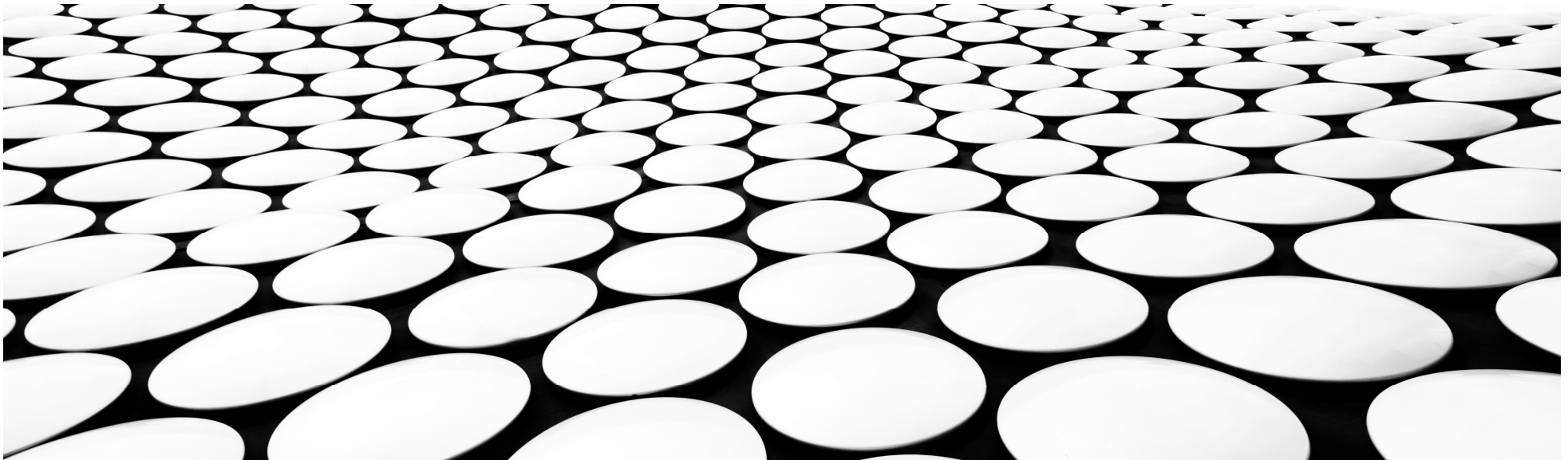


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# CONVEX HULL ALGORITHM

KYLE KOLODZIEJ



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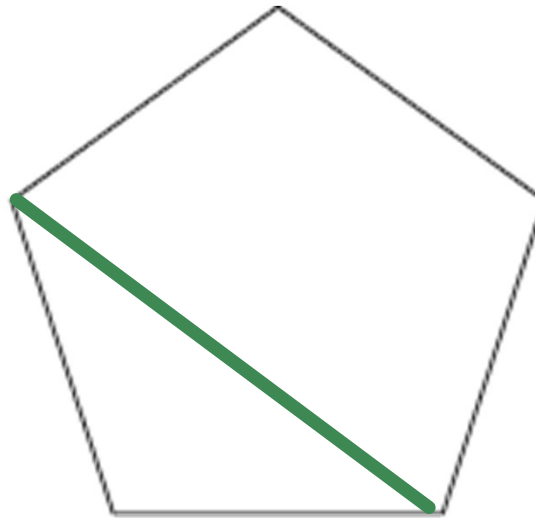
## OVERVIEW OF PRESENTATION

1. Convex Polygon
2. Convex Hull
3. Jarvis' March/Gift Wrapping
4. Graham's Scan
5. Jarvis' March vs. Graham's Scan
6. Real-World Application

