Line Assignment

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I. Question

Class 11, Exercise 10.1, Q(9): **Without using distance formula, show that points (– 2, – 1), (4, 0), (3, 3) and (–3, 2)are the vertices of a parallelogram.**

II. Solution

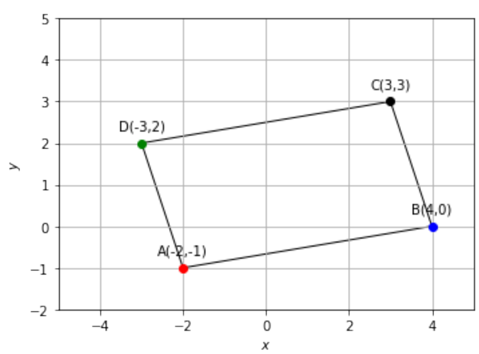

Figure 1: paralellogram ABCD

We can prove that the points are the vertices of a parallelogram if $\vec{AB} \parallel \vec{DC}$, $\vec{BC} \parallel \vec{AD}$ and $\vec{AB} = \vec{DC}$, $\vec{BC} = \vec{AD}$

Theorm:if θ is the angle between $\vec{a}$ and $\vec{b}$,then

$$|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}|sin\theta$$

corollary: The two non-zero vectors $\vec{a}$ and $\vec{b}$ are parallel to each other,if their product is a zero vector

Consider Parallelogram ABCD,where

$\vec{A} = \begin{pmatrix} -2 \\ -1 \end{pmatrix}$ $\vec{B} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}$

$\vec{C} = \begin{pmatrix} 3 \\ 3 \end{pmatrix}$ $\vec{D} = \begin{pmatrix} -3 \\ 2 \end{pmatrix}$

let

$\vec{P} = \vec{B} - \vec{A} = \begin{pmatrix} 6 \\ 1 \end{pmatrix}$ $\vec{Q} = \vec{C} - \vec{D} = \begin{pmatrix} 6 \\ 1 \end{pmatrix}$

$\vec{R} = \vec{A} - \vec{C} = \begin{pmatrix} 1 \\ -3 \end{pmatrix}$ $\vec{S} = \vec{A} - \vec{D} = \begin{pmatrix} 1 \\ -3 \end{pmatrix}$

A. *proof for $\vec{P} \parallel \vec{Q}$ and $\vec{P} = \vec{Q}$*

$$\vec{P} \times \vec{Q} = |(\vec{B} - \vec{A}) \times (\vec{C} - \vec{D}))| = \begin{vmatrix} \hat{i} & \hat{j} \\ 6 & 1 \\ 6 & 1 \end{vmatrix} = 0\hat{k}$$

so,according to corollary $\vec{P}||\vec{Q}$

$$||\vec{P}|| = \sqrt{\begin{pmatrix} 1 & 6 \end{pmatrix} \begin{pmatrix} 1 \\ 6 \end{pmatrix}} = \sqrt{37} \tag{1}$$

$$||\vec{Q}|| = \sqrt{\begin{pmatrix} 1 & 6 \end{pmatrix} \begin{pmatrix} 1 \\ 6 \end{pmatrix}} = \sqrt{37} \tag{2}$$

form equation (1) and (2) $\vec{P} = \vec{Q}$

B. *proof for $\vec{R} \parallel \vec{S}$ and $\vec{R} = \vec{S}$*

$$\vec{R} \times \vec{S} = |(\vec{B} - \vec{C}) \times (\vec{A} - \vec{D})| = \begin{vmatrix} \hat{i} & \hat{j} \\ 1 & -3 \\ 1 & -3 \end{vmatrix} = 0\hat{k}$$

so,according to corollary $\vec{R}||\vec{S}$

$$||\vec{R}|| = \sqrt{\begin{pmatrix} 1 & -3 \end{pmatrix} \begin{pmatrix} 1 \\ -3 \end{pmatrix}} = \sqrt{10} \tag{3}$$

$$||\vec{S}|| = \sqrt{\begin{pmatrix} 1 & -3 \end{pmatrix} \begin{pmatrix} 1 \\ -3 \end{pmatrix}} = \sqrt{10} \tag{4}$$

form equation (3) and (4) $\vec{R} = \vec{S}$

since the opposite lines are parallel and equal the points (– 2, – 1), (4, 0), (3, 3) and (–3, 2) forms the vertices of a parallelogram

CONSTRUCTION

Symbol	**Value**	**Description**
$\vec{A}$	$\begin{pmatrix} -2 \\ -1 \end{pmatrix}$	Vertex A
$\vec{B}$	$\begin{pmatrix} 4 \\ 0 \end{pmatrix}$	Vertex B
$\vec{C}$	$\begin{pmatrix} 3 \\ 3 \end{pmatrix}$	Vertex C
$\vec{D}$	$\begin{pmatrix} -3 \\ 2 \end{pmatrix}$	Vertex D
$\vec{P}$	$\begin{pmatrix} 1 \\ 6 \end{pmatrix}$	vector AB
$\vec{Q}$	$\begin{pmatrix} 1 \\ 6 \end{pmatrix}$	vector DC
$\vec{R}$	$\begin{pmatrix} 1 \\ -3 \end{pmatrix}$	vector BC
$\vec{S}$	$\begin{pmatrix} 1 \\ -3 \end{pmatrix}$	vector AD

Get the python code of the figures from

https://github.com/kkousar/KOUSAR_FWC/blob/main/matrices/line/code/line.py