

Lab 3: Inverse Dynamics Joint Control

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1 Introduction

The focus of this lab 3 is to implement friction compensation and inverse dynamics control laws. The friction compensation acted to reduce mechanical friction effects on all three joints. The inverse dynamics control served to look at the robustness of control methods for systems with varying payloads, which we tested by the addition of a heavy mass to the end-effector of our robot.

2 Friction Compensation

Newton's first Law of Motion states that any object in motion will stay in motion unless an external force acts upon it. One of the biggest forces that the CRS robot faces (other than gravity) is that of friction from the internal gears, chains, and motors. Although our previous control systems usually compensated somewhat for friction, it was never perfect. In all cases, friction caused more control effort to be extended. In order to allow our control systems to deal primarily with disturbances rather than friction forces, we implemented a friction compensation system. Ideally, this friction compensation would allow one to set a joint in motion and have the joint continue rotating forever (ignoring mechanical limits, gravity, and other external forces of course).

The mechanical friction present in each joint was accounted for by determining both the Coulomb (static) and viscous (dynamic) friction for each joint. In a further attempt to accurately characterize this, we measured these friction coefficients for joints moving in the positive direction and the negative direction. These coefficients are present in Equations 2.1 and 2.2.

The friction compensation was added independently for each joint and acted linearly with respect to joint velocity. Once a minimum velocity (0.05 m/s) was met. A slope of 3.6 was

implemented between minimum velocities, as given in the lab manual. The implementation we had is present in Listing 1. This calculated compensation was then added to the τ control effort sent to each motor.

```

1 void friction_compensation() {
2     int i = 0;
3     for(i = 0; i < 3; i++) {
4         if (Omega[i] > minimum_velocity[i]) {
5             u_fric[i] = Viscous_positive[i]*Omega[i] + Coulomb_positive[i];
6         } else if (Omega[i] < -minimum_velocity[i]) {
7             u_fric[i] = Viscous_negative[i]*Omega[i] + Coulomb_negative[i];
8         } else {
9             u_fric[i] = slope_between_minimums[i]*Omega[i];
10        }
11    }
12 }

```

Listing 1: Friction Compensation Method

$$\begin{aligned}
 Viscous_1^+ &= 0.130 & Viscous_1^- &= 0.074 \\
 Viscous_2^+ &= 0.250 & Viscous_2^- &= 0.210 \\
 Viscous_3^+ &= 0.210 & Viscous_3^- &= 0.330
 \end{aligned} \tag{2.1}$$

$$\begin{aligned}
 Coulomb_1^+ &= 0.300 & Coulomb_1^- &= -0.390 \\
 Coulomb_2^+ &= 0.250 & Coulomb_2^- &= -0.600 \\
 Coulomb_3^+ &= 0.400 & Coulomb_3^- &= -0.600
 \end{aligned} \tag{2.2}$$

3 PD Plus Feedforward Control Law

3.1 Trajectory Tracking without Weight

Figure 1 shows the trajectory tracking response of the PD plus feedforward controller. The scale of the trajectory makes the error of the response difficult to see, with rise time at 330 milliseconds and the full trajectory at 4 seconds long.

Figure 2 shows a zoomed in picture of the response, at the peak, which gives a clearer idea of the errors.

Figure 3 shows the error over time of the feedforward trajectory tracking without weight. The response shows that the error tracking is different at the top and bottom of the trajectory response. While Joint 1 has no steady state error, the response takes a different

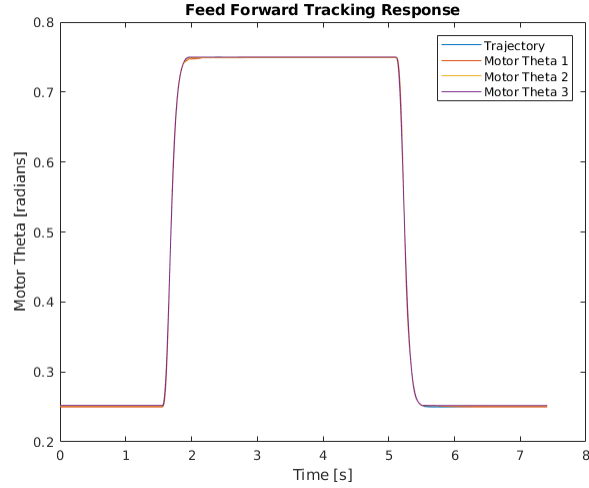


Figure 1: Feed Forward Trajectory Tracking without Weight

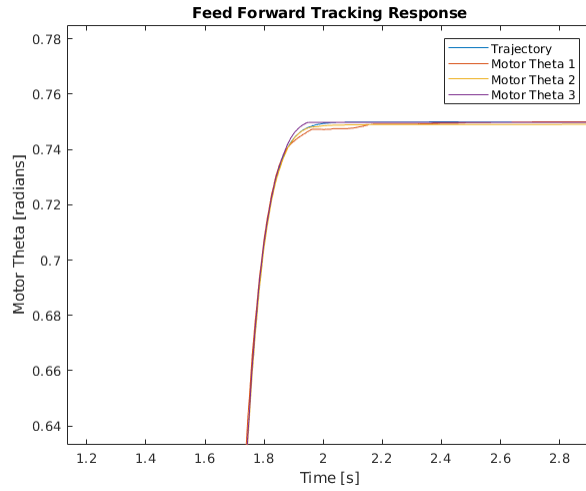


Figure 2: Close Up Feed Forward Trajectory Tracking without Weight

amount of time to settle to that state. The step response also shows a difference between the two movements, with the downwards movement from 0.75 radians to 0.25 radians having a greater error for Joint 1 than the upwards movement. This is not a gravitational effect, since the link actuated by Joint 1 is not acted upon by gravity in the axis of actuation. Joints 2 and 3 have steady state error on both sides of the trajectory, not reaching the desired point on either side. Without integral control, the control signal doesn't accumulate. The magnitude of the error is greater at the bottom of the response for both Joints 2 and 3.

