Practical Course MATLAB/SIMULINK

Session 2: Symbolic Math Toolbox

Lecture Objectives & Preparation

- Which MathWorks products are covered?
  - Symbolic Math Toolbox
- What skills are learnt?
  - Usage of symbolic objects within MATLAB
  - Symbolic calculus including differentiation, integration and vector analysis
  - Solving symbolic algebraic equations and equation systems
  - Solving symbolic differential equations and symbolic Laplace transform
- How to prepare for the session?
  - MathWorks Tutorials:
    - https://de.mathworks.com/help/symbolic/getting-started-with-symbolic-math-toolbox.html
    - https://de.mathworks.com/help/symbolic/create-symbolic-numbers-variables-and-expressions.html
    - https://de.mathworks.com/help/symbolic/create-symbolic-functions.html

Lecture Outline

1. Introduction
2. Symbolic Objects
   2.1. Variables, Numbers, Expressions & Functio
   2.2. Assumptions & Simplification
   2.3. Variable Precision Arithmetic
3. Calculus
   3.1. Differentiation & Integration
   3.2. Vector Analysis
   3.3. Series & Limits
   3.4. Transforms

Lecture Outline

4. Equation Solving
    4.1. Algebraic Equations
    4.2. Ordinary Differential Equations (ODEs)
5. Symbolic Plotting Functions
6. Code Generation
7. List of Useful Commands
8. Self-Assessment

1. Introduction

Symbolic vs Numeric

SYMBOLIC	vs.	NUMERIC
▪ “you write it down”		▪ “you (or a computer) compute it”
▪ The symbol π stands for an infinite number		▪ A finite number must be used to approximate π
▪ $y = \sin^2 x + \cos^2 x$ can be simplified to $y = 1$		▪ $y = \sin^2 x + \cos^2 x$ can be computed for different values of x. The result is always close to 1, depending on the numerical precision.
▪ Computation of indefinite integrals		▪ Computation of definite integrals
▪ **Results are always exact**		▪ **Results are always approximate**

Symbolic Computation Software

MATLAB Symbolic Math Toolbox

Scope of this course

MuPAD

A computer algebra system originally developed at the University of Paderborn.

(Part of the Symbolic Math Toolbox since 2008.)

Interface

Mathematica

https://upload.wikimedia.org/wikipedia/de/b/bf/MathematicaSpikeyVersion8.png

Interface

Maple

Maple™

https://www.maplesoft.com/images2015/resources/Maple/Maple_2015_logo.jpg

Key Features of the Symbolic Math Toolbox

- **Analytic** manipulation and solving of mathematic expressions:

Simplification	Differentiation	Series	Transforms
Equation Solving	Integration	Limits	Vector Analysis

- Perform **variable-precision arithmetic** (**VPA**):

Exact computations	VPA	Double-precision floating-point arithmetic
1/3	0.33 or 0.33333	$3FD5\ 5555\ 5555\ 5555_{16}$

- **Generate code** from symbolic expressions for MATLAB, Simulink, Simscape, C, Fortran, MathML, and TeX.

2. Symbolic Objects

2.1. Variables, Numbers, Expressions & Functions

Symbolic Objects sym and symfun

Symbolic Variables

`rho, s`

MATLAB data type: sym

Symbolic Numbers

`sqrt(2), 1/3`

MATLAB data type: sym

Symbolic Expressions

`distance = rho*exp(s)`

MATLAB data type: sym

Symbolic Functions

`distance(rho,s) = rho*exp(s)`

MATLAB data type: symfun

Creating Symbolic Variables

- There are two ways to **create symbolic variables**:

```
x = sym('x');
y = sym('y');
```

or

```
syms x y
```

- MATLAB language vector and matrix notation **extends to symbolic variables**:

```
A = x.^((0:2)'*(0:2));
```

```
A =
[1,   1,   1]
[1,   x, x^2]
[1, x^2, x^4]
```

Very Important: in the MATLAB command window, **symbolic results are not indented**!

Creating Symbolic Matrices

- To create **matrices of symbolic variables**, either **create** the matrix elements and **assemble** them, or use the functionality of the sym command:

```
syms a b c d
A = [a,b; c,d];
B = sym('b', 2);
C = sym('val%d%d', [2,3]);
```

Only when using the first method, are the **individual matrix elements** are created in the workspace.

```
A =
[a, b]
[c, d]
B =
[b1_1, b1_2]
[b2_1, b2_2]
C =
[val11, val12, val13]
[val21, val22, val23]
```

Symbolic Matrices Operations

- **Operations** can be performed on **symbolic matrices.** For example:

```
>>  A = sym('a',2); det(A)
```

```
ans  =

a1_1*a2_2 – a1_2*a2_1
```

```
>>  B = sym(‘b',2); A - B
```

```
ans  =

[a1_1 - b1_1, a1_2 - b1_2]

[a2_1 - b2_1, a2_2 - b2_2]
```

2.2. Assumptions & Simplification

Assumptions

It is possible to define **assumptions** about symbolic variables.

- Make a first assumption with either the sym or assume commands:

```
I = sym('I','integer');
P = sym('P','positive');
Q = sym('Q','rational');
syms R
assume(R,'real');
```

- To add **further assumptions** after the first one, use assumeAlso:

```
assumeAlso(I ~= 5);
```

- To check the **current** assumptions:

```
assumptions
```

```
[0 < P, in(Q, 'rational'), in(R, 'real'), I ~= 5, in(I, 'integer')]
```

- To **clear** the assumptions on a symbolic variable:

```
assume(R, 'clear')
```

Symbolic Numbers

Symbolic numbers have **no floating-point approximations** – they are exact! Symbolic numbers are created in a similar way to variables:

- For a symbolic **number**:

```
sum_of_angles_of_a_triangle = sym('pi');
one_third = sym('1/3');
two_fifths = sym('2/5');
one_third + two_fifths
```

```
ans =

11/15
```

- For a symbolic **expression**:

```
s = str2sym('sqrt(2)')
s_numeric = double(s)
```

```
s =
2^(1/2)
s_numeric =
     1.4142
```

Numeric to Symbolic Conversion

It is also possible to **convert numeric values to symbolic numbers**.

