and data cleaning. We explain our approach separately for each asset class; Table 14 summarises the various carry metrics.

Table 14: How to measure Carry across asset classes

Asset Class	Chosen Metric	Alternatives
FX	$\frac{Spot_t - Fwd_t^{1M}}{Fwd_t^{1M}}$	1. $r_t^* - r_t$ 2. $\frac{1}{T_2 - T_1} \cdot \frac{Fut_t^{T_1} - Fut_t^{T_2}}{Fut_t^{T_2}}$
Equity Indices	$\frac{1}{T_2 - T_1} \cdot \frac{Fut_t^{T_1} - Fut_t^{T_2}}{Fut_t^{T_2}}$ Seasonally adjusted (12 months)	$\frac{Spot_t - Fut_t^{1M,intrpl}}{Fut_t^{1M,intrpl}}$ Seasonally adjusted (12 months)
Commodities	$\frac{1}{T_2 - T_1} \cdot \frac{Fut_t^{T_1} - Fut_t^{T_2}}{Fut_t^{T_2}}$ Seasonally adjusted (12 months)	$\frac{Fut_t^{T_1} - Fut_t^{T_1+1y}}{Fut_t^{T_1+1y}}$
Government Bonds	$\frac{Spot_t^{9Y11M,intrpl} - Fut_t^{1M;10Y,Synth}}{Fut_t^{1M;10Y,Synth}}$	$\frac{1}{T_2 - T_1} \cdot \frac{Fut_t^{T_1} - Fut_t^{T_2}}{Fut_t^{T_2}}$

FX: accessing the foreign exchange market is typically achieved either directly at the spot market or alternatively via forward contracts. For this reason, our chosen metric for the slope of the futures/forward curve makes use of spot prices and one-month forward prices, all collected from Bloomberg. As alternatives, we could theoretically use (a) the 3-month Libor rate differential between the foreign and the USD markets or (b) the front and first back futures contracts (maturing after T_1 and T_2 days), adjusting the slope by the difference in days-to-maturity $(T_2 - T_1)$, so that we can allow for cross-sectional ranking between currencies. The first alternative (rate differential) results in carry metrics that are very largely correlated with our preferred metric; the correlations range from 88.1% for NZD up to 99.2% for JPY. This is largely expected due to the covered interest rate parity. The second alternative is not tested as there are no good quality historical data for the back futures contracts for most currencies; the liquidity of these back contracts is almost non-existent and only picks up a few days before maturity, when roll-overs from the front contract start taking place.

Equity Indices: we have two options. Our preferred metric is the slope of the futures curve, as estimated by the front and first back futures contracts, adjusted by the difference in days-to-maturity $(T_2 - T_1)$. Additionally, we apply a seasonality adjustment, which is trivially a 12-month moving average filter of the raw carry metric, following the documentation of strong seasonal patterns within equity indices and commodities, but not within FX or government bond markets (see Keloharju, Linnainmaa, and Nyberg, 2016 and Baltas, 2016). Baz *et al.* (2015) and Koijen *et al.* (2017), who also look at multi-asset carry strategies, similarly seasonally-adjust the carry metrics for equity indices and commodities. An alternative for equity indices would be to use the front and the first back futures contracts in order to estimate (using interpolation) the futures price for a maturity of one month and compare this with the prevailing spot price of the index; seasonal adjustment should also apply. One important observation (and warning) here is that these two carry metrics would have different values and even different signs, if the futures curve happens to be humped in the short end.

Commodities: given that commodities are generally accessed via futures contracts, we have one option, and that is to estimate the slope of the curve using the front and first back futures contracts, adjusted by the difference in days-to-maturity $(T_2 - T_1)$. As for equity indices, a 12-month moving average filter is applied in order to eliminate any seasonal patterns. An alternative definition would estimate the slope using the front futures contract and the contract expiring one year after (these contracts are relatively liquid in the commodity markets). This estimate is by construction free from any seasonal patterns and therefore no further adjustment is required.

Government Bonds: the quality of historical futures data, except for the front contract, is very poor for government bonds. This does not really allows us to estimate the slope of the curve using the front and first back futures contracts, adjusted by the difference in days-to-maturity ($T_2 - T_1$), except for a subset of our universe and for a subset of our sample period. Instead, we follow Koijen *et al.* (2017) and estimate the slope of the curve using the spot price of a bond with maturity of 9 years and 11 months and a synthetic 10-year bond futures price with maturity of one month. To achieve this, we make use of zero-coupon yield data from Bloomberg for maturities of 9 and 10 years. The 9 years and 11 months yield is therefore trivially estimated using linear interpolation:

$$y_t^{9Y11M,intrpl} = \frac{1}{12} \cdot y_t^{9Y} + \frac{11}{12} \cdot y_t^{10Y} \quad (21)$$

The spot bond price with maturity of 9 years and 11 months is therefore:

$$Spot_t^{9Y11M,intrpl} = \frac{1}{\left(1 + y_t^{9Y11M,intrpl}\right)^{9+\frac{11}{12}}} \quad (22)$$

Finally, the futures price of a 10-year bond with maturity of one month is trivially equal to the respective bond price accrued to the risk-free rate (r_t) for a month:

$$Fut_t^{1M;10Y,Synth} = \frac{1 + r_t}{(1 + y_t^{10Y})^{10}} \quad (23)$$

For the risk-free rate we use the 3-month Libor rate for the respective country. Estimating the slope of the futures curve should then be straightforward.

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