Algorithmic Operation Research

Homework 1
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Fall 2019

1. Let $C \subseteq \mathbb{R}^n$ be a convex set with $x_1, \dots x_k \in C$ and let $\theta_1, \dots, \theta_k \in \mathbb{R}$ satisfy $\theta_i \geq 0$ and $\theta_1 + \cdots + \theta_k = 1$. Show that $\theta_1 x_1 + \cdots + \theta_k x_k \in C$.

2. Show that a set is convex if and only if its intersection with any line is convex.

3. Show that a set is affine if and only if its intersection with any line is affine.

4. A set C is midpoint convex, if whenever two points $a, b \in C$, the average or midpoint $(a + b)/2$ is in C. Obviously, a convex set is midpoint convex. Prove that if C is closed and midpoint convex, then C is convex.

5. Show that the convex hull of a set S is the intersection of all convex sets that contain S.(The same method can be used to show that the conic, or affine, or linear hull of a set S is the intersection of all conic sets, or affine sets, or subspaces that contain S.

6. What is the distance between two parallel hyperplanes $\{x \in \mathbb{R}^n : a^T x = b_1\}$ and $\{x \in \mathbb{R}^n : a^T x = b_2\}$?

7. Let a and b be distinct points in $\mathbb{R}^n$. Show that the set of all points that are closer (in Euclidean norm) to a than b is a halfspace.