- For highest accuracy, be sure to use sym **subexpressions** instead of using sym on an entire expression:

```
s1 = 1/sym(234567)
s2 = sym(1/234567)
s3 = sym('1/234567')
```

Note: The string represents a number and can be entered without str2sym.

```
s1 =
1/234567
s2 =
5033067825897979/1180591620717411303424
s3 =
1/234567
```

This ratio is not exactly 1/234567, but it can be represented exactly by a double-precision floating point number.

- When using sym on an **entire expression**, the expression is converted to a **floating-point number first** and this floating-point number then is converted to a symbolic number.

Numeric to Symbolic Conversion Technique

The conversion technique can be chosen by specifying a **second function argument** , 'r', 'f', 'e', or 'd'.

- 'r' stands for **rational** and is the **default**:

```
s_rational = sym(0.1,'r')
```

```
s_rational =
1/10
```

For modest sized integers p and q, floating point numbers obtained by evaluating expressions of the form:

- p/q
- $p \cdot \pi/q$
- $\sqrt{p}$
- 2^q
- 10^q

are converted to the **corresponding symbolic form**.

Numeric to Symbolic Conversion Technique

- 'f' stands for **floating-point**:

```
s_float = sym(0.1,'f')
```

```
s_float =
3602879701896397/36028797018963968
```

- 'e' stands for **estimate error**. The 'r' form is **supplemented** by a term involving the **variable eps**, which estimates the **difference** between the **theoretical** rational expression and its **actual** floating-point value.

```
s_eps = sym(0.1,'e')
```

eps is the distance from 1.0 to the next largest double-precision number: eps = 2^-52

```
s_eps =
eps/40 + 1/10
```

Numeric to Symbolic Conversion Technique

- 'd' stands for **decimal**. This 32-digit result does not end in a string of zeros but is an **accurate decimal representation** of the floating-point number **nearest** to 0.1.

```
s_decimal = sym(0.1,'d')
```

```
s_decimal =
0.10000000000000000555111512312578
```

- The number of **significant decimal digits** can be changed using the digits command.

```
s_decimal = sym(0.1,'d')
```

```
s_decimal =
0.1
```

Note the different result in the numeric to symbolic conversion!

Non-exact Floating-point Approximations

The following example illustrates possible consequences of **non-exact floating-point approximations**.

- The signum or sign function is defined as follows:

$$\operatorname{sgn}(x) = \begin{cases} 1 & \text{for } x > 0 \\ 0 & \text{for } x = 0 \\ -1 & \text{for } x < 0 \end{cases}$$

- Thus, in theory, $\operatorname{sgn}(\sin(\pi)) = 0$.

Non-exact Floating-point Approximations

- However:

```
numeric_incorrect  = sign(sin(pi))
symbolic_incorrect = sign(sym(sin(pi)))
symbolic_correct   = sign(sin(sym(pi)))
```

```
numeric_incorrect=
     1
symbolic_incorrect =
1
symbolic_correct =
0
```

In this case, sin(pi) is computed with floating point accuracy before being converted to symbolic. The floating-point approximation is not exactly zero! This is the cause for the incorrect result.

Symbolic Expression & Substitution

Symbolic variables can be **combined** into **symbolic expressions**.

- For Example:

```
syms A omega t
d = A*sin(omega*t);
e = d^2 + (A*cos(omega*t))^2
```

```
e =
A^2*cos(omega*t)^2 + A^2*sin(omega*t)^2
```

- In symbolic expressions, each symbolic variable can be **substituted** by a **numeric value** or **by another symbolic variable**:

```
f = subs(e, [omega,t], [A,2])
```

```
f =
A^2*cos(2*A)^2 + A^2*sin(2*A)^2
```

f can be simplified. The next slide shows how...

Expression Simplification

- **Simplifying** the previous expression yields:

```
syms A omega t
e = (A^2*cos(omega*t)^2 + A^2*sin(omega*t)^2)^(1/2);
simplify(e)
```

```
ans =
(A^2)^(1/2)
```

- Simplified expressions are always **mathematically equivalent** to initial expressions! Therefore, the result is **not** A. If we add the **assumption** assume(A>0), the result would indeed be A.

```
syms A omega t;
assume(A>0);
e = (A^2*cos(omega*t)^2 + A^2*sin(omega*t)^2)^(1/2);
simplify(e)
```

```
ans =
A
```

Expression Simplification

- The toolbox can simplify **expressions** and **functions** with:
  - polynomials,
  - trigonometric,
  - logarithmic, and
  - other functions (e.g., Gamma or Bessel function)

- simplify(f,'Steps',n) with a positive integer n > 0 can **increase simplification** of a complex expression f.

Symbolic Functions

- Symbolic functions are created by specifying the **function variables** and the **function itself**:

```
syms A omega t
f(A,omega,t) = A*sin(omega*t);
```

- A **generic function** and its variables can be created simply by:

```
syms g(x,y)
```

- Symbolic functions can be **evaluated** as follows, **without** using the command subs:

```
f(3, 0.1*t, t)
```

```
ans =
3*sin(t^2/10)
```

- To find **all symbolic variables** in expressions, functions and matrices, use:

```
symvar(f)
```

```
ans =
[A, omega, t]
```

2.3. Variable-Precision Arithmetic

Variable Precision Arithmetic

- Symbolic numbers can be approximated with **varying accuracy**, trading off **accuracy for code performance**. Default accuracy, defined in **significant decimal digits**, can be retrieved or set with the digits command.

```
old_digits_setting = digits;
digits(3);
```

Variable-Precision Arithmetic

- The **approximation** is done using the vpa command:

```
v1 = vpa('1/2')
v2 = vpa('1/3000')
v3 = vpa(str2sym('sqrt(2)'), 20)
```

The second argument of the vpa command substitutes the digits accuracy.

```
v1 =
0.5
v2 =
3.33e-4
v3 =
1.4142135623730950488
```

Accuracy cannot be read from individual numbers. (Accuracy is 3 here as well.)

3. Calculus

3.1. Differentiation & Integration

Differentiation

- **Symbolic expressions** or **functions** can be differentiated as follows:

```
syms A omega t x y
f = A*sin(omega*t);
g(x,y) = x^2 + x*y^2;
df_dt = diff(f,t)
ddf_dtdt = diff(f,t,2)
ddg_dxdy = diff(g,x,y)
```

```
df_dt =
A*omega*cos(omega*t)

ddf_dtdt =
-A*omega^2*sin(omega*t)

ddg_dxdy(x,y) =
2*y
```

The function or expression character is conserved.