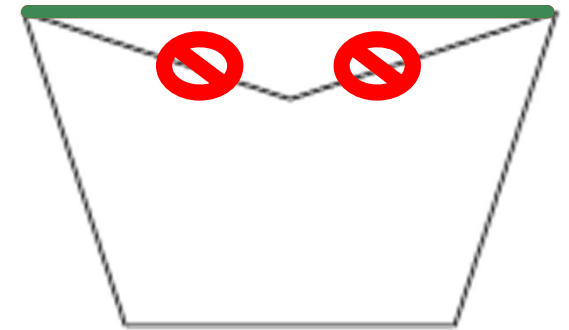
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## CONVEX POLYGON

- Simple polygon (not self-intersecting)
- Interior angles are all less than or equal to 180 degrees
- Line segments from two vertices stay in its boundaries



*convex polygon*



*concave polygon*

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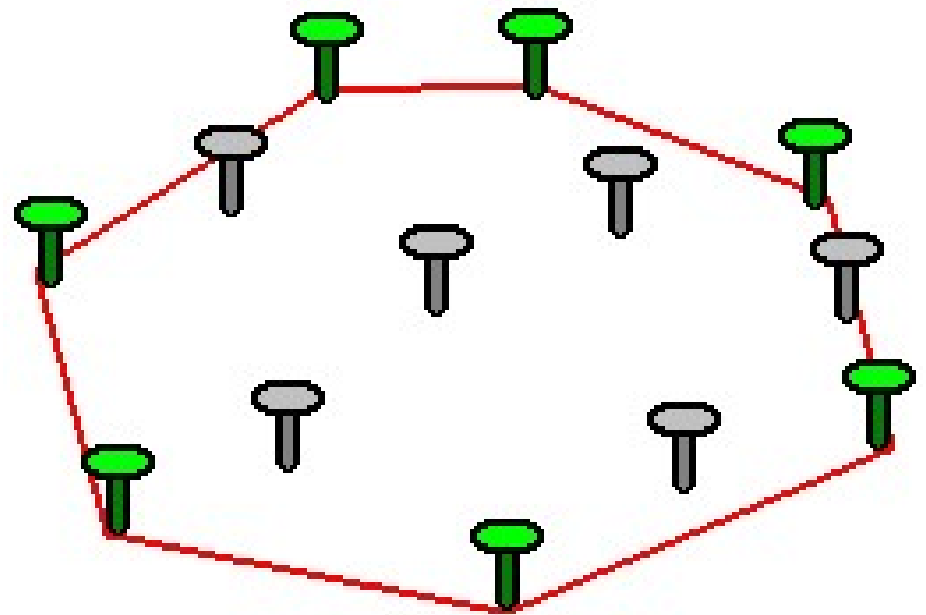
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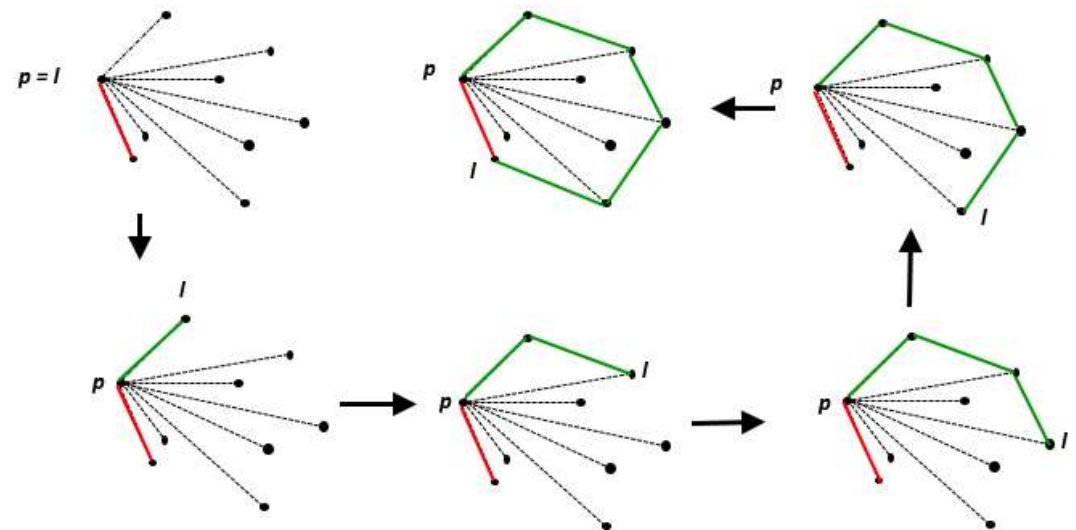
## WHAT IS A CONVEX HULL

