

Kyle Kodan

(S161 HW #5)

<u>(1) P   Q   <math>P \rightarrow Q</math>   <math>Q \rightarrow P</math></u>
T   T   F   F
T   F   T   F
F   T   F   T
F   F   T   T

<u>P   Q   <math>P \leftrightarrow Q</math>   <math>((P \rightarrow Q) \vee (\neg P \wedge Q))</math></u>
T   T   F   F F F
T   F   T   T T F
F   T   F   F F T
F   F   T   F T F

<u>(2) smoke   fire   <math>(\text{smoke} \rightarrow \text{fire}) \rightarrow (\neg \text{smoke} \rightarrow \neg \text{fire})</math></u>
T   T   T   T F T F
T   F   F   T F T T
F   T   T   F T F F
F   F   T   T T T T

neither

<u>smoke   fire   heat   <math>(\text{smoke} \rightarrow \text{fire}) \rightarrow ((\neg \text{smoke} \wedge \text{heat}) \rightarrow \text{fire})</math></u>
T   T   T   T T T
T   T   F   T T T
T   F   T   F T F
T   F   F   F T F
F   T   T   T T T
F   T   F   T T T
F   F   T   T F T
F   F   F   T F T

valid

<u>smoke   heat   fire   <math>((\text{smoke} \wedge \text{heat}) \rightarrow \text{fire}) \leftrightarrow ((\text{smoke} \rightarrow \text{fire}) \vee (\text{heat} \rightarrow \text{fire}))</math></u>
T   T   T   T T T
T   T   F   T F T
T   F   T   F T T
T   F   F   T T T
F   T   T   T T T
F   T   F   T F T
F   F   T   T T T
F   F   F   T T T

valid

(3) a)  $(\text{mythical} \rightarrow \text{immortal}) \wedge (\neg \text{mythical} \rightarrow (\neg \text{immortal} \wedge \text{mammal}))$   
 $(\text{immortal} \vee \text{mammal}) \rightarrow \text{horned}$   
 $\text{horned} \rightarrow \text{magical}$

b)  $(\text{mythical} \vee \text{immortal})$   
 $\text{mythical} \vee (\neg \text{immortal} \wedge \text{mammal})$   
 $\hookrightarrow (\text{mythical} \vee \neg \text{immortal}) \wedge (\text{mythical} \vee \text{mammal})$   
 $\neg(\text{immortal} \wedge \neg \text{mammal}) \vee \text{horned}$   
 $\hookrightarrow (\neg \text{immortal} \vee \text{horned}) \wedge (\neg \text{mammal} \vee \text{horned})$   
 $\neg \text{horned} \vee \text{magical}$

c) 1.  $\neg \text{mythical} \vee \text{immortal}$   
2.  $\text{mythical} \vee \neg \text{immortal}$   
3.  $\text{mythical} \vee \text{mammal}$   
4.  $\neg \text{immortal} \vee \text{horned}$   
5.  $\neg \text{mammal} \vee \text{horned}$   
6.  $\neg \text{horned} \vee \text{magical}$

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7.  $\neg \text{mythical}$  7.  $\neg \text{magical}$   
8.  $\neg \text{immortal}$  2, 7 8.  $\neg \text{horned}$  7, 6  
9.  $\text{mammal}$  3, 7 9.  $\neg \text{mammal}$  8, 5  
10.  $\text{horned}$  9, 5 10.  $\neg \text{immortal}$  8, 4  
11.  $\text{magical}$  10, 6 11.  $\neg \text{mythical}$  9, 3  
[cannot conclude if mythical] 12.  $\neg \text{mythical}$  10, 1  
13. contradiction 11, 12

[unicorns are magical!]

7.  $\neg \text{horned}$   
8.  $\neg \text{mammal}$  7, 5  
9.  $\neg \text{immortal}$  7, 4  
10.  $\text{mythical}$  8, 3  
11.  $\neg \text{mythical}$  9, 1  
12. contradiction 10, 11  
[unicorns are horned!]