Differentiation

- **Specify** the variable to be differentiated by! The diff(f) command would **otherwise** assume the **default variable**, given by symvar(f,1). Whatever it is...

Integration

- Similarly, you can perform **indefinite** and **definite integrations** on the functions f, g from above:

```
F = int(f, t)
Gdef = int(g, x, -3, 1)
```

```
F =
-(A*cos(omega*t))/omega

Gdef(y) =
28/3 - 4*y^2
```

The integration constant is missing from indefinite integration

Integration

- By default, the integration function also considers **special cases**:

```
int(x^t, x)
int(x^t, x, 'IgnoreSpecialCases', true)
```

A special case arises when

t = -1

```
ans=
piecewise([t == -1, log(x)], [t ~= -1, x^(t+1)/(t+1)])

ans =
x^(t+1)/(t+1)
```

3.2. Vector Analysis

Vector Analysis

There are various functions for **vector analysis**, including:

- gradient

```
syms x y z
m = x*y*z;
n = gradient(m, [x,y,z])
```

```
n =
y*z
x*z
x*y
```

Vector Analysis

- divergence

```
p = [x, 2*y^2, 3*z^3];
q = divergence(p, [x,y,z])
```

```
Q =
9*z^2 + 4*y + 1
```

- Other functions for vector analysis:
  - curl
  - hessian
  - jacobian
  - laplacian
  - potential
  - vectorPotential

3.3. Series & Limits

Series & Limits

Examples of operations on series include:

- **Symbolic summations:**

```
syms x k
s = symsum(x^k, k, 0, inf)
```

```
s =
piecewise([1 <= x, Inf], [abs(x) < 1, -1/(x-1)])
```

Remember that the geometric series is only finite for x<1

- **Other** functions for **series** and **limits:**
  - cumprod
  - cumsum
  - pade
  - rsums
  - symprod
  - limit

Taylor Series

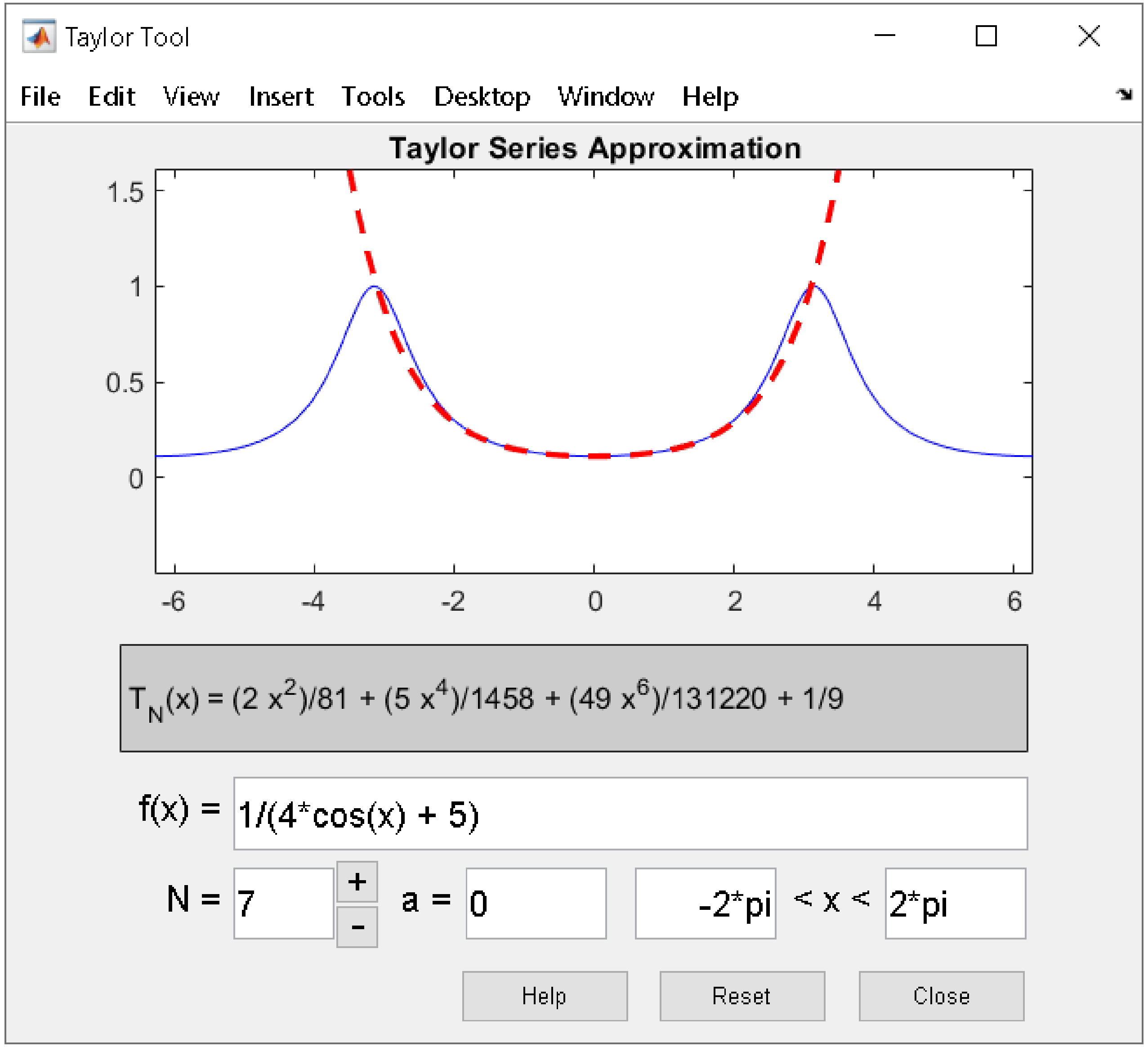
- **Taylor** series:

```
f = 1/(5 + 4*cos(x));
T = taylor(f, 'Order', 8)
```

```
T =
(49*x^6)/131220 + (5*x^4)/1458 + (2*x^2)/81 + 1/9
```

- **Taylor** series can also be **graphically** manipulated:

```
taylortool(f)
```



3.4. Transforms

Transforms

- The symbolic toolbox can perform the **transforms**:
  - Fourier fourier
  - Laplace laplace
  - Z-Transforms ztrans

```
syms x s
f = 1/sqrt(x);
Lf = laplace(f, x, s)
```

```
Lf =
pi^(1/2)/s^(1/2)
```