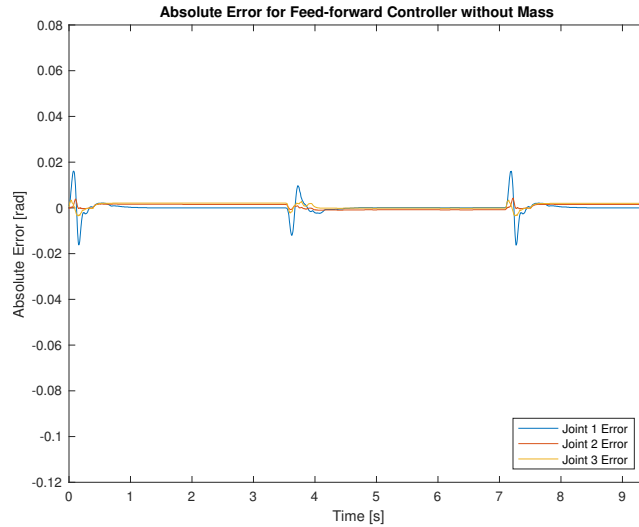


Figure 3: Error for Feed-Forward Controller without Mass Added

3.2 Trajectory Tracking with Weight

When the mass is added to the robot the performance of the controller response declines, although the steady state error decreases for all three joints. The full response for all three joints is given by Figure 4. For Joints 2 and 3, the error goes to zero at both sides of the response, while for Joint 1 it only goes to zero at the top of the response.

The response is still hard to see with the full step up and down, so a zoomed in view of the peak of the upwards movement is provided by 5

The error signal for all three joints is provided by Figure 6. Similar to the response without the added weight, the error for the trajectory tracking on the upwards movement is less for all three joints. On the upwards motion, the error response for all joints is approximately the same value as the error for Joint 1 without the mass, about 0.02 radians. On the downwards movement, the error is about five times larger, reaching a magnitude of 0.1 radians. This difference in response for Joints 2 and 3 can be explained by the force of gravity working on the mass. Gravity is against the motion of mass on the upwards motion and with the mass on the downwards motion. Joint 1 error can be partially explained by the inertia of the mass. The mass is moving faster on the downswing which causes the joint to move farther in that direction.

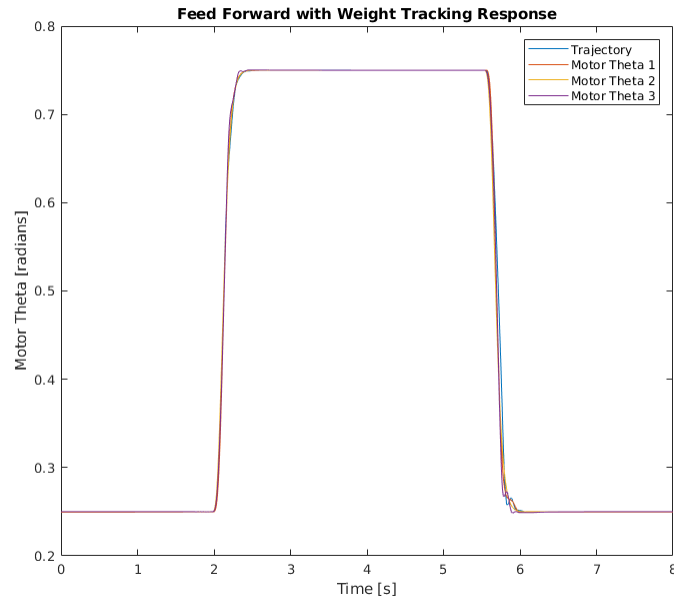


Figure 4: Feed Forward Trajectory Tracking with Weight

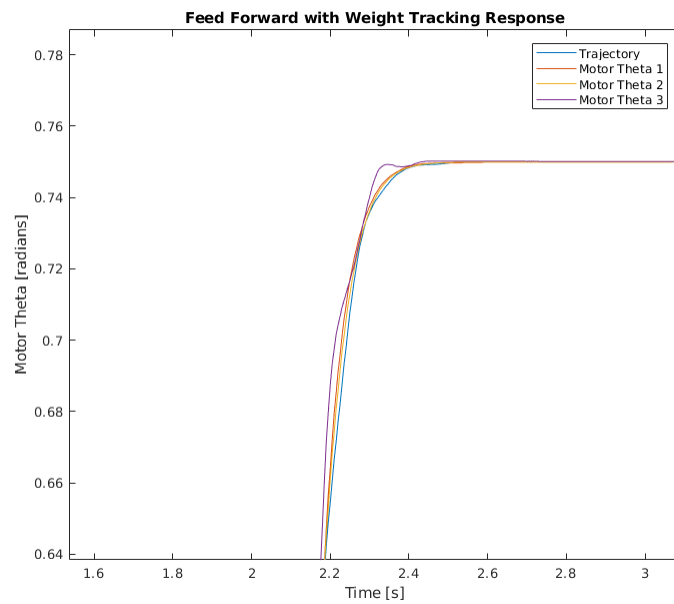


Figure 5: Close Up Feed Forward Trajectory Tracking with Weight

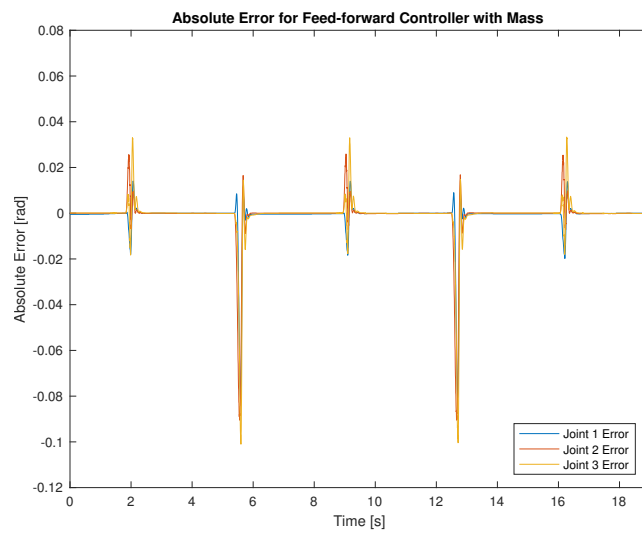


Figure 6: Error for Feed-Forward Controller with Mass Added

4 Inverse Dynamics Control Law

Implementing a control law based on inverse dynamics can be thought of as a cascaded, two-loop control system. The "inner loop" inverts the dynamics of the robot to linearize the system, while the "outer loop" implements a PD controller. To expand on this topic, outer loop acts as a PD controller on the acceleration $\ddot{\theta}$. The inner loop is based on the standard $\tau = D(\theta)\ddot{\theta} + C(\theta, \dot{\theta}) + g(\theta)$ dynamics equation, with the inertia matrix, Coriolis matrix, and gravity matrix respectively. However, inverting these terms to isolate $\ddot{\theta}$ means that a direct relationship between α_θ and $\ddot{\theta}$ can be found, as shown in Equation 4.3, with all other dynamics cancelled out. This α_θ (the control effort from the outer loop) is then able to be used in the dynamics Equation 4.2 in place of $\ddot{\theta}$ with respect to the inertia matrix $D(\theta)$. This allows us to utilize PD control while still taking into account the non-linearity of the actual robot's system response. As such, it leads to a more robust controller, as the dynamics that are inverted in the inner loop can be easily modified based on mass added to the system, while not requiring the PD control gains to be modified.

