Linear Programming Model

Assignment 1

1.) Back Savers is a company that produces backpacks primarily for students. They are considering offering some combination of two different models—the Collegiate and the Mini. Both are made out of the same rip-resistant nylon fabric. Back Savers has a long-term contract with a supplier of nylon and receives a 5000 square-foot shipment of the material each week. Each Collegiate requires 3 square feet while each Mini requires 2 square feet. The sales forecasts indicate that at most 1000 Collegiates and 1200 Minis can be sold per week. Each Collegiate requires 45 minutes of labor to produce and generates a unit profit of $32. Each Mini requires 40 minutes of labor and generates a unit profit of $24. Back Savers has 35 laborers that each provides 40 hours of labor per week. Management wishes to know what quantity of each type of backpack to produce per week.

a. Clearly define the decision variables.
b. What is the Objective Function?
c. What are the constraints?
d. Write down the full mathematical formulation for this LP problem.

Solution: **a.)** To set up a mathematical formulation for this LP problem, my decision variables for two different models of backpacks are: -

- Let **x** be the decision variable for Collegiate backpacks.
- Let **y** be the decision variable for Mini backpacks.

b.) The objective is to maximize profits; therefore, the objective function for this linear programming problem will be: -

Z = 32x + 24y (where Z is the maximum profit, x and y are the decision variables)

Since we cannot make negative backpacks, our restrictions of decision statements for both the models of the backpack are: -

- For Collegiate model: 0<= x <= 1000
- For Mini model: 0<= y <= 1200

c.) Our constraints are the amount of material and how many labor hours we have each week. As mentioned, we have 5000 sqft of nylon and a total of 35*40= 1400 labor hours for making our (Collegiate and Mini) backpacks.

So, as the mentioned collegiate model requires 3 sqft, whereas the mini model requires 2sqft of nylon; our material constraint will be:

5000>= 3x+2y

And in the same way, the constraint for labor hours will be:

$1400 >= 45/60\ x + 40/60\ y = \quad 1400 >= 3/4\ x + 2/3\ y$

d.) $Z = 32\ x + 24\ y$ (Objective Function, where x and y are decision variable)

$0 <= x <= 1000$ (Restrictions of decision statements)

$0 <= y <= 1200$ (“)

Subject to,

$5000 >= 3x + 2y$ (Constraint for material)

$1400 >= 3/4x + 2/3y$ (Constraint for labour hours)

2.) The Weigelt Corporation has three branch plants with excess production capacity. Fortunately, the corporation has a new product ready to begin production, and all three plants have this capability, so some of the excess capacity can be used in this way. This product can be made in three sizes-large, medium, and small--that yield a net unit profit of $420, $360, and $300, respectively. Plants 1, 2, and 3 have the excess capacity to produce 750, 900, and 450 units per day of this product, respectively, regardless of the size or combination of sizes involved. The amount of available inprocess storage space also imposes a limitation on the production rates of the new product. Plants 1, 2, and 3 have 13,000, 12,000, and 5,000 square feet, respectively, of in-process storage space available for a day's production of this product. Each unit of the large, medium, and small sizes produced per day requires 20, 15, and 12 square feet, respectively. Sales forecasts indicate that if available, 900, 1,200, and 750 units of the large, medium, and small sizes, respectively, would be sold per day. At each plant, some employees will need to be laid off unless most of the plant’s excess production capacity can be used to produce the new product. To avoid layoffs if possible, management has decided that the plants should use the same percentage of their excess capacity to produce the new product. Management wishes to know how much of each of the sizes should be produced by each of the plants to maximize profit.

a. Define the decision variables

b. Formulate a linear programming model for this problem.

Solution: The decision variables for Weigelt Corporation are: -

xL1= Number of large units produced per day at Plant 1

xM1= Number of medium units produced per day at Plant 1

xS1= Number of small units produced per day at Plant 1

xL2= Number of large units produced per day at Plant 2

xM2= Number of medium units produced per day at Plant 2

$xS2$= Number of small units produced per day at Plant 2

$xL3$= Number of large units produced per day at Plant 3

$xM3$= Number of medium units produced per day at Plant 3

$xS3$= Number of small units produced per day at Plant 3

Denoting **Z** as the total net profit, the linear programming model will be

Maximum Profit**: $Z = 420xL1 + 360xM1 + 300xS1 + 420xL2 + 360xM2 + 300xS2 + 420xL3 + 360xM3 + 300xS3$**

Subject to,

<u>Production capacity per day Constraint</u>

$xL1 + xM1 + xS1 <= 750$

$xL2 + xM2 + xS2 <= 900$

$xL3 + xM3 + xS3 <= 450$

<u>Storage space Constraint</u>

$20xL1 + 15xM1 + 12xS1 <= 13000$

$20xL2 + 15xM2 + 12xS3 <= 12000$

$20xL3 + 15xM3 + 12xS3 <= 5000$

<u>Sales Forecast Constraint</u>

$xL1 + xL2 + xL3 <= 900$

$xM1 + xM2 + xM3 <= 1200$

$xS1 + xS2 + xS3 <= 750$

<u>To avoid Layoffs</u>

$1/750\ (xL1 + xM1 + xS1) – 1/900\ (xL2 + xM2 + xS2) = 0$

$1/750\ (xL1 + xM1 + xS1) – 1/450\ (xL3 + xM3 + xS3) = 0$ and,

$xL1 >= 0, xM1 >= 0, xS1 >= 0, xL2 >= 0, xM2 >= 0, xS2 >= 0, xL3 >= 0, xM3 >= 0, xS3 >= 0.$