- … and their **inverse**, ifourier, laplace and iztrans.

```
iLf = ilaplace(Lf, s, x)
```

```
iLf =
1/x^(1/2)
```

4. Equation Solving

Symbolic vs Numeric Solvers

SYMBOLIC SOLVER	vs.	NUMERIC SOLVER
`solve`		`vpasolve`
Returns exact solutions, which can then be approximated using `vpa`		Returns approximate solutions, whose precision can be controlled using `digits`
Returns a general form of the solution, providing insight into the solution.		Returns all/the first numeric solution(s) of polynomial/nonpolynomial equations.
Search ranges can be specified using inequalities.		Search ranges and starting points can be specified.
Can return parameterized solutions		Does not return parameterized solutions.
Equations may comprise parameters and inequalities		Runs faster than the symbolic solver.

4.1. Algebraic Equations

Algebraic Equation Definition and Solving

- An **algebraic equation** can be **defined** and **solved** using the solve command:

```
syms a b c x
eqn = a*x^2 + b*x + c == 0;
sol_x = solve(eqn, x)
```

```
sol_x =
-(b + (b^2 - 4*a*c)^(1/2))/(2*a)
-(b - (b^2 - 4*a*c)^(1/2))/(2*a)
```

- With the **second argument** of the solve function, specify the **variable to solve for**! Otherwise, solve(eqn) would solve for the **default variable**, given by symvar(eqn,1).

```
sol_b = solve(eqn, b)
```

```
sol_b =
-(a*x^2 + c)/x
```

Algebraic Equation with Multiple Solutions

- The solve function does **not** return **all solutions** by default:

```
syms x
sol_x = solve(cos(x) == -sin(x), x)
```

```
sol_x =
-pi/4
```

Algebraic Equation with Multiple Solutions

- To return **all solutions** along with the **parameters** in the solution and the **conditions** on the solution, set the ReturnConditions option to true.

```
[sol_x, parameters, conditions] = solve(cos(x) == -sin(x), x, 'ReturnConditions', true)
```

```
sol_x =
pi*k - pi/4
parameters =
k
conditions =
in(k, 'integer')
```

Algebraic Equation with Multiple Solutions

- Moreover, it is possible to use the **parameters** and **conditions** to find solutions under **additional conditions**. For instance, to find values of x ∈ $(0,2\pi)$:

```
assume(conditions)
sol_k = solve(0 < sol_x, sol_x < 2*pi, parameters)
```

It is possible to hand over multiple equations and even multiple variables!

```
sol_k =
1
2
```

- To find values of x corresponding to these values of k, use subs to **substitute** for k in sol_x:

```
x_values = subs(sol_x, sol_k)
```

```
x_values =
(3*pi)/4
(7*pi)/4
```

System of Algebraic Equations

- When solving for **multiple variables** at a time, solve returns a **structure of solutions**. For example, for the system:

$$u^2 - v^2 = a^2$$
$$u + v = 1$$
$$a^2 - 2a = 3$$

```
syms u v a
S = solve(u^2 - v^2 == a^2, u + v == 1, a^2 - 2*a == 3, [a, u, v])
```

```
S =

    a: [2x1 sym]
    u: [2x1 sym]
    v: [2x1 sym]
```

Specifying the variables is optional when the number of equations equals the number of variables.

System of Algebraic Equations

```
S =
        a: [2x1 sym]
        u: [2x1 sym]
        v: [2x1 sym]
```

- The system of equations has **2 solutions**. The second solution can be **visualized** as follows.

```
s2 = [S.a(2), S.u(2), S.v(2)]
```

```
s2 =
[3, 5, -4]
```

System of Linear Equations

- A systems of **linear algebraic equations** can also be solved using **matrix division.**

```
syms u v x y
equations = [x + 2*y == u, 4*x + 5*y == v];
[A,b] = equationsToMatrix(equations, x, y)
S = A\b
```

Remember: this expression solves $A \cdot S = b$ for S.

```
A =
[1, 2]
[4, 5]
b =
 u
 v
S =
 (2*v)/3 - (5*u)/3
      (4*u)/3 - v/3
```

Numerical Solving of Algebraic Equations

- Equations can be solved **numerically** using vpasolve. This is especially useful to find **polynomial roots**. For example, to solve $X^2 - 2 = 0$:

```
syms f(x)
f(x) = x^2 - 2;
sol_x = vpasolve(f)
```

> vpasolve(f) is equivalent to vpasolve(f == 0)

```
sol_x =
-1.4142135623730950488016887242097
 1.4142135623730950488016887242097
```

- Again, digits can be used to **adjust** the **desired precision**.

Numerical Solving: Cases of Multiple Solutions

- **Non-polynomial equations** can be solved **numerically** as well. Consider the function $f(x) = e^x + e^{-x} - 10$, with a minimum on zero and **two** (symmetrical) **roots**.

```
syms f(x)
f(x) = exp(x) + exp(-x) - 10;
sol_x = vpasolve(f)
```

```
sol_x =
2.2924316695611776878007873113480
```

- Note that only **one root** has been found! To find the **second root**, provide an **initial guess**:

```
sol_x = vpasolve(f, -2)
```

```
sol_x =
-2.2924316695611776878007873113480
```

Numerical Solving: Cases of Multiple Solutions

- The initial guess can also be a range:

```
sol_x = vpasolve(f, [-3, 3])
```

```
sol_x =
-2.2924316695611776878007873113480
```

We still get the negative solution.

