circle Assignment

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I. QUESTION

Q(6), C , Section-A, Chapter-8:If a circle passes through the point (a,b)and cuts the circle $x^2+y^2 = k^2$ orthogonally, then the equation of the locus of its center is.
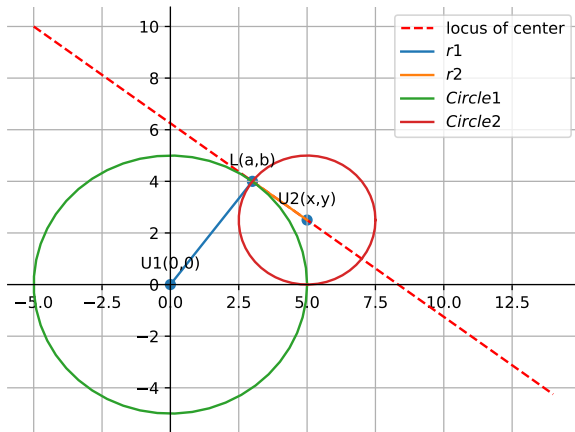
II. SOLUTION

Figure 1: a circle passes through the point L and cuts the circle $x^2 + y^2 = k^2$ orthogonally

With the given circle equation $x^2 + y^2 = k^2$, we can find out centre U_1 and radius r_1 of Circle-1
STEP-1

Centre of Circle-1,

$$\vec{U_1} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{1}$$

Radius of Circle-1,

$$r_1 = k \tag{2}$$

STEP-2

let,the center of the circle which passes through the point L and cuts the circle $x^2 + y^2 = k^2$ orthogonally is:

$$\vec{U_2} = \begin{pmatrix} x \\ y \end{pmatrix} \tag{3}$$

$$\vec{L} = \begin{pmatrix} a \\ b \end{pmatrix} \tag{4}$$

Radius of Circle be r_2
As both the circles are orthogonal, we get:

$$||\vec{U_2} - \vec{U_1}||^2 = r_1^2 + r_2^2 \tag{5}$$

where

$$\implies ||\vec{U_2} - \vec{U_1}||^2 = ||\vec{U_2}||^2 + ||\vec{U_1}||^2 - 2\vec{U_1}^\mathsf{T}\vec{U_2}$$

$$= \vec{U_2}\vec{U_2}^\mathsf{T}$$

$$= \begin{pmatrix} x \\ y \end{pmatrix} \begin{pmatrix} x & y \end{pmatrix} \tag{6}$$

$$\implies r_1^2 = k^2 \tag{7}$$

$$\implies r_2^2 = ||\vec{U_2} - \vec{L}||^2$$

$$= (U_2 - L)(U_2 - L)^\mathsf{T} \tag{8}$$

substitute equation (6),(7),(8) in equation (5)
$$\implies ||\vec{U_2} - \vec{U_1}||^2 = r_1^2 + r_2^2$$
$$\implies \begin{pmatrix} x \\ y \end{pmatrix} \begin{pmatrix} x & y \end{pmatrix} = k^2 + (U_2 - L)(U_2 - L)^\mathsf{T} \text{ by}$$
solving the above equation we get,
$$\implies x^2 + y^2 = x^2 + a^2 - 2ax + y^2 + b^2by + k^2$$
$$\implies a^2 + b^2 + k^2 - 2ax - 2by = 0$$

$$\implies 2ax + 2by - (a^2 + b^2 + k^2) = 0 \tag{9}$$

equation (9) is the required equation

CONSTRUCTION

Symbol	Value	Description
$\vec{U}_1$	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	center of given circle
r_1	k	radius of given circle
$\vec{U}_2$	$\begin{pmatrix} x \\ y \end{pmatrix}$	center of circle 2
$\vec{L}$	$\begin{pmatrix} a \\ b \end{pmatrix}$	a point on circle 2
r_2	$||\vec{U}_2 - \vec{L}||^2$	radius of circle 2

Get the python code of the figures from

https://github.com/kkousar/KOUSAR_FWC/blob/main/circle_Assignment/code/circle.py