Effect of Drag Coefficient on Projectile Motion: A Numerical Simulation Using the Runge-Kutta Method

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Abstract

In many introductory models, projectile motion is examined by neglecting air resistance. This approximation might be theoretically useful; however, it does not reflect real-world projectile motion.

In this study, air resistance is included in the model and the motion of the object is examined by numerical simulation.

Motion equations are solved by using the fourth-order Runge-Kutta method.

Different objects are modelled by different drag coefficients and all of the simulations are executed under the same initial conditions.

The results of the simulations show a significant effect of air resistance on flight time, maximum height, horizontal range.

Introduction

Projectile motion is one of the main topics of classical mechanics and is generally taught by neglecting air resistance. This assumption makes obtaining analytical solutions easier.

However, the motions observed in the real world show deviations from this ideal model. Interaction with air changes the objects' speed and trajectory in a way that cannot be ignored.

In situations where air resistance is proportional to the square of velocity, motion equations are not linear and hard to solve analytically.
Therefore, numerical methods offer an effective tool for investigating these types of problems.
In this report, the effect of air resistance on projectile motion is investigated with the help of a numerical simulation.

Physical Model

The motion of the object is modelled by using Newton's second law.
There are two main forces acting on the object: gravitational force and drag force.

Gravitational force is downward and of constant magnitude.
Drag force depends on the velocity of the object and acts in the opposite direction of the motion.

In this study, drag force is modelled as being proportional to the square of the object's velocity.
The motion equations obtained under these forces consist of nonlinear differential equations that describe the time-varying velocity and position.

Numerical Method

Because of the air resistance force's nonlinearity, the motion equations cannot be solved analytically.
As a result of this problem, the equations are solved with the help of a numerical integration method.

In this study, fourth-order Runge-Kutta method is used.
This method provides a higher accuracy than simple first-order methods.

With the help of Runge-Kutta method, the objects' velocity and position are updated by small time intervals.
As a result, the evolution of the motion over time has been calculated with stability and precision.

Simulation Setup

All of the simulations were executed by using a fixed time step.
Gravitational acceleration and air density were assumed to be constant.

To isolate the effect of air resistance, all of the simulations were executed with the same initial velocity, launch angle and mass.

In the simulations, the mass of the object was taken as 0.43 kg, the initial velocity as 35 m/s, the angle of launch as 20 degrees, the area as 0.038 m^2, the air density as 1.225 kg/m^3, the gravitational acceleration as 9.8 m/s^2 and the dt (small time interval) as 0.0001 s.

These parameters were kept the same for all object types.
Only drag coefficient was changed to reflect different shapes.

This approximation allows a fair comparison for different shapes.

Results

At the end of the simulations, quantities like flight time, maximum height, horizontal range and impact speed are obtained.
A significant difference is observed between the objects with different drag coefficients.
The numerical values obtained are summarized in Table 1.

Table 1

Type of Object	**Drag Coefficient (Cd)**	**Flight Time (s)**	**Maximum Height (m)**	**Horizontal Range (m)**	**Time Up (s)**	**Time Down (s)**	**Impact Speed (m/s)**
Sphere	0.47	1.925 s	4.643 m	37.312 m	0.883 s	1.042 s	14.839 m/s
Flat Plate	1.28	1.560 s	3.138 m	21.494 m	0.678 s	0.882 s	9.427 m/s
Cube	1.05	1.636 s	3.431 m	24.247 m	0.719 s	0.917 s	10.332 m/s
Cylinder	0.82	1.730 s	3.805 m	27.976 m	0.771 s	0.959 s	11.575 m/s
Cone	0.50	1.905 s	4.552 m	36.226 m	0.871 s	1.033 s	14.445 m/s

Objects with higher drag coefficients reach lower maximum heights and shorter horizontal ranges.
Flight time and impact speed also vary depending on the drag coefficient.

Discussion

The presence of air resistance creates an asymmetry between the object's upward and downward motion.
During the upward motion, gravity and air resistance act on the object together and decrease the velocity significantly.

During the downward motion, as the velocity of the object approaches a certain value, air resistance increases and is balanced by gravity.
This situation causes the object to approach terminal velocity and extends the descent time.

Moreover, motion in horizontal is also affected by air resistance.
As a result, when air resistance is neglected, the horizontal range is estimated more than what it is.

Conclusion

In this study, projectile motion under air resistance is simulated with the help of a numerical method.
Obtained results show that air resistance has a significant effect on the motion of the object.

Numerical simulation method offers an effective tool in situations where analytical solutions are not possible. This approach can be extended to investigate more complex physical effects.