- It is also possible to make a **random initial guess**. We will randomly get either solution.

```
sol_x = vpasolve(f,'Random',true)
```

```
sol_x =
-2.2924316695611776878007873113480
```

- To avoid this, **combine** the **range** with a **random initial guess**.

```
vpasolve(f,[-3 3],'Random',true)
```

4.2. Ordinary Differential Equations (ODEs)

ODEs

- ODEs can be **solved** using dsolve. For example,

$$\frac{dx(t)}{dt} = -5 \cdot x(t)$$
$$\Rightarrow x(t) = C \cdot e^{-5t}$$

```
syms x(t)
x(t) = dsolve(diff(x,t) == -5*x)
```

```
x(t) =
C5*exp(-5*t)
```

ODEs with Initial Condition

```
syms x(t)
x(t) = dsolve(diff(x,t) == -5*x)
```

```
x(t) =
C5*exp(-5*t)
```

- Note that dsolve works with **integration constants.** They can be directly eliminated by specifying **initial conditions**:

```
syms x(t)
x(t) = dsolve(diff(x,t) == -5*x, x(0) == -3)
```

```
x(t) =
-3*exp(-5*t)
```

Higher order ODEs

- Higher order ODEs require **multiple initial conditions**. To be able to specify these, create **additional symbolic functions**, like Du in this case.

```
syms u(b)
Du   = diff(u,b);
u(b) = dsolve(diff(u, b, b) == cos(2*b) - u, u(0) == 1, Du(0) == 0)
```

```
u(b) =
1 - (8*sin(x/2)^4)/3
```

- Nonlinear ODEs can have **multiple solutions**, even if initial conditions are specified.

```
syms x(t)
x(t) = dsolve((diff(x,t) + x)^2 == 1, x(0) == 0)
```

```
x(t) =
exp(-t) - 1
1 - exp(-t)
```

Systems of Ordinary Differential Equations

- **Systems of differential equations** can be solved much like systems of algebraic equations.

```
syms f(t) g(t)
S = dsolve(diff(f,t) == 3*f + 4*g, diff(g,t) == -4*f + 3*g)
```

```
S =

    g: [1x1 sym]
    f: [1x1 sym]
```

- It is also possible to **retrieve** f and g directly:

```
[f(t), g(t)] = dsolve(diff(f,t) == 3*f + 4*g, diff(g,t) == -4*f + 3*g)
```

```
f(t) =
C2*cos(4*t)*exp(3*t) + C1*sin(4*t)*exp(3*t)
g(t) =
C1*cos(4*t)*exp(3*t) - C2*sin(4*t)*exp(3*t)
```

Systems of Ordinary Differential Equations

- Systems of differential equations can also be solved in **matrix form**:

$$\dot{x} = Ax + B$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ t \end{bmatrix}$$

```
syms x1(t) x2(t)
A = [1 2; -1 1];
B = [1; t];
x = [x1; x2];
[x1, x2] = dsolve(diff(x) == A*x + B)
```

Systems of Ordinary Differential Equations

```
syms x1(t) x2(t)
A = [1 2; -1 1];
B = [1; t];
x = [x1; x2];
[x1, x2] = dsolve(diff(x) == A*x + B)
```

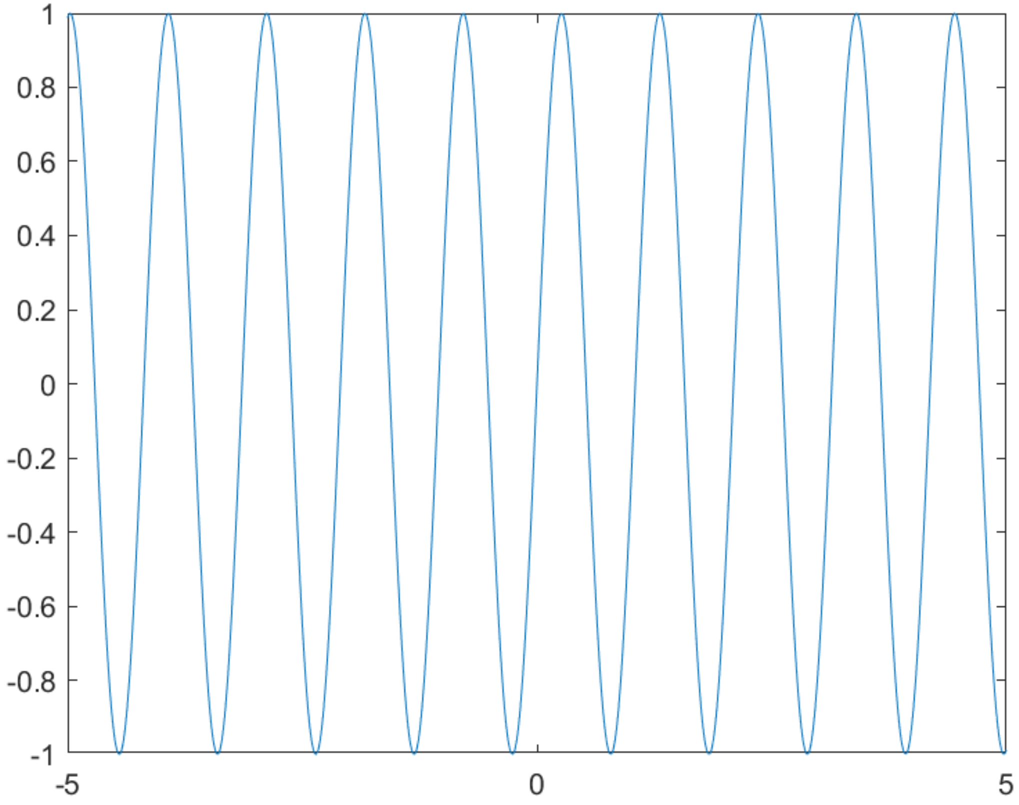
```
x1 =

2^(1/2)*exp(t)*cos(2^(1/2)*t)*(C3 + (exp(-t)*(4*sin(2^(1/2)*t) + 2^(1/2)*cos(2^(1/2)*t)
+ 6*t*sin(2^(1/2)*t) + 6*2^(1/2)*t*cos(2^(1/2)*t)))/18) +
2^(1/2)*exp(t)*sin(2^(1/2)*t)*(C2 - (exp(-t)*(4*cos(2^(1/2)*t) - 2^(1/2)*sin(2^(1/2)*t)
+ 6*t*cos(2^(1/2)*t) - 6*2^(1/2)*t*sin(2^(1/2)*t)))/18)


x2 =

exp(t)*cos(2^(1/2)*t)*(C2 - (exp(-t)*(4*cos(2^(1/2)*t) - 2^(1/2)*sin(2^(1/2)*t) +
6*t*cos(2^(1/2)*t) - 6*2^(1/2)*t*sin(2^(1/2)*t)))/18) - exp(t)*sin(2^(1/2)*t)*(C3 +
(exp(-t)*(4*sin(2^(1/2)*t) + 2^(1/2)*cos(2^(1/2)*t) + 6*t*sin(2^(1/2)*t) +
6*2^(1/2)*t*cos(2^(1/2)*t)))/18)
```