$$\begin{aligned}
 p_1 &= m_2 l_{c2}^2 + m_2 l_2^2 + I_2 &= 0.0300 \\
 p_2 &= m_3 l_{c3}^2 + I_3 &= 0.0128 \\
 p_3 &= m_3 l_2 l_{c3} &= 0.0076 \\
 p_4 &= m_2 l_{c2} + m_3 l_2 &= 0.0753 \\
 p_5 &= m_3 l_{c3} &= 0.0298
 \end{aligned} \tag{4.1}$$

The parameters given in Equation 4.1 defined the inverse dynamics of the robot without any mass added to the end-effector. These parameters are utilized in the matrices shown as part of the inner loop control in Equation 4.2. The outer loop PD control is dictated by the α values calculated from Equation 4.4 and 4.5. The PD gains used in this calculation are shown in Equation 4.6. We implemented the inverse dynamics control on Joints 2 and 3, however kept feed-forward control on Joint 1 (as the z-axis rotation was reasonably linear). Figure 7 shows the program flow.

$$\tau = D(\theta)\alpha_\theta + C(\theta, \dot{\theta})\dot{\theta} + g(\theta) \tag{4.2}$$

$$\alpha_\theta = \ddot{\theta} \tag{4.3}$$

$$\alpha_{\theta_2} = \ddot{\theta}_2^d + K_{P2} \times (\theta_2^d - \theta_2) + K_{D2} \times (\dot{\theta}_2^d - \dot{\theta}_2) \tag{4.4}$$

$$\alpha_{\theta_3} = \ddot{\theta}_3^d + K_{P3} \times (\theta_3^d - \theta_3) + K_{D3} \times (\dot{\theta}_3^d - \dot{\theta}_3) \tag{4.5}$$

$$\begin{array}{ll}
 k_{p,1} = 0 & k_{d,1} = 0 \\
 k_{p,2} = 10000 & k_{d,2} = 200 \\
 k_{p,3} = 10000 & k_{d,3} = 200
 \end{array} \tag{4.6}$$

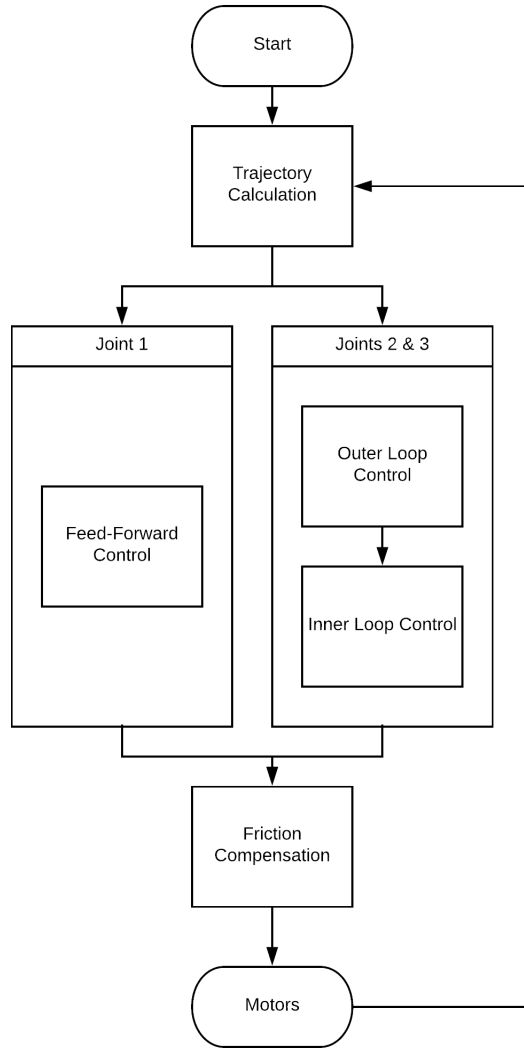


Figure 7: Program Execution Flowchart

4.1 Smooth Trajectory

Instead of utilizing the same cubic trajectory as in Lab 2, we utilized a discrete approximation of the transfer function shown in Equation 4.7 given by a Matlab function that was provided by the lab instructors. This trajectory was much smoother than the cubic we previously used. The position trajectory is shown in Figure 8, the velocity in Figure 9, and the acceleration in Figure 10.

$$\text{Transfer Function} = \left(\frac{30}{s + 30}\right)^6 \quad (4.7)$$

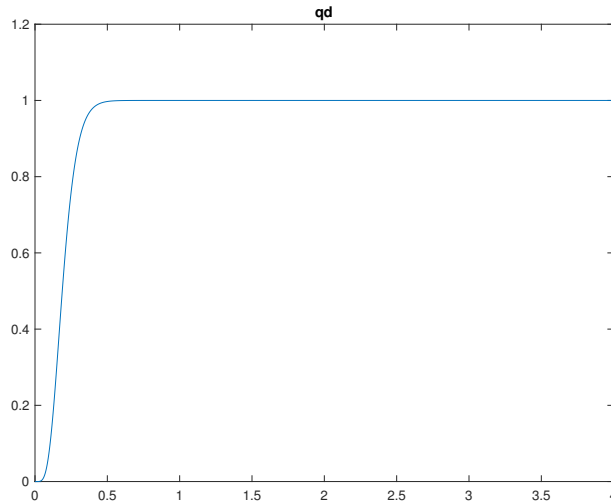


Figure 8: Desired Trajectory Position

4.2 Trajectory Tracking without Weight

The inverse dynamics step response with the smoothed trajectory is shown in Figure 11.

Since the motor response tracks very closely to the desired trajectory, it is helpful to see the zoomed in version of the response, at the peak of the response. Figure 12 gives this perspective with the scale given by the axes.