- Given a set  $P$  of  $n$  points in  $d$ -dimensional space, it finds the smallest convex polygon containing all the points of set  $P$
- Rubber band and nail example
- Equivalents
  - Smallest convex set containing all points
  - Smallest convex polygon containing all points



# CONVEX HULL ALGORITHMS

- Jarvis' March a.k.a. Gift Wrapping
- Graham's Scan
- Quickhull
- Divide and Conquer
- Monotone Chain a.k.a. Andrew's Algorithm
- Chan's Algorithm
- <https://github.com/kkolodziej39/ConvexHull>



The execution of jarvis's March

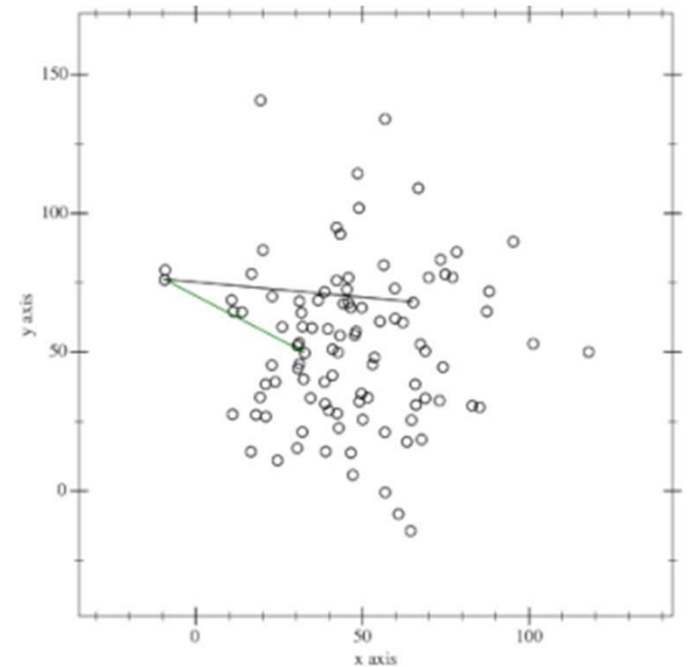
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## JARVIS' MARCH (GIFT WRAPPING)

- Start with an extreme point that will be in the convex hull
- Using this as a point of reference, add the point that is leftmost (most counterclockwise)
- Repeat until at initial point
- Time complexity of  $O(nh)$ 
  - $N$  = Number of input points
  - $H$  = Number of hull points
- Worst case of  $O(n^2)$ 
  - All points are hull points





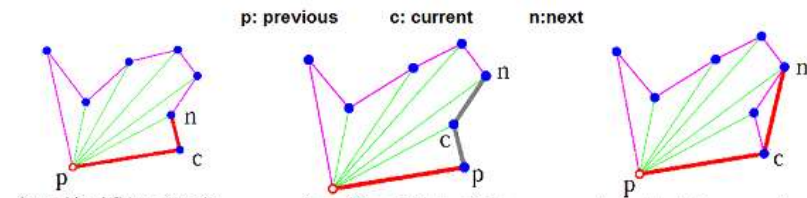
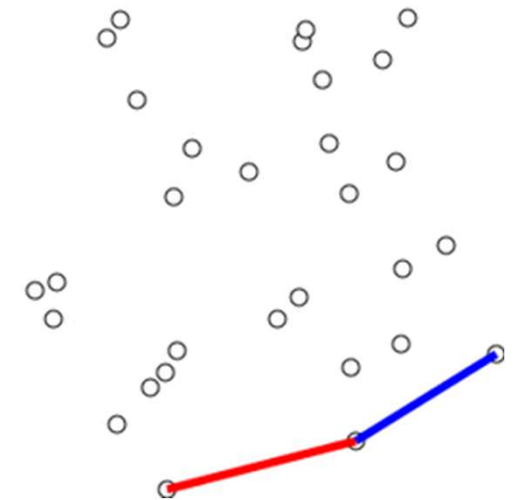
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# GRAHAM'S SCAN