5. Symbolic Plotting Functions

Symbolic Plotting Functions

- Apart from the standard MATLAB plotting functions, there are also **symbolic plotting functions**. Here are very simple example (corresponding figure on the bottom left):

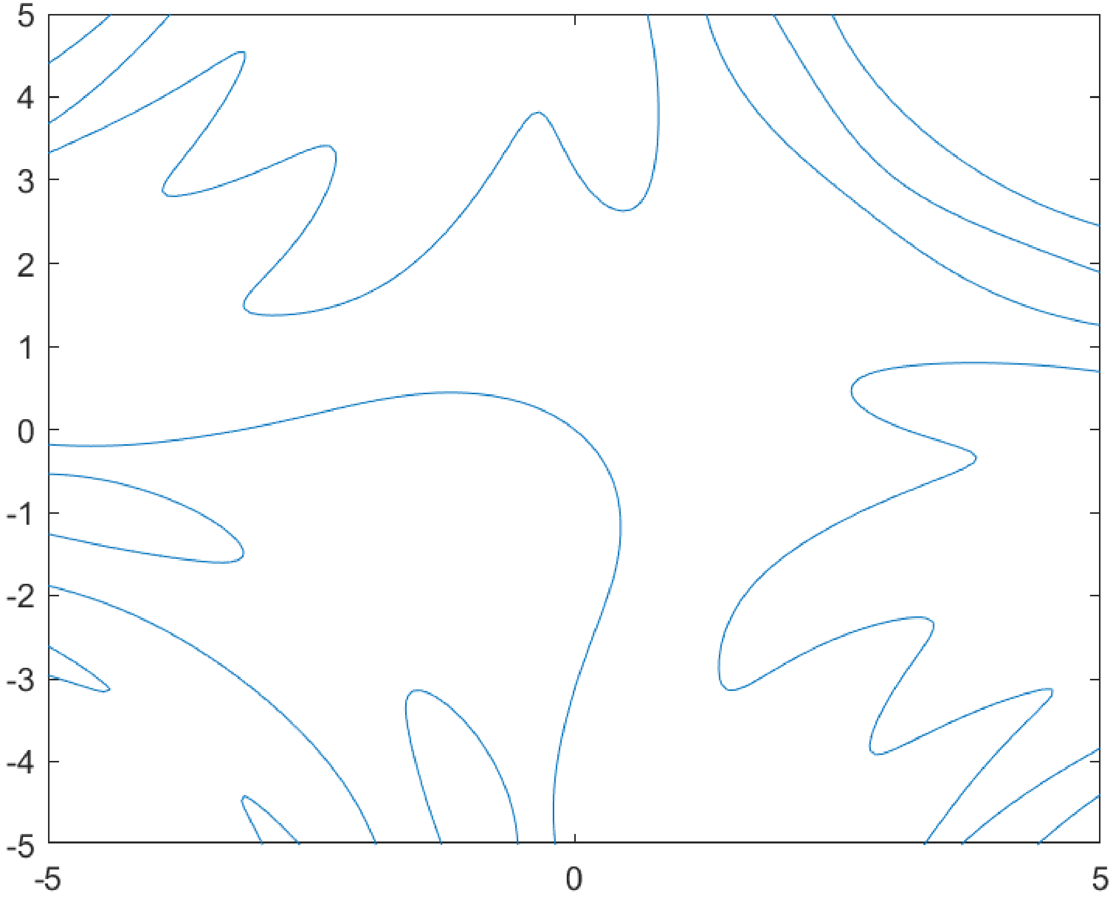
```
syms x;
fplot(sin(6*x));
```



Symbolic Plotting Functions

- With fimplicit, it is possible to **combine two variables** and plot an implicit equation:

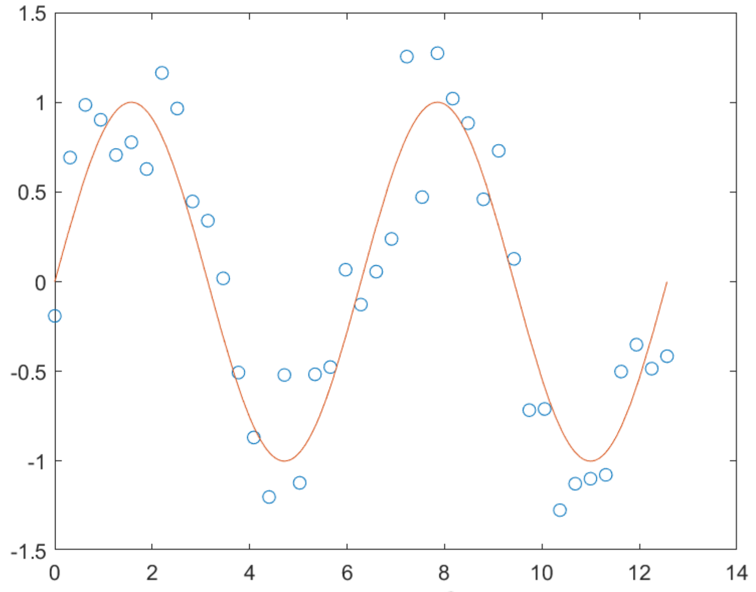
```
syms x y;
fimplicit(sin(x) + sin(y) == sin(x*y));
```



Symbolic Plotting Functions

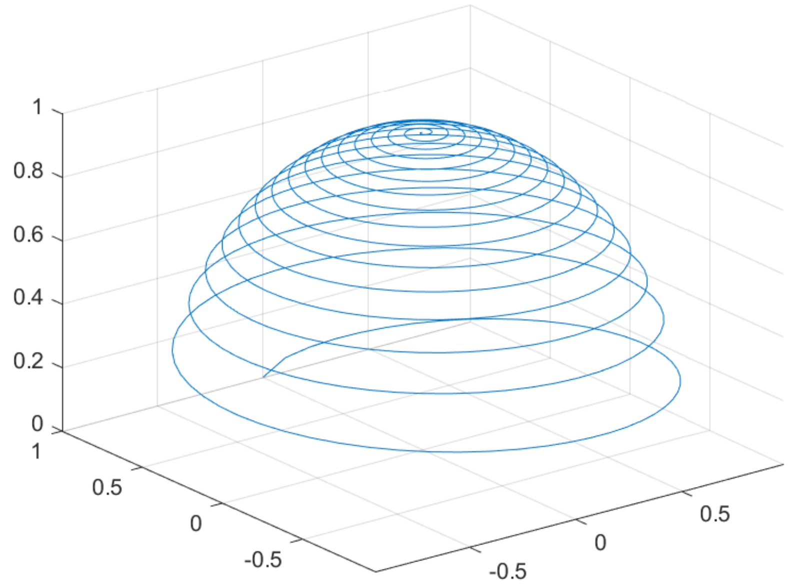
- Numeric and symbolic plots can be **combined**:

```
x = 0:pi/10:4*pi;
y = sin(x) + (-1).^randi(10, 1, 41).*rand(1, 41)./2;
syms t;
figure; plot(x,y,'o'); hold on;
fplot(sin(t),[0, 4*pi]);
```