The inverse dynamics controller performs nearly identically to the PD plus feedforward controller without the mass attached. The magnitude of the maximum error response for both motions for Joints 2 and 3 were approximately the same for each controller. The values

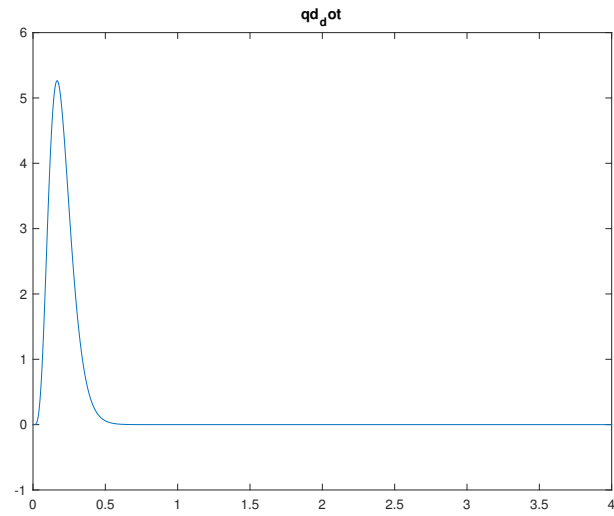


Figure 9: Desired Trajectory Velocity

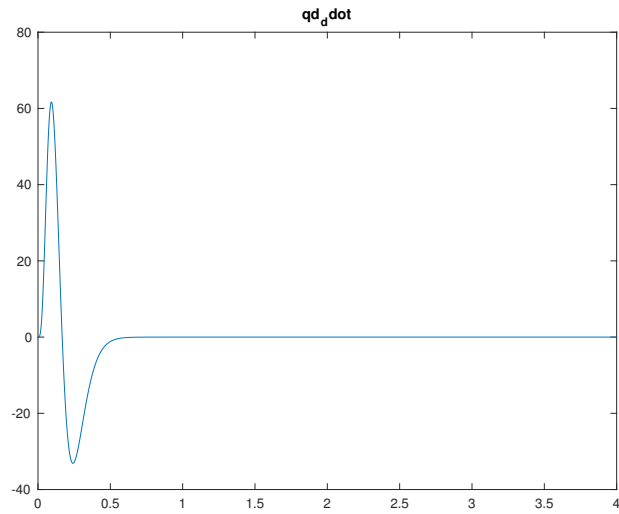


Figure 10: Desired Trajectory Acceleration

were all under 0.005 radians. The steady state error for both sides of the response were about the same as well, less than or equal to 0.002 radians.

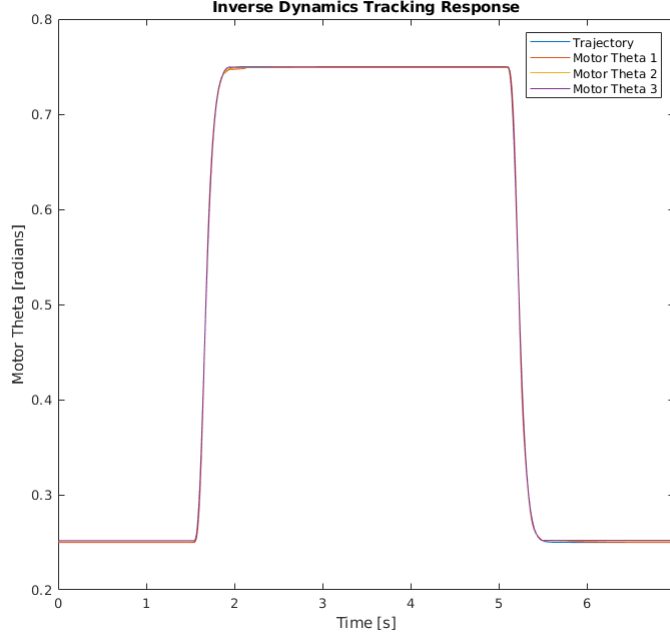


Figure 11: Inverse Dynamics Trajectory Tracking without Weight

4.3 Trajectory Tracking with Weight

$$\begin{aligned}
 p_1 &= 0.0466 \\
 p_2 &= 0.0388 \\
 p_3 &= 0.0284 \\
 p_4 &= 0.1405 \\
 p_5 &= 0.1298
 \end{aligned}
 \tag{4.8}$$

The inverse dynamics step response with the weight added to the end effector is shown by Figure 14. When the weight is added to the end effector, and the dynamical parameters are changed to account for the new equations of motion, the response is managed much better than the PD plus feedforward controller on the upwards motion. There is still significant error on the downwards motion for Joint 2. We tuned the controller response exclusively for the upwards motion response, so we did not account for this error. Joint 3 was tuned to the same parameters, but did not exhibit this behavior.

Since the motor response tracks very closely to the desired trajectory as compared to other controllers, it is helpful to see the Figure 15.

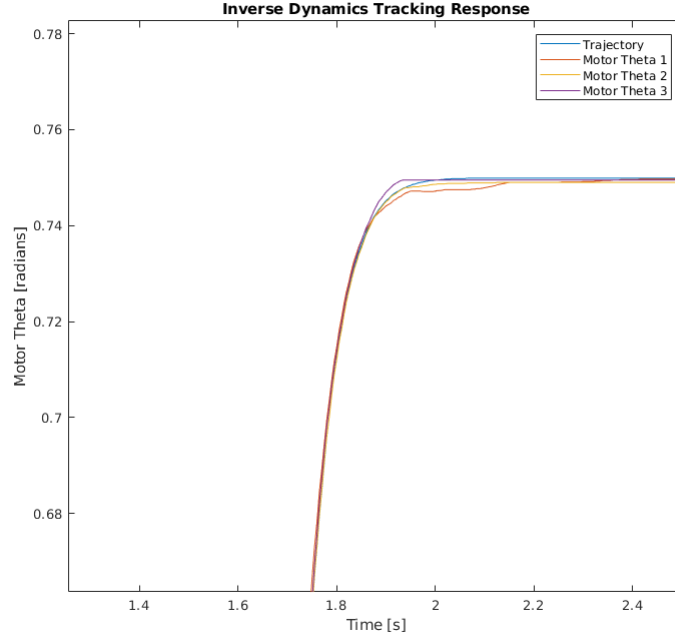


Figure 12: Close Up Inverse Dynamics Trajectory Response

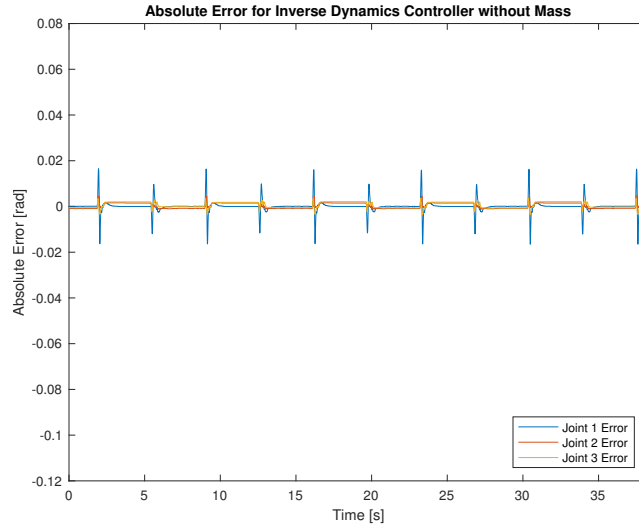


Figure 13: Error for Inverse Dynamics Controller with Mass Added

The inverse dynamics controller reduces the maximum error of the response for Joints 2 and 3 on the upwards motion by an order of magnitude as compared to the PD plus feedforward controller with the attached mass. There is, however, slight steady state error for the two

