Line Assignment

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I. QUESTION

Class 11, Exercise 10.1, Q(9): **Without using distance formula, show that points (– 2, – 1), (4, 0), (3, 3) and (–3, 2)are the vertices of a parallelogram.**

II. SOLUTION

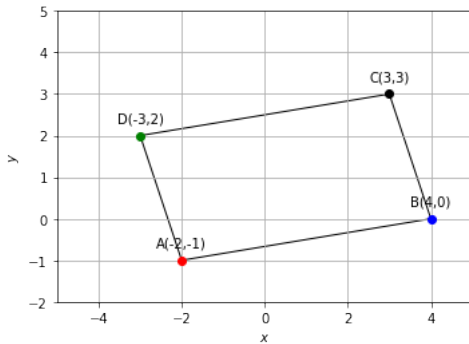Figure 1: paralellogram ABCD

We can prove that the points are the vertices of a parallelogram if **AB ∥ DC** ,and **BC ∥ AD**.

Theorm:if θ is the angle between $\vec{a}$ and $\vec{b}$,then
$$|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}|cos\theta$$
corollary: The two non-zero vectors $\vec{a}$ and $\vec{b}$ are parallel to each other,if their product is a zero vector

Consider Parallelogram ABCD,where

$$\vec{A} = \begin{pmatrix} -2 \\ -1 \end{pmatrix} \quad \vec{B} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}$$

$$\vec{C} = \begin{pmatrix} 3 \\ 3 \end{pmatrix} \quad \vec{D} = \begin{pmatrix} -3 \\ 2 \end{pmatrix}$$

let

$$\vec{P} = \vec{B} - \vec{A} = \begin{pmatrix} 6 \\ 1 \end{pmatrix} \quad \vec{Q} = \vec{C} - \vec{D} = \begin{pmatrix} 6 \\ 1 \end{pmatrix}$$

$$\vec{R} = \vec{A} - \vec{C} = \begin{pmatrix} 1 \\ -3 \end{pmatrix} \quad \vec{S} = \vec{A} - \vec{D} = \begin{pmatrix} 1 \\ -3 \end{pmatrix}$$

A. *proof for **AB ∥ DC***

$$\vec{P} \times \vec{Q} = |(\vec{B} - \vec{A}) \times (\vec{C} - \vec{D}))| = \begin{vmatrix} \hat{i} & \hat{j} \\ 6 & 1 \\ 6 & 1 \end{vmatrix} = 0\hat{k}$$

$$|\vec{P} \times \vec{Q}| = ||(\vec{B} - \vec{A}) \times (\vec{C} - \vec{D})|| = 0$$

so,according to corollary AB∥CD

B. *proof for **BC ∥ AD***

$$\vec{P} \times \vec{Q} = |(\vec{B} - \vec{C}) \times (\vec{A} - \vec{D})| = \begin{vmatrix} \hat{i} & \hat{j} \\ 1 & -3 \\ 1 & -3 \end{vmatrix} = 0\hat{k}$$

$$|\vec{P} \times \vec{Q}| = ||(\vec{B} - \vec{C}) \times (\vec{A} - \vec{D})|| = 0$$
so,according to corollary BC∥AD

since the opposite lines are parallel the points (– 2, – 1), (4, 0), (3, 3) and (–3, 2) forms the vertices of a parallelogram

CONSTRUCTION

Symbol	Value	Description
$\vec{A}$	$\begin{pmatrix} -2 \\ -1 \end{pmatrix}$	Vertex A
$\vec{B}$	$\begin{pmatrix} 4 \\ 0 \end{pmatrix}$	Vertex B
$\vec{C}$	$\begin{pmatrix} 3 \\ 3 \end{pmatrix}$	Vertex C
$\vec{D}$	$\begin{pmatrix} -3 \\ 2 \end{pmatrix}$	Vertex D
$\vec{P}$	$\begin{pmatrix} 1 \\ 6 \end{pmatrix}$	vector AB
$\vec{Q}$	$\begin{pmatrix} 1 \\ 6 \end{pmatrix}$	vector DC
$\vec{R}$	$\begin{pmatrix} 1 \\ -3 \end{pmatrix}$	vector BC
$\vec{S}$	$\begin{pmatrix} 1 \\ -3 \end{pmatrix}$	vector AD

Get the python code of the figures from

https://github.com/kkousar/KOUSAR_FWC/blob/main/matrices/line/code/line.py