- Start with point known to be on the convex hull
- Sort the rest in angular order around this point
- Add to the hull if previous, current, and next point form a counterclockwise orientation
- Delete chain of vertices from the hull if previous, current, and next point do not form counterclockwise orientation
- Time complexity of  $O(n \lg n)$ 
  - $N$  = Number of input points



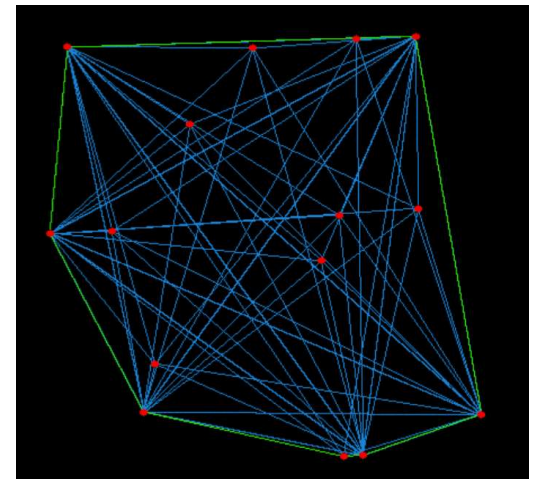
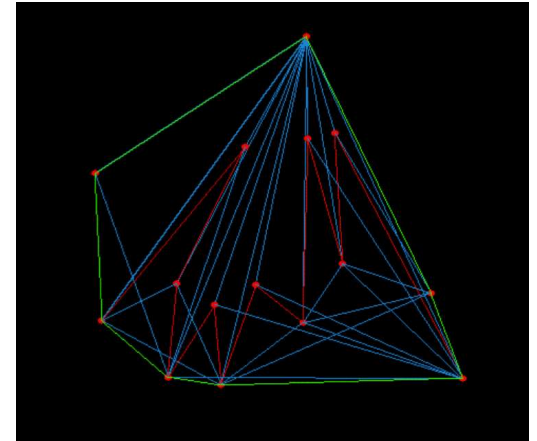
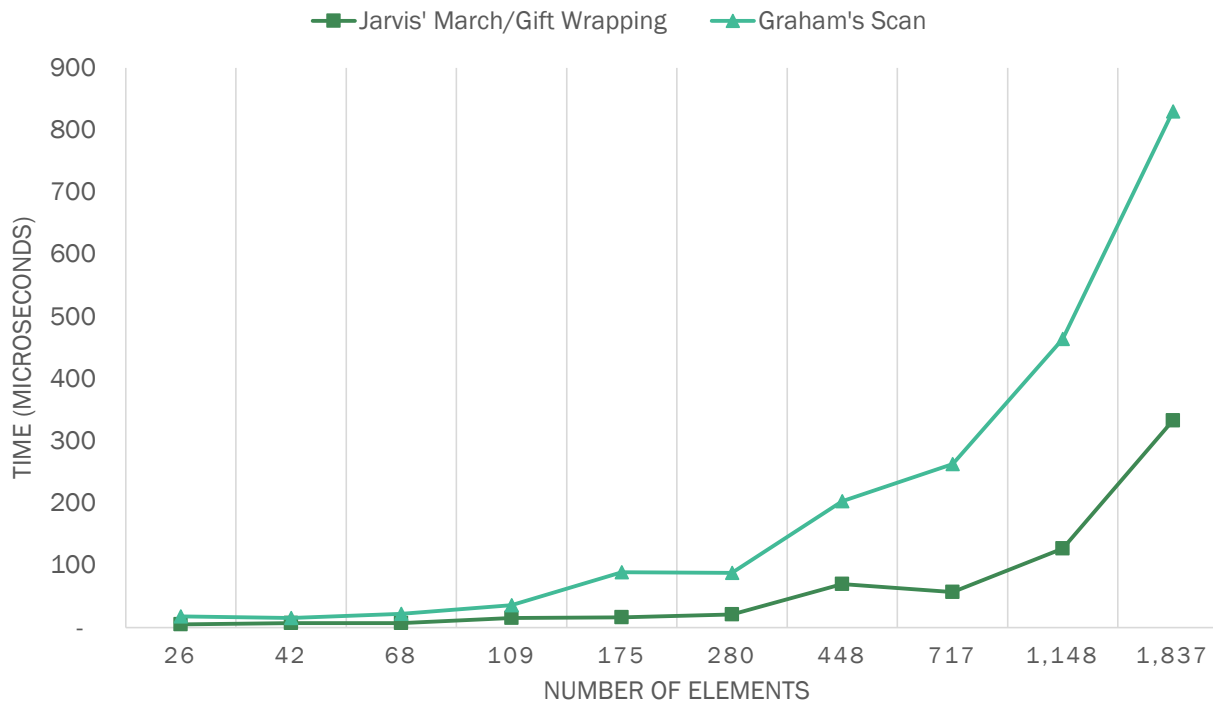
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# JARVIS' MARCH VS. GRAHAM'S SCAN