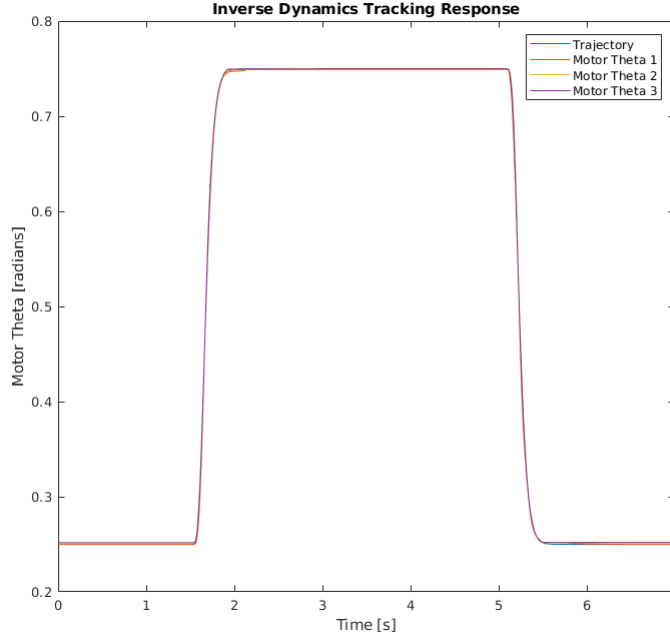


Figure 14: Inverse Dynamics Trajectory Tracking with Weight

joints, while the feedforward controller settled to zero. This response shows that the inverse dynamics controller has a better response, but does not account for all non-linearities of the system. There is also no integral control being used on this controller to remove steady state error.

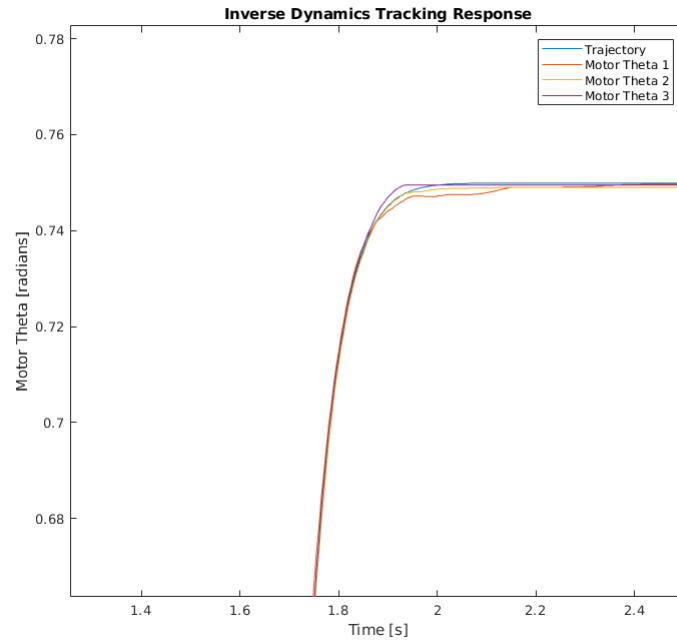


Figure 15: Close Up Inverse Dynamics Trajectory Response with Weight

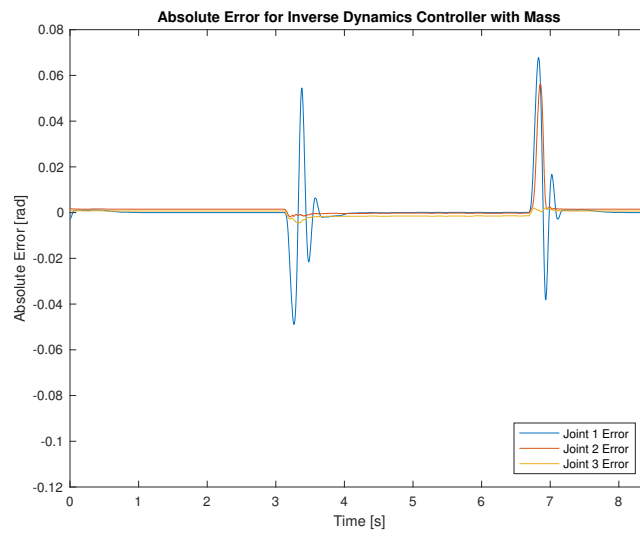


Figure 16: Error for Inverse Dynamics Controller without Mass Added

5 Inverse Dynamics vs. PD Control

The inverse dynamics controller was able to properly compensate for added mass, where the feed-forward was unable to achieve the same. This shows that the parameters given were able to accurately characterize the system and account for the added payload mass.

The inverse dynamics controller is more robust to changes in end-effector/payload mass. Both the feed-forward and inverse dynamics controllers saw a reduction in overshoot by about 50% when a large mass was added to the end-effector. This was to be expected, as the force due to gravity from the added mass dampened the overshoot effect. However, the inverse dynamics controller performed significantly better for undershoot. The feed-forward controller saw undershoot values increase almost 7-fold, however the inverse dynamics controller saw no discernible increase in overshoot, even with the added mass.

6 Conclusion

This lab included the implementation of friction compensation and inverse dynamics control. We explored the advantages of inverse dynamics control by adding mass to the end-effector of the robot (thereby changing the dynamics) and seeing how the inverse dynamics controller response differed with respect to a standard feed-forward controller. The superior robustness provided by inverse dynamics control methods was clearly seen in that regard.