- **3-D symbolic plots** are possible as well:

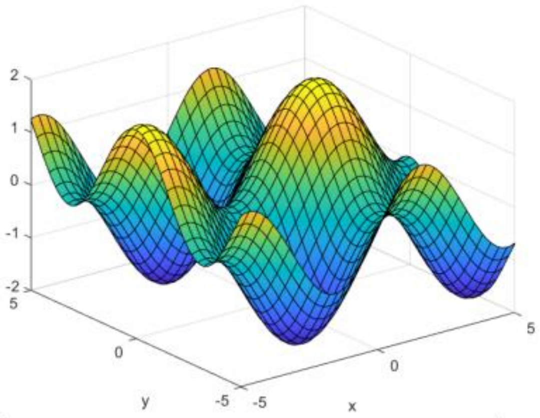
```
syms t;
x = (1-t)*sin(100*t);
y = (1-t)*cos(100*t);
z = sqrt(1 – x^2 – y^2);
fplot3(x, y, z, [0 1]);
```



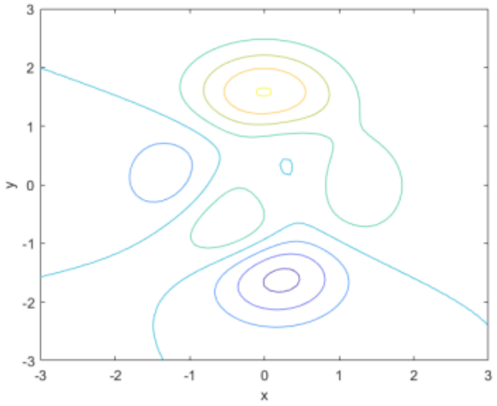
Other Symbolic Plotting Functions

- Examples for **other** symbolic plots:

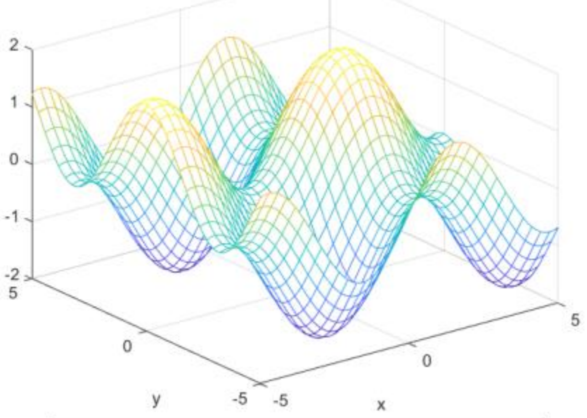
`fsurf`



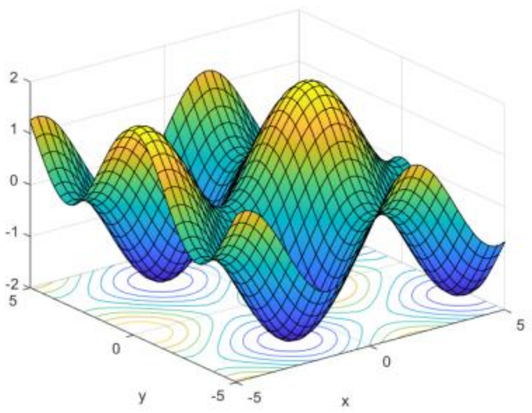
`fcontour`



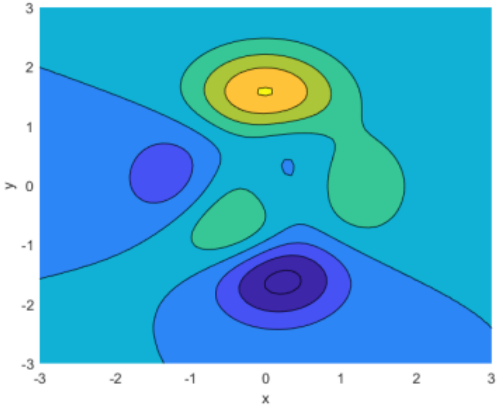
`fmesh`



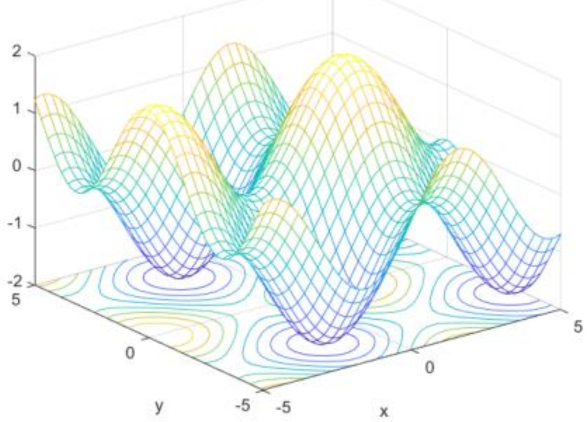
`fsurf` with `'ShowContours', 'on'`



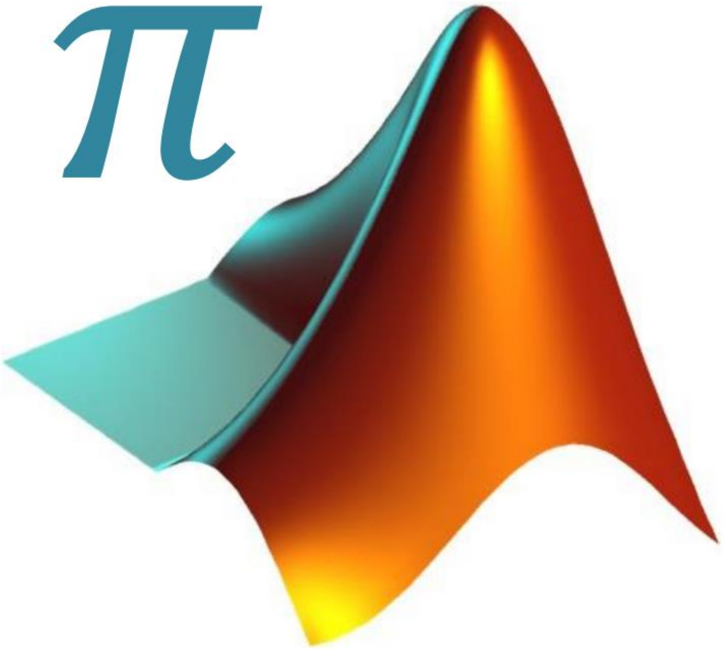
`fcontour` with `'Fill', 'on','LineColor','k'`



`fmesh` with `'ShowContours', 'on'`



6. Code Generation



Code Generation: MATLAB Functions

- **Symbolic expressions and functions** can be **converted** into code (MATLAB, Simulink, Simscape, C, Fortran, MathML, and TeX):

```
syms x y;
z = 30*x^4/(x*y^2 + 10) - x^3*(y^2 + 1)^2;
matlabFunction(z,'file','MExample.m');
```

- The **file** created has the following content:

```
function z = MExample(x,y)
%MEXAMPLE
%    Z = MEXAMPLE(X,Y)

%    This function was generated by the Symbolic Math Toolbox version 8.3.
%    01-Oct-2019 15:29:18

t2 = y.^2;
z = (x.^4.*3.0e+1)./(t2.*x+1.0e+1)-x.^3.*(t2+1.0).^2;
```

By default, **optimized** code is generated. (This intermediate variable makes the code more efficient.)

Code Generation: Simulink Blocks

- Similarly, blocks can be created in a Simulink model.
    - First, the **model** needs to be **opened** (here simulink_system).
    - Then, a **block** (type: MATLAB function) can be **created** (here named pythagoras).

```
syms a b;
c = sqrt(a^2 + b^2);
matlabFunctionBlock('simulink_system/pythagoras',c);
```

Code Generation: C-Code

- It is also possible to generate **optimized** C-Code:

```
syms x y;
z = 30*x^4/(x*y^2 + 10) - x^3*(y^2 + 1)^2;
ccode(z,'file','CExample');
```

- It is also possible to generate **optimized** C-Code:

```
t2 = y*y;
t0 = ((x*x*x*x)*3.0E+1)/(t2*x+1.0E+1)-(x*x*x)*pow(t2+1.0,2.0);
```

7. List of Useful Commands

Code Generation: C-Code

Command	Explanation	Slide #	Command	Explanation	Slide #
`sym`	Create symbolic object	9	`hessian`	Hessian matrix	-
`syms`	Create symbolic object	9	`jacobian`	Jacobian matrix	-
`assume`	Make/clear an assumption	11	`laplacian`	Laplacian of scalar function	-
`assumeAlso`	Add assumptions	11	`potential`	Potential of vector field	-
`assumptions`	Show assumptions	11	`vectorPotential`	Vector potential of vec. field	-
`subs`	Substitute	18	`symsum`	Sum of series	26
`simplify`	Simplify expression/function	19	`symprod`	Product of series	-
`symvar`	Show symbolic variables	20	`cumsum`	Cumulative sum	-
`digits`	Show/set signif. decim. digits	16, 21	`cumprod`	Cumulative product	-
`vpa`	Variable Precision Arithmetic	21	`taylor`	Taylor series expansion	26
`diff`	Differentiate	23	`taylortool`	Taylor series calculator	26
`int`	Integrate	24	`pade`	Padé approximant	-
`gradient`	Compute gradient	25	`rsums`	Riemann sums (interactive)	-
`divergence`	Compute divergence	25	`limit`	Compute limit	-
`curl`	Curl of vector field	-	`sympref`	Symbolic preferences	-

Code Generation: C-Code

Command	Explanation	Slide #
`laplace`	Laplace transform	27
`ilaplace`	Inverse Laplace transform	27
`fourier`	Fourier transform	-
`ifourier`	Inverse Fourier transform	-
`ztrans`	Z-transform	-
`iztrans`	Inverse Z-transform	-
`solve`	Symbolic solver	31
`vpasolve`	Numeric solver	36
`equationsToMatrix`	Convert set of linear equations to matrix form	35
`dsolve`	ODE solver	39

Command	Explanation	Slide #
`fplot`	Symbolic 2D plot	45
`fplot3`	Symbolic 3D plot	46
`ezpolar`	Symbolic polar plot	-
`fsurf`	Symbolic surface plot	47
`fmesh`	Symbolic mesh plot	47
`fcontour`	Symbolic contour plot	47
`fimplicit`	Implicit function 2D plot	45
`fimplicit3`	Implicit function 3D plot	-
`matlabFunction`	Generate MATLAB function	49
`matlabFunctionBlock`	Generate MATLAB function block for Simulink	50
`ccode`	Generate C-Code	50

8. Self-assessment

Self-assessment

- What are the differences between numeric and symbolic computation?
- How can you tell, simply looking at the Command Window, that an output is symbolic?
- What two commands allows you to create symbolic variables? Symbolic matrices?
- What command do you use to convert an expression (string) into a symbol?
- Write down the lines of code to:
  - Define a symbolic variable r
  - Assume it to be real
  - Then assume it to be positive
  - Finally, clear the assumptions on r
- Do you need the subs command to evaluate a symbolic function?
- How does the digits command affect the output of the vpa command?

Self-assessment

- What famous number is given by the following series?

$$\sqrt{12}\sum_{k=0}^{\infty}\frac{(-3)^{-k}}{2k+1}$$

You should find it to be equal to:

$$\int_{-\infty}^{+\infty}\frac{dx}{1+x^2}$$

- What additional symbolic function can it be useful to define when solving higher order differential equations?
- What function do you use to plot implicit functions? Plot the equation of a hyperbola:

$$x^2-y^2=1$$