HD-RACE

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1 Cooking up Our Own Private Tone Mapping Operator: OOPTMO

Notation:

- $\Omega \subset \mathbb{Z}^2$: image support;
- $i \in \Omega$: fixed pixel position;
- $I_c(i)$: original HDR image intensity in the chromatic channel $c \in \{R, G, B\}$, $I_c(i) \in [0, +\infty)$. The omission of c must be interpreted as the repetition on the corresponding formula on each chromatic channel separately. We also have the option of working only on the V channel, where the V value of the pixel i is
$$V(i) = \max_{c \in \{R,G,B\}} I_c(i),$$
notice that the channel which provides the highest value for the computation of V varies, in general, with the pixel i.
- The Naka-Rushton (NR) formula is:
$$\tilde{I}(i) = \frac{I(i)}{I(i) + \mu}, \tag{1}$$
where μ is an estimation of the average intensity, which can be the arithmetic μ_a or geometric μ_g one, but it can also be computed as $\mu_a^p \mu_g^{1-p}$, $p \in [0, 1]$, which is the convex combination in the logarithmic domain of the two average values. In particular, when $p = 1/2$, we obtain $\sqrt{\mu_a \mu_g}$, the geometric average of μ_a and μ_g.
- $S(i)$: *localized* spray with n points centered in i. The density of the points in the spray decreases linearly with the distance from i. The image is padded symmetrically with respect to all the edges and the radius of the spray is taken to be the minimum between the width and the height of the image, so that all the spray points fall into the padded image.

Steps of the algorithm:

1. RSR:
$$L^{\mathrm{RSR}}(i) = \frac{I(i)}{\max\limits_{j \in S(i)} I(j)}, \qquad \forall i \in \Omega. \tag{2}$$
Many variants of RSR can be considered...smart, light, etc. Let's pay attention to which one is used. **Attention to the number of points and sprays that are used!**

   L^{RSR} will produce very pleasantly saturated colors in the brightest part of the image.

2. NR-RSR:
$$L^{\text{NR-RSR}}(i) = \frac{\tilde{I}(i)}{\max\limits_{j \in S(i)} \tilde{I}(j)}, \qquad \forall i \in \Omega. \tag{3}$$
$L^{\text{NR-RSR}}$ will work nicely for pixels with 'intermediate' brightness.

3. NR-ACE: for this step, it is essential[1] that the image range is contained in $[0, 1]$, for this reason we apply ACE on $\tilde{I}$:
$$L^{\text{NR-ACE}}(i) = \frac{1}{n-1} \sum_{j \in S(i) \setminus \{i\}} s_{\alpha_{S(i)}}(\tilde{I}(i) - \tilde{I}(j)), \qquad \forall i \in \Omega, \tag{4}$$
where
$$s_{\alpha_{S(i)}}(\tilde{I}(i) - \tilde{I}(j)) = \begin{cases} 0 & \tilde{I}(i) - \tilde{I}(j) \leq -\frac{1}{2\alpha_{S(i)}} \\ \frac{1}{2} + \alpha_{S(i)}[\tilde{I}(i) - \tilde{I}(j)] & |\tilde{I}(i) - \tilde{I}(j)| < \frac{1}{2\alpha_{S(i)}}, \\ 1 & \tilde{I}(i) - \tilde{I}(j) \geq \frac{1}{2\alpha_{S(i)}} \end{cases} \tag{5}$$
and $\alpha_{S(i)} \geq 1$for all $i \in S(i)$ is a local pixel-dependent slope built as follows
$$\alpha_{S(i)} = \max_{j \in S(i)} G_\sigma * I(j) \Big/ \min_{j \in S(i)} G_\sigma * I(j), \tag{6}$$
where $G_\sigma * I(j)$ is a blurred version of the original HDR image obtained by convolution with a Gaussian (what's the standard deviation of it? Is it an important parameter? It seems not). This procedure strongly reduces the effects of possible outliers on the ratio.

   With this choice $\alpha_{S(i)}$ is *illuminant-independent*, i.e. independent under the scaling by a positive constant λ representing the color of a global and uniform illuminant, and it will be big in areas covered by the spray in which the variation between the minimal and the maximal intensities is large. This will allow a very efficient detail rendition of this kind of areas, which are well-known to be particularly difficult to tone-map.

   $L^{\text{NR-ACE}}$ will work optimally on the dimmest part of the image, but it will result in washed-out pixels on the brightest parts.

[1]Otherwise, the definition of the sigmoid function would not make sense.

4. After applying RSR and HD-ACE we obtain images with values between 0 and 1, thus it makes sense to combine them, to obtain HD-RACE, via convex combinations with suitably selected coefficients $\beta_{S(i)}, \gamma_{S(i)} \in [0,1]$:

$$L^{\text{HD-RSR}}(i) = \beta(i)L^{\text{RSR}}(i) + (1-\beta(i))L^{\text{NR-RSR}}(i), \qquad \forall i \in \Omega, \tag{7}$$

$$L^{\text{HD-RACE}}(i) = \gamma(i)L^{\text{HD-RSR}}(i) + (1-\gamma(i))L^{\text{HD-ACE}}(i), \qquad \forall i \in \Omega. \tag{8}$$

5. Towards the determination of the coefficients $\beta(i)$ and $\gamma(i)$ (**STILL TO BE FORMALIZED**). Let $\langle I \rangle$ be the original image luminance:

$$\langle I \rangle(i) = \frac{1}{3}\sum_{c\in\{R,G,B\}} I_c(i), \qquad \forall i \in \Omega.$$

To avoid problems with very small values, we compute the logarithmic luminance after blurring the original luminance via convolution with a Gaussian:

$$\ell(i) := \text{Log}(G_\sigma * \langle I \rangle)(i), \qquad \forall i \in \Omega,$$

where $\text{Log} \equiv \log_{10}$.

Let us now define the *normalized logarithmic luminance* as

$$\bar{\ell}(i) := \ell(i) - \min_{i\in\Omega} \ell(i)$$

and the normalized logarithmic luminance range as

$$\mu := \max_{i\in\Omega} \bar{\ell}(i) = \max_{i\in\Omega} \ell(i) - \min_{i\in\Omega} \ell(i),$$

then a **tentative expression** for $\beta(i)$ is:

$$\beta(i) = e^{-\frac{(\bar{\ell}(i)-\mu)^2}{2\sigma^2}},$$

$\beta(i) = 1$ for all the pixels of position i such that $\ell(i) = \mu$ and it will decrease to 0 for pixels with lower luminance. The decreasing speed is tuned by the value of σ, that must be selected properly...

A similar expression for $\gamma(i)$ is attended, for a different definition of μ, of course.

6. Once we get nice images, we can try to speed up the algorithm by using the techniques developed at FER. . .