A lab3.c code

```
1  #include <tistdtypes.h>
2  #include <coecsl.h>
3  #include "user_includes.h"
4  #include "math.h"
5
6  /*
7   * 1 for inverse dynamics
8   * 2 for feed-forward only (all joints)
9   */
10 int CONTROLMODE = 2;
11
12 // These two offsets are only used in the main file user-CRSRobot.c You just
   need to create them here and find the correct offset and then these offset
   will adjust the encoder readings
13 //float offset_Enc2_rad = -0.37;
14 //float offset_Enc3_rad = 0.27;
15
16 float offset_Enc2_rad = -0.427257;
17 float offset_Enc3_rad = 0.230558;
18
19
20 // Your global variables.
21
22 long mycount = 0;
23
24 #pragma DATA_SECTION(whattoprint, ".my_vars")
25 float whattoprint = 0.0;
26
27 #pragma DATA_SECTION(theta1array, ".my_arrs")
28 float theta1array[100];
29
30 #pragma DATA_SECTION(theta2array, ".my_arrs")
31 float theta2array[100];
32
33 #pragma DATA_SECTION(theta3array, ".my_arrs")
34 float theta3array[100];
35
36
37 long arrayindex = 0;
38
39 float printtheta1motor = 0;
40 float printtheta2motor = 0;
41 float printtheta3motor = 0;
42
43 float x_pos = 0;
44 float y_pos = 0;
45 float z_pos = 0;
46
47 float fric_on = 0.0;
48
```



```

49 // inverse kinematics
50 float theta_1 = 0;
51 float theta_2 = 0;
52 float theta_3 = 0;
53
54 float motor_theta_1 = 0;
55 float motor_theta_2 = 0;
56 float motor_theta_3 = 0;
57
58 // Assign these float to the values you would like to plot in Simulink
59 float Simulink_PlotVar1 = 0;
60 float Simulink_PlotVar2 = 0;
61 float Simulink_PlotVar3 = 0;
62 float Simulink_PlotVar4 = 0;
63
64 //Controller Parameters
65 //Proportional
66 float kp[3] = {110, 130, 55};
67
68 //Derivative
69 float kd[3] = {2,2,1.7};
70
71 //Integral
72 float ki[3] = {550,600,185};
73
74 // velocity filtering
75 float Theta_old[3] = {0,0,0};
76 float Omega_old1[3] = {0,0,0};
77 float Omega_old2[3] = {0,0,0};
78 float Omega[3] = {0,0,0};
79
80 //Integral Estimation
81 float Ik[3] = {0,0,0};
82 float Ik_old[3] = {0,0,0};
83 float e_old[3] = {0,0,0};
84
85 float integral_threshold = 0.1;
86
87
88 // current positions
89 float theta_motor[3] = {0,0,0};
90
91 //current tau
92 float t[3] = {0,0,0};
93
94 //feedforward control
95 float theta_d[3] = {0,0,0};
96 float theta_dot_d[3] = {0,0,0};
97 float theta_ddot_d[3] = {0,0,0};
98
99 float J[3] = {0.0167, 0.03, 0.0128};
100

```

```

101 // friction compensation
102 float minimum_velocity[3] = {0.05, 0.05, 0.05};
103 float u_fric[3] = {0,0,0};
104 float Viscous_positive[3] = {0.130,0.2500,0.21};
105 float Viscous_negative[3] = {0.074,0.21,0.33};
106 float Coulomb_positive[3] = {0.300,0.25,0.4};
107 float Coulomb_negative[3] = {-0.390,-0.6,-0.6};
108 float slope_between_minimums[3] = {3.6,3.6,3.6};
109
110 //Controller Parameters
111 //Proportional
112 float kp_inv[3] = {0, 10000, 10000};
113
114 //Derivative
115 float kd_inv[3] = {0,200,200};
116
117 //alpha values
118 float a_theta[3] = {0, 0, 0};
119
120 float mystep = 0.25;
121
122
123
124
125
126 //This function calculates the motor positions given a desired x,y,and z
    position
127 void inverse_kinematics(float x, float y, float z){
128
129     //The DH angles are calculated using a geometric analysis of the possible
    configurations of the robot
130     theta_1 = atan2(y,x);
131     //The base DH angle is the inverse tangent between the x and y
    coordinates
132     theta_3 = 3.14159 - acos(-(pow(x,2)+pow(y,2)+pow(z-10,2)-200)/200);
133     //The elbow DH angle is calculated using the law of cosines
134     theta_2 = -(atan2(z-10,sqrt(pow(x,2)+pow(y,2))) + theta_3/2);
135     //The shoulder DH angle is calculated using Pythagoras' Theorem and the
    half angle formula
136
137     //The DH angles must be converted to motor theta angles using appropriate
    transformation
138     theta_d[0] = theta_1;
139     theta_d[1] = theta_2 + PI/2;
140     theta_d[2] = theta_3 + theta_2;
141 }
142
143 void filter_velocity() {
144     int i;
145     for(i = 0; i < 3; i++) {
146         Omega[i] = (theta_motor[i] - Theta_old[i])/0.001;
147         Omega[i] = (Omega[i] + Omega_old1[i] + Omega_old2[i])/3.0;
148     }
149 }

```

```

145         Theta_old[i] = theta_motor[i];
146
147         Omega_old2[i] = Omega_old1[i];
148         Omega_old1[i] = Omega[i];
149     }
150 }
151
152 void estimate_integral() {
153     int i;
154     for(i=0;i<3;i++){
155         float e_k = theta_d[i] - theta_motor[i];
156         Ik[i] = Ik_old[i] + (e_old[i] + e_k)/2*0.001;
157         Ik_old[i] = Ik[i];
158         e_old[i] = e_k;
159     }
160 }
161
162
163
164 void pid_control() {
165     int i = 0;
166     for (i = 0; i < 3; i++){
167         t[i] = kp[i]*(theta_d[i]-theta_motor[i]) - kd[i]*Omega[i];
168         if (fabs(theta_d[i] - theta_motor[i]) < integral_threshold){
169             estimate_integral();
170             t[i] = t[i] + ki[i]*Ik[i];
171         } else {
172             Ik[i] = 0;
173             Ik_old[i] = 0;
174         }
175
176
177         if (t[i] >= 5) {
178             t[i] = 5;
179             Ik[i] = Ik_old[i];
180         }
181         else if (t[i] <= -5) {
182             t[i] = -5;
183             Ik[i] = Ik_old[i];
184         }
185     }
186 }
187
188 //void trajectory(float time){
189 //    //lemniscate
190 //    float x_d = 5*sqrt(2)*cos(PI*time)*sin(PI*time)/(sin(PI*time)*sin(PI*
191 //    time) + 1) + 14;
192 //    float y_d = 5*sqrt(2)*cos(PI*time)/(sin(PI*time)*sin(PI*time) + 1);
193 //    float z_d = 3*sin(PI*time)+ 10;
194 //
195 //    inverse_kinematics(x_d,y_d,z_d);

