

Line Assignment

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I. QUESTION

Class 11, Exercise 10.1, Q(9): Without using distance formula, show that points $(-2, -1)$, $(4, 0)$, $(3, 3)$ and $(-3, 2)$ are the vertices of a parallelogram.

A. proof for $\vec{P} \parallel \vec{Q}$ and $\vec{P} = \vec{Q}$

$$\vec{P} \times \vec{Q} = |(\vec{B} - \vec{A}) \times (\vec{C} - \vec{D})| = \begin{vmatrix} \hat{i} & \hat{j} \\ 6 & 1 \\ 6 & 1 \end{vmatrix} = 0\hat{k}$$

so, according to corollary $\vec{P} \parallel \vec{Q}$

II. SOLUTION

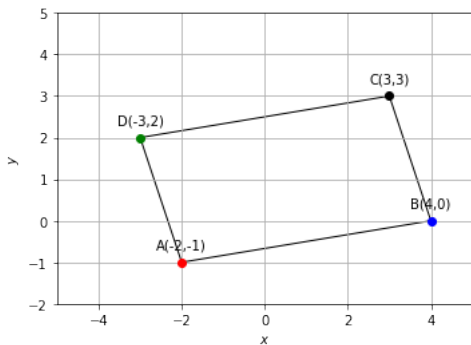


Figure 1: parallelogram ABCD

We can prove that the points are the vertices of a parallelogram if $\vec{AB} \parallel \vec{DC}$, $\vec{BC} \parallel \vec{AD}$ and $\vec{AB} = \vec{DC}$, $\vec{BC} = \vec{AD}$

Theorem: if θ is the angle between \vec{a} and \vec{b} , then

$$|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}|\sin\theta$$

corollary: The two non-zero vectors \vec{a} and \vec{b} are

parallel to each other, if their product is a zero vector

Consider Parallelogram ABCD, where

$$\vec{A} = \begin{pmatrix} -2 \\ -1 \end{pmatrix} \quad \vec{B} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}$$

$$\vec{C} = \begin{pmatrix} 3 \\ 3 \end{pmatrix} \quad \vec{D} = \begin{pmatrix} -3 \\ 2 \end{pmatrix}$$

let

$$\vec{P} = \vec{B} - \vec{A} = \begin{pmatrix} 6 \\ 1 \end{pmatrix} \quad \vec{Q} = \vec{C} - \vec{D} = \begin{pmatrix} 6 \\ 1 \end{pmatrix}$$

$$\vec{R} = \vec{A} - \vec{C} = \begin{pmatrix} -5 \\ -4 \end{pmatrix} \quad \vec{S} = \vec{D} - \vec{B} = \begin{pmatrix} -7 \\ 2 \end{pmatrix}$$

$$||\vec{P}|| = |1| + |6| = 7 \quad (1)$$

$$||\vec{Q}|| = |1| + |6| = 7 \quad (2)$$

from equation (1) and (2) $\vec{P} = \vec{Q}$

B. proof for $\vec{R} \parallel \vec{S}$ and $\vec{R} = \vec{S}$

$$\vec{R} \times \vec{S} = |(\vec{B} - \vec{C}) \times (\vec{A} - \vec{D})| = \begin{vmatrix} \hat{i} & \hat{j} \\ 1 & -3 \\ 1 & -3 \end{vmatrix} = 0\hat{k}$$

so, according to corollary $\vec{R} \parallel \vec{S}$

$$||\vec{R}|| = |1| + |-3| = 4 \quad (3)$$

$$||\vec{S}|| = |1| + |-3| = 4 \quad (4)$$

from equation (3) and (4) $\vec{R} = \vec{S}$

since the opposite lines are parallel and equal the points $(-2, -1)$, $(4, 0)$, $(3, 3)$ and $(-3, 2)$ forms the vertices of a parallelogram

CONSTRUCTION

Symbol	Value	Description
\vec{A}	$\begin{pmatrix} -2 \\ -1 \end{pmatrix}$	Vertex A
\vec{B}	$\begin{pmatrix} 4 \\ 0 \end{pmatrix}$	Vertex B
\vec{C}	$\begin{pmatrix} 3 \\ 3 \end{pmatrix}$	Vertex C
\vec{D}	$\begin{pmatrix} -3 \\ 2 \end{pmatrix}$	Vertex D
\vec{P}	$\begin{pmatrix} 1 \\ 6 \end{pmatrix}$	vector AB
\vec{Q}	$\begin{pmatrix} 1 \\ 6 \end{pmatrix}$	vector DC
\vec{R}	$\begin{pmatrix} 1 \\ -3 \end{pmatrix}$	vector BC
\vec{S}	$\begin{pmatrix} 1 \\ -3 \end{pmatrix}$	vector AD

Get the python code of the figures from

https://github.com/kkousar/KOUSAR_FWC/blob/main/matrices/line/code/line.py