NUMBER OF ELEMENTS VS TIME OF EACH IMPLEMENTATION



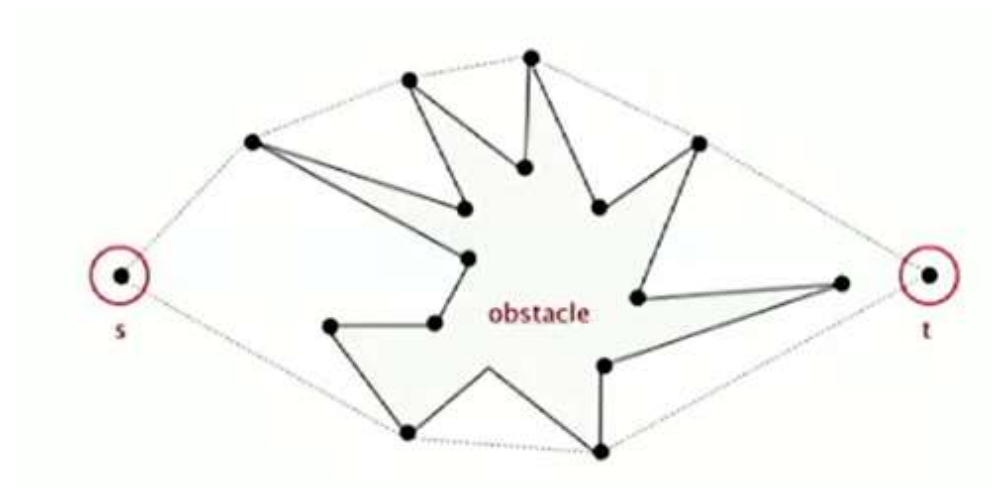
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## REAL-WORLD APPLICATION

- Geometry
- Collision detection and path planning
- Shape analysis/detection
- Nuclear leak evacuation
- Tracking diseases
- Cooking
  - <http://veronising.blogspot.com/2008/04/cooking-for-nerds-ingredient-polyhedron.html>



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