```

```

196 //}
197
198 //void get_trajectory(float time) {
199 //    float i = 0;
200 //    for (i = 0; i < 3; i++){
201 //        if((time >= 0) && (time < 1)) {
202 //            theta_d[i] = 1.5*pow(time,2) -pow(time,3);
203 //            theta_dot_d[i] = 3*time - 3*pow(time,2);
204 //            theta_ddot_d[i] = 3 - 6*time;
205 //        }
206 //        else if ((time >= 1) && (time <=2 )) {
207 //            theta_d[i] = -2 + 6*time - 4.5*pow(time,2) + pow(time,3);
208 //            theta_dot_d[i] = 6 - 9*time +3*pow(time,2);
209 //            theta_ddot_d[i] = -9 +6*time;
210 //        }
211 //        else {
212 //            theta_d[i] = 0;
213 //            theta_dot_d[i] = 0;
214 //            theta_ddot_d[i] = 0;
215 //        }
216 //    }
217 //}
218 //}
219
220 void feedforward_control() {
221
222     int joint_limit = 1;
223
224     if(CONTROLMODE == 1) {
225         joint_limit = 1;
226     }
227     else if(CONTROLMODE == 2) {
228         joint_limit = 3;
229     }
230
231     int i = 0;
232     for (i = 0; i < joint_limit; i++){ // ONLY OPERATE ON JOINT 1 WHILE USING
INVERSE DYNAMICS CONTROL LOOPS
233         t[i] = kp[i]*(theta_d[i]-theta_motor[i]) + kd[i]*(theta_dot_d[i] -
Omega[i]) + J[i]*theta_ddot_d[i];
234         if (fabs(theta_d[i] - theta_motor[i]) < 0.05){
235             estimate_integral();
236             t[i] = t[i] + ki[i]*Ik[i];
237         } else {
238             Ik[i] = 0;
239             Ik_old[i] = 0;
240         }
241         if (t[i] >= 5) {
242             t[i] = 5;
243             Ik[i] = Ik_old[i];
244         }
245         else if (t[i] <= -5) {

```

```

246         t[i] = -5;
247         Ik[i] = Ik_old[i];
248     }
249 }
250
251 }
252
253 void friction_compensation() {
254     int i = 0;
255     for(i = 0; i < 3; i++) {
256         if (Omega[i] > minimum_velocity[i]) {
257             u_fric[i] = Viscous_positive[i]*Omega[i] + Coulomb_positive[i];
258         } else if (Omega[i] < -minimum_velocity[i]) {
259             u_fric[i] = Viscous_negative[i]*Omega[i] + Coulomb_negative[i];
260         } else {
261             u_fric[i] = slope_between_minimums[i]*Omega[i];
262         }
263     }
264 }
265
266
267 void inverse_dynamics_outer_loop() {
268     int joint = 1; // only implement on Joint 2 and 3
269     for(joint = 1; joint < 3; joint++) {
270         a_theta[joint] = theta_ddot_d[joint]
271             + kp_inv[joint]*(theta_d[joint]-theta_motor[joint])
272             + kd_inv[joint]*(theta_dot_d[joint]-Omega[joint]);
273     }
274 }
275
276 void inverse_dynamics_inner_loop() {
277     // tau = D(theta)a_theta + C(theta, theta_dot)theta_dot + G(theta)
278
279     float sintheta32 = sin(theta_motor[2] - theta_motor[1]);
280     float costheta32 = cos(theta_motor[2] - theta_motor[1]);
281
282     float p1 = 0.0466;
283     float p2 = 0.0388;
284     float p3 = 0.0284;
285     float p4 = 0.1405;
286     float p5 = 0.1298;
287
288
289
290     // t[1] = a_theta[1]*(p1-p3*sintheta32/*Omega[1]*/)
291     //         + Omega[1]*p3*costheta32*Omega[1]
292     //         - p4*9.81*sin(theta_motor[1]);
293     //
294     // t[2] = a_theta[2]*(p2-p3*sintheta32)
295     //         - Omega[2]*p3*costheta32*Omega[2]
296     //         - p5*9.81*cos(theta_motor[2]);
297

```

```

298     t[1] = p1*a_theta[1] - p3*sintheta32*a_theta[2]
299           - Omega[2]*p3*costheta32*Omega[2]
300           - p4*9.81*sin(theta_motor[1]);
301
302     t[2] = -p3*sintheta32*a_theta[1] + p2*a_theta[2]
303           + Omega[1]*p3*costheta32*Omega[1]
304           - p5*9.81*cos(theta_motor[2]);
305
306 }
307
308
309
310 //This function calculates the forward kinematics of the manipulator given the
    motor positions
311
312 void forward_kinematics(float motor1, float motor2, float motor3){
313
314     //The forward kinematics function uses the translational vector from the
    full DH matrix calculated in Robotica
315     x_pos = 10*cos(motor1)*(cos(motor3)+sin(motor2));
316     y_pos = 10*sin(motor1)*(cos(motor3)+sin(motor2));
317     z_pos = 10*(1+cos(motor2)-sin(motor3));
318
319 }
320
321
322 //////////////////////////////////////////////////
323
324 typedef struct steptraj-s {
325     long double b[7];
326     long double a[7];
327     long double xk[7];
328     long double yk[7];
329     float qd_old;
330     float qddot_old;
331     int size;
332 } steptraj-t;
333
334 steptraj-t trajectory = {1.0417207161648879e-11L,6.2503242969893271e-11L
    ,1.5625810742473319e-10L,2.0834414323297759e-10L,1.5625810742473319e-10L
    ,6.2503242969893271e-11L,1.0417207161648879e-11L,
335     1.0000000000000000e+00L,-5.8226600985221673e+00L
    ,1.4126404426217572e+01L,-1.8278500308471997e+01L,1.3303686800870629e+01L
    ,-5.1641897532443632e+00L,8.3525893381702754e-01L,
336     0,0,0,0,0,0,0,
337     0,0,0,0,0,0,0,
338     0,
339     0,
340     7};
341
342 // this function must be called every 1ms.

```

```

343 void implement_discrete_tf(steptraj_t *traj, float step, float *qd, float *
    qd_dot, float *qd_ddot) {
344     int i = 0;
345
346     traj->xk[0] = step;
347     traj->yk[0] = traj->b[0]*traj->xk[0];
348     for (i = 1; i < traj->size; i++) {
349         traj->yk[0] = traj->yk[0] + traj->b[i]*traj->xk[i] - traj->a[i]*traj->
yk[i];
350     }
351
352     for (i = (traj->size-1); i > 0; i--) {
353         traj->xk[i] = traj->xk[i-1];
354         traj->yk[i] = traj->yk[i-1];
355     }
356
357     *qd = traj->yk[0];
358     *qd_dot = (*qd - traj->qd_old)*1000; //0.001 sample period
359     *qd_ddot = (*qd_dot - traj->qddot_old)*1000;
360
361     traj->qd_old = *qd;
362     traj->qddot_old = *qd_dot;
363 }
364
365 // to call this function create a variable that steps to the new positions you
    want to go to, pass this var to step
366 // pass a reference to your qd variable your qd_dot variable and your
    qd_double_dot variable
367 // for example
368 // implement_discrete_tf(&trajectory, mystep, &qd, &dot, &ddot);
369
370 ///////////////
371
372
373
374
375
376
377
378 // This function is called every 1 ms
379 void lab(float theta1motor, float theta2motor, float theta3motor, float *taul,
    float *tau2, float *tau3, int error) {
380
381     // theta_motor = {theta1motor, theta2motor, theta3motor};
382     theta_motor[0] = theta1motor;
383     theta_motor[1] = theta2motor;
384     theta_motor[2] = theta3motor;
385
386     filter_velocity();
387
388
389 // position_d((mycount%2000)/1000.0);

```

```

390 //
391 //     feedforward_control();
392
393 //     float time = (mycount%2000)/1000.0;
394
395
396 // 1) Calculate the desired trajectory
397 //     trajectory(time);
398
399     if((mycount % 4000)==0) {
400         mystep = 0.75;
401     }
402
403     if((mycount % 8000)==0) {
404         mystep = 0.25;
405     }
406
407     float qd;
408     float dot;
409     float ddot;
410
411     implement_discrete_tf(&trajectory , mystep, &qd, &dot, &ddot);
412
413     int joint_idx = 0;
414     for(joint_idx = 0; joint_idx < 3; joint_idx++) {
415         theta_d[joint_idx] = qd;
416         theta_dot_d[joint_idx] = dot;
417         theta_ddot_d[joint_idx] = ddot;
418     }
419
420 // 2) Given measured thetas, calculate actual states, error, error_dot,
421 //     theta_dot
422
423 // 3) Calculate the outer loop control to come up with values for
424 //     a_theta_2 and a_theta_3
425     inverse_dynamics_outer_loop();
426
427 // 4) Calculate the inner loop control to find control effort to apply to
428 //     joint 2 and 3
429     inverse_dynamics_inner_loop();
430
431 // 5) Calculate Lab 2 feed-forward control for joint 1 to find control
432 //     effort to apply.
433     feedforward_control(); // currently only active on joint 1
434
435     *tau1 = t[0];
436     *tau2 = t[1];
437     *tau3 = t[2];

```



```

437 // 6) Calculate friction compensation control effort given the velocities
438 of joint 1,2, and 3
439 friction_compensation();
440
441 // 7) Add the friction compensation to the control efforts calculated in 3
442 and 4 above
443
444 *tau1 = *tau1 + fric_on*u_fric[0];
445 *tau2 = *tau2 + fric_on*u_fric[1];
446 *tau3 = *tau3 + fric_on*u_fric[2];
447
448 // 8) Write control efforts to PWM outputs to drive each joint
449
450 Simulink_PlotVar1 = qd;
451 Simulink_PlotVar2 = theta_motor[0];
452 Simulink_PlotVar3 = theta_motor[1];
453 Simulink_PlotVar4 = theta_motor[2];
454
455 //Motor torque limitation(Max: 5 Min: -5)
456
457 // save past states
458 if ((mycount%50)==0) {
459     thetalarray[arrayindex] = thetalmotor;
460     theta2array[arrayindex] = theta2motor;
461
462     if (arrayindex >= 100) {
463         arrayindex = 0;
464     } else {
465         arrayindex++;
466     }
467 }
468
469 }
470
471 /*
472 * Forward Kinematics
473 *
474 * based on motor angles
475 */
476
477 forward_kinematics(thetalmotor, theta2motor, theta3motor);
478
479 /*
480 * Inverse Kinematics
481 *
482 * based on {x,y,z} pos calculated above
483 */
484
485 inverse_kinematics(x_pos, y_pos, z_pos);
486

```

```

487
488
489     if ((mycount%500)==0) {
490         if (whattoprint > 0.5) {
491             serial_printf(&SerialA , "I love robotics\n\r");
492         } else {
493             printthetalmotor = thetalmotor;
494             printtheta2motor = theta2motor;
495             printtheta3motor = theta3motor;
496
497
498             SWI_post(&SWI_printf); //Using a SWI to fix SPI issue from sending
too many floats.
499         }
500         GpioDataRegs.GPBTOGGLE.bit.GPIO34 = 1; // Blink LED on Control Card
501         GpioDataRegs.GPBTOGGLE.bit.GPIO60 = 1; // Blink LED on Emergency Stop
Box
502     }
503
504
505     mycount++;
506 }
507
508 void printing(void){
509 //     serial_printf(&SerialA , "%.2f %.2f,%.2f  \n\r",printthetalmotor ,
printtheta2motor,printtheta3motor);
510 //     serial_printf(&SerialA , "x: %.2f,   y: %.2f,   z: %.2f  \n\r",x_pos,y_pos,
z_pos);
511 //     //serial_printf(&SerialA , "Estimated IK solution: theta1: %.2f,   theta2:
%.2f,   theta3: %.2f  \n\r",theta_1,theta_2,theta_3);
512 //     serial_printf(&SerialA , "Estimated IK solution: motor_theta1: %.2f,
motor_theta2: %.2f,   motor_theta3: %.2f  \n\r",motor_theta_1,
motor_theta_2 ,motor_theta_3);
513
514 //     serial_printf(&SerialA , "thetalmotor:  %.2f \n\r",thetalmotor);
515 //     serial_printf(&SerialA , "theta_motor[0]:  %.2f \n\r",theta_motor[0]);
516 }

```

Listing